

CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES



CDMA06/07

Alternative Perspectives on Optimal Public
Debt Adjustment*

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SEPTEMBER 2011

ABSTRACT

We compare alternative optimal public debt adjustment strategies in a New Keynesian economy. We find that the unconditionally optimal policy is consistent with a gradual adjustment in public debt towards its mean value at a speed determined by the rate of time preference of agents. To a second-order approximation in a stochastic setting, debt follows a unit root process with a negative drift under the 'timeless-perspective' approach but converges to an unconditional mean different from the non-stochastic steady state in the unconditionally optimal economy. Overall, increases in public debt are shown to be optimally reduced by half only after approximately two decades at best.

JEL Classification: C61, E61, E62, E63.

Keywords: Optimal Monetary and Fiscal Policy, Unconditionally Optimal Policy, Timeless Perspective, Public Debt Dynamics, Second-Order Approximation.

* Acknowledgements: The paper benefited from comments and advice received at different stages from Huixin Bi, Charles Brendon, Tatiana Damjanovic, Martin Ellison, Sean Holly, Kaushik Mitra, Antonio Mele, Ioana R. Moldovan, Charles Nolan, Simon Wren-Lewis, audiences at Birkbeck College, at the 2010 RES conference, in an MMF workshop in Birmingham, the editor of the journal and two anonymous referees.

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1. Introduction

Maintaining fiscal solvency requires matching outstanding government liabilities by a discounted stream of future budget surpluses. When inflation or lump sum taxes cannot be used as a costless means to deal with inherited debt or to restore solvency following shocks, appropriate use of public debt facilitates smoothing of distortions over time. If governments borrowed excessively in the past or borrow at present to partially absorb the consequences of shocks, the question arises: How to deal with the higher stock of public debt?

In pursuit of new answers to this question, we introduce the concept of unconditional optimality in the sense of Damjanovic et al. (2008) to fiscal policy in an otherwise standard New Keynesian economy. The policy we examine is also the optimal continuation policy proposed by Jensen and McCallum (2010), which is the best policy on average for all possible initial conditions in a dynamic economy. We find that the speed of debt reduction consistent with the unconditionally optimal policy is determined by the rate of time preference of agents.

The prevailing wisdom is that it is optimal to allow permanent increases in debt and taxes following structural shocks under nominal rigidity.¹ This result has been derived in first-order-accurate models, in which welfare is defined over a conditional welfare measure that discounts future welfare losses, whilst taking into account the impact of current policy decisions on past expectation formation.² The optimality of this strategy rests on a version of the tax smoothing argument according to which it is best to keep debt and taxes permanently higher to avoid a more abrupt short-term reaction in taxes and hence prices. One implication of this policy is that any inherited level of liabilities should be validated by the policy

¹See Benigno and Woodford (2004, 2006) and Schmitt-Grohé and Uribe (2004a).

²We do not consider discretionary policies in this paper given that they are clearly inferior in terms of welfare to policies that take into account the impact of current decisions on the past under standard conditions.

maker. With an unconditional objective, the strategy under which taxes are kept permanently higher cannot be optimal. Intuitively, long-term outcomes implicitly receive a higher weight, given that the policy maker effectively maximizes an undiscounted sequence of period utility functions, which would make the strategy of permanently higher level of taxes that would have to accompany a permanently higher level of debt very costly. Instead, we observe a gradual reduction in public debt to its steady state value. This is a shared feature with the optimal policy under discretion, as is the related fact that the unconditionally optimal policy involves less inertia in the conduct of policy relative to optimal policies consistent with a unit root for debt and taxes.³

We also examine second-order-accurate optimal strategies. Adam (2010) has argued that in such a case, higher debt generates larger risks to the budget and the distortive tax rate, which in turn renders a gradual reduction in debt optimal. We show that in a stochastic economy, public debt follows a unit root process with a drift under the second-order-accurate timelessly optimal strategy. The corresponding unconditionally optimal result for public debt involves convergence to an unconditional mean different from the non-stochastic steady state. In our analysis, these results are mainly driven by the impact of uncertainty in the economy on (the utility value of) the firms' marginal revenue. Expected marginal revenues are a key factor in the firms' price-setting decision, and hence their responsiveness to uncertainty in the economy is an important second-order effect that has to be considered in an environment in which stabilization of (relative) prices is a primary concern. When marginal revenue is convex in uncertainty, as is the case in the unconditionally optimal economy, optimal inflation stabilization requires a small reduction in the mean level of output relative to the steady state.

³Leith and Wren-Lewis (2007) provide a detailed analysis of optimal fiscal policy under discretion in a New Keynesian framework.

The economy thus moves into a territory where marginal revenue is less responsive to uncertainty about government spending. The fiscal policy that implements this then involves on average higher taxes relative to the steady state, which finance on average higher debt in the economy. By contrast, marginal revenue responds to increases in uncertainty at a falling rate under timeless perspective, and the converse of this argument holds. However, we argue that the adjustment in second-order-accurate stochastic settings and the underlying intuition might be model- and shock-dependent.

We also show that the time-series properties of public debt from the first-order-accurate analysis are restored if second-order-accurate economies are treated as deterministic.

