NVO-1163-175

MASTER

ERC

ENVIRONMENTAL RESEARCH CORPORATION

ALEXANDRIA • VIRGINIA

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "persons acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

Classification approved by: RWK NVO-1163-175

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

AMPLIFICATION OF RAYLEIGH WAVES IN A SURFACE LAYER OF VARIABLE THICKNESS

RECEIVED BY DTIE AUG 11 1969

by

J.R. Murphy
A.H. Davis

February 4, 1969

Environmental Research Corporation
813 North Royal Street
Alexandria, Virginia

Prepared under
Contract AT(29-2)-1163
for the
Nevada Operations Office
U.S. Atomic Energy Commission

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

ENVIRONMENTAL RESEARCH CORPORATION
A SUBSIDIARY OF COMMUNICATIONS & SYSTEMS, INCORPORATED
813 NORTH ROYAL STREET • P. O. BOX 1061 • ALEXANDRIA, VIRGINIA 22313 • PHONE 703 549-2910

IN REPLY REFER TO: ERC-13312

February 6, 1969

Mr. Donald H. Edwards, Director
Effects Safety Division
Nevada Operations Office
U. S. Atomic Energy Commission
P. O. Box 14100
Las Vegas, Nevada 89114

Dear Mr. Edwards:

Enclosed for your review is one copy of Environmental Research Corporation's report NVO-1163-175 entitled "Amplification of Rayleigh Waves in a Surface Layer of Variable Thickness."

Your attention is directed to the fact that this report completes the formulation of a mathematical model applicable to the analysis of localized amplification of seismic energy.

We would appreciate your comments and instructions for final distribution of this report.

Very truly yours,

ENVIRONMENTAL RESEARCH CORPORATION

O. Allen Israelsen
Technical Director

JRM:bls
Enclosure
cc: Mr. L. R. West, w/enclosure

TABLE OF CONTENTS

LIST OF APPENDICES

LIST OF ILLUSTRATIONS

ABSTRACT

This report presents a method of calculating the relative amplitudes of Rayleigh waves as a function of frequency at two observation points located on surface layers with differing thickness or physical properties. The analytic approach centers on a solution of the boundary value problem which allows the amplitude as a function of depth to be written in terms of a single unknown; the free surface amplitude. Then, under the assumption of equality of energy flux at the two horizontal positions of interest, the ratio of the free surface amplitudes at the two locations can be computed as a function of frequency. A computer program which has been written to compute these amplitude ratios is described and a series of theoretical amplification curves are presented for models considered to be representative of the geology in and around the Nevada Test Site. Recommendations are made that the analytic model presented in this report be tested empirically using data from the Las Vegas valley instrumentation line.

CHAPTER 1

INTRODUCTION

1.1 SCOPE OF THE REPORT

The effect of a low-velocity surface layer on the amplitude of observed seismic motion has been investigated in a number of studies at ERC. From a theoretical point of view the problem is best treated separately for different wave modes. Body waves (P, SV, SH) have been discussed in Reference 1 and Love waves in Reference 2. This report completes the treatment of surface waves by presenting a method for calculating the relative amplitudes of Rayleigh waves at two stations located on surface layers with differing thickness or physical properties. The ratio of these amplitudes, referred to here as the amplification, is a function of the frequency of the incident wave. Some calculated amplification curves are presented in Chapter 3 for models considered to be typical of Nevada Test Site (NTS) geology.

1.2 SEPARATION OF WAVE MODES

In applying an analysis of this type to actual seismograms, it is first necessary to identify the various wave modes. This is not an easy task. Our current procedure is to separate the seismogram into P-wave, S-wave and surface-wave windows on the basis of experienced judgment (References 1 and 9). The Rayleigh waves are then assumed to correspond to the radial and vertical components of the surface wave window, the transverse component corresponding to Love waves. This identification is only approximate but should suffice as a reasonable first attempt. The ratio of the Fourier amplitude spectra, smoothed if necessary, of corresponding surface wave windows at two stations should then correspond to the theoretical amplification curves calculated by the methods described here. Actual comparisons with observation have not yet been completed and will be discussed in a later report.

1.3 OUTLINE OF THE PROCEDURE FOR RAYLEIGH WAVES

Although the mathematical formulation of the problem for Rayleigh waves is similar in principle to that for Love waves (Reference 2), it is considerably more complex. Only recently has an explicit expression been published for the energy density of a Rayleigh wave in a layer on a halfspace (Reference 3) and this contained several errors. Here a very brief description of the method will be given to illustrate its range of applicability; the details will be found in Chapter 2 and the Appendices.

The wave equation and the appropriate boundary conditions can be solved to give a dispersion equation, relating phase velocity to wave number, and an expression for the amplitude of the Rayleigh wave at any point in the ground in terms of the (unknown) surface amplitude. From the dispersion equation one can obtain the group velocity of the wave; from the amplitude one can form an expression for the energy density in the wave in terms of the surface amplitude. It is then necessary to assume that the energy flux in the wave - that is, the product of group velocity times energy density - is the same at both stations of interest. Equating the expressions for the energy flux allows one to solve for the ratio of the surface amplitudes at the two stations. The assumption on the energy flux implies that no energy is lost by the wave because of spreading, scattering or absorption between the two stations. Hence the method is limited to stations which are closely spaced relative to their distance from the source, and between which there are no sharp geological discontinuities. As an example, the formulation may be applicable to adjacent stations along the Las Vegas Valley (SE) instrumentation line.

CHAPTER 2

MATHEMATICAL FORMULATION

In this chapter we will consider the mathematical formulation for Rayleigh waves propagating in a surface layer overlying a semi-infinite halfspace. The geometrical configuration is shown in Figure 1. In this figure α represents the compressional (P) wave velocity, β represents the shear (S) wave velocity and ρ represents the density. Assume a plane waves solution given by (Reference 4)

$$\varphi_1 = [Ae^{-\nu_1 z} + Be^{\nu_1 z}]\, e^{i(\omega t-kx)}$$

$$\psi_1 = [Ce^{-\nu_1' z} + De^{\nu_1' z}]\, e^{i(\omega t-kx)} \qquad (1)$$

$$\varphi_2 = Ee^{-\nu_2 z}\, e^{i(\omega t-kx)}$$

$$\psi_2 = Fe^{-\nu_2' z}\, e^{i(\omega t-kx)}$$

where the φ_i, ψ_i are the compressional and shear wave displacement potentials which satisfy the wave equations

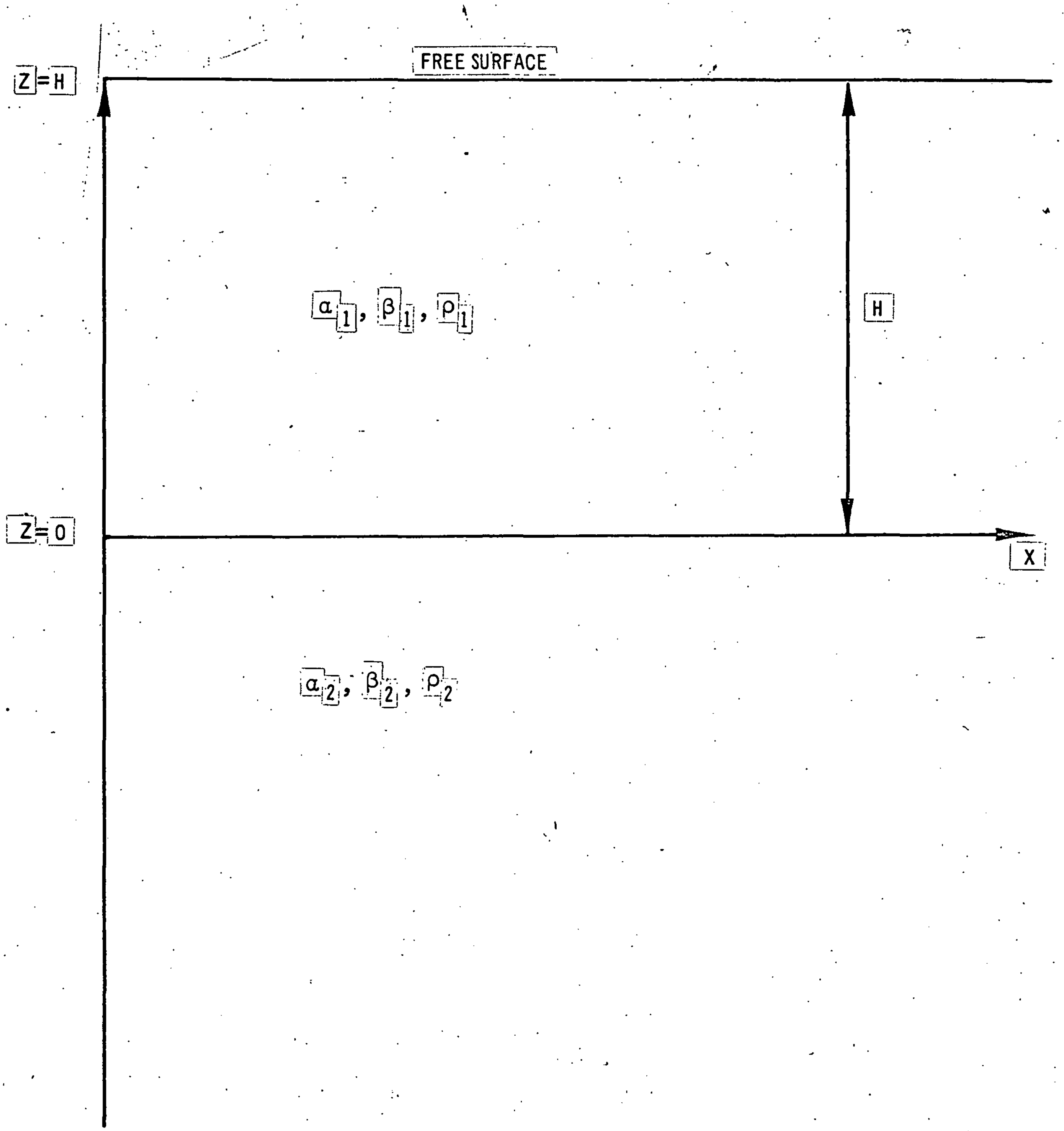

Figure 1. Mathematical Model for Rayleigh Waves.

$$\nabla^2 \varphi_i = \frac{1}{\alpha_i^2} \frac{\partial^2 \varphi_i}{\partial t^2}$$

$$\nabla^2 \psi_i = \frac{1}{\beta_i^2} \frac{\partial^2 \psi_i}{\partial t^2}. \qquad (2)$$

In the above expressions A, B, C, D, E and F are amplitude constants, ω is the circular frequency, k is the wave number, $\nu_i^2 = k^2 - k_{\alpha_i}^2$ where $k_{\alpha_i} = \omega/\alpha_i$; $\nu_1'^2 = k_{\beta_1}^2 - k^2$, $\nu_2'^2 = k^2 - k_{\beta_2}^2$ where $k_{\beta_i} = \omega/\beta_i$.

