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Letter to the Editor

Amplitudes of solar-like oscillations: a new scaling relation

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ABSTRACT

Solar-like oscillations are excited by near-surface convection and are being observed in growing numbers of stars using ground- and space-based telescopes. We have previously suggested an empirical scaling relation to predict their amplitudes. This relation has found widespread use but it predicts amplitudes in F-type stars that are higher than observed. Here we present a new scaling relation that is based on the postulate that the power in velocity fluctuations due to p-mode oscillations scales with stellar parameters in the same way as the power in velocity fluctuations due to granulation. The new relation includes a dependence on the damping rate via the mode lifetime and should be testable using observations from the CoRoT and *Kepler* missions. We also suggest scaling relations for the properties of the background power due to granulation and argue that both these and the amplitude relations should be applicable to red giant stars.

Key words. asteroseismology – stars: oscillations – stars: general

1. Introduction

When the first attempts to detect solar-like oscillations were being made, we suggested an empirical scaling relation to predict their amplitudes (Kjeldsen & Bedding 1995, hereafter Paper I). This relation (and its various modifications – see Sect. 2) has found widespread use in asteroseismology as observations of solar-like oscillations have accumulated. It has also been applied to cases in which p-mode oscillations are regarded as noise, namely searches for exoplanets (e.g., O'Toole et al. 2008; Niedzielski et al. 2009; Benedict et al. 2010) and even for their moons (Simon et al. 2011).

The steady flow of results over recent years from ground-based observations (e.g., reviews by Aerts et al. 2008; Bedding & Kjeldsen 2008), together with the flood of data now arriving from the CoRoT and *Kepler* space missions (e.g., Michel et al. 2008; Gilliland et al. 2010) makes it timely to revisit the amplitude scaling relation. That is the aim of this Letter.

2. The empirical relation

The empirical relation in Paper I was based on theoretical models by Christensen-Dalsgaard & Frandsen (1983), which were the only ones available at the time. Based on these models, we suggested that the velocity amplitudes of solar-like oscillations scale from star to star according to

$$A_{\rm vel} \propto \frac{L}{M}, \tag{1}$$

where L is the stellar luminosity and M is the mass.

Subsequently, model calculations seemed to show that a better match might be obtained with

$$A_{\rm vel} \propto \left(\frac{L}{M}\right)^{s}, \tag{2}$$

with $s \approx 1.5$ (Houdek et al. 1999) or with $s \approx 0.7$ (Samadi et al. 2005, 2007a,b; see also Houdek 2006).

Meanwhile, observations indicated that amplitudes of F-type stars fall below both of these relations. To account for this, Kjeldsen & Bedding (2001) suggested a modified scaling relation that included the effective temperature. This was

$$A_{\rm vel} \propto \frac{1}{g} \propto \frac{L}{M T_{\rm eff}{}^{4}}. \tag{3}$$

However, this modification was not based on any physical grounds. In this Letter we use simple physical arguments to derive a revised scaling relation for the amplitudes of solar-like oscillations.

3. A revised approach

Solar-like oscillations are excited by convection, which is also the process responsible for granulation. Our basic assumption is that the power in velocity fluctuations due to p-mode oscillations scales with stellar parameters in the same way as the power in velocity fluctuations due to granulation. The latter is a strong function of frequency and we will evaluate it at $\nu_{\rm max}$, the frequency at which the p-mode oscillations are centred. Our task, therefore, is to find a scaling relation for granulation power in velocity at $\nu_{\rm max}$.

In doing this, we can take advantage of the established scaling relation for $\nu_{\rm max}$. This is based on the suggestion by Brown et al. (1991) that $\nu_{\rm max}$ might be expected to be a fixed fraction of the acoustic cutoff frequency (see also Paper I):

$$\nu_{\rm max} \propto \nu_{\rm ac} \propto \frac{M T_{\rm eff}{}^{3.5}}{L}. \tag{4}$$

This relation agrees quite well with observations (Bedding & Kjeldsen 2003) and also with model calculations (Chaplin et al. 2008), and has recently been given a theoretical basis by Belkacem et al. (2011).