Overall, when a gradual debt reduction is an element of an optimal debt adjustment strategy, the prescribed rate of reduction is very slow. At best, the rate of reduction should be in line with the rate of time preference of agents, implying a half life for the deviations from mean of public debt and also the debt-to-GDP ratio of 69 quarters under a standard parameterization. In terms of the speed of adjustment, our results echo the findings of Siu (2004) and Kirsanova and Wren-Lewis (2006). Such a slow adjustment rate is also not at odds with some of the empirical evidence. Friedman (2005), for example, finds a half life of 85 quarters for the response of the debt-to-GDP ratio to a shock to itself on postwar US data.

The rest of the paper is organized as follows. Section 2 sets out the microfoundations of a standard New Keynesian economy with endogenous fiscal dynamics. In section 3, we introduce and define the concept of unconditional optimality in general terms. In section 4, we summarize the key results from our numerical exercise, and put them into a broader context. Finally, section 5 concludes.

2. The model

In this section, we briefly set out the microeconomic foundations of our economy. The model is a standard New Keynesian economy with endogenous fiscal dynamics. We present the key relationships in their non-linear form.

2.1. Consumers

Our model economy is inhabited by an infinite number of identical households of measure one. The representative household derives positive utility from total consumption C of differentiated goods and incurs disutility from supplying labour h , which is captured by the utility function

$$U_t^j = E_t \sum_{T=t}^{\infty} \beta^{T-t} u_T; \quad (2.1)$$

$$u_t = U(C_t) - \Lambda(h_t(j)) dj. \quad (2.2)$$

$0 < \beta < 1$ is the subjective discount rate. U and Λ are functions that satisfy standard properties of continuity and regarding their first and second derivatives. There are j industries producing j industry-specific goods in the economy. Each household supplies labour to a single industry. We assume there exist perfect capital markets that enable insurance across households against idiosyncratic uncertainty, and that the initial level of asset holdings of each household is identical. Social welfare in this economy will be given by the expression

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(Y_T; \widehat{G}_T) - \int_0^1 \Lambda(h_T(j)) dj \right]. \quad (2.3)$$

We assume the following specific functional forms

$$U(Y_t; \widehat{G}_t) = \frac{(Y_t - G_t)^{1-\tilde{\sigma}^{-1}}}{1 - \tilde{\sigma}^{-1}}, \quad (2.4)$$

$$\Lambda(h_t(j)) = \frac{h_t(j)^{1+\omega_w}}{1+\omega_w}, \quad (2.5)$$

where $\tilde{\sigma} > 0$ and $\omega_w > 0$ are constants. In the social welfare function, Y_t denotes aggregate demand, while \widehat{G}_t stands for a shock to government expenditures, which is the only source of disturbance in our model.⁴ The shock is observed after individual (and policy) decisions have been made in the economy. Consumption of individual goods is aggregated into a total consumption index using a standard Dixit-Stiglitz (1977) aggregator

$$C_t = \left[\int_0^1 c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (2.6)$$

in which $\varepsilon > 0$ is a constant and represents the elasticity of substitution across goods in the goods market. Minimization of an expenditure function subject to (2.6) yields an expression for the optimal consumption of good j . A standard income identity then implies the demand function

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon} \quad (2.7)$$

in which p represents the price of individual goods. The aggregate price index P is given by

$$P_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}. \quad (2.8)$$

Furthermore, we assume a decreasing-returns-to-scale production technology so that

$$y_t(j) = h_t(j)^{1/\alpha}, \quad (2.9)$$

⁴We use a shock to government spending to illustrate our main point because its fiscal consequences are most obvious from among the shocks. It is also through such a shock the closest we can get to modelling a ‘fiscal stimulus’ in our framework, which is the context much of the debate about debt adjustment has been taking place in. We have used $\widehat{G} = (G - \bar{G})/\bar{Y}$ in which \bar{Y} stands for steady-state aggregate output.

with $\alpha > 1$. This setup allows us to express the total disutility from supplying labour as

$$\int_0^1 \Lambda(h_t(j)) dj = \frac{1}{1 + \omega_w} Y_t^{\alpha(1 + \omega_w)} \delta_t \quad (2.10)$$

in which δ_t refers to price dispersion and is given by

$$\delta_t = \int_0^1 \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon(1 + \omega)} dj. \quad (2.11)$$

We have used $\omega = \alpha(1 + \omega_w) - 1$.

The representative household maximizes (2.3) subject to a standard intertemporal constraint

$$P_t C_t + B_t = (1 + i_{t-1}) B_{t-1} + (1 - \tau_t) \int_0^1 w_t(j) h_t(j) dj + P_t \Psi_t,$$

equating after-tax wage and dividend income Ψ together with asset returns to consumption and change in assets B . w and i are period nominal wage and interest rates, respectively. τ denotes the proportional tax rate levied on wage income. This problem yields the Euler equation that defines the stochastic asset pricing kernel in our model

$$Q_{t,t+1} = \frac{1}{(1 + i_t)} = \beta E_t \frac{U_C(Y_{t+1}; \widehat{G}_{t+1})}{U_C(Y_t; \widehat{G}_t)} \frac{P_t}{P_{t+1}}. \quad (2.12)$$

We also obtain the expression for the equilibrium wage rate

$$\frac{w_t(j)}{P_t} = \frac{\Lambda_h(h_t(j))}{U_C(Y_t; \widehat{G}_t) (1 - \tau_t)}, \quad (2.13)$$

where lower-case subscripts denote the respective first derivatives.