The boundary conditions for the present problems are that:

(a) the normal and radial components of stress vanish at the free surface;

$$P_{zz_1} = 0 \qquad (3)$$

$$P_{zx_1} = 0 \qquad (4)$$

(b) the normal and radial components of stress and displacement are continuous across the boundary;

$$\left.\begin{aligned} P_{zz_1} &= P_{zz_2} \qquad (5) \\ P_{zx_1} &= P_{zx_2} \qquad (6) \\ w_1 &= w_2 \qquad (7) \\ u_1 &= u_2 \qquad (8) \end{aligned}\right\} \quad \text{at } z = 0$$

where

$$P_{zz_i} = (\lambda_i + 2\mu_i)\, \nabla^2\varphi_i - 2\mu_i \left(\frac{\partial^2 \varphi_i}{\partial x^2} - \frac{\partial^2 \psi_i}{\partial x \partial z} \right) \tag{9}$$

$$P_{zx_i} = \mu_i \left(2\, \frac{\partial^2 \varphi_i}{\partial x \partial z} - \frac{\partial^2 \psi_i}{\partial z^2} + \frac{\partial^2 \psi_i}{\partial x^2} \right) \tag{10}$$

$$w_i = \frac{\partial \varphi_i}{\partial z} + \frac{\partial \psi_i}{\partial x} \tag{11}$$

$$u_i = \frac{\partial \varphi_i}{\partial x} - \frac{\partial \psi_i}{\partial z} \tag{12}$$

in which λ_i, μ_i are the Lamé constants.

Substituting equations (1) into the boundary conditions (3) - (8) we obtain six equations in the six unknowns A, B, C, D, E and F. Taking the determinant of the coefficients and equating to zero we obtain the period equation in the form given by Sezawa and Kanai (Reference 5):

$$4\,\frac{\nu_1\nu_1'}{k^2}\left(2-\frac{k_{\beta_1}^{\;2}}{k^2}\right)\eta-\frac{\nu_1\nu_1'}{k^2}\left\{4\theta+\left(2-\frac{k_{\beta_1}^{\;2}}{k^2}\right)^2\zeta\right\}$$

$$\cdot\cosh\nu_1H\cos\nu_1'H+\frac{\nu_1}{k}\,\xi\left\{\frac{4\nu_2\nu_1'^2}{k^3}\right.$$

$$\left.+\frac{\nu_2'}{k}\left(2-\frac{k_{\beta_1}^{\;2}}{k^2}\right)^2\right\}\cosh\nu_1H\sin\nu_1'H+\frac{\nu_1'}{k}\,\xi \qquad (13)$$

$$\cdot\left\{-\frac{4\nu_2'\nu_1^2}{k^3}+\frac{\nu_2}{k}\left(2-\frac{k_{\beta_1}^{\;2}}{k^2}\right)^2\right\}\sinh\nu_1H$$

$$\cdot\cos\nu_1'H+\left\{-\frac{4\nu_1^2\nu_1'^2}{k^4}\,\zeta+\left(2-\frac{k_{\beta_1}^{\;2}}{k^2}\right)^2\theta\right\}$$

$$\cdot\sinh\nu_1H\sin\nu_1'H=0,$$

where

$$\xi = \frac{\mu_1 k_{\beta_2}^2 k_{\beta_1}^2}{\mu_2 k^4}, \quad \zeta = \frac{4\nu_2\nu_2'}{k^2}\left(\frac{\mu_1}{\mu_2} - 1\right)^2 - \alpha^2,$$

$$\eta = \frac{2\nu_2\nu_2'}{k^2}\left(\frac{\mu_1}{\mu_2} - 1\right)\beta - \alpha\gamma, \quad \theta = \frac{\nu_2\nu_2'}{k^2}\beta^2 - \gamma^2,$$

$$\alpha = \frac{2\mu_1}{\mu_2} - \left(2 - \frac{k_{\beta_2}^2}{k^2}\right), \quad \beta = \frac{\mu_1}{\mu_2}\left(2 - \frac{k_{\beta_1}^2}{k^2}\right) - 2, \tag{14}$$

$$\gamma = \frac{\mu_1}{\mu_2}\left(2 - \frac{k_{\beta_1}^2}{k^2}\right) - \left(2 - \frac{k_{\beta_2}^2}{k^2}\right).$$

Equation (13) provides an implicit relationship between the phase velocity c $(= \omega/k)$ and the wave number k. Once c has been determined as a function of k, the group velocity U can be determined by numerical differentiation. Thus

$$U = c + k\frac{dc}{dk}. \tag{15}$$

From equation (13) it can be shown that at the long-wave limit $(k \to 0)$, the phase velocity c approaches the velocity of Rayleigh waves on a halfspace with the properties of the underlying medium ($\approx 0.92\ \beta_2$), and at the short-wave limit ($k \to \infty$) c approaches the velocity of Rayleigh waves on a halfspace with the properties of the layer ($\approx 0.92\ \beta_1$). Thus, the range of existence of Rayleigh waves in the fundamental mode is given by

$$0.92\ \beta_1 < c < 0.92\ \beta_2. \qquad (16)$$

Therefore, ν_2, ν_2' are always real quantities and ν_1, ν_1' may be either real or imaginary depending on the value of c. When ν_1 is imaginary ($c > \alpha_1 > \beta_1$), the hyperbolic functions in equation (13) go over to the corresponding trigonometrics and when ν_1' is imaginary ($c < \beta_1 < \alpha_1$), the trigonometrics go over into the corresponding hyperbolic functions.

Returning now to the six simultaneous equations in the six unknowns A, B, C, D, E and F, it is possible by successive elimination to express five of the variables in terms of the sixth and consequently to write the displacements u and w in terms of the one remaining unknown. Thus, schematically,

$$
\begin{aligned}
u_1 &= A \cdot f_1(z) \\
w_1 &= A \cdot f_2(z) \\
u_2 &= A \cdot f_3(z) \\
w_2 &= A \cdot f_4(z)
\end{aligned} \tag{17}
$$

where the $f_i(z)$ are functions of the elastic parameters of the layer and the halfspace, the thickness of the layer and the frequency of the plane wave under consideration. Now, evaluating u_1 and w_1 at the free surface $z = H$, we have

$$
\begin{aligned}
u_1\big|_{z=H} &\equiv \bar{u}_1 = A\,[f_1(z)]_{z=H} \\
w_1\big|_{z=H} &\equiv \bar{w}_1 = A\,[f_2(z)]_{z=H} \,.
\end{aligned} \tag{18}
$$

Therefore,

$$
A = \frac{\bar{u}_1}{[f_1(z)]_{z=H}} = \frac{\bar{w}_1}{[f_2(z)]_{z=H}} \tag{19}
$$

and consequently the expressions for the displacements can be written in terms of the free surface amplitudes as

$$u_1 = \frac{\bar{u}_1}{[f_1(z)]_{z=H}} \cdot f_1(z) = \frac{\bar{w}_1}{[f_2(z)]_{z=H}} \cdot f_1(z)$$

$$w_1 = \frac{\bar{u}_1}{[f_1(z)]_{z=H}} \cdot f_2(z) = \frac{\bar{w}_1}{[f_2(z)]_{z=H}} \cdot f_2(z)$$

$$u_2 = \frac{\bar{u}_1}{[f_1(z)]_{z=H}} \cdot f_3(z) = \frac{\bar{w}_1}{[f_2(z)]_{z=H}} \cdot f_3(z) \qquad (20)$$

$$w_2 = \frac{\bar{u}_1}{[f_1(z)]_{z=H}} \cdot f_4(z) = \frac{\bar{w}_1}{[f_2(z)]_{z=H}} \cdot f_4(z) .$$

The interest in the present case lies in relating the free surface amplitudes at two different horizontal positions where the thickness and the elastic parameters of the surface layer may be different at the two locations. The situation is presented graphically in Figure 2. Now, if the slope of the line joining the points P_1 and P_2 is

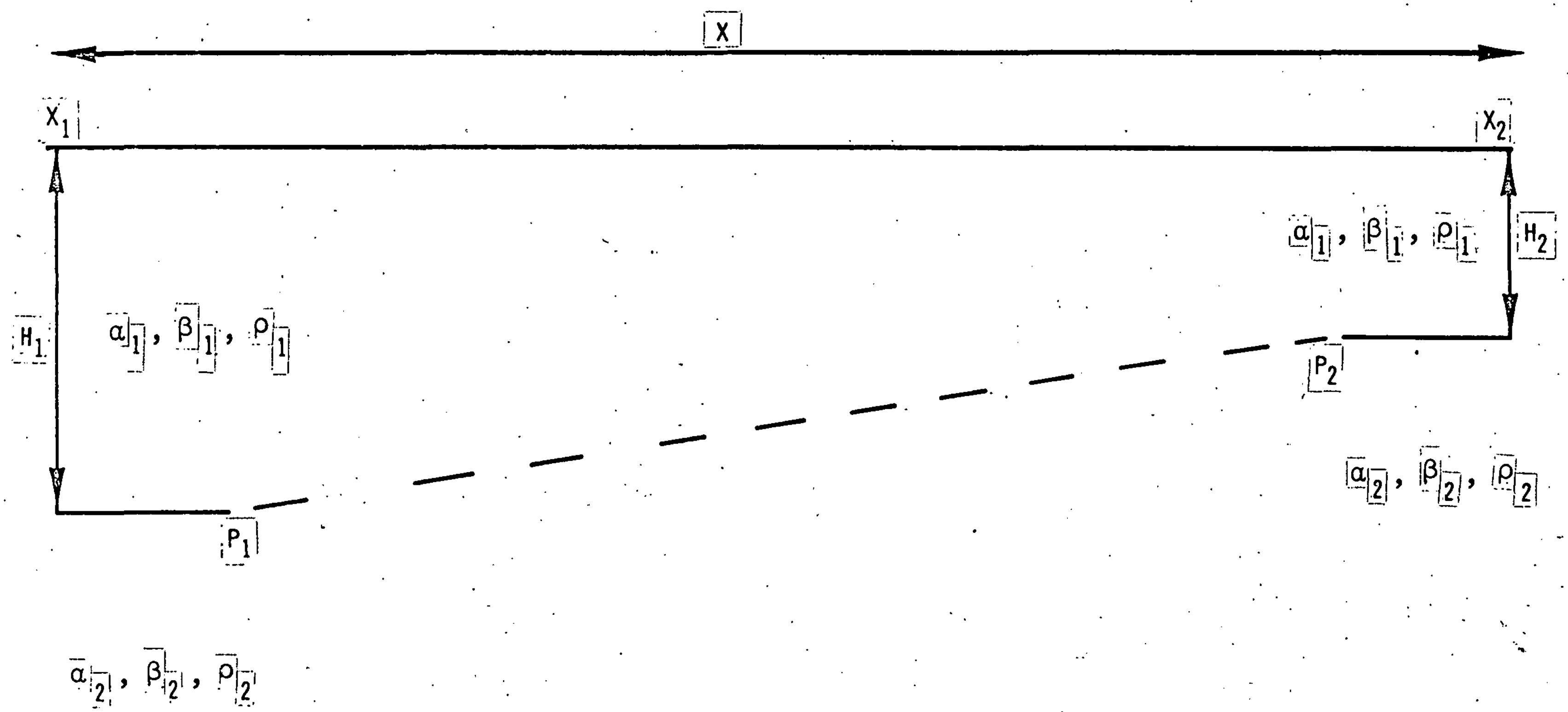

Figure 2. Model for Variable Thickness Surface Layer.

