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America West Airlines Develops Efficient Boarding Strategies

Menkes H. L. van den Briel, J. René Villalobos, Gary L. Hogg
Department of Industrial Engineering, Arizona State University, PO Box 875906, Tempe, Arizona 85287-5906
{menkes@asu.edu, rene.villalobos@asu.edu, ghogg@asu.edu}

Tim Lindemann, Anthony V. Mulé
Airport Services, America West Airlines, 111 West Rio Salado Parkway, Tempe, Arizona 85281
{tim.lindemann@americawest.com, anthony.mule@americawest.com}

In September 2003, America West Airlines implemented a new aircraft boarding strategy that reduces the airline's average passenger boarding time by over two minutes, or approximately 20 percent, for full and nearly full flights. The strategy, developed by a team of Arizona State University and America West Airline's personnel, is a hybrid between traditional back-to-front boarding and outside-inside boarding used by other airlines. Field observations, numerical results of analytical models, and simulation studies provided information that resulted in an improved aircraft-boarding strategy termed *reverse pyramid*. With the new boarding strategy, passengers still have personal seat assignments, but rather than boarding by rows from the back to the front of the airplane, they board in groups minimizing expected passenger interference in the airplane. The analytical, simulation, and implementation results obtained show that the method represents a significant improvement in terms of boarding time over traditional pure back-to-front, outside-inside boarding strategies.

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A traditional metric used by commercial airlines to measure the efficiency of their operations is airplane turnaround time. Usually turnaround time (or turn time) is measured by the time between an airplane's arrival and its departure. Recently, commercial airlines have paid a great deal of attention to turnaround time because they believe it affects the overall success of an airline. One of the main factors cited for the success of discount (or no-frills) airlines is the quick turnaround of their airplanes, which helps them achieve high airplane utilization (Allen 2000, Michaels 2003). Thus, they make efficient use of their primary capital investment, the aircraft. Long turnaround times decrease revenue-producing flying time, while short turnaround times please customers and can increase airlines' revenues.

Some factors that determine turnaround time include passenger deplaning, baggage unloading, fueling, cargo unloading, airplane maintenance, cargo loading, baggage loading, and passenger boarding. While improving any of these factors can decrease turnaround time, a factor that is particularly difficult to shorten is passenger-boarding time. Airlines have little control over passenger-boarding time because they have limited control over passengers. Furthermore, passengers expect levels of service corresponding to the airline and the class of service they pay for, from no preassigned seats on a discount airline to boarding preference in first class on a full-service airline. Therefore, while airlines want to speed up the passengers boarding airplanes, they have been cautious in making changes to increase operational efficiency.

America West Airlines has made efforts to improve its turnaround performance. We worked on a joint project between America West and Arizona State University to cut passenger-boarding times for America West's narrow-body passenger airplanes, such as the Airbus A320 and the Boeing 737, which have a central aisle and rows of three seats on both sides of the aisle. The project included gathering data, developing and solving mathematical programming and simulation models, and validating and implementing the results.

America West Airlines

America West Airlines is a major US carrier based in Phoenix, Arizona, from which it serves more destinations nonstop than any other carrier. America West has hubs at Phoenix Sky Harbor International Airport in Phoenix and McCarran International Airport in Las Vegas, Nevada. The airline's modern, fuel-efficient fleet consists of Airbus A320s, Airbus A319s, Boeing 757s, and Boeing 737s. By the end of 2006, America West expects to take delivery of new 110-seat Airbus A318s. The average age of the planes in America West's fleet is about 10 years.

America West is the only major airline not only to survive, but also to thrive since the US airline industry was deregulated in 1978. The carrier began service on August 1, 1983, with three airplanes and 280 employees. It grew rapidly and, by 1990, had become a major airline, with annual revenues of over $1 billion.

Today, America West is a low-fare, full-service airline. Its coast-to-coast route system includes 90+ destinations across the United States, Mexico, Canada, and Central America, with more than 800 daily departures. It uses its Phoenix and Las Vegas hubs as gateways for travel throughout its route network. America West Express provides regional service through code-sharing agreements with Mesa Airlines, and Air Midwest, which are wholly owned subsidiaries of Mesa Air Group, one of the largest regional airlines in the world. These regional carriers channel traffic to America West's hubs. Through the agreement with Mesa Air Group, America West plans to extend its route system and enhance its flight schedule as America West Express increases its regional jet fleet to 77 airplanes by 2005.

Project Objectives

America West Airlines asked members of the industrial engineering department of Arizona State University to take a critical look at the existing boarding procedures and to propose new strategies.

Our task consisted of recommending a boarding strategy that would minimize the average boarding time, thus improving turnaround times and utilization of the aircraft fleet.