2.2. Firms

Firms maximize profits with wages being the only cost item in their accounts. The t -period profit function of a firm producing good j can be written as follows

$$\Psi_t(k) = (1 + \bar{\tau}^s) p_t(j) y_t(j) - w_t(j) y_t(j)^\alpha - T. \quad (2.14)$$

The constant $\bar{\tau}^s$ stands for a time- and state-invariant subsidy received by the firms from the government as a compensation to eliminate the distortions arising from taxation and excess market power. The inclusion of such a subsidy is conceptually useful, as it ensures that inflation is zero in the Ramsey steady state.⁵ Here, we also include a steady-state lump-sum tax T on the private sector. This is a parametric assumption that ensures that the government runs a surplus in the steady state.

We assume pricing according to Calvo (1983), with γ being the probability of leaving prices unchanged in a given period. The firm is choosing the optimal price and the intertemporal first-order condition—which defines price dispersion (and hence implicitly also inflation) as a function of discounted streams of marginal revenues and costs—can be written as

$$\frac{K_t}{F_t} = \left(\frac{1 - \gamma \Pi_t^{\varepsilon-1}}{1 - \gamma} \right)^{\frac{1+\varepsilon\omega}{1-\varepsilon}}, \quad (2.15)$$

with

$$K_t = \frac{\mu\alpha Y_t^{1+\omega}}{(1 + \bar{\tau}^s)(1 - \tau_T)} + \gamma\beta E_t K_{t+1} \Pi_{t+1}^{\varepsilon(1+\omega)} \quad (2.16)$$

and

$$F_t = U_C \left(Y_t; \widehat{G}_t \right) Y_t + \gamma\beta E_t F_{t+1} \Pi_{t+1}^{\varepsilon-1}. \quad (2.17)$$

⁵Relaxing this assumption and allowing for trend inflation or a suboptimal (inefficient) steady state is a natural extension of the analysis. It is, however, associated with significant losses in terms of the clarity of the analysis. The main results from this paper concerning long-term debt dynamics would still hold, as we argue in section 4.3 of the paper.

The constant $\mu = \varepsilon / (\varepsilon - 1)$ stands for the price mark-up over marginal cost and $\Pi_t = P_t / P_{t-1}$. We have used the fact that the evolution of the price level is given by

$$P_t = [(1 - \gamma) p_t^{*1-\varepsilon} + \gamma P_{t-1}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \quad (2.18)$$

and the implicit definition of inflation implied by this evolution of the price level. The variable p_t^* above is the (common) optimal price chosen by the optimizing firms in period t . The law of motion for price dispersion defined in (2.11) is given by

$$\delta_t = \gamma \Pi_t^{\varepsilon(1+\omega)} \delta_{t-1} + (1 - \gamma) \left(\frac{1 - \gamma \Pi_t^{\varepsilon-1}}{1 - \gamma} \right)^{-\frac{\varepsilon(1+\omega)}{1-\varepsilon}}. \quad (2.19)$$

2.3. Government

Monetary and fiscal authorities, the two branches of the central government, coordinate their actions to ensure that social welfare given by (2.3) is maximized. The government raises revenue via distortionary taxes on wage income to finance exogenous government spending G and the steady-state subsidy. It also collects the lump-sum tax from the private sector. It issues one-period nominal bonds to bridge the gap between taxation and spending. The government therefore faces a flow budget constraint

$$B_t = (1 + i_{t-1}) B_{t-1} - P_t \Delta_t \quad (2.20)$$

where B denotes the volume of one-period nominal bonds issued by the fiscal authority and Δ is the primary budget surplus which can be expressed as follows

$$\Delta_t = \tau_t \int_0^1 \frac{w_t(j)}{P_t} h_t(j) dj - G_t - \bar{\tau}^s \int_0^1 \frac{p_t(j)}{P_t} y_t(j) dj + T.$$

This constraint can be re-written as

$$\frac{b_t}{(1 + i_t)} = \frac{b_{t-1}}{\Pi_t} - \Delta_t \quad (2.21)$$

with $b = (1 + i)B/P$. This flow budget constraint implies the following sustainability condition

$$\frac{b_{t-1}}{\Pi_t} U_C \left(Y_t; \widehat{G}_t \right) = \Delta_t U_C \left(Y_t; \widehat{G}_t \right) + \beta E_t \frac{b_t}{\Pi_{t+1}} U_C \left(Y_{t+1}; \widehat{G}_{t+1} \right) \quad (2.22)$$

which requires the current value of outstanding real liabilities to be offset by the discounted sum of future primary surpluses, all priced in marginal utility terms. We have used (2.12) to substitute for the period interest rates. We also assume that government policies are such that a no-Ponzi condition and a transversality condition on debt are satisfied.

2.4. Equilibrium

Equilibrium in this model is given by state-contingent paths for endogenous variables $\{Y_T, b_T, \tau_T, \Pi_T, \delta_T, i_T, w_T\}_{T=t}^{\infty}$ that satisfy (2.12), (2.13) with $w = \int_0^1 w(j) dj$, (2.15), (2.16), (2.17), (2.19) and (2.22), given values for b_{t-1}, δ_{t-1} .

The standard way to proceed from here is to set up an optimal policy problem, which involves finding the government policy consistent with the equilibrium of the economy such that maximizes social welfare (2.3). Under the timeless perspective approach to optimal policy, the Lagrangian contains terms that constrain the policy maker to implement the long-run optimum in the initial period. We solve such a problem numerically, and present first- and second-order-accurate results in section 4 of the paper.