small (i.e., $H_1 - H_2' << x$), then the Rayleigh wave at each location can be considered to be propagating in a layer existing for all x with the properties of the layer at that station (Reference 6). Then the amplitudes as a function of depth at each horizontal position can be expressed in terms of their respective free surface amplitudes as in (20). In order to relate the free surface amplitudes at the two positions we assume that the energy flux is equal at the two locations. That is, we assume that no energy has been lost from the system as the wave train propagates between x_1 and x_2. Thus,

$$U_{x=x_1} E_{x=x_1} = U_{x=x_2} E_{x=x_2} \tag{21}$$

where U is the group velocity and E is the energy density given by

$$E = \frac{\omega^2}{2} \int_{-\infty}^{H} \rho \left(|u|^2 + |w|^2 \right) dz \tag{22}$$

in which the vertical bars denote the modulus of the expression. Thus, schematically, we have

$$UE\Big|_{x=x_1} = |\bar{u}_{x=x_1}|^2 \, U_{x=x_1} \frac{\omega^2}{2} \frac{\int_{-\infty}^{H} \rho\left(|u_{x=x_1}|^2 + |w_{x=x_1}|^2\right) dz}{|\bar{u}_{x=x_1}|^2}$$

$$UE\Big|_{x=x_2} = |\bar{u}_{x=x_2}|^2 \, U_{x=x_2} \frac{\omega^2}{2} \frac{\int_{-\infty}^{H} \rho\left(|u_{x=x_2}|^2 + |w_{x=x_2}|^2\right) dz}{|\bar{u}_{x=x_2}|^2} \qquad (23)$$

where the multiplication and division by the $\bar{u}$'s is introduced to cancel the common factor $\bar{u}$ in the numerator (see equation (20)). Therefore, using equations (21) and (23), we can express the ratio of the free surface amplitudes at $x=x_1$ and $x=x_2$ as

$$\frac{|\bar{u}_{x=x_1}|}{|\bar{u}_{x=x_2}|} = \sqrt{\frac{U_{x=x_2} \dfrac{\int_{-\infty}^{H} \rho\left(|u_{x=x_2}|^2 + |w_{x=x_2}|^2\right) dz}{|\bar{u}_{x=x_2}|^2}}{U_{x=x_1} \dfrac{\int_{-\infty}^{H} \rho\left(|u_{x=x_1}|^2 + |w_{x=x_1}|^2\right) dz}{|\bar{u}_{x=x_1}|^2}}} \,. \qquad (24)$$

Similarly, for the vertical component,

$$\frac{|\bar{w}_{x=x_1}|}{|\bar{w}_{x=x_2}|} = \sqrt{\frac{U_{x=x_2} \dfrac{\int_{-\infty}^{H} \rho(|u_{x=x_2}|^2 + |w_{x=x_2}|^2)\, dz}{|\bar{w}_{x=x_2}|^2}}{U_{x=x_1} \dfrac{\int_{-\infty}^{H} \rho(|u_{x=x_1}|^2 + |w_{x=x_1}|^2)\, dz}{|\bar{w}_{x=x_1}|^2}}} \qquad (25)$$

The energy integrals in equations (24) and (25) are evaluated in detail in Appendix I. Having evaluated the integrals, equations (24) and (25) in conjunction with equations (13) and (15) can be used to compute the ratio of the vertical and horizontal free surface displacements (also velocity and acceleration) for any two horizontal positions as a function of frequency.

CHAPTER 3

PARAMETRIC STUDY OF THE MATHEMATICAL MODEL

In this chapter the variation in the horizontal (U) and vertical (W) amplification with variations in the parameters of the model will be analyzed. In general, one is free to consider, at both horizontal positions, variations in the thickness and elastic parameters of the layer and variations in the elastic parameters of the halfspace. In the present case it will be assumed that the elastic parameters of both the layer and the halfspace are the same at both observation points. This assumption is made because it is felt that any appreciable change in the elastic parameters from one horizontal position to the next would, in nature, correspond to the existence of a discontinuity of rock type which would probably lead to a violation of the equality of energy flux condition of Chapter 2. This decision leaves two sets of elastic parameters (one for the layer and one for the halfspace) and the two layer thicknesses to be varied. A further simplification can be made if the results are presented versus a dimensionless quantity defined by the wavelength divided by the thickness at position 1. In this case only the ratio of the thickness at position 2 to that at position 1 need be considered and not the individual values of the thicknesses. This mode of presentation is not used in this report since the frequency range considered with regard to surface waves is fixed at 0.1 - 0.8 Hz in the distance ranges of interest with regard to prediction. Generally, our interest centers on the effects of relatively thin, low velocity alluvial layers overlying Paleozoic rocks. Consequently, only layer thicknesses over the range of 0.2 - 2 km, shear wave velocities for the layer in the range of 0.75 - 1.5 km/sec and shear wave velocities for the halfspace in the range of 1.5 - 2.5 km/sec will be considered. The compressional wave velocities are determined from the shear wave velocities assuming that Poisson's ratio, $\nu = 0.25$ (i.e. $\beta \approx 0.6\ \alpha$) and the densities of the layer and the halfspace are held constant at 1.5 and 2.5 gm/cm^3 respectively.

The amplification curves as a function of frequency are shown in Figure 3 for the "standard" model defined by the elastic parameters and layer thicknesses shown on the figure. This model is the standard in the sense that as one of the parameters is varied the values of the other parameters are held constant at the values given in the standard model. The effects of variations in layer thicknesses are displayed in Figures 4-8. The effects of variations in the shear wave velocity of the layer are shown in Figures 9 and 10 and the effects of variations in the shear wave velocity of the halfspace are shown in Figures 11 and 12.

A general observation which can be made based on these figures is that significant amplifications do occur within the frequency range of interest. More specifically, as indicated in Figures 4-8, variations in layer thickness for a fixed velocity cause the frequency of the peak amplification to shift in such a way that as the thickness decreases the peak amplification shifts to higher frequencies. This can best be understood by considering the dimensionless parameter wavelength divided by layer thickness mentioned previously. In terms of this parameter, the amplification depends only on the ratio of the layer thicknesses and not on the actual thickness. For a given ratio of layer thicknesses the peak amplification will always occur at the same value of this parameter. Consequently, as the layer thickness decreases the wavelength must decrease, corresponding to a shift to higher frequency. Variations in the elastic parameters of the layer lead to a modification of both the amplitude and frequency of the peak amplification as indicated in Figures 9 and 10. The change in frequency of the peak amplification is related to changes in the effective wavelength similar to that noted for variations in layer thickness. The variation in the magnitude of the peak amplification is related to the magnitude of the impedance mismatch at the boundary. These interpretations are supported by Figures 11 and 12 which indicate that a change in the elastic parameters of the halfspace leads to a change in the amplitude but not the frequency of the peak amplification.

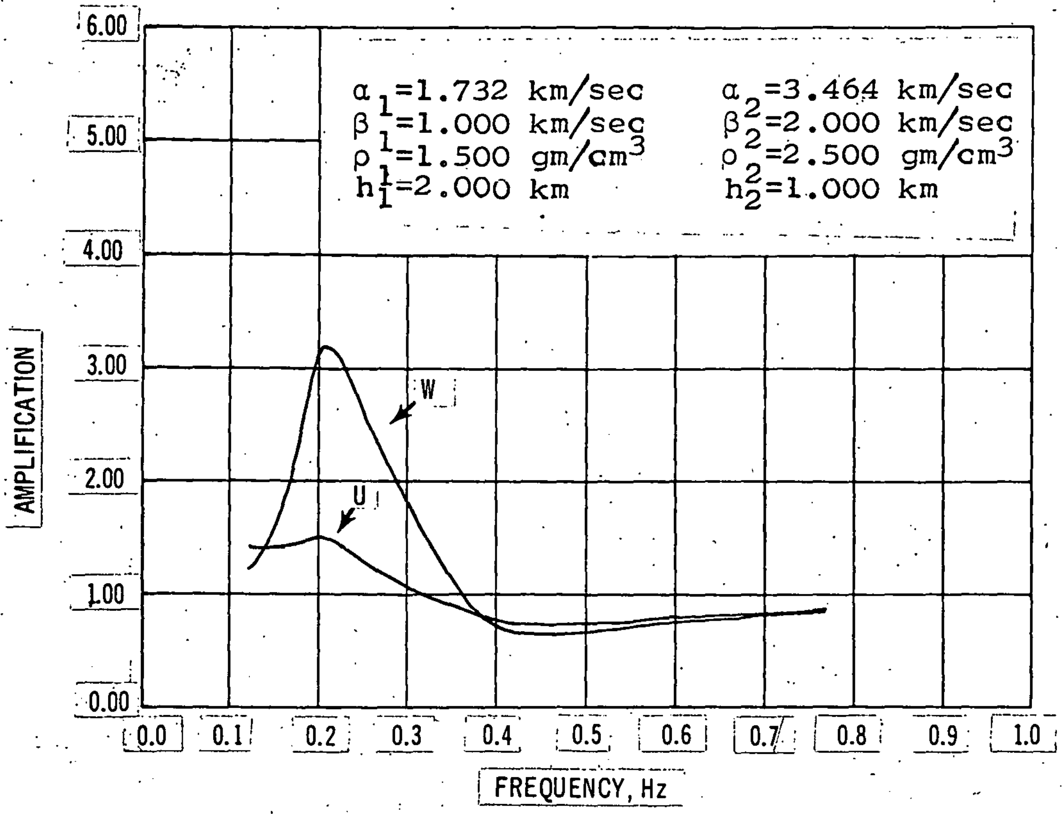

Figure 3. Parameter Study; Standard Model.

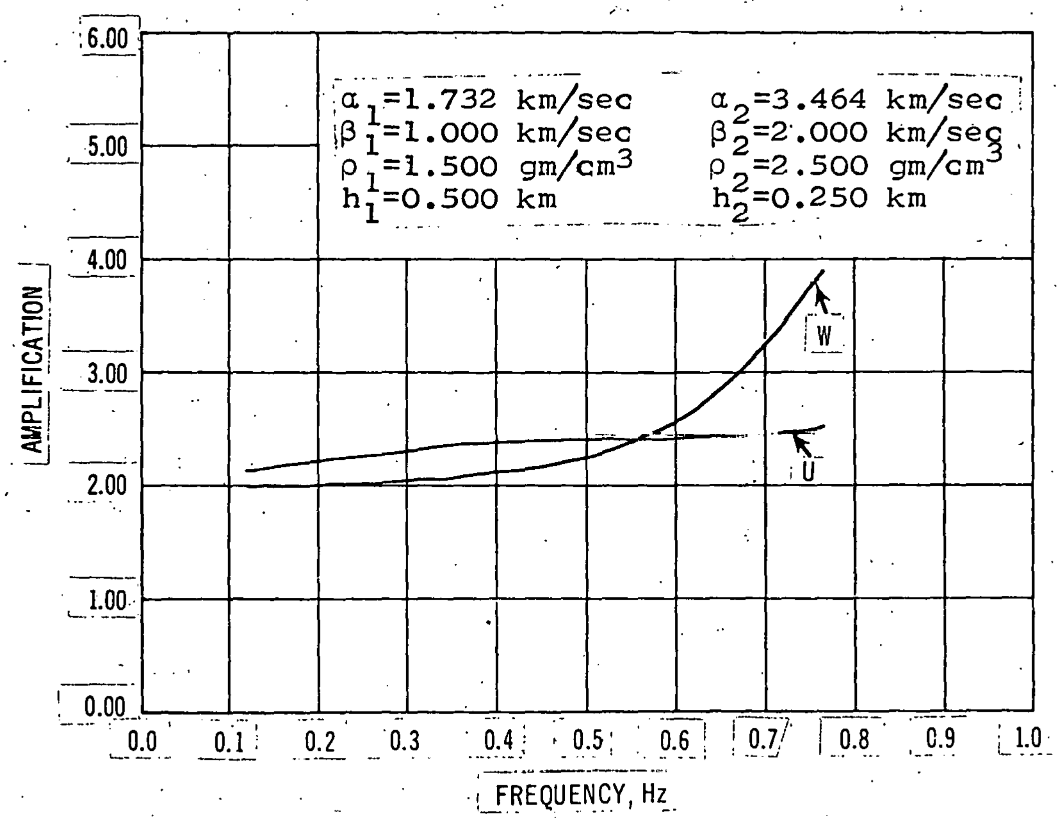

Figure 4. Parameter Study; h_1=0.500 km, h_2=0.250 km.

