

# ALTERNATIVE LOOP RINGS

Edgar G. GOODAIRE

*Department of Mathematics and Statistics  
Memorial University of Newfoundland, St. John's  
Newfoundland, Canada*

Eric JESPERS

*Department of Mathematics and Statistics  
Memorial University of Newfoundland, St. John's  
Newfoundland, Canada*

César Polcino MILES

*Instituto de Mathemática e Estatística  
Universidade de São Paulo,  
São Paulo, Brasil*



1996

ELSEVIER  
Amsterdam - Lausanne - New York - Oxford - Shannon - Tokyo

# Contents

Preface	ix
Introduction	1
Chapter I. Alternative Rings	5
1. Fundamentals	5
2. The real quaternions and the Cayley numbers	13
3. Generalized quaternion and Cayley-Dickson algebras	15
4. Composition algebras	23
5. Tensor products	38
Chapter II. An Introduction to Loop Theory and to Moufang Loops	49
1. What is a loop?	49
2. Inverse property loops	60
3. Moufang loops	63
4. Hamiltonian loops	68
5. Examples of Moufang loops	80
Chapter III. Nonassociative Loop Rings	85
1. Loop rings	85
2. Alternative loop rings	90
3. The LC property	94
4. The nucleus and centre	100
5. The norm and trace	103
Chapter IV. RA Loops	107
1. Basic properties of RA loops	107
2. RA loops have LC	119
3. A description of an RA loop	122

Chapter V. The Classification of Finite RA Loops	125
1. Reduction to indecomposables	125
2. Finite indecomposable groups	131
3. Finite indecomposable RA loops	138
4. Finite RA loops of small order	143
Chapter VI. The Jacobson and Prime Radicals	147
1. Augmentation ideals	149
2. Radicals of abelian group rings	153
3. Radicals of loop rings	159
4. The structure of a semisimple alternative algebra	166
Chapter VII. Loop Algebras of Finite Indecomposable RA Loops	173
1. Primitive idempotents of commutative rational group algebras	174
2. Rational loop algebras of finite RA loops	182
Chapter VIII. Units in Integral Loop Rings	195
1. Trivial torsion units	196
2. Bicyclic and Bass cyclic units	200
3. Trivial units	204
4. Trivial central units	212
5. Free subgroups	213
Chapter IX. Isomorphisms of Integral Alternative Loop Rings	223
1. The isomorphism theorem	225
2. Inner automorphisms of alternative algebras	228
3. Automorphisms of alternative loop algebras	233
4. Some conjectures of H. J. Zassenhaus	236
Chapter X. Isomorphisms of Commutative Group Algebras	247
1. Some results on tensor products of fields	248
2. Semisimple abelian group algebras	252
3. Modular group algebras of abelian groups	258
4. The equivalence problem	261
Chapter XI. Isomorphisms of Loop Algebras of Finite RA Loops	267
1. Semisimple loop algebras	267

2. Rational loop algebras	271
3. The equivalence problem	282
Chapter XII. Loops of Units	289
1. Reduction to torsion loops	289
2. Group identities	294
3. The centre of the unit loop	309
4. Describing large subgroups	314
5. Examples	320
Chapter XIII. Idempotents and Finite Conjugacy	333
1. Central idempotents	333
2. Nilpotent elements	346
3. Finite conjugacy	348
Bibliography	369
Index	377
Notation	383