3.1. Granulation power in velocity

To estimate the velocity fluctuations caused by granulation, we use the standard model due to Harvey (1985) for the power density spectrum:

$$P_{\rm vel}(\nu) = \frac{4\sigma^2_{\rm vel}\tau_{\rm gran}}{1+(2\pi\nu\tau_{\rm gran})^2}. \tag{5}$$

Here, $\sigma_{\rm vel}$ is the rms of the velocity fluctuations due to granulation and $\tau_{\rm gran}$ specifies the granulation timescale. Similar components at lower frequencies arising from mesogranulation, supergranulation and active regions are not relevant here. Towards low frequencies the power density spectrum described by Eq. (5) becomes flat and approaches a value of $4\sigma^2_{\rm vel}\tau_{\rm gran}$. Towards higher frequencies it drops to half power at frequency $(2\pi\tau_{\rm gran})^{-1}$, and falls off as a power law with a slope of -2.

3.1.1. A relation for $\tau_{\rm gran}$

To derive a scaling relation for the granulation timescale as a function of stellar parameters, we assume that the vertical speed of the convection cells is proportional to the sound speed, $c_{\rm s}$, and that the vertical distance travelled scales with the pressure scale height, $H_{\rm P}$. Hence we have

$$\tau_{\rm gran} \propto \frac{H_{\rm P}}{c_{\rm s}}. \tag{6}$$

As argued in Paper I (Sect. 2.1), the ideal gas law implies that the sound speed near the surface scales approximately as

$$c_{\rm s} \propto \sqrt{T_{\rm eff}}. \tag{7}$$

Meanwhile, the pressure scale height (Sect. 3.2 of Paper I, see also Kippenhahn & Weigert 1990) scales approximately as

$$H_{\rm P} \propto \frac{T_{\rm eff}}{g} \propto \frac{L}{M{T_{\rm eff}}^3}. \tag{8}$$

We therefore have

$$\tau_{\rm gran} \propto \frac{L}{M{T_{\rm eff}}^{3.5}}. \tag{9}$$

Comparing with Eq. (4) shows that the granulation timescale scales inversely with the acoustic cutoff frequency:

$$\tau_{\rm gran} \propto \frac{1}{\nu_{\rm ac}} \propto \frac{1}{\nu_{\rm max}}, \tag{10}$$

a relation that we have already used for pipeline-processing of data from the *Kepler* mission (Huber et al. 2009).

3.1.2. A relation for $P_{\rm vel}(\nu_{\rm max})$

To evaluate Eq. (5) at $\nu_{\rm max}$, we note that $\nu_{\rm max} \gg 1/\tau_{\rm gran}$ in the Sun and so the proportionalities in Eqs. (4) and (10) ensure this will remain true for other stars. We can therefore replace the denominator in Eq. (5) by 1 to get

$$P_{\rm vel}(\nu_{\rm max}) \propto \sigma^2_{\rm vel}\tau_{\rm gran} \propto \frac{\sigma^2_{\rm vel}}{\nu_{\rm max}}. \tag{11}$$

For observations of an unresolved star, the velocity fluctuations arise from a large number of granules on its surface (assumed to behave in a statistically independent manner) and we therefore expect the rms of these fluctuations to scale inversely with the square root of the number of granules. We also assume that the vertical speed of the granules is proportional to the sound speed, $c_{\rm s}$. Therefore, the rms variation in velocity due to granulation should scale as

$$\sigma_{\rm vel} \propto \frac{c_{\rm s}}{\sqrt{n}}. \tag{12}$$

To estimate n, we assume the diameter of the granules to be proportional to the pressure scale height of the atmosphere (Schwarzschild 1975; Antia et al. 1984; Freytag et al. 1997). In this case, the number occupying the surface of a star scales as

$$n \propto \left(\frac{R}{H_{\rm P}}\right)^2, \tag{13}$$

where R is the stellar radius.