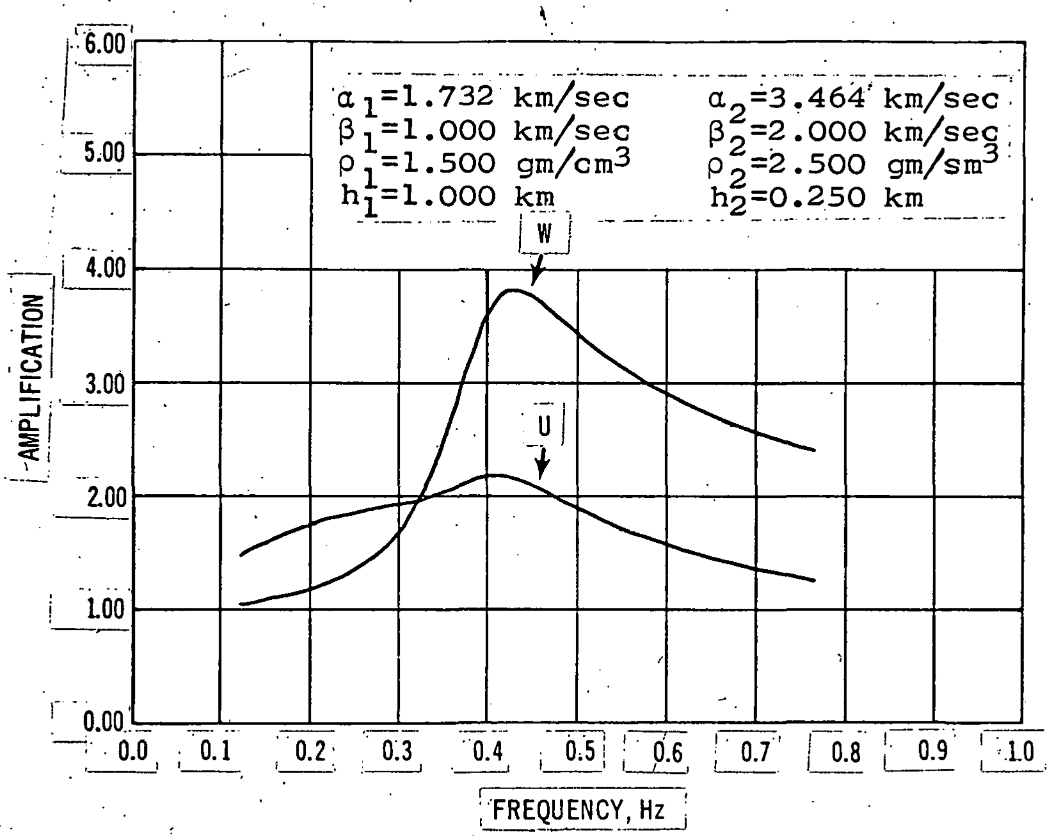

Figure 5. Parameter Study; h_1=1.000 km, h_2=0.250 km.

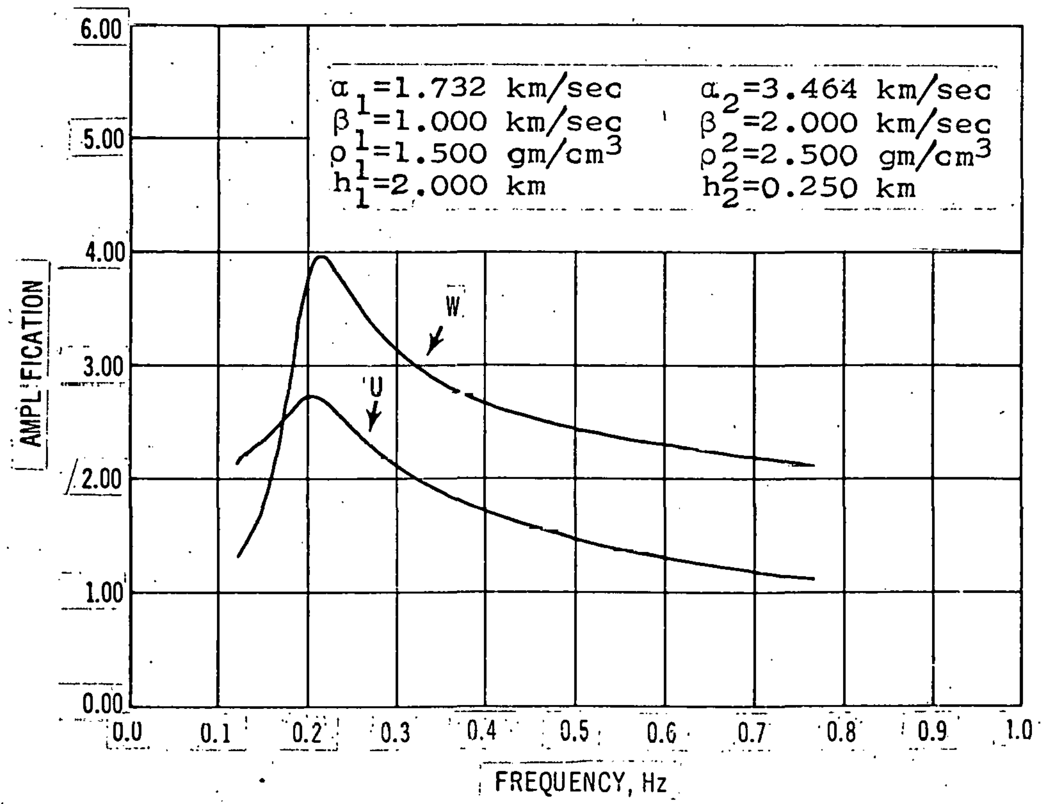

Figure 6. Parameter Study; h_1=2.000 km, h_2=0.250 km.

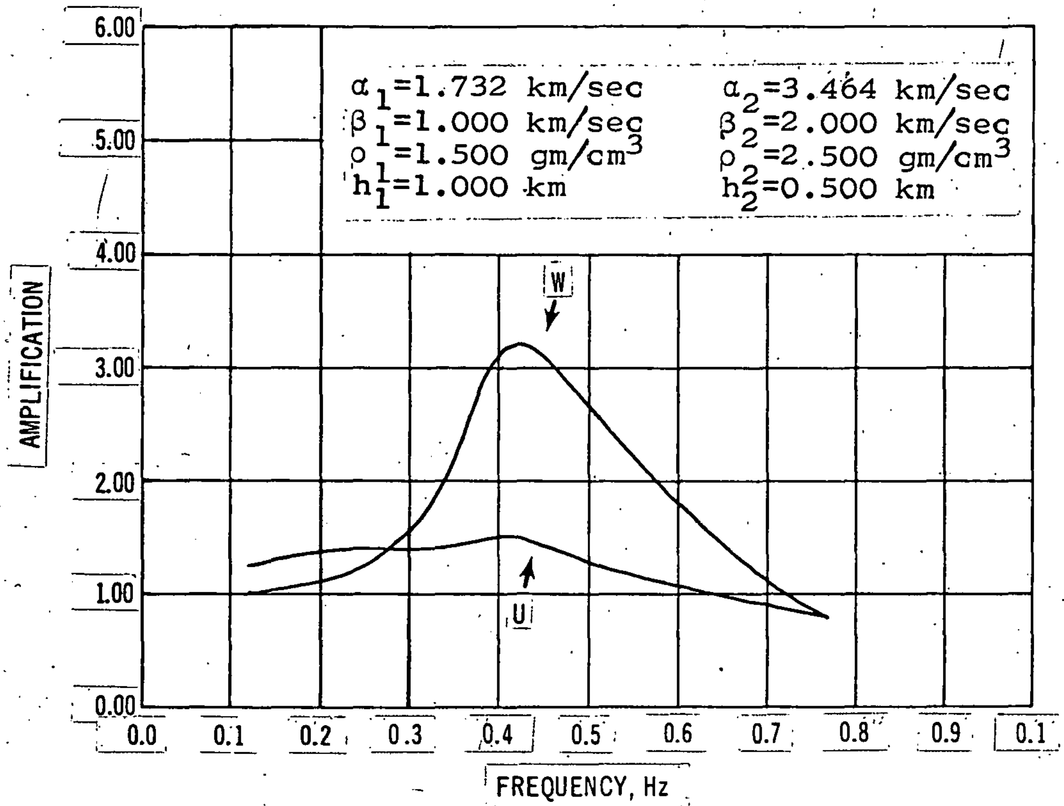

Figure 7. Parameter Study; h_1=1.000 km, h_2=0.500 km.

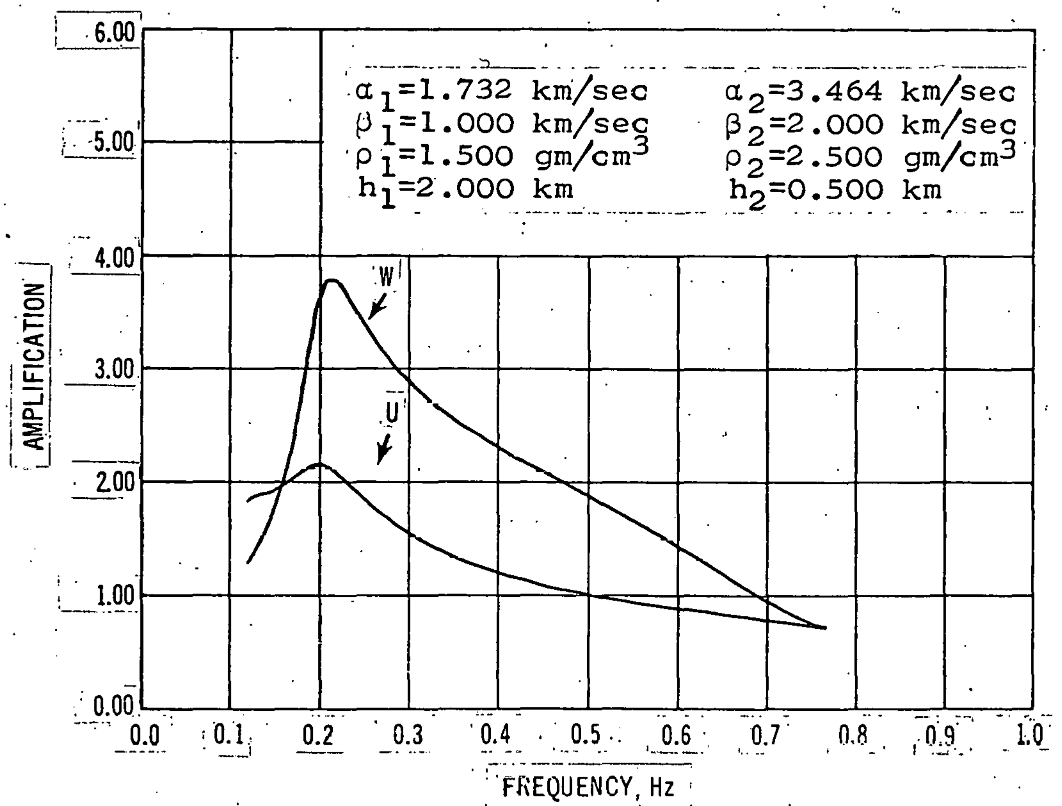

Figure 8. Parameter Study; h_1=2.000 km, h_2=0.500 km.