Like most commercial airlines, America West traditionally boarded passengers in groups of those sitting in contiguous rows, ordering these groups from the back to the front of the airplane (back-to-front approach), after boarding special groups (usually first-class and special-needs passengers). The logic behind this boarding procedure was that freeing the passengers making the journey to the back of the airplane from aisle obstacles would minimize congestion in the aircraft aisle. However, the back-to-front approach created congestion in a reduced area of the aisle among passengers of the same group, impeding their access to overhead bins for stowing carry-on luggage and making it difficult for them to reach their assigned seats. We conjectured that a different boarding approach, with the groups composed of passengers dispersed throughout the airplane, might perform better. This conjecture was the basis for our recommendation that America West should replace the existing back-to-front groups with groups made up of a widespread cross-section of the plane. Other airlines have experimented with such alternatives as the outside-in approach; but we found no formal and comprehensive analysis of this approach or of the back-to-front approach in the open literature.

Previous Works and Project Strategy

Marelli et al. (1998) described a simulation-based analysis performed for Boeing. They designed the passenger enplane/deplane simulation (PEDS) to test different boarding strategies and different interior configurations on a Boeing 757 airplane. PEDS showed that by boarding from the outside in, that is, window-seats first, middle seats second, and aisle seats last, airlines could reduce boarding times significantly.

Shuttle by United was one of the first airlines to actually employ the outside-in strategy. While Kimes and Young (1997) reported that the airline implemented the method with a good degree of success, United Airlines later discontinued the method and replaced it with its current approach: boarding all premium-class customers first, economy-plus customers second, customers seated in the last 10 rows of economy third, and all remaining customers fourth.

Van Landeghem and Beuselinck (2002) conducted another simulation-based study on airplane boarding that showed that the fastest way to get people on an airplane would be to board them individually by their row and seat number by calling each one of the passengers individually to board the aircraft. Although this approach seems impractical, the authors claimed that it could halve total boarding time. In their study, they analyzed many alternative boarding patterns. One pattern that seemed practical and efficient was boarding passengers by half-row, that is, by splitting each row into a starboard-side group and a port-side group and then boarding the half-rows one by one.

While traditional computer-based simulation studies are good tools for testing the performance of already identified alternatives, they do not provide efficient mechanisms for constructing the most promising alternatives. For this reason, we decided to use analytical models to analyze the problem. Surprisingly, in the airline industry, which has a very rich background in operations research applications, we found only simulation-based solutions for analyzing and improving passenger airplane boarding. One exception is a study by Bachmat et al. (2005) that approaches the airplane-boarding problem from a physicist's point of view. They constructed a model based on space-time geometry and random matrix theory that captures the asymptotic behavior of airplane boarding. Their results are qualitatively in line with ours.

Our analysis of the problem consisted of six phases: (1) We developed a simple integer-programming model to understand the problem and to create general patterns for efficient boarding strategies. To make the problem more tractable, we used the minimization of passenger interferences as our objective in lieu of the minimization of boarding time. (2) We tested these patterns using simulation models that incorporated more details of an actual airplane-boarding procedure. (3) We improved and refined these models to accommodate practical factors and implementation limits, such as several passengers traveling together and the processing speed of the gate agent. (4) We analyzed the results of the simulation models and the analytical models to determine the best boarding procedures to recommend. (5) We tested and fine-tuned the recommended procedures. (6) Finally, we implemented and validated the proposed boarding procedures.

Problem Analysis

Boarding interference is defined as an instance of a passenger blocking another passenger's access to his or her seat. We assume that there is a correspondence between minimizing the expected number of passenger interferences and minimizing the boarding time. A passenger blocked by another passenger takes longer to reach his or her seat than one who has free access. Therefore, as the number of passengers facing interference during the boarding process increases, the total boarding time increases. Thus, by minimizing passenger interferences, we shorten individual passengers' seating times, which will ultimately shorten overall boarding times.

We defined two types of interferences: *seat interferences* and *aisle interferences*. Seat interferences occur when passengers seated close to the aisle block other passengers seated in the same row. Consider, for example, an aircraft with rows progressively numbered from front to back and seats labeled A to F from left to right. A passenger sitting in seat 7C (the aisle seat in row 7) could block the passenger seeking seat 7A (the window seat) and will have to stand in the aisle for the passenger in 7A to be seated. The interference is even worse when passenger 7A arrives and passengers 7B and 7C are seated.

Aisle interferences occur when passengers stowing luggage in overhead bins block other passengers' access to seats. For example, if passenger 9A boards the airplane just before passenger 14C, passenger 9A will block passenger 14C's progress down the aisle as he or she stores luggage.

We developed a model to minimize expected boarding interferences. The decision is to assign each passenger boarding the airplane to a boarding group, to minimize boarding interferences. The objective function includes all the different interferences that could possibly occur during boarding. Each of these interferences has a certain penalty, and the sum of all the penalties related to a particular seat assignment determines the objective value. The constraints guarantee that every seat is assigned to exactly one group and that every group is assigned a particular number or range of numbers of total seats.