3. The unconditionally optimal policy

One encounters different perspectives on optimality in the optimal policy literature. Kim et al. (2005), for instance, have argued in favour of defining

optimal policy over conditional expectations to allow policies to deliver different stochastic steady states. Indeed, this has been the perspective taken in the most recent papers on optimal monetary and fiscal policy interactions. There is, however, an alternative perspective going back to Taylor (1979) and Whiteman (1986), also used in Rotemberg and Woodford (1997) and Erceg et al. (2000) to evaluate alternative policy options, and most recently treated extensively in the context of monetary policy design in Damjanovic et al. (2008) and Jensen and McCallum (2010). Whilst constraining the analysis to stationary outcomes, this perspective seeks to identify the policy that is optimal on average for all possible initial conditions in an economy.

The treatment of initial conditions is particularly important for welfare in environments in which expectations of future policy determine current and past outcomes. The essence of the debate in the literature on alternative concepts of optimality is about weighing the welfare effects of dealing with initial conditions against the welfare effects of responding optimally to shocks, which includes allowing for non-stationary responses.

When deriving optimal policy over conditional expectations in forward-looking frameworks, one either has to assume full commitment to the optimal Ramsey plan, which implies a time-varying policy rule and often rather unrealistic policy prescriptions, or bypass the time-inconsistency problem of Kydland and Prescott (1977) by imposing some restriction on the nature of policy in the initial period. This may either take the form of a commitment to a specific outcome such as a price level, as in Schmitt-Grohé and Uribe (2004a), or to an appropriate policy rule ensuring continuation, as in Benigno and Woodford (2004). In effect, formulating optimal policies this way implies that the derived policy rule is associated with optimal responses to shocks but will be suboptimal for a transitory period if the system starts from non-zero initial conditions, as explained lucidly in Woodford

(2003).⁶

By contrast, if optimal policy is formulated over unconditional expectations, the policy response will be suboptimal throughout but will be ‘optimally suboptimal’ according to Jensen and McCallum (2010). This is because the economy does not respond to shocks optimally in the long-run under the unconditionally optimal policy. However, the policy partially exploits non-zero initial conditions and the fact that they are given, which involves a welfare gain. Whilst ensuring continuation through a time-invariant policy rule, it is closer to the full commitment outcome in the initial period, and to discretion in general, where the latter also implies that the incentive to deviate from the unconditionally optimal policy is smaller.

Having reviewed the arguments, let us state that it is not the purpose of this paper to contribute to the discussion on the relative merits of the alternative perspectives on optimality, nor is the intention to conduct a comparative welfare analysis. We wish to concentrate on the implications of these alternative concepts for optimal debt dynamics.

Turning to a more formal definition of the unconditional perspective on optimality, let us denote the historical realizations of the shock to government spending as $G^T = \{G_s\}_{s=0}^T$, which have a marginal distribution F^T , for all T .⁷ More generally, we define $G_t^T = \{G_s\}_{s=t}^T$ for $0 \leq t \leq T$. Let $F_t^T(G_t^T|G^{t-1})$, in which $0 \leq t \leq T$, denote the conditional distribution of G_t^T given G^{t-1} . The social

⁶Otherwise, the conditionally optimal policy—in a strict sense—would be different for different initial conditions. This dependence on initial conditions, also identified in Soderlind (1999), could cause a substantial degree of ambiguity in the ranking of alternative policies.

⁷For simplicity, we consider, as in much of the analysis in the rest of the paper, that the disturbance G is i.i.d. Damjanovic et al. (2008) set out the same problem for autocorrelated shocks.

welfare function (2.3) can then be written as

$$U_t = \int \sum_{T=t}^{\infty} \beta^{T-t} \tilde{U}(C_T(G_t^T), h_T(G_t^T)) dF_t^T(G_t^T | G^{t-1}).$$

This is the conditional objective the literature traditionally looks at.

Now, let F be the time-invariant joint distribution of G^t for all t . Let us also assume that the probability distribution of an endogenous variable x , $F(x(G^t))$, is time-invariant. The objective the unconditionally optimal policies will aim to maximize is then given as follows.

Definition 3.1. *The unconditional expectation of the social welfare function U_t , denoted $\tilde{E}U_t$, is defined as*

$$\tilde{E}U_t = \int U_t(G^t) dF(G^{t-1}).$$

Definition 3.2. *The unconditionally optimal policy is a pair of sequences $\{i_T, \tau_T\}_{T=t}^{\infty}$ consistent with the maximum value of $\tilde{E}U_t$, whilst satisfying the constraints (2.15), (2.16), (2.17), (2.19) and (2.22). Equation (2.12) then defines the optimal interest rate and (2.13) gives us the optimal real wage dynamic.*

Intuitively, the unconditionally optimal policy thus maximizes social welfare on average for all possible histories of shocks. It also implies an asymptotic distribution of initial conditions for endogenous state variables. The relevant welfare ranking criterion for policies in the class is then defined over this distribution of initial conditions.