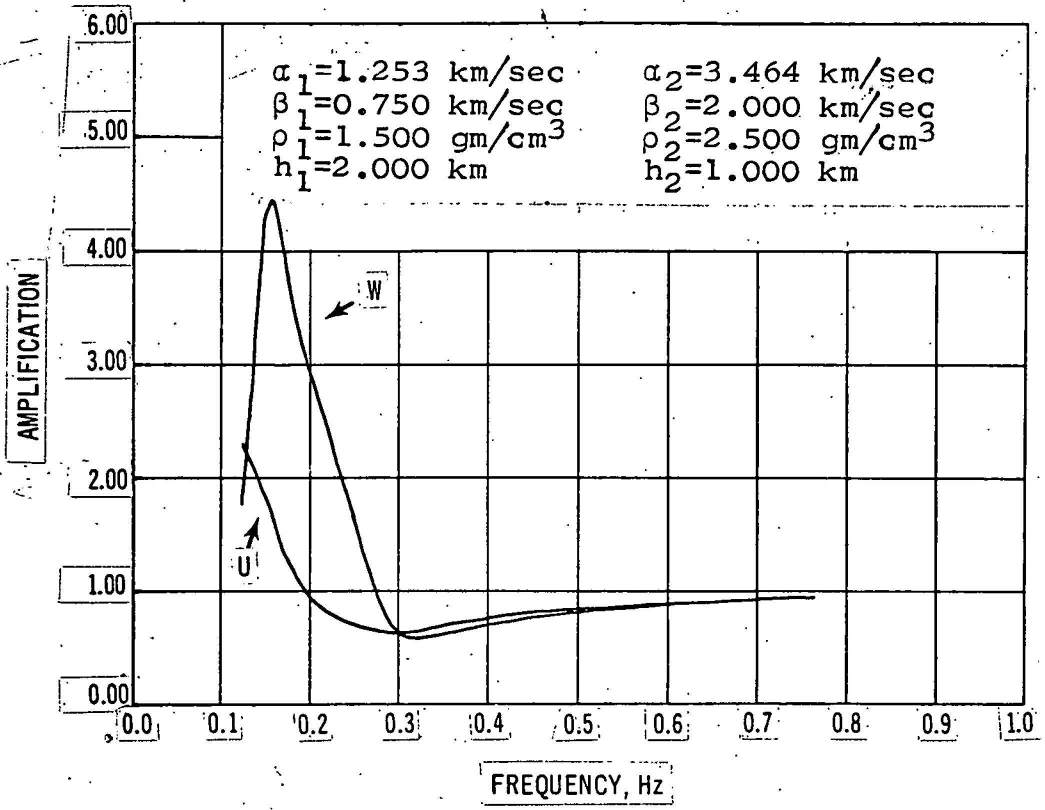

Figure 9. Parameter Study; α_1=1.253 km/sec, β_1=0.750 km/sec.

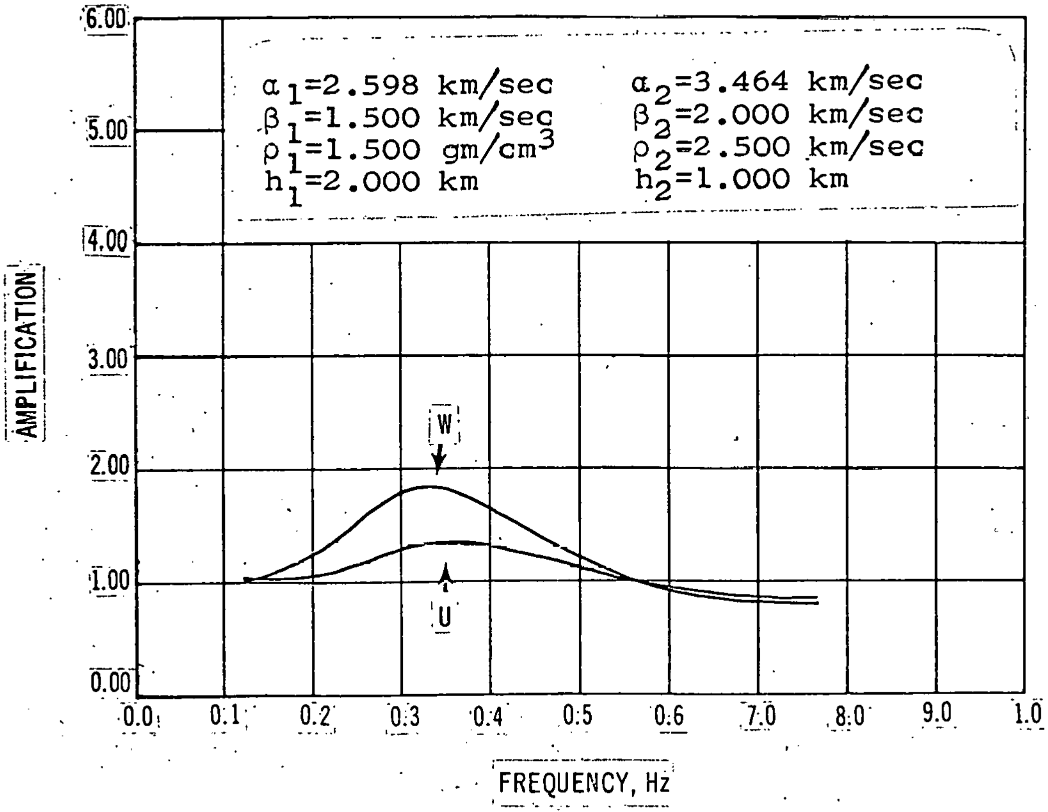

Figure 10. Parameter Study; α_1=2,598 km/sec; β_1=1.500 km/sec.

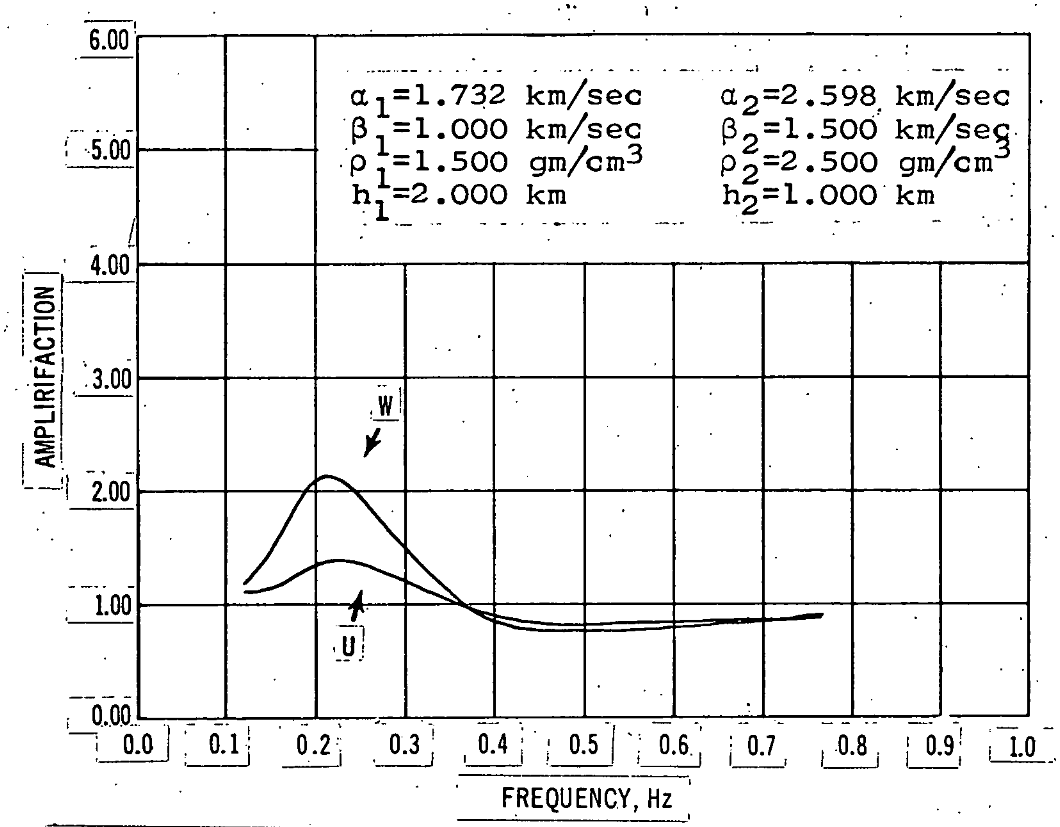

Figure 11. Parameter Study; α_2=2.598 km/sec, β_2=1.500 km/sec.

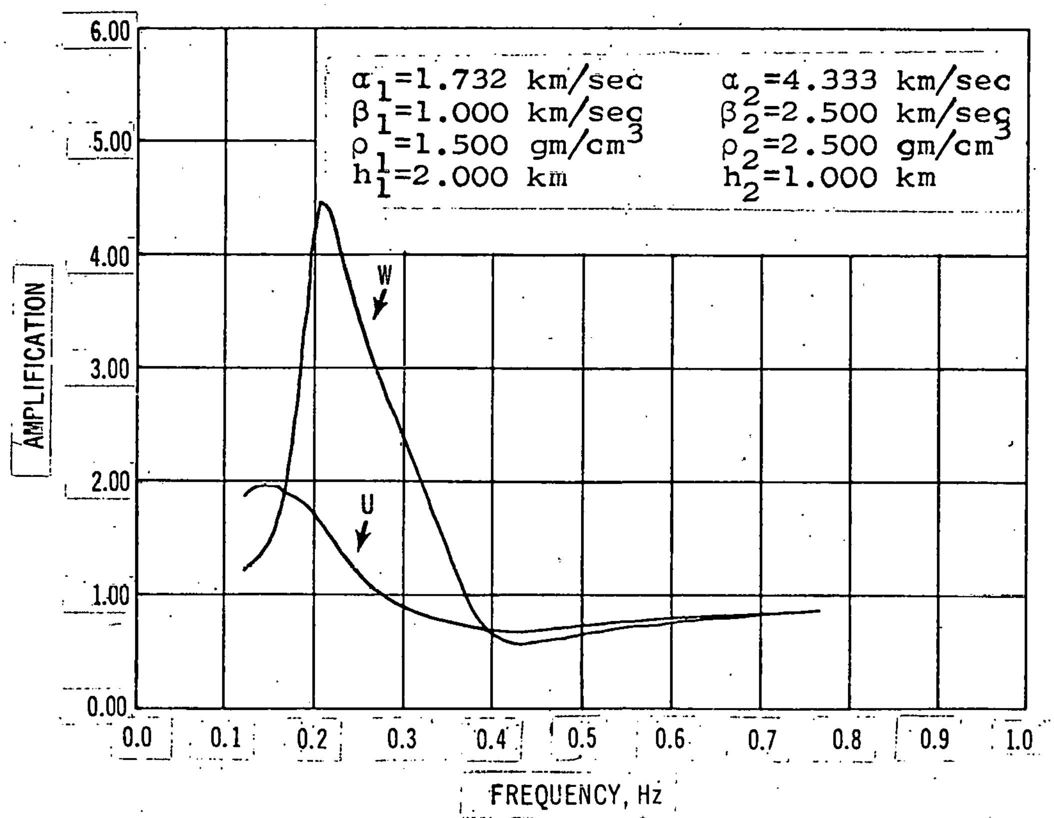

Figure 12. Parameter Study; α_2=4.333 km/sec, β_2=2.500 km/sec.

CHAPTER 4

SUMMARY AND RECOMMENDATIONS

4.1 SUMMARY

The present work, together with References 1 and 2, completes the formulation of a mathematical model for describing the effect of a low-velocity surface layer on observed seismic motions for all the principal wave modes and for the entire frequency range of interest. The examples presented in Chapter 3 show that significant amplifications of Rayleigh waves can be expected under conditions prevailing at numerous NTS instrumentation sites. The amplifications are markedly frequency dependent; the nature of this dependence is determined by the thickness and physical properties of the layer.