The interference model is a nonlinear assignment problem with quadratic and cubic terms in the objective function. Such assignment problems belong to the

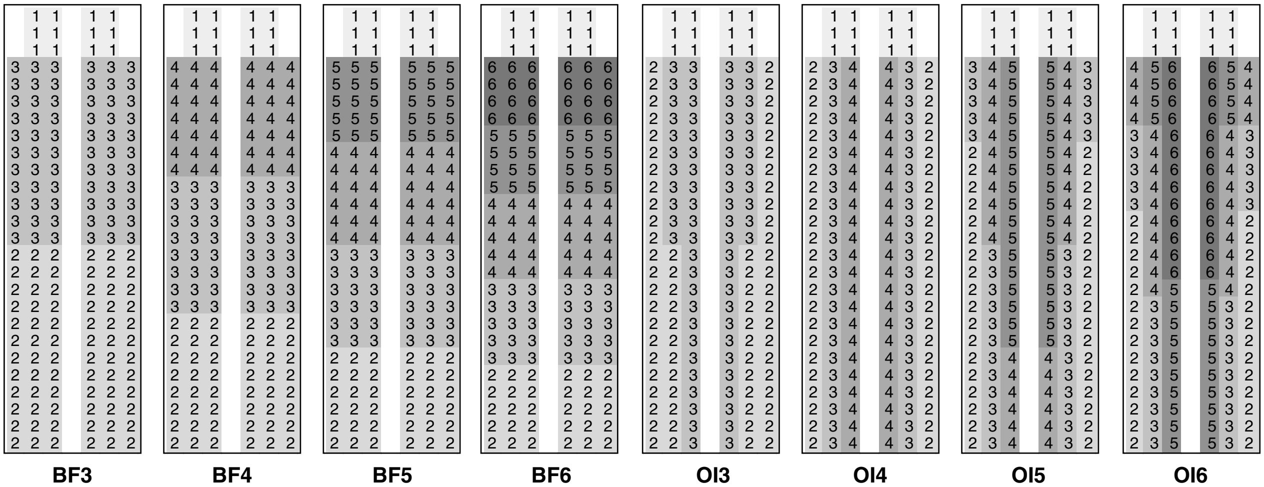

Figure 1: The eight boarding patterns analyzed are presented in this figure. Each schematic shows the seat layout of an Airbus A320 airplane. The number shown on each seat is the boarding group to which the seat is assigned. Back-to-front boarding strategies are identified by BF, and the boarding strategies found by MINLP that minimize expected passenger interference and tend to board passengers from the outside in are identified by OI. The number following the boarding-strategy identification indicates the number of boarding groups. The OI5 and OI6 patterns are also referred to as *reverse-pyramid* boarding strategies.

NP-hard complexity class. In fact, if the model contained only quadratic terms, it would be a quadratic assignment problem, a type of problem generally known to be NP-hard. We used MINLP, a mixed-integer nonlinearly constrained optimization solver (Leyffer 1999). MINLP uses a branch-and-bound algorithm in which each node corresponds to a continuous nonlinearly constrained optimization problem. This algorithm is effective in solving nonconvex MINLP problems, but being a heuristic approach, it does not give any guarantees that a global solution will be found.

We compared boarding patterns for different numbers of boarding groups; four using the traditional back-to-front (BF) boarding pattern and four using the boarding patterns obtained from MINLP (Figure 1, Table 1). The MINLP solutions have a tendency to board outside-inside (OI). For both the BF and OI strategies, we fixed the first boarding group to contain first-class seats only. We divided the economy class section in the BF strategies into groups of similar size; for the OI strategies, we let MINLP decide on the most efficient size for each group.

The OI boarding strategies are also referred to as the *reverse-pyramid strategies*. The reverse-pyramid boarding patterns actually take on characteristics of multiple strategies. Interestingly, as the number of groups increases, the way in which boarding passengers are distributed changes its form. That is, it moves from a mixture of window/middle and middle/aisle seats contained in the same groups with three groups, to a strictly laminar form with four groups, to patterns that mix back-to-front with outside-in strategies with five or more groups.

Boarding window seats before middle seats and middle seats before aisle seats as in the reverse-pyramid strategies reduced the number of expected seat interferences significantly. Additionally, the number of expected aisle interferences in the reverse-pyramid strategies is below the number expected in the BF strategies (Table 1). The numbers favor the reverse-pyramid strategies over the BF strategies, but the numbers can be a little misleading. In our model, we used unit weights for each interference type. That is, each type of interference was assumed to be of equal importance. It might be the case, however, that aisle interferences should be weighted more heavily than seat interferences, and maybe aisle interferences that occur within groups should be weighted more heavily than aisle interferences that occur between groups. It is very difficult to estimate these weights