Let $\phi_i, i = 1, 2, 3, 4, 5$ be the Lagrange multipliers associated with the constraints (2.15), (2.16), (2.17), (2.19) and (2.22) respectively. It follows from what we have defined above that for any stationary endogenous variable x , $\tilde{E}x_T = \tilde{E}x_t$ for all $T > t$. The law of iterated expectations holds and hence

$\tilde{E}x_t = \tilde{E}E_t x_t$. Another important property of the unconditional expectations of variables with invariant distribution is that for any endogenous variable x_t , it holds that $\tilde{E}E_t \phi_{iT} x_{T+1} = \tilde{E}\phi_{it-1} x_t$ for all $T \geq t$.⁸ Note also that maximizing $\tilde{E}U_t$ is, given the above properties, equivalent to minimizing

$$L_t = -\tilde{E}E_t u_t. \quad (3.1)$$

This is because maximizing a discounted stream of variables that are constant in expectation is equivalent to maximizing a period utility function.⁹ Moreover, since the policy that maximizes welfare in every state of nature will also maximize welfare in a (weighted) sum of those states, i.e. in the unconditional expectation, it is sufficient to evaluate policies according to the term inside the unconditional expectation operator in (3.1). The solution from here onwards again follows standard steps used in optimal control problems.

4. Numerical results

To analyze the dynamic implications of alternative perspectives on optimality in the context of our model, we conduct several numerical exercises. We solve the policy problems defined in the previous sections using the procedure of Schmitt-Grohé and Uribe (2004b). We shall look at the dynamics of the economy implied by first- and second-order approximations to the optimality conditions, and concentrate on optimal debt dynamics.

⁸These properties follow from the definition of stationarity. See, for instance, Hamilton (1994, pp. 45-6 and 261-2).

⁹We have dropped the scaling factor $1/(1-\beta)$ which, of course, has no effect on the relative ranking of alternate policies within the class of stationary policies.

Parameter		Value
Discount factor	β	0.99
Coefficient of relative risk aversion	$\tilde{\sigma}^{-1}$	0.13
Elasticity of substitution in the goods sector	ε	10.0
Production function parameter	α	1.25
Calvo-pricing parameter	γ	0.65
Inverse Frisch elasticity	ω_w	0.18
Steady-state tax on wage income	$\bar{\tau}$	0.30
Share of private consumption on Y	\bar{C}/\bar{Y}	0.80

Table 4.1: Parameter values

4.1. Parameterization

We parameterize the model using the values in Table 4.1. The values for $\tilde{\sigma}^{-1}$ and $\omega_w = (1 + \omega) / \alpha - 1$ are consistent with Rotemberg and Woodford (1997). We set the lump-sum steady-state transfer from the private sector to the government so that there is a primary surplus consistent with a 40 percent public debt-to-GDP ratio in the steady state. In the stochastic simulations, the standard deviation of the distribution from which the government spending shock is drawn is assumed to be one percent.

4.2. Numerical exercises

We conduct three exercises. In all, we look at the consequences of a single non-inertial innovation to the government spending-to-GDP ratio.¹⁰ First, we look at the optimal dynamics when the optimality conditions under both the timeless and unconditional perspective on optimal policy are log-linearized. Second, we simulate the second-order-accurate non-stochastic optimal economies. Third, we

¹⁰One advantage of considering a non-inertial shock is that it is easy to disentangle endogenous inertia from the effects of serial correlation in the shock. Otherwise, the latter might dominate for a considerable length of time. Also, note that the convergence properties are the same in response to initial debt and hence we do not treat this question separately.

look at optimal dynamics in second-order-accurate stochastic economies.

Result 1 *In the log-linearized unconditionally optimal economy, public debt converges to its non-stochastic steady state at a rate determined by the rate of time preference.*

In other words, half of the response in public debt to the shock is undone only after approximately 17 years. We also see from Figure 4.1 that the dynamic of the unconditionally optimal economy involves more short-term volatility than under the economy under the timelessly optimal plan.¹¹ This follows from the fact that under the unconditionally optimal perspective, the degree to which tax smoothing can be implemented is limited. This is intuitive and follows from the way formulation of unconditionally optimal policies implied by the ranking criterion (3.1) differs from the formulation of policies optimal from a timeless perspective. When deriving optimal responses to shocks over conditional expectations, one discounts future welfare losses arising from deviations in public debt, the tax rate and hence output from their steady-state levels. The benefits of short-term stability outweigh the (discounted) costs of permanent future deviations. Hence, optimal tax smoothing involves a permanent tax increase, which makes it possible to achieve more stability in the short term through more extensive use of debt finance. In the case of the alternative class of policies we examine, the intertemporal ‘terms of trade’ are different. The unconditional welfare measure is defined so that future welfare losses are undiscounted and thus receive an equal weight. This makes the policy of a permanent shift in public debt and the tax rate an unattractive strategy. Debt and taxes are thus brought back to their steady

¹¹The short-term dynamics is little changed when we later consider higher-order approximations. Hence, we do not reproduce this figure for higher-order approximations, and concentrate instead on debt dynamics. Also, note that one on the vertical axes denotes a one-percent deviation from the non-stochastic steady state.

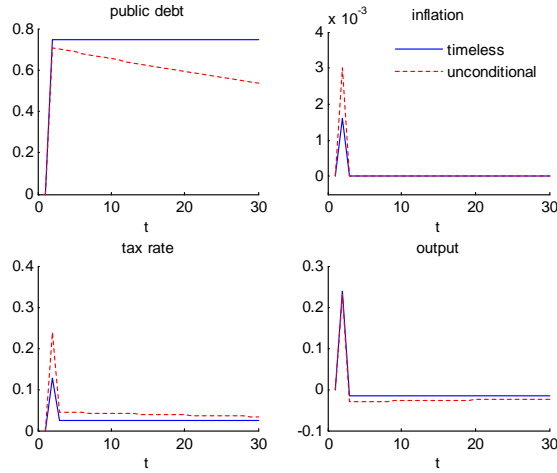


Figure 4.1: Optimal dynamics under timeless perspective and unconditionally optimal dynamics in the log-linearized economy

state level at the cost of somewhat higher short-term volatility.