4.2 RECOMMENDATIONS

a. The model developed here should be verified as outlined in Chapter 1 using available data. Data from pairs of adjacent stations along the Las Vegas instrumentation line appear to be suitable for this purpose. At the same time comparisons with the results for Love waves can be made to check the consistency of the predictions. If the present models prove to be inadequate, various refinements are possible, such as the proper treatment of successive arrivals of separate wave trains.

b. If the empirical verification of the model is satisfactory, techniques will be developed for applying the method to routine prediction procedures. The primary area of application would be frequency dependent quantities, such as PSRV curves. Given adequate information on the geophysical characteristics at the sites of interest, it should be possible to predict variations in such curves from station to station.

REFERENCES

1. Davis, A.H. and Murphy, J.R.; "Amplification of Seismic Body Waves by Low-Velocity Surface Layers," NVO-1163-130; AEC; 1967.

2. Schultz, A.; "Magnification of Love Waves," NVO-1163-80; AEC; 1967.

3. DeNoyer, J.; "Determination of the Energy in Body and Surface Waves (Part I)," Bull. Seism. Soc. of Am.; 48; 355-368; 1958.

4. Ewing, W.M., Jardetzky, W.S., and Press, F.; Elastic Waves in Layered Media; McGraw-Hill Book Company, Inc.; New York; 1957.

5. Seyawa, K. and Kanai, K., "The M_2 Seismic Waves," Bull. Earthquake Res. Inst., 13, 471, 1935.

6. Herrera, I., "A Perturbation Method for Elastic Wave Propagation, 1. Nonparallel Boundaries," J. Geophys. Res., 69, 3845, 1964.

7. Henrici, P., Elements of Numerical Analysis; John Wiley & Sons, Inc., New York; 1964.

8. Seyawa, K. and Kanai, K., "Discontinuity in the Dispersion Curves of Rayleigh Waves," Bull. Earthquake Res. Inst., 13, 237, 1934.

9. Hays, W. W.; "Identification of Elastic Wave Types on Seismograms from Underground Nuclear Explosions," NVO-1163-157; AEC; 1968.

APPENDIX I

EXPLICIT EVALUATION OF THE ENERGY INTEGRALS

The energy integrals in equations (24) and (25) can be broken down into a sum of four integrals (Reference 3) with

$$E = \frac{\omega^2}{2}\left[\rho_1 \int_o^H |u_1|^2 \, dz + \rho_1 \int_o^H |w_1|^2 \, dz + \rho_2 \int_{-\infty}^o |u_2|^2 \, dz + \rho_2 \int_{-\infty}^o |w_2|^2 \, dz\right] \qquad \text{(I-1)}$$

where

$$u_1 = -i \frac{\nu_1' A}{\varphi k^2} [X_1 \cosh \nu_1 z + X_2 \sinh \nu_1 z + X_3 \cos \nu_1' z + X_4 \sin \nu_1' z]$$

$$w_1 = \frac{A}{\varphi k} [Z_1 \cosh \nu_1 z + Z_2 \sinh \nu_1 z + Z_3 \cos \nu_1' z + Z_4 \sin \nu_1' z] \qquad \text{(I-2)}$$

$$u_2 = i \frac{A}{\varphi k} \frac{\mu_1 \nu_1' k_{\beta_1}^2}{\mu_2 k^3} \left[P e^{\nu_2 z} + \frac{\nu_2'}{k} Q e^{\nu_2' z} \right]$$

$$w_2 = \frac{A}{\varphi k} \frac{\mu_1 \nu_1' k_{\beta_1}^2}{\mu_2 k^3} \left[\frac{\nu_2}{k} P e^{\nu_2 z} + Q e^{\nu_2' z} \right]$$

and

$$X_1 = \frac{2\nu_1 \nu_2'}{k^2} \xi \cos \nu_1' H + \frac{2\nu_1 \nu_1'}{k^2} \zeta \sin \nu_1' H - \left(\frac{k_{\beta_1}^2}{k^2} - 2 \right) \eta \sinh \nu_1 H$$

$$X_2 = 2\theta \cos \nu_1' H - \frac{2\nu_2 \nu_1'}{k^2} \xi \sin \nu_1' H + \left(\frac{k_{\beta_1}^2}{k^2} - 2 \right) \eta \cosh \nu_1 H$$

$$X_3 = - \frac{2\nu_1 \nu_1'}{k^2} \eta \sin \nu_1' H + \frac{\nu_1 \nu_2'}{k^2} \left(\frac{k_{\beta_1}^2}{k^2} - 2 \right) \xi \cosh \nu_1 H + \left(\frac{k_{\beta_1}^2}{k^2} - 2 \right) \theta \sinh \nu_1 H \qquad \text{(I-3)}$$

$$X_4 = \frac{2\nu_1\nu_1'}{k^2}\,\eta\,\cos\nu_1'H + \frac{\nu_1\nu_1'}{k^2}\left(\frac{k_{\beta_1}^{\;2}}{k^2} - 2\right)\zeta\,\cosh\nu_1 H$$

$$- \frac{\nu_2\nu_1'}{k^2}\left(\frac{k_{\beta_1}^{\;2}}{k^2} - 2\right)\xi\,\sinh\nu_1 H$$

$$Z_1 = \frac{2\nu_1\nu_1'}{k^2}\,\theta\,\cos\nu_1'H - \frac{2\nu_2\nu_1\nu_1'^{\,2}}{k^2}\,\xi\,\sin\nu_1'H$$

$$+ \frac{\nu_1\nu_1'}{k^2}\left(\frac{k_{\beta_1}^{\;2}}{k^2} - 2\right)\eta\,\cosh\nu_1 H$$

$$Z_2 = \frac{2\nu_1^{\,2}\nu_1'\nu_2'}{k^4}\,\xi\,\cos\nu_1'H + \frac{2\nu_1^{\,2}\nu_1'^{\,2}}{k^4}\,\zeta\,\sin\nu_1'H$$

$$- \frac{\nu_1\nu_1'}{k^2}\left(\frac{k_{\beta_1}^{\;2}}{k^2} - 2\right)\eta\,\sinh\nu_1 H$$

$$Z_3 = -\frac{2\nu_1\nu_1'}{k^2}\,\eta\,\cos\nu_1'H - \frac{\nu_1\nu_1'}{k^2}\left(\frac{k_{\beta_1}^{\;2}}{k^2} - 2\right)\zeta\,\cosh\nu_1 H$$

$$+ \frac{\nu_2\nu_1'}{k^2}\left(\frac{k_{\beta_1}^{\;2}}{k^2} - 2\right)\xi\,\sinh\nu_1 H$$

$$Z_4 = -\frac{2\nu_1\nu_1'}{k^2}\eta \sin \nu_1'H + \frac{\nu_1\nu_2'}{k^2}\left(\frac{k_{\beta_1}^{\ 2}}{k^2} - 2\right)\xi \cosh \nu_1 H$$

$$+ \left(\frac{k_{\beta_1}^{\ 2}}{k^2} - 2\right)\theta \sinh \nu_1 H$$

$$P = \frac{2\nu_2'\nu_1}{k^2}\beta \cos \nu_1'H + \frac{2\nu_1\nu_1'}{k^2}\alpha \sin \nu_1'H$$

$$- \frac{2\nu_2'\nu_1}{k^2}\left(2 - \frac{c^2}{\beta_1^{\ 2}}\right)\left(\frac{\mu_1}{\mu_2} - 1\right)\cosh \nu_1 H$$

$$+ \left(2 - \frac{c^2}{\beta_1^{\ 2}}\right)\gamma \sinh \nu_1 H$$

$$Q = -\frac{2\nu_1}{k}\gamma \cos \nu_1'H - 4\,\frac{\nu_2\nu_1\nu_1'}{k^3}\left(\frac{\mu_1}{\mu_2} - 1\right)\sin \nu_1'H$$

$$+ \frac{\nu_1}{k}\left(2 - \frac{c^2}{\beta_1^{\ 2}}\right)\alpha \cosh \nu_1 H$$

$$- \frac{\nu_2}{k}\left(2 - \frac{c^2}{\beta_1^{\ 2}}\right)\beta \sinh \nu_1 H$$

$$\varphi = \frac{\nu_1' k_{\alpha_2} k_{\beta_1}^2}{k^5} \left[- \frac{2\nu_2'\nu_1}{k^2} \left\{ 2 \frac{\mu_2}{\mu_1} - \left(2 - \frac{k_{\beta_1}^2}{k^2} \right) \right\} \cosh \nu_1' H \right.$$

$$- 2 \frac{\nu_1 \nu_1'}{k^2} \left\{ \frac{\mu_2}{\mu_1} \left(2 - \frac{k_{\beta_2}^2}{k^2} \right) - 2 \right\} \sinh \nu_1' H$$

$$+ 2 \frac{\nu_2' \nu_1}{k^2} \left(2 - \frac{k_{\beta_1}^2}{k^2} \right) \left(\frac{\mu_2}{\mu_1} - 1 \right) \cosh \nu_1 H$$

$$\left. - \left(2 - \frac{k_{\beta_1}^2}{k^2} \right) \left\{ \frac{\mu_2}{\mu_1} \left(2 - \frac{k_{\beta_2}^2}{k^2} \right) - \left(2 - \frac{k_{\beta_1}^2}{k^2} \right) \right\} \sinh \nu_1 H \right]$$

and as usual $i = \sqrt{-1}$.

In the remaining portion of the derivation it will be assumed that both ν_1 and ν_1' are real (i.e., $\beta_1 < c < \alpha_1$). The results for the other two cases are similar except that the trigonometric functions go over to hyberbolics and the hyperbolics go over to trigonometrics.

From equations (I-2) we have,

$$|u_1|^2 = u_1 \cdot u_1^* = \frac{A^2 \nu_1'^2}{\varphi^2 k^4} \Big[X_1^2 \cosh^2 \nu_1 z + X_2^2 \sinh^2 \nu_1 z$$

$$+ X_3^2 \cos^2 \nu_1' z + X_4^2 \sin^2 \nu_1' z + 2X_1X_2 \cosh \nu_1 z \sinh \nu_1 z$$

$$+ 2X_1X_3 \cosh \nu_1 z \cos \nu_1' z + 2X_1X_4 \cosh \nu_1 z \sin \nu_1' z \tag{I-4}$$

$$+ 2X_2X_3 \sinh \nu_1 z \cos \nu_1' z + 2X_2X_4 \sinh \nu_1 z \sin \nu_1' z$$

$$+ 2X_3X_4 \cos \nu_1' z \sin \nu_1' z \Big]$$

$$|w_1|^2 = w_1 \cdot w_1^* = \frac{A^2}{\varphi^2 k^2} \Big[Z_1^2 \cosh^2 \nu_1 z + Z_2^2 \sinh^2 \nu_1 z$$

$$+ Z_3^2 \cos^2 \nu_1' z + Z_4^2 \sin^2 \nu_1' z + 2Z_1Z_2 \sinh \nu_1 z \cosh \nu_1 z$$

$$+ 2Z_1Z_3 \cosh \nu_1 z \cos \nu_1' z + 2Z_1Z_4 \cosh \nu_1 z \sin \nu_1' z \tag{I-5}$$

$$+ 2Z_2Z_3 \sinh \nu_1 z \cos \nu_1' z + 2Z_2Z_4 \sinh \nu_1 z \sin \nu_1' z$$

$$+ 2Z_3Z_4 \cos \nu_1' z \sin \nu_1' z \Big]$$

$$|u_2|^2 = u_2 \cdot u_2^* = \frac{A^2}{\varphi^2 k^2} \frac{\mu_1^2 \nu_1'^2 k_{\beta_1}^4}{\mu_2^2 k^6} \left[P^2 e^{2\nu_2 z} + \frac{2\nu_2'}{k} PQe^{(\nu_2+\nu_2')z} + \frac{\nu_2'^2}{k^2} Q^2 e^{2\nu_2' z} \right] \tag{I-6}$$

$$|w_2|^2 = w_2 \cdot w_2^* = \frac{A^2}{\varphi^2 k^2} \frac{\mu_1^2 \nu_1'^2 k_{\beta_1}^4}{\mu_2^2 k^6} \left[\frac{\nu_2^2}{k^2} P^2 e^{2\nu_2 z} + \frac{2\nu_2}{k} PQe^{(\nu_2+\nu_2')z} + Q^2 e^{2\nu_2' z} \right] \tag{I-7}$$

in which the asterisk (*) denotes the complex conjugate of the quantity.