	BF3	BF4	BF5	BF6	OI3	OI4	OI5	OI6
Seat interferences								
First class [xx]	3	3	3	3	3	3	3	3
First class [x] → [x]	0	0	0	0	0	0	0	0
Economy class [xxx]	69	69	69	69	0	0	0	0
Economy class [xx] → [x]	0	0	0	0	12	0	0	0
Economy class [x] → [xx]	0	0	0	0	11	0	0	0
Economy class [x] → [x] → [x]	0	0	0	0	0	0	0	0
Aisle interferences								
Within groups								
Same row same side	5	7	9	11	2.33	1	1	1
Same row different side	8	11	14	17	5.33	5	6	7
Different rows	67	64	61	58	69.67	70	69	68
Between groups								
Same row same side	0	0	0	0	0.02	0.04	0.06	0.06
Same row different side	0	0	0	0	0.02	0.04	0.06	0.06
Different rows	1	1	1	1	1.31	1.96	2.29	2.56
Total seat interferences	72	72	72	72	26	3	3	3
Total aisle interferences	81	83	85	87	78.68	78.04	78.40	78.68
Total interferences	153	155	157	159	104.69	81.04	81.40	81.68

Table 1: We show the number of expected passenger interferences by boarding strategy. We indicate the type of seat interferences by the boarding order of the passengers. We use squared brackets, [ ], to group together the passengers boarding in the same group, that is, [xxx] indicates that three passengers in the same half-row board in the same group. We use an arrow to indicate a precedence relation between groups. For instance, the notation [x] → [xx] means that a single passenger sitting in a half-row boards alone in a group followed by the other two passengers who board together in a subsequent group. We divide aisle interferences into interferences that occur within groups and those that occur between groups.

and determine how important each interference type is compared to other interference types.

We used simulation to validate the analytical model and to obtain a finer level of detail. America West personnel filmed a number of actual airplane boardings. They used two cameras, one inside the airplane and one inside the jet bridge leading to the plane. We retrieved data on the time between passengers, walking speed, interference time, and time to store luggage in the overhead bins by analyzing the tapes.

We built the simulation model in ProModel 2001 and simulated each of the boarding strategies (Figure 1) 100 times (Table 2). Then, we obtained enough replicates for all the strategies we tested to give 95 percent confidence intervals of less than 60 seconds. The model showed that the strategies based on the solutions of the interference model are better than the back-to-front approach. The average number of seat interferences matches well with the number produced by the interference model. The number

	BF3	BF4	BF5	BF6	OI3	OI4	OI5	OI6
Average seat interferences	70.76	72.11	73.36	72.22	26.05	2.94	2.94	2.94
Average aisle	53.41	53.36	52.74	52.27	46.95	42.02	42.92	42.64
Interferences								
Average total	124.17	125.47	126.10	124.49	73.00	44.96	45.86	45.58
Interferences								
Average boarding time (seconds)	1,436.76	1,460.68	1,473.69	1,491.68	1,412.79	1,376.07	1,382.71	1,387.80

Table 2: We show the simulation results by boarding strategy. The data represents the average of 100 runs. This number of runs is sufficient to obtain a 95 percent confidence interval of at most 60 seconds on total boarding time. We computed the number of aisle interferences by taking into account only those aisle interferences caused by the passenger immediately ahead in the boarding process of any passenger boarding the aircraft.

of aisle interferences in the simulation, however, is significantly lower than that produced by the interference model. The reason for this difference is that the way we defined aisle interferences in the analytical model does not always result in an aisle interference in the simulation model. For example, when two passengers are expected to interfere in the aisle, in the simulation model, they might enter the airplane with enough of a gap that the first clears the aisle before the second arrives. As the time between passenger boardings decreases, the chance of the passengers interfering with each other increases (Figure 2). As the time between passengers decreases, the outside-in strategy with six groups tends to perform increasingly better than the back-to-front strategy with six groups. A plateau seems to exist where further reductions in the time between passengers boarding stops being beneficial in terms of reducing the overall boarding time. The bottleneck for the outside-in strategy appears at a much lower time between passengers (that is, at a higher passenger throughput rate) than for the back-to-front strategy. Because the gate agents determine the time between passenger boardings, the airline can shorten boarding times by expediting the process at the gate.

From the videotapes, we determined that a single gate agent processed about 6.7 passengers per minute (or one passenger every nine seconds). By using two or more gate agents or having an extra agent to check passengers' identification documents, the airline can speed up the boarding process without changing much. Based on the information provided by the data and the analytical and simulation models, we estimated that America West could cut its boarding times by as much as 37 percent by using two agents and the OI strategy with six groups. With only one agent, we estimated that it could reduce times by around 25 percent.

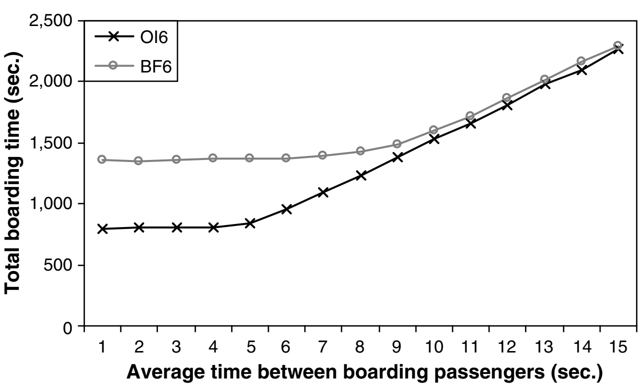

Figure 2: We show boarding time performance of the OI6 and BF6 boarding strategies for various average times between boarding passengers. The average time between passengers may decrease if passenger throughput at the gate increases.