Result 2 *Optimal debt dynamics in second-order-accurate deterministic models do not differ substantially from those in the linearized optimal economies.*

We simulated the optimal economies of sections 2 and 3 using second-order approximations around their non-stochastic steady state, assuming that the agents have perfect foresight. Figure 4.2 shows that the optimal reactions to a one-period increase in government spending barely differ from those obtained in the linearized economy. The conclusions concerning the optimal speed of debt reduction in the unconditionally optimal economy remain unchanged.

Result 3 *In the second-order-accurate stochastic optimal economy, public debt follows a unit root process with a drift under timeless perspective but converges to a new unconditional mean in the unconditionally optimal economy. The implied autocorrelation coefficient of debt is approximately the discount factor in the*

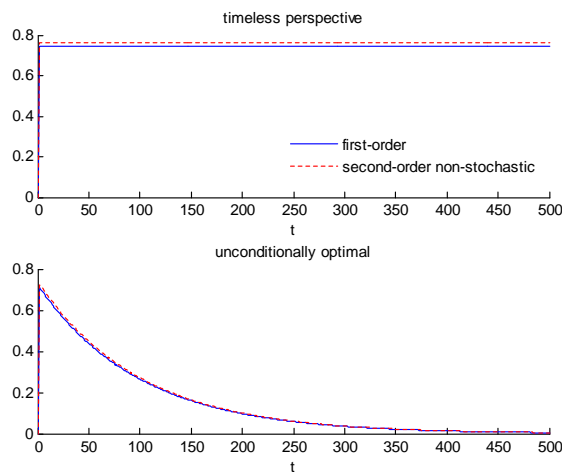


Figure 4.2: Debt dynamics in the log-linearized and in the second-order accurate non-stochastic economy

unconditionally optimal economy and even closer to unity in the baseline timelessly optimal economy.

As established in Schmitt-Grohé and Uribe (2004b), and also shown in Gomme and Klein (2011), the second-order-accurate solution to the dynamic of a variable in a stochastic economy contains a deterministic drift term accounting for the impact of the presence of uncertainty on the mean of the optimal decision rules. Thereby, in a stochastic setting, the unconditional means of variables may be different from their non-stochastic steady state values, whereas they coincide (in stationary models) under first-order approximation. Adam (2010) reports the drift terms associated with public debt as the parameter determining optimal debt reduction.

Whilst we see debt reduction being consistent with the timelessly optimal policy in our model too, the process for debt does not represent convergence to

zero.¹² The influence of drift terms comes on top of an economy that has a unit root. This conclusion is based on the information in the previous exercise, and is also confirmed when we plot the optimal response in public debt for an economy with more volatile shocks in the bottom panel of Figure 4.3. Moreover, Table 4.2 tells us that the drift term associated with debt is positive. Yet we still see a gradual reduction in debt being consistent with optimal policy.¹³

Interestingly also, debt converges back to a new unconditional mean which is higher than the steady state in an unconditionally optimal economy, as also seen from Figure 4.3, whilst the drift term associated with the optimal debt dynamic has the same sign as in the timelessly optimal economy.

We find that the key factor underlying these dynamics is the curvature of the firms' marginal revenue function F with respect to uncertainty. Marginal revenue in the timelessly optimal economy is concave with respect to uncertainty about the level of spending, whilst it is convex in the unconditionally optimal economy. From the perspective of price stabilization in the face of uncertainty, it is thus optimal to move into a territory in which changes in uncertainty have less of an effect on marginal revenue by reducing (raising) the level of output below (above) its steady-state level in the unconditionally (timelessly) optimal economy. The corresponding optimal tax policy features a permanently higher (lower) tax rate, which finances a permanently higher (lower) level of public debt relative to the steady state.

¹²We have assumed that the transversality condition is satisfied, albeit demonstrating this is not necessarily straightforward. One might also have concerns about the accuracy of approximation under such non-stationary dynamics. As in Benigno and Woodford (2004, footnote 26), one can impose a suitable distribution on the disturbances to ensure the economy remains in the neighbourhood of the steady state. An interesting line of thought is whether the policy maker would wish to fine-tune his steady-state subsidization policy in the light of the level effects of uncertainty. We abstract from such considerations in the analysis given that the steady-state subsidy is motivated by analytical rather than fundamental reasons.

¹³Note that the source of disturbance in our model are government spending shocks, whilst Adam (2010) looks at the consequences of productivity shocks.