Substituting equations (I-4) through (I-7) into equation (22) we find:

$$E = \frac{\omega^2}{2} \int_{-\infty}^{H} (|u|^2 + |w|^2)\, dz = \frac{\omega^2}{2} \left[\rho_1 \int_0^H |u_1|^2\, dz + \rho_1 \int_0^H |w_1|^2\, dz + \rho_2 \int_{-\infty}^{0} |u_2|^2\, dz + \rho_2 \int_{-\infty}^{0} |w_2|^2\, dz \right]$$

$$= \frac{\omega^2 A^2}{2\varphi^2 k^2} \left[\frac{\rho_1 \nu_1'^2}{k^2} \left\{ \frac{H}{2} \left(X_1^2 - X_2^2 + X_3^2 + X_4^2 \right) \right.\right.$$

$$+ \frac{1}{2\nu_1} (X_1^2 + X_2^2) \sinh \nu_1 H \cosh \nu_1 H ,$$

$$+ \frac{1}{2\nu_1'} (X_3^2 - X_4^2) \sin \nu_1' H \cos \nu_1' H + \frac{X_1 X_2}{\nu_1} \sinh^2 \nu_1 H \qquad \text{(I-8)}$$

$$+ \frac{X_3 X_4}{\nu_1'} \sin^2 \nu_1' H + 2 \left[X_2 X_3 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) \right.$$

$$\left. - X_1 X_4 \left(\frac{\nu_1'}{\nu_1^2 + \nu_1'^2} \right) \right] \left(\cosh \nu_1 H \cos \nu_1' H - 1 \right)$$

$$+ 2 \left[X_2 X_4 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) + X_1 X_3 \left(\frac{\nu_1'}{\nu_1^2 + \nu_1'^2} \right) \right] \cosh \nu_1 H \sin \nu_1' H$$

$$+ 2 \left[X_1 X_3 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) - X_2 X_4 \left(\frac{\nu_1'}{\nu_1^2 + \nu_1'^2} \right) \right] \sinh \nu_1 H \cos \nu_1' H$$

$$+ 2 \left[X_1 X_4 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) + X_2 X_3 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) \right] \sinh \nu_1 H \sin \nu_1' H \Big\}$$

$$+ \rho_1 \Big\{ \frac{H}{2} (Z_1^2 - Z_2^2 + Z_3^2 + Z_4^2)$$

$$+ \frac{1}{2\nu_1} (Z_1^2 + Z_2^2) \sinh \nu_1 H \cosh \nu_1 H$$

$$+ \frac{1}{2\nu_1'} (Z_3^2 - Z_4^2) \sin \nu_1' H \cos \nu_1' H$$

$$+ \frac{Z_1 Z_2}{\nu_1} \sinh^2 \nu_1 H + \frac{Z_3 Z_4}{\nu_1'} \sin^2 \nu_1' H$$

$$+ 2 \left[Z_2 Z_3 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) - Z_1 Z_4 \left(\frac{\nu_1'}{\nu_1^2 + \nu_1'^2} \right) \right] (\cosh \nu_1 H \cos \nu_1' H - 1)$$

$$+ 2 \left[Z_2 Z_4 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) + Z_1 Z_3 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) \right] \cosh \nu_1 H \sin \nu_1' H$$

$$+ 2 \left[Z_1 Z_3 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) - Z_2 Z_4 \left(\frac{\nu_1'}{\nu_1^2 + \nu_1'^2} \right) \right] \sinh \nu_1 H \cos \nu_1' H$$

$$+ 2 \left[Z_1 Z_4 \left(\frac{\nu_1}{\nu_1^2 + \nu_1'^2} \right) + Z_2 Z_3 \left(\frac{\nu_1'}{\nu_1^2 + \nu_1'^2} \right) \right] \sinh \nu_1 H \sin \nu_1' H \Big\}$$

$$+ \rho_2 \frac{\mu_1^2 \nu_1'^2 k_{\beta_1}^4}{\mu_2^2 k^6} \left[\frac{P^2}{2\nu_2} + \frac{\nu_2'}{2k^2} Q^2 + \frac{2\nu_2'}{k(\nu_2 + \nu_2')} PQ \right]$$

$$+ \rho_2 \frac{\mu_1^2 \nu_1'^2 k_{\beta_1}^4}{\mu_2^2 k^6} \left[\frac{\nu_2}{2k^2} P^2 + \frac{Q^2}{2\nu_2'} + \frac{2\nu_2}{k(\nu_2 + \nu_2')} PQ \right] \Bigg]$$

Now, replacing the amplitude constant A in (I-7) by either $\bar{u}_1$ or $\bar{w}_1$ gives a computational form in terms of known quantities which can be used to evaluate the expressions for the amplification given by equations (24) and (25).

APPENDIX II

DESCRIPTION OF THE COMPUTER PROGRAM

A computer program has been written for the CDC 3600 computer which performs the calculations needed to compute the amplitude ratios given by equations (24) and (25). The program has been written using the complex arithmetic features under Fortran IV. The operations performed in the program can be grouped into four main sections: (1) solution of the dispersion equation (i.e. equation (13)) for the frequency range of interest, (2) determination of the group velocity as a function of frequency by a numerical differentiation of the phase velocity, (3) numerical evaluation of the energy integrals and (4) outputting of the dispersion and amplification curves in plotted and printed form. Some of the details involved in these operations will now be briefly outlined.

(1) Solution of the Dispersion Equation

In this initial section of the program the phase velocity is computed as a function of frequency by locating the roots of equation (13). Computationally, an initial value of frequency is specified and then the value of the phase velocity which satisfies equation (13) for that frequency is determined using standard numerical techniques. Output from this portion of the program has been checked against the results obtained by several other investigators (References 4 and 5) and the comparisons have been satisfactory.

(2) Determination of the Group Velocity

In this section the group velocity is calculated by a differentiation of the phase velocity using a version of equation (15). Using the relationship between k and ω (i.e. $k = \omega/c$) equation (15) can be rewritten:

$$\frac{1}{U} = \frac{1}{c} - \frac{\omega}{c^2}\frac{dc}{d\omega} \qquad \text{(II-1)}$$

in which the only unknown (at a given frequency) is $dc/d\omega$. The numerical differentiation scheme used is such that the j^{th} value of $dc/d\omega$ is given by (Reference 7):

$$\left(\frac{dc}{d\omega}\right)_j = \frac{8\,(c_{j+1} - c_{j-1}) - c_{j+2} + c_{j-2}}{12\,(\omega_j - \omega_{j-1})} \qquad \text{(II-2)}$$

This portion of the program was also checked against the results of other investigators as in (1).

(3) Evaluation of the Energy Integrals

Once the phase and group velocities have been determined as functions of frequency, the energy integrals given by equation (I-8) can be evaluated. Computationally, the integral is evaluated for the first horizontal position as a function of frequency and the results are stored internally. The program then recycles to compute the phase and group velocities and evaluate the energy integral for the second horizontal position. The ratios of the free surface amplitudes at the two horizontal positions are then calculated using equations (24) and (25). This portion of the computational scheme has been checked in two ways. First, the results of an intermediate computational step (i.e., the ratio of the horizontal to vertical free surface amplitudes at a given horizontal position) have been compared with those published in the literature by Sezawa and Kanai (Reference 8). Secondly, a check has been made against the theoretical asymptote of the energy integral. Specifically, it would be anticipated that at very long wavelengths the computed output would be equal to the result which would be computed analytically for a Rayleigh wave on a halfspace with the elastic properties of the halfspace used in the two-layer formulation. Both of these checks were satisfactorily completed using output obtained from the program.

(4) Output Specifications

At the present time the options are available to both print and plot the dispersion and amplification curves as a function of frequency. Examples of typical plotted output are shown in Figures II-1 - II-3 for the standard model described in Chapter 3.

The program in its present form occupies 55,000 memory locations and the average run time is about two seconds for each frequency.

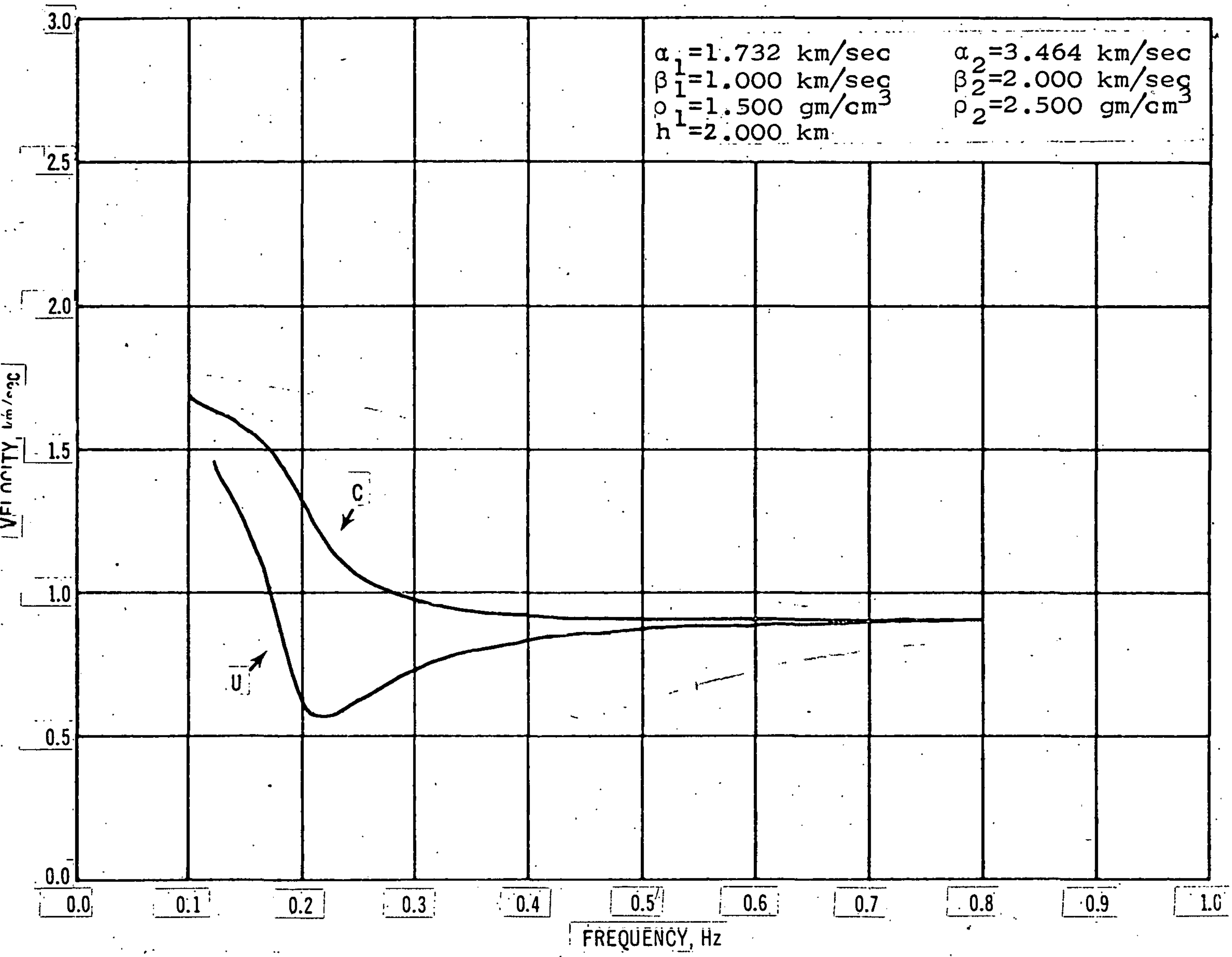

Figure II-1. Plot Illustration; Dispersion Curves for Position 1.