Implementation

We implemented the results of the project first in a pilot and then systemwide. In the pilot implementation, our main objective was to validate and fine tune the results of our analysis. In the pilot implementation, we used the reverse-pyramid boarding strategy with six boarding groups (OI6) in all the America West boarding gates at the Los Angeles International (LAX) airport (Tables 3 and 4).

The average savings obtained by using the new strategy were 26 percent using one gate agent and 39 percent using two agents, almost exactly matching those predicted by the simulation model.

America West management decided to implement the new boarding strategies systemwide. It implemented reverse-pyramid group boarding patterns in 80 percent of its airports in September 2003. It has not yet employed this method in Las Vegas McCarran International Airport (LAS) because of that airport's unique network infrastructure, which prints boarding passes. Las Vegas is the largest airport in the America West system that still boards passengers using the traditional back-to-front method.

	1BF6	2BF6	1OI6	2OI6
Average number of passengers	123.70	131.90	121.20	135.60
Average total boarding time (seconds)	1,462.70	1,476.60	1,460.00	1,025.00
Average time between passengers	11.82	11.19	12.05	7.56

Table 3: We show the average results of a field study using the BF6 and OI6 boarding strategies. The total boarding time is the time it takes for all passengers to get seated. The headings 1BF6 (1OI6) and 2BF6 (2OI6) denote the BF6 (OI6) boarding strategy using one or two boarding agents, respectively. The numbers shown represent the average of 10 (eight for 1OI6) airplane boardings. All data points for the BF6 strategy were obtained at Phoenix Sky Harbor International Airport (PHX), and those for the OI6 strategy were obtained at Los Angeles International Airport (LAX).

	1BF6	2BF6	1OI6	2OI6
Average total boarding time	1,217.70	1,246.70	887.70	788.10
Average time between passengers	9.84	9.45	7.32	5.81

Table 4: We show the average results of a field study using the BF6 and OI6 boarding strategies. The numbers are from the same data set as those in Table 3 but exclude the passengers that do not board with the bulk of the passengers, such as preboarding and late passengers.

Departure delays have decreased significantly since the implementation of the reverse-pyramid boarding strategy. An average decrease of 21.0 percent in departure delays (Figure 3) was observed in the first three months after incorporating the new boarding strategy at all of America West's airports except Las Vegas McCarran International Airport. At Phoenix's Sky Harbor Airport, the largest hub in the America West system, the average decrease in boarding time delays was 60.1 percent (Figure 4).

In addition to the quantifiable benefits, America West has realized nonquantifiable benefits with this boarding method:

(1) The airline's customers can easily understand when to queue up for boarding, and

(2) The airport agents can easily remember where they are in the boarding process because they are calling group numbers instead of row sections.

Also, agents can preferentially board passengers for special seats on the airplane, such as the bulkhead seats. Passengers in the bulkhead seats have traditionally been last in boarding. These seats have no under-seat storage for carry-on luggage. When passengers assigned to the bulkhead seats board last, they often find the overhead bins full. Giving these passengers boarding priority grants them first access to the overhead-bin storage space.

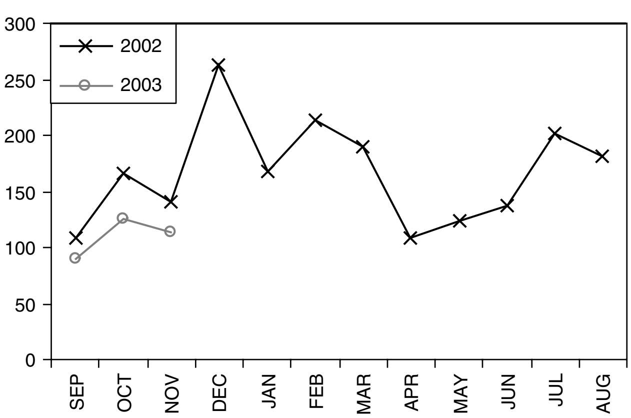

Figure 3: This figure shows total departure delays at America West Airlines in hours per month. The data does not include Las Vegas McCarran International Airport (LAS).