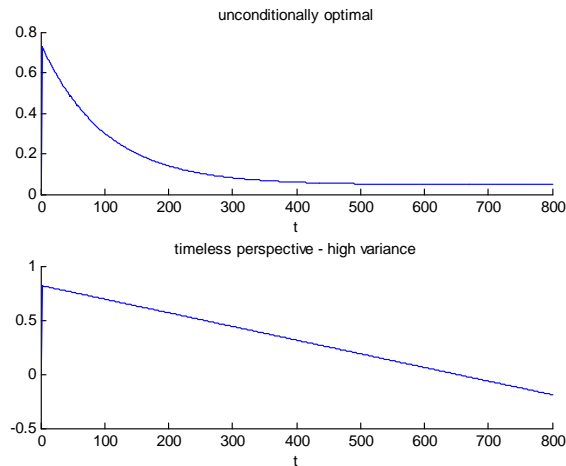


Figure 4.3: Second-order-accurate optimal debt dynamics in the stochastic economy

The rate of debt adjustment is in either case very slow. The serial correlation coefficient of the debt series under timeless perspective exceeds 0.9999, whilst it is again approximately the discount factor in the unconditionally optimal economy.¹⁴

4.3. General remarks

The results concerning optimal debt dynamics under first-order approximation require little qualification if one were to consider more complicated models in the family of models to which our simple setup belongs. The general intuition concerning debt dynamics outlined below would be little changed, should we consider models with sticky wages in addition to prices, different indexation mechanisms to introduce more real or nominal inertia or in which no steady-

¹⁴The coefficient under timeless perspective falls when the standard deviation of the shock is increased. For example, it falls to 0.998 on average over the first twenty years of adjustment if the standard deviation of the distribution from which the shock is drawn is increased five times (as shown in the bottom panel in Figure 4.3).

Variable	Timeless perspective	Unconditionally optimal
Y	0.142	0.267
K	-0.308	-0.361
F	-0.081	0.059
Π	-0.021	-0.039
τ	-1.624	-3.075
b	0.445	0.925
δ	0	0

Table 4.2: Constant terms associated with the impact of uncertainty (x $1/10^4$)

state subsidy would exist or be of practical use, and hence the optimal steady state would be associated with non-zero inflation.¹⁵

A straightforward way of detecting what happens to debt in the linearized model is to inspect the dynamic properties of the Lagrange multiplier associated with the fiscal solvency constraint (2.22). Such a constraint is present in all models with endogenous fiscal dynamics. The standard result from the literature using welfare measures defined over conditional expectations is that this shadow price is a Martingale. When the objective is unconditional, the shadow price follows an autoregressive process. The value of the shadow price is non-zero if the solvency condition binds when shocks hit the economy, which is the case under nominal rigidity and distortive taxation. This means that welfare is enhanced if debt finance different from the steady-state level of debt is available following shocks. The optimal solution will then entail a deviation in debt from its steady state value. The unit-root property of the Lagrange multiplier under timeless perspective implies maintaining a higher debt level will be optimal, whilst the

¹⁵See Horvath (2007) for analysis including sticky wages in an environment with an inefficient zero-inflation steady state. Schmitt-Grohé and Uribe (2006) and Chugh (2006) only report serial correlations for public debt, showing that debt follows a near-random-walk process under nominal rigidity. Although Schmitt-Grohé and Uribe (2006) do not comment explicitly on the issue of stationarity, the figures seem to suggest a level shift under optimal policy.

autoregressive nature under unconditional optimality tells us that the positive contribution of debt increments to welfare will slowly vanish over time, and hence debt will slowly converge back to its mean value. Short-term dynamics of public debt might be temporarily dominated by the certain inertial elements elsewhere in the model such as habits, price and wage indexation schemes as well as inertial shocks. Once these influences die out, one would observe debt staying permanently higher under timeless perspective, and a smooth convergence determined by the rate of time preference back to the mean under unconditional optimality.

In terms of policy, such debt dynamic is a consequence of the policy maker placing a smaller weight in the policy rule (by a factor of β) on lagged variables under the unconditionally optimal policy relative to the policy optimal from a timeless perspective. Since our model differs from Benigno and Woodford (2004) only to the extent that the steady state is assumed to be efficient, this result is easily shown if one takes the linear-quadratic problem from their paper and solves it using the alternative policy objectives.¹⁶ Hence, the two policies differ in the extent policy makers take into account the effect of their current decisions on expectation formation in the past. This is a general point also mentioned in Jensen and McCallum (2010).

Clearly, the conclusions concerning the speed of debt adjustment in the first-order accurate model would not be affected by the type of shock considered either, as these enter the approximated model in an additive fashion and hence, do not influence dynamics. Adam (2010) shows that whilst the optimal steady state of the economy and the dynamics of other variables in the system are affected, the optimal unit-root result for debt survives when government spending is endogenized. We have explained above why this happens and how it would

¹⁶It can be shown that the correct linear-quadratic problem defined over unconditional expectations has the same functional form as the ‘naive’ approach of taking the unconditional expectation of the linear-quadratic problem defined over conditional expectations.

change, should one adopt the unconditional perspective on optimal policy.

Finally, the results from the second-order-accurate stochastic simulation, in particular the magnitude and the sign of the drift terms, appear to be model-sensitive and also dependent on the type of shock considered. The intuition behind our results driven by government spending shocks in a simple setup differs from the intuition given by Adam (2010) in a more complex framework perturbed by productivity shocks. We have repeated the simulations with productivity shocks instead of government spending shocks in our framework, and found the drift term associated with public debt to be negative and debt also falling at a slow speed, as in Adam (2010). But the analysis in this paper also tells us that looking at the drift term associated with debt might not be sufficient to fully account for its dynamics. Also, the new mean to which the unconditionally optimal economy converges to following the productivity shock was below the non-stochastic steady state, which is different from the convergence seen following government spending shocks. It would perhaps be interesting to investigate this issue further in the context of a medium-scale macroeconomic model.