Combining Eqs. (12) and (13), we therefore have

$$\sigma_{\rm vel} \propto \frac{c_{\rm s}H_{\rm P}}{R} \propto \frac{L^{0.5}}{M{T_{\rm eff}}^{0.5}}, \tag{14}$$

where we have eliminated R using $L \propto R^2{T_{\rm eff}}^4$ and eliminated $H_{\rm P}$ using Eq. (8).

Using Eqs. (4) and (14), we can write Eq. (11) as

$$P_{\rm vel}(\nu_{\rm max}) \propto \frac{L^2}{M^3{T_{\rm eff}}^{4.5}}. \tag{15}$$

This is the scaling relation for granulation power in velocity at $\nu_{\rm max}$.

3.2. Oscillation amplitudes in velocity

The amplitudes of solar-like oscillations depend on both the excitation and the damping rate, where the latter is given by the mode lifetime. We postulate that the squared amplitude of p-mode oscillations in velocity is proportional to the mode lifetime multiplied by the velocity power density of the granulation at $\nu_{\rm max}$:

$${A_{\rm vel}}^2 \propto P_{\rm vel}(\nu_{\rm max})\,\tau_{\rm osc}. \tag{16}$$

This gives

$$A_{\rm vel} \propto \frac{L{\tau_{\rm osc}}^{0.5}}{M^{1.5}{T_{\rm eff}}^{2.25}}, \tag{17}$$

which is our new scaling relation for velocity amplitudes to replace Eq. (1). It now includes a strong dependence on $T_{\rm eff}$, and also a weak dependence on mode lifetime. Stars with longer mode lifetimes will show larger amplitudes, all other things being equal. Efforts to establish a scaling relation for $\tau_{\rm osc}$ have been made (Chaplin et al. 2009; Baudin et al. 2011) and the wealth of new data from CoRoT and *Kepler* will hopefully confirm one soon.

4. Comparison with observations

The first point to note is that correlated variations in mode amplitude and lifetime are seen in the Sun over the solar cycle. Chaplin et al. (2000) analysed velocity observations of the p-mode oscillations in the Sun over the declining phases of activity cycle 22 and found a 24% increase in mode linewidths (that is,