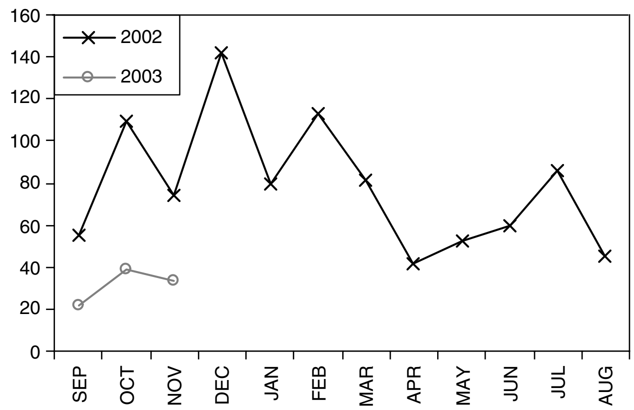

Figure 4: We present total departure delays at America West Airlines in hours per month. The data includes only Phoenix Sky Harbor International Airport (PHX).

Conclusions

The models and implementation results show that outside-in boarding outperforms traditional back-to-front boarding. Although the models make several simplifying assumptions, they provide a good analysis of the factors affecting the boarding process. Based on the results provided by the integer programming and simulation models, we developed a new boarding strategy; a hybrid between the traditional back-to-front and outside-in boarding strategies termed the *'reverse-pyramid' approach*. This approach was implemented in pilot form in the America West flights departing Los Angeles international airport. During this pilot implementation, America West estimated that boarding times were reduced by up to 39 percent. After the success of this pilot implementation, America West management decided to implement the reverse-pyramid boarding strategy systemwide. While it is too early to draw general conclusions, preliminary information indicates that the implementation has been successful. For instance, America West has observed a significant increase in the number of on-time departures from its Phoenix hub.

While the results available show that our method is an excellent alternative for reducing passenger-boarding time, additional research is needed to fine tune and improve its implementation. For instance, we did not consider the saturation of storage space for carry-on luggage explicitly in the analytical model. This and other factors should be explored further to determine the best possible boarding strategies. We hypothesize that the best boarding strategy depends on such factors as airplane design and the profile of the passengers boarding the plane. These factors could be analyzed in future projects.

Appendix

In our mathematical programming model for the airplane-boarding problem, we considered an airplane with one aisle and three seats on either side. For brevity, we considered only the economy-class section, but the model could easily be extended to include the business or first-class section.

Let N represent the set of rows and $M = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}\}$ represent the set of seat positions in the airplane. In addition, let the seats on the left side of the aisle be represented by $L = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\} \in M$ and those on the right side by $R = \{\mathrm{D}, \mathrm{E}, \mathrm{F}\} \in M$, such that A and F are window seats, B and E are middle seats, and C and D are aisle seats. Now, given a row number $i \in N$ and a seat position $j \in M$, we can uniquely identify and represent all seat locations in the airplane by the pair (i, j).

By assigning seats to groups (with fixed seat assignments, this is similar to assigning passengers to groups), we can create different boarding patterns. For the airplane-boarding problem, we want to assign each seat (i, j) to a boarding group k, $k \in G$, with G representing the set of groups. Let us define the decision variable $x_{ijk} = 1$ if seat (i, j) is assigned to group k and $x_{ijk} = 0$ otherwise, for all $i \in N$, $j \in M$, and $k \in G$.

In our formulation of the airplane-boarding problem, the equation numbers indicate the purpose of each set of expressions. In the objective function, we have different penalties for each type of interference. We represent seat-interference penalties by λ_s and aisle-interference penalties by λ_a. The penalties associated with the different types of interferences capture their relative contributions to the total delay of the boarding procedure.

Minimize

$$z = \lambda_1^s \sum_{i\in N} \sum_{k\in G} x_{iAk}x_{iBk}x_{iCk} + \lambda_1^s \sum_{i\in N} \sum_{k\in G} x_{iFk}x_{iEk}x_{iDk} \tag{1a}$$

$$+\lambda_2^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAk}x_{iBk}x_{iCl}$$

$$+\lambda_3^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAk}x_{iBl}x_{iCk}$$

$$+\lambda_4^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAl}x_{iBk}x_{iCk} \tag{1b}$$

$$+\lambda_2^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFk}x_{iEk}x_{iDl} + \lambda_3^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFk}x_{iEl}x_{iDk}$$

$$+\lambda_4^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFl}x_{iEk}x_{iDk} + \lambda_5^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAk}x_{iBl}x_{iCl}$$

$$+\lambda_6^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAl}x_{iBk}x_{iCl}$$

$$+\lambda_7^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAl}x_{iBl}x_{iCk} \tag{1c}$$

$$+\lambda_5^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iDk}x_{iEl}x_{iFl} + \lambda_6^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFl}x_{iEk}x_{iDl}$$

$$+\lambda_7^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFl}x_{iEl}x_{iDk}$$

$$+\lambda_8^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAl}x_{iBm}x_{iCk}$$

$$+\lambda_9^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAk}x_{iBl}x_{iCm}$$

$$+\lambda_{10}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAm}x_{iBl}x_{iCk} \tag{1d}$$

$$+\lambda_{11}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAk}x_{iBm}x_{iCl}$$

$$+\lambda_{12}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAm}x_{iBk}x_{iCl}$$