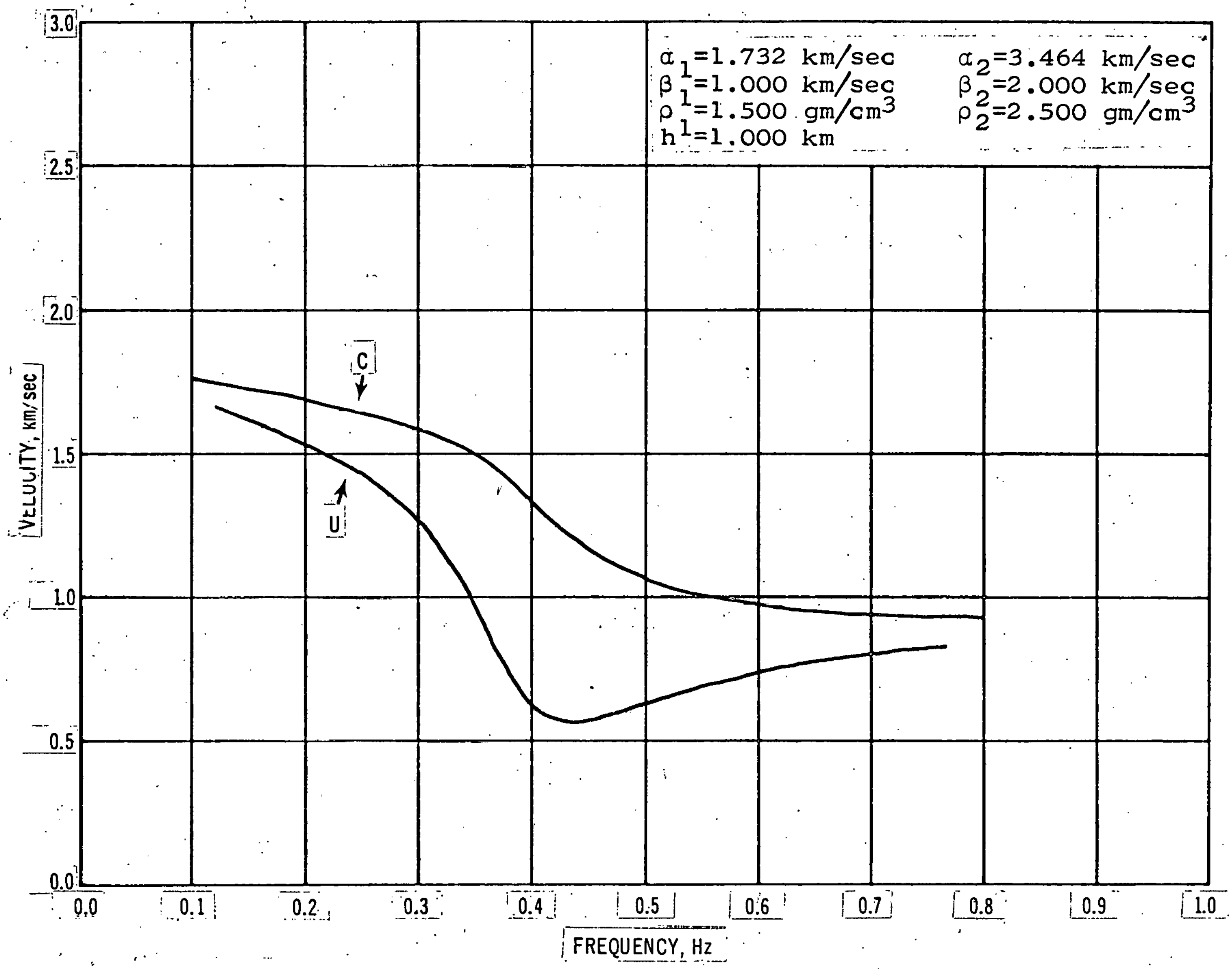

Figure II-2. Plot Illustration; Dispersion Curves for Position 2.

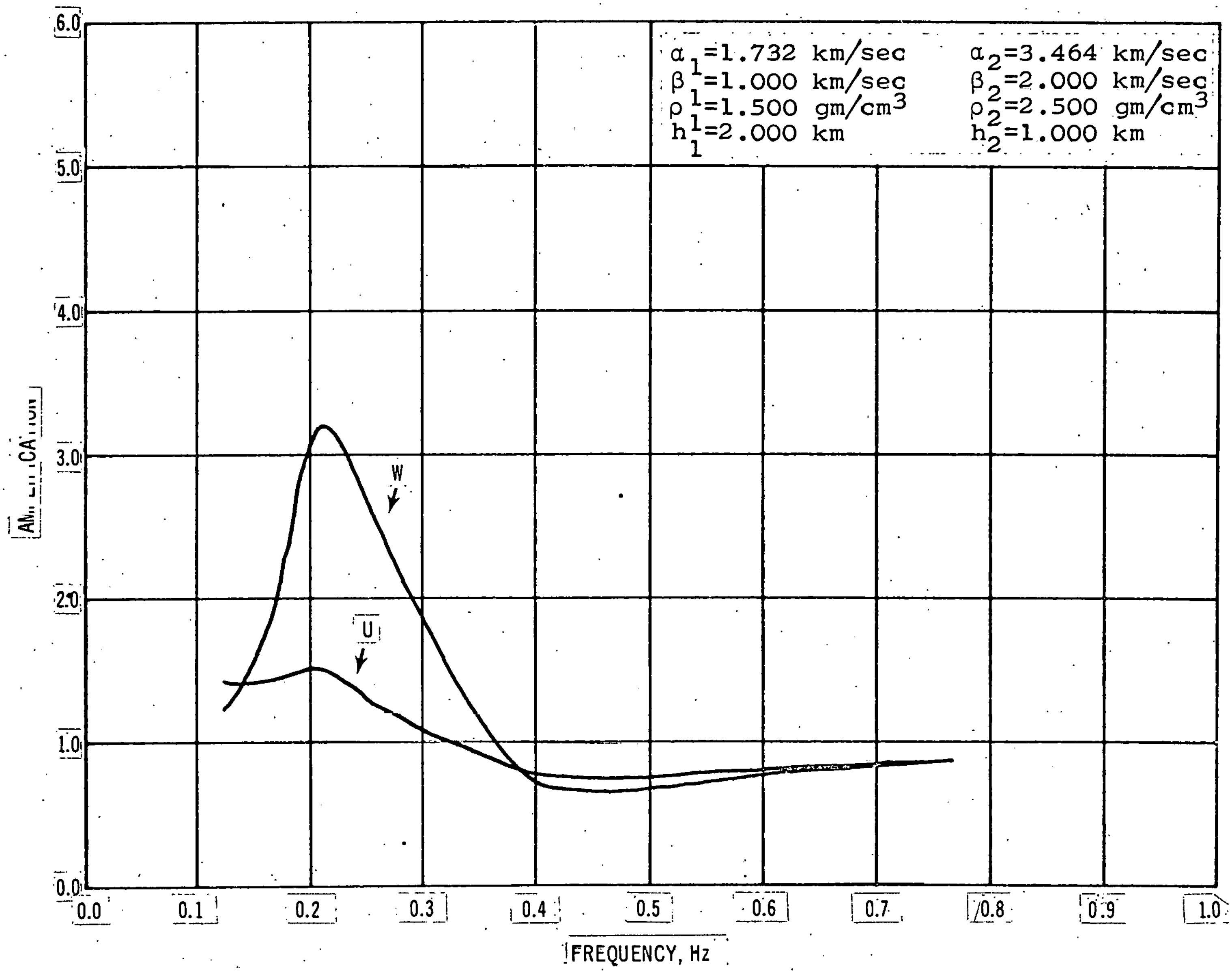

Figure II-3. Plot Illustrations; Amplification Curves for Position 1 Relative to Position 2.

Distribution - NVO-1163-175

Mr. D. H. Edwards, AEC/NVOO, Las Vegas, Nevada (3 copies)
Dr. G. H. Higgins, LRL, Livermore, California (1 copy)
Dr. H. L. Reynolds, LRL, Livermore, California (1 copy)
Dr. G. C. Werth, LRL, Livermore, California (1 copy)
Dr. J. W. Hadley, LRL, Livermore, California (1 copy)
Dr. M. B. Biles, AEC/DOS, Hq., Washington, D.C. (1 copy)
Mr. J. S. Kelly, AEC/DPNE, Hq., Washington, D.C. (1 copy)
Mr. R. R. Loux, AEC/NVOO, Las Vegas, Nevada (1 copy)
Mr. J. G. Philip. AEC, Berkeley, California (1 copy)
Maj. Gen. E. B. Giller, AEC/DMA, Hq., Washington, D.C. (2 copies)
Mr. P. L. Russell, USBM, Denver, Colorado (1 copy)
Mr. Wendell Mickey, USC&GS, Rockville, Maryland (1 copy)
Dr. W. E. Ogle, LASL, Los Alamos, New Mexico (1 copy)
Mr. R. W. Newman, LASL, Los Alamos, New Mexico (1 copy)
Dr. M. L. Merritt, Org. 5412, Sandia Corp., Albuquerque, New Mexico (1 copy)
Dr. W. D. Weart, Org. 5412, Sandia Corp., Albuquerque, New Mexico (1 copy)
DTIE, Oak Ridge, Tennessee (1 copy)
Dr. J. R. Banister, Sandia Corp., Albuquerque, New Mexico (1 copy)
Dr. W. S. Twenhofel, USGS, Denver, Colorado (1 copy)
Dr. John A. Blume, John A. Blume & Associates, San Francisco, California (2 copies)
Mr. Richard Hamburger, AEC/DPNE, Hq., Washington, D.C. (1 copy)
Dr. Fred Holzer, LRL, Livermore, California (1 copy)
Dr. R. H. Sproull, ARPA, Washington, D.C. (1 copy)
Mr. L. E. Rickey, H&N, Las Vegas, Nevada (1 copy)
Mr. K. W. King, USC&GS, Las Vegas, Nevada (2 copies)
Dr. L. B. Werner, Isotopes, Inc., Palo Alto, California (2 copies)
Mr. T. F. Thompson, San Francisco, California (1 copy)
Dr. N. M. Newmark, University of Illinois, Urbana, Illinois (1 copy)
Mr. S. D. Wilson, Shannon & Wilson, Inc., Seattle, Washington (1 copy)
Dr. D. U. Deere, University of Illinois, Urbana, Illinois (1 copy)
Dr. L. S. Jacobsen, AEC, Los Angeles, California (1 copy)
Dr. G. B. Maxey, University Station, Reno, Nevada (1 copy)
Mr. L. G. vonLossberg, Sheppard T. Powell & Associates, Baltimore, Maryland (1 copy)
Dr. Benhamin Grote, TCD-B, DASA, Sandia Base, Albuquerque, New Mexico (2 copies)
Environmental Research Corporation, Las Vegas, Nevada (2 copies)
Environmental Research Corporation, Alexandria, Virginia (3 copies)