5. Conclusions

We have looked at the question of optimal debt adjustment in a New Keynesian economy from the angle of two different concepts of optimality. We have shown that the conventional result of keeping debt and taxes permanently at a different level following shocks no longer holds if one considers the unconditionally optimal policy, or if second-order considerations are brought into play in a stochastic setting. We found that the speed of debt reduction consistent with optimal policy is likely to be very slow, with the half life of debt adjustment exceeding 17 years. Given this slow speed of adjustment, it might be interesting to consider appropriate institutional arrangements to implement such optimal plans as a

distinct and credible strategy within a class of policies that includes more costly alternatives with a similar dynamic for public debt.

Throughout the paper, we assumed that fiscal solvency is always satisfied, and that the interest rate on public debt does not carry a risk premium. Allowing for the possibility of default and increases in the risk premium following increases in public debt might affect the optimal debt adjustment following shocks. An interesting avenue for future research is thus to consider optimal debt adjustment in the context of the recently developed literature on fiscal limits and sovereign default.

References

- [1] Adam, K., 2010. Government Debt and Optimal Monetary and Fiscal Policy, CEPR Discussion Paper No. 8064, October 2010. (forthcoming in the European Economic Review)
- [2] Benigno, P., Woodford, M., 2004. Optimal monetary and fiscal policy: A linear-quadratic approach, in: Gertler, M., Rogoff, K. (Eds.), NBER Macroeconomics Annual 2003, pp. 271-332.
- [3] Benigno, P., Woodford, M., 2006. Optimal inflation targeting under alternative fiscal regimes, NBER Working Paper No. 12158.
- [4] Calvo, G., 1983. Staggered prices in a utility-maximising framework, *Journal of Monetary Economics* 12, pp. 383-98.
- [5] Chugh, S.K., 2006. Optimal fiscal and monetary policy with sticky wages and sticky prices, *Review of Economic Dynamics* 9, pp. 683-714.
- [6] Damjanovic, T., Damjanovic, V., Nolan, C., 2008. Unconditionally optimal monetary policy, *Journal of Monetary Economics* 55 (3), pp. 491-500.
- [7] Dixit, A. , Stiglitz, J.E., 1977. Monopolistic competition and optimal product diversity, *American Economic Review* 67, pp. 297-308.
- [8] Erceg, C.J., Henderson, D.W., Levin, A.T., 2000. Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics* 46, pp. 281-313.
- [9] Friedman, B. M., 2005. Deficits and Debt in the Short and Long Run, in Kopcke, R. W., Tootell G. M. B. and Triest, R. K. (Eds.), *The Macroeconomics of Fiscal Policy*, MIT, Cambridge.

- [10] Gomme, P., Klein, P., 2011. Second-order approximation of dynamic models without the use of tensors, *Journal of Economic Dynamics and Control*, 35, pp. 604-615.
- [11] Hamilton, J.D., 1994. *Time Series Analysis*, Princeton University Press.
- [12] Horvath, M., 2007. *Optimal Monetary and Fiscal Policy in Economies with Multiple Distortions*, Ph.D. Thesis, University of St Andrews.
- [13] Jensen, C., McCallum, B.T., 2010. Optimal Continuation vs Timeless Perspective, *Journal of Money, Credit and Banking*, 42(6), pp. 1093-1107.
- [14] Kim, J., Kim, S., Schaumburg, E., Sims, C.A., 2005. Calculating and using second order accurate solutions of discrete dynamic equilibrium models, Working Paper, Tufts University
- [15] Kirsanova, T., Wren-Lewis, S., 2006. Optimal fiscal feedback on debt in an economy with nominal rigidities, *CDMA Conference Papers 2006*.
- [16] Kydland, F.E., Prescott, E.C., 1977. Rules rather than discretion: The inconsistency of optimal plans, *The Journal of Political Economy* 85 (3), pp. 473-492.
- [17] Leith, C., Wren-Lewis, S., 2007. *Fiscal Sustainability in a New Keynesian Model*, Department of Economics Discussion Paper Series No. 310, University of Oxford.
- [18] Rotemberg, J.J., Woodford, M., 1997. An optimization-based econometric framework for the evaluation of monetary policy, in: Bernanke, B.S., Rotemberg, J.J. (Eds.), *NBER Macroeconomics Annual 12*, pp. 297-346.

- [19] Schmitt-Grohé, S., Uribe, M., 2004a. Optimal fiscal and monetary policy under sticky prices, *Journal of Economic Theory* 114, pp. 198-230.
- [20] Schmitt-Grohé, S., Uribe, M., 2004b. Solving dynamic general equilibrium models using a second-order approximation to the policy function, *Journal of Economic Dynamics and Control*, 28, pp. 755-775.
- [21] Schmitt-Grohé, S., Uribe, M., 2006. Optimal fiscal and monetary policy in a medium-scale macroeconomic model, in Gertler, M., Rogoff, K. (Eds.), *NBER Macroeconomics Annual*, pp. 383-425.
- [22] Siu, H.E., 2004. Optimal fiscal and monetary policy with sticky prices, *Journal of Monetary Economics* 51, pp. 575-607.
- [23] Soderlind, P., 1999. Solution and Estimation of RE Macromodels with Optimal Policy, *European Economic Review*, 43, 813-823.
- [24] Taylor, J. B., 1979. Estimation and control of a macroeconomic model with rational expectations, *Econometrica* 47 (5), pp. 1267-1286.
- [25] Whiteman, C., 1986. Analytical policy design under rational expectations, *Econometrica* 54 (6), pp. 1387-1405.
- [26] Woodford, M., 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

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