$$+\lambda_8^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFl}x_{iEm}x_{iDk}$$

$$+\lambda_9^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFk}x_{iEl}x_{iDm}$$

$$+\lambda_{10}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFm}x_{iEl}x_{iDk}$$

$$+\lambda_{11}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFk}x_{iEm}x_{iDl}$$

$$+\lambda_{12}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFm}x_{iEk}x_{iDl}$$

$$+\lambda_1^a \sum_{i\in N} \sum_{u,v\in L:v\neq u} \sum_{k\in G} x_{iuk}x_{ivk}$$

$$+\lambda_1^a \sum_{i\in N} \sum_{u,v\in R:u\neq v} \sum_{k\in G} x_{iuk}x_{ivk} \tag{2a}$$

$$+2\lambda_2^a \sum_{i\in N} \sum_{u,v\in M:u\in L,v\in R} \sum_{k\in G} x_{iuk}x_{ivk} \tag{2b}$$

$$+\lambda_3^a \sum_{a,b\in N:a<b} \sum_{u,v\in M} \sum_{k\in G} x_{auk}x_{bvk} \tag{2c}$$

$$+\lambda_4^a \sum_{i\in N} \sum_{u,v\in R} \sum_{k,l\in G:k<l} x_{iuk}x_{ivl}$$

$$+\lambda_4^a \sum_{i\in N} \sum_{u,v\in L} \sum_{k,l\in G:k<l} x_{iuk}x_{ivl} \tag{2d}$$

$$+\lambda_5^a \sum_{i\in N} \sum_{u\in L,v\in R} \sum_{k,l\in G:k<l} x_{iuk}x_{ivl}$$

$$+\lambda_5^a \sum_{i\in N} \sum_{u\in R,v\in L} \sum_{k,l\in G:k<l} x_{ivl}x_{iuk} \tag{2e}$$

$$+\lambda_6^a \sum_{a,b\in N:a<b} \sum_{u,v\in M} \sum_{k,l\in G:k<l} x_{auk}x_{bvl} \tag{2f}$$

subject to

$$\sum_{k\in G} x_{ijk} = 1 \quad \text{for all } i\in N,\ j\in M, \tag{3}$$

$$\sum_{i\in N}\sum_{j\in M} x_{ijk} \geq C_{\min} \quad \text{for all } k\in G, \tag{4}$$

$$\sum_{i\in N}\sum_{j\in M} x_{ijk} \leq C_{\max} \quad \text{for all } k\in G, \tag{5}$$

$$x_{ijk}\in\{0,1\} \quad \text{for all } i\in N,\ j\in M,\ k\in G. \tag{6}$$

Expressions (1a)–(1d) are associated with seat interferences. Seat interferences may occur when agents assign all seats on the same side in one row to the same boarding group (1a); when they assign two seats on the same side and row to one boarding group and the third seat to a later (1b) or earlier (1c) boarding group; or when they assign all seats on the same side and row to different boarding groups (1d).

We represent aisle interferences by expressions (2a)–(2f). Where (2a)–(2c) represent the aisle interferences that take place within a group; (2d)–(2f) are the aisle interferences that take place between two consecutive groups. Aisle interferences can occur when the two passengers are seated in the same row and on the same side (2a) and (2d), in the same row and on different sides (2b) and (2e), and in different rows (2c) and (2f) (in this case, the side does not matter). We could have gathered expressions (2a)–(2c), similarly (2d)–(2f), into one expression. However, we could not then have applied different penalties to these seemingly different interferences.

The constraints grouped under (3) represent the assignment restrictions. They ensure that each seat is assigned to only one boarding group. Constraints (4) and (5) restrict the group size to at least $C_{\min}$ and at most $C_{\max}$ seats assigned to each group. Finally, (6) are the binary constraints.

Determination of Penalties

We could use various procedures to determine penalty values; for example, we could use historical data and estimate the contributions of each type of interference to the total delay. However, we used probabilistic expectations (Table 5). Our main assumption in determining these expectations was that all boarding positions for a particular passenger within the group are equally likely. That is, a particular passenger within a group can take any of the different boarding positions in that group with the same probability. Based on these assumptions, we computed the expected number of interferences and used it as the penalty value.

For example, suppose that three passengers seated in the same row on the same side of the aisle at positions A (window), B (middle), and C (aisle) are assigned to the same boarding group. These passengers could board the plane in six ways (ABC, ACB, BAC, BCA, CAB, CBA). Based on our assumptions, these boarding patterns are equally likely; however,

Penalty	Passenger order	E (No. of interferences)
λ_1^s	[window, middle, aisle]	1.5
λ_2^s	[window, middle] → [aisle]	0.5
λ_3^s	[window, aisle] → [middle]	1.5
λ_4^s	[middle, aisle] → [window]	2.5
λ_5^s	[window] → [middle, aisle]	0.5
λ_6^s	[middle] → [window, aisle]	1.5
λ_7^s	[aisle] → [window, middle]	2.5
λ_8^s	[window] → [aisle] → [middle]	1
λ_9^s	[middle] → [window] → [aisle]	1
λ_{10}^s	[middle] → [aisle] → [window]	2
λ_{11}^s	[aisle] → [window] → [middle]	2
λ_{12}^s	[aisle] → [middle] → [window]	3