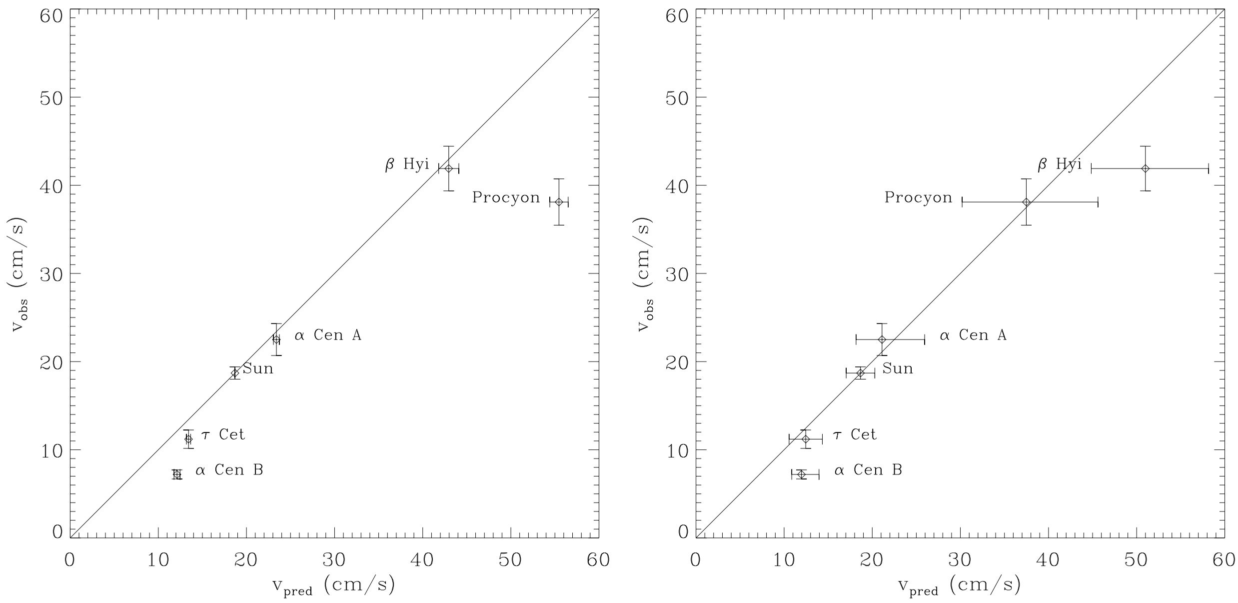

Fig. 1. Velocity amplitudes of solar-like oscillations compared to the $(L/M)^{0.7}$ scaling (*left*) and also compared to the new relation given by Eq. (17) (*right*).

a decrease in τ_{osc}) that was matched by an identical decrease in modal velocity powers (that is, in ${A_{vel}}^2$). This agrees exactly with expectations from Eq. (17) and with the inferences of Chaplin et al. (2000), who showed (by analogy with a damped, stochastically driven oscillator) that the observed in changes of the mode lifetimes and amplitudes could be explained by changes in the damping alone (i.e., without the need to alter the net forcing of the modes over the solar cycle). This also leads us to suggest that variations in oscillation amplitudes in other stars during their activity cycles, as recently reported by García et al. (2010) for the star HD 49933 from CoRoT observations, would also be due to changes in the damping. That is, the input power stays constant but the mode lifetime changes, and so the amplitude changes according to Eq. (17).

We now compare Eq. (17) with observations of solar-like oscillations. We are limited to those stars with measured velocity amplitudes and for which reliable mode lifetimes have been determined. These are α Cen A and B (Kjeldsen et al. 2005), τ Cet (Teixeira et al. 2009), Procyon (Bedding et al. 2010), and β Hyi (Bedding et al. 2007). Observed amplitudes were estimated using the method described by Kjeldsen et al. (2008), which involves smoothing the power spectrum.

To compare with other stars, we need a value for the mode lifetime in the Sun at maximum power. To establish this, we averaged the linewidths for the 5 central values in Table 1 of Chaplin et al. (2001), giving $\tau_{osc,\odot} = 2.88$ d. We then used Eq. (17) to calculate expected velocity amplitudes for those stars with published measured mode lifetimes. The results are shown in Fig. 1. Most of these stars have similar effective temperatures to the Sun and there is little to choose between the two relations. However, for the F-type star Procyon ($T_{eff} = 6500$ K), which probes the domain where the old relation is known to fail, the new relation gives much better agreement. Clearly more stars with measured mode lifetimes are needed to confirm the result. Detailed tests should soon be possible using the large number of stars now being observed by the CoRoT and *Kepler* space missions. However, those missions are performing photometry and so we need to discuss amplitudes measured in intensity.

5. Intensity measurements

5.1. Oscillation amplitudes in intensity

The amplitudes of oscillations in velocity are directly related to the velocity fluctuations from granulation, because the physical motion of convective cells is what drives the oscillations. The same is not true when the oscillations are observed in intensity. The intensity variations arise primarily from temperature variations that are caused by the compression and expansion of the atmosphere during the oscillation cycle.