Table 5: We present the expected seat interferences for different passenger boardings. Squared brackets, [ ], in the second column indicate passengers boarding in the same group. The order in which passengers board the airplane within a group is arbitrary. An arrow indicates a precedence relation between groups. The first column gives the corresponding seat interference penalty.

the interferences they cause are different. For example, in the boarding pattern ABC, the window passenger (A) boards the plane first, then the passenger sitting in the middle seat (B) boards, and then the passenger sitting in the aisle seat (C). This represents the best-case scenario (zero penalties) because none of the three would have to get up once seated.

In the boarding pattern ACB, the window-seat passenger again boards first, then the aisle-seat passenger, and then the middle-seat passenger. In this case, there is one interference, the aisle-seat passenger must get up for the middle-seat passenger. Because all of the boarding patterns are equally likely (1/6 probability), we can determine the interferences associated with each pattern to obtain its expected value. We use the same procedure when the three passengers on the same side of a row are assigned to different groups. For instance, if the window-seat and the aisle-seat passengers are assigned to one boarding group and the middle-seat passenger is assigned to another boarding group that boards after the first group, we will have one interference if the aisle-seat passenger boards before the window-seat passenger and none if the window-seat passenger boards first. Because the probability of each of the two boarding patterns is 0.5 and because we know that the middle-seat passenger will be assigned to a different group, causing an interference of one with probability of one, the total expected interferences, and the penalty, for this case is 1.5.

In a similar way, we can calculate the expected aisle interferences (Table 6). For aisle interferences within a boarding group, we look for the number of ways two passengers can interfere with each other. Consider a boarding group of size s_1. One type of aisle interference occurs when a passenger seated in a higher row number enters the airplane right behind a lower-row-number passenger. If we assume the size of their boarding group to be equal to s_1, then there are $(s_1-1)(s_1-2)!$ out of $s_1!$ ways the passengers could have boarded the airplane such that these two passengers would interfere with each other. To be more specific, there are (s_1-1) positions for the two passengers to board one after another, leaving $(s_1-2)!$ ways for the remaining passengers in this group to board, with $s_1!$ total ways to board s_1 passengers. Hence, the probability that two passengers with different row numbers will interfere is equal to $1/s_1(=(s_1-1)(s_1-2)!/s_1!)$. Thus, if m passengers have a lower row number than the higher-row-number passenger, the expected value for this type of aisle interference is m/s_1.

For aisle interferences between groups, we look at the probability that the first passenger in one group has a higher row number than the last passenger in the preceding group. The probability that a passenger will board first or last in a group of size s_1 is equal to $1/s_1$. Because boarding groups can be of different sizes, the probability that a passenger will be last in a group of size s_1 is equal to $1/s_1$, and the probability that a passenger will be first in a group of size s_2 is equal to $1/s_2$. Hence, the probability of the two passengers boarding first and last in their groups is $1/(s_1 s_2)$. Hence, if m passengers have lower row numbers than the higher-row-number passenger, the expected value for this type of aisle interference is equal to $m/(s_1 s_2)$. Table 6 summarizes the aisle interference penalties.

Penalty	Description	E (No. of interferences)
$\lambda_1^a, \lambda_2^a, \lambda_3^a$	Within groups	$1/s_1$
$\lambda_4^a, \lambda_5^a, \lambda_6^a$	Between groups	$1/(s_1 s_2)$

Table 6: We present the expected aisle interferences for two passengers within the same and between two consecutive groups. We use s_1 and s_2 to represent the sizes of the groups in which the two passengers board. The first column gives the corresponding aisle interference penalty.

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Anthony V. Mulé, Senior Vice President, Customer Service, America West Airlines, 4000 E. Sky Harbor Boulevard, Phoenix, Arizona 85034, writes: "For over two years we have worked with Dr. Villalobos and his team to improve the aircraft boarding procedures at America West Airlines. Some of the results of these efforts are presented in the paper "America West Airlines Develops Efficient Boarding Strategies." In particular, the boarding strategies described in the paper have been implemented successfully at America West Airlines. Currently, we board 800+ of our daily departures using group-boarding patterns that are very similar to the reverse-pyramid pattern described in the paper. We are currently working at using the reverse-pyramid boarding method at the Las Vegas International Airport, which is the only major airport in the America West system not using it.

"We estimate that the implementation of the boarding methods described in the paper has been a complete success. For instance, we estimate a reduction in boarding time of two minutes has been achieved.

"Summarizing, we are very pleased with the results obtained from the aircraft boarding project and we look forward to continue partnering with Arizona State University to solve additional problems faced by the aviation industry."