In Paper I we argued that for an adiabatic sound wave, the fractional change in bolometric luminosity (integrated over all wavelengths) is related to the velocity amplitude:

$$A_{bol} \propto \frac{A_{vel}}{\sqrt{T_{eff}}}. \tag{18}$$

In practice, observations are made in a certain wavelength range. The intensity amplitude observed at a wavelength λ, assuming this wavelength is reasonably close to the peak of the blackbody spectrum (see Sect. 2.2 in Paper I), is

$$A_\lambda \propto \frac{A_{vel}}{\lambda {T_{eff}}^r} \tag{19}$$

with $r = 1.5$ (adopting the notation of Huber et al. 2010). In fact, a fit to observations of pulsating stars indicated $r = 2.0$ (Fig. 1 in Paper I). This difference presumably reflects that fact that stellar oscillations are not adiabatic. We note that some authors have chosen to adopt $r = 1.5$ (e.g., Michel et al. 2008; Mosser et al. 2010).

Combining Eqs. (17) and (19) gives our new scaling relation for intensity amplitudes:

$$A_\lambda \propto \frac{L {\tau_{osc}}^{0.5}}{\lambda M^{1.5} {T_{eff}}^{2.25+r}}, \tag{20}$$

which can be tested with observations from CoRoT and *Kepler*.

5.2. Granulation power in intensity

For completeness, we also make some comments about the intensity fluctuations caused by granulation. These arise because of surface brightness variations from the contrast between cool and dark regions (e.g., Trampedach et al. 1998; Svensson & Ludwig 2005; Ludwig 2006; Ludwig et al. 2009). We can estimate the granulation power in intensity by again using the Harvey (1985) model:

$$P_{\rm int}(\nu) = \frac{4\sigma_{\rm int}^2 \tau_{\rm gran}}{1 + (2\pi\nu\tau_{\rm gran})^2}. \tag{21}$$

The granulation timescale, $\tau_{\rm gran}$, is the same as before, but we need a scaling relation for the rms of the intensity fluctuations, $\sigma_{\rm int}$. This should depend on the total number of granules that are visible on the surface:

$$\sigma_{\rm int} \propto \frac{1}{\sqrt{n}}. \tag{22}$$

Choosing once again to measure the power at $\nu_{\rm max}$, we can use the same arguments as before (see Eq. (11)) to get

$$P_{\rm int}(\nu_{\rm max}) \propto \frac{\sigma_{\rm int}^2}{\nu_{\rm max}}. \tag{23}$$

Using Eqs. (8), (13) and (22), we therefore have

$$P_{\rm int}(\nu_{\rm max}) \propto \frac{L^2}{M^3 {T_{\rm eff}}^{5.5}}. \tag{24}$$

This scaling relation for the granulation background can be tested and refined with data from CoRoT and *Kepler*. However, we should note that the intensity fluctuations caused by granulation depend on the contrast between cool and dark regions, which in turn depends on the opacities and the amount of limb darkening (Freytag et al. 1997; Svensson & Ludwig 2005; Ludwig 2006). We might therefore expect some additional dependence on $T_{\rm eff}$ and also on metallicity.

6. Red giants

The original scaling relations discussed in Sect. 2 were based on models of stars on or close to the main sequence. However, they have also been applied to observations of red giants (e.g., Edmonds & Gilliland 1996; Gilliland 2008; Zechmeister et al. 2008; Stello & Gilliland 2009; Huber et al. 2010; Mosser et al. 2010; Baudin et al. 2011), despite the fact that model calculations of red giants show poor agreement (Houdek & Gough 2002). The new relations proposed in this Letter might reasonably be expected to apply to red giants. However, it should be kept in mind that red giants show additional $l = 1$ mixed modes that may affect the total amount of measured oscillation power (e.g., Dupret et al. 2009; Huber et al. 2010). In addition, the coolest red giants have molecular absorption features in their spectra that are sensitive to temperature and would produce larger intensity amplitudes than expected under the black-body assumption (Sect. 5.1). For the case of granulation background, on the other hand, the relations in Eqs. (10) and (24) appear to be confirmed by data from *Kepler* (Mathur et al. 2011).

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