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# An Adaptive Disturbance Decoupling Perspective to Longitudinal Platooning

Di Liu, Bart Besselink, Simone Baldi, Wenwu Yu, Harry L. Trentelman

***Abstract*—Despite the progress in the field of longitudinal formations of automated vehicles, only recently an interpretation of longitudinal platooning has been given in the framework of disturbance decoupling, i.e. the problem of making a controlled output independent of a disturbance. The appealing feature of this interpretation is that the disturbance decoupling approach naturally yields a decentralized controller that guarantees stability and string stability. In this work, we further exploit the disturbance decoupling framework and we show that convergence to a stable, string stable and disturbance decoupled behavior can be achieved even in the presence of parametric uncertainty of the engine time constant. We refer to this framework as adaptive disturbance decoupling.**

## I. Introduction

The topics of longitudinal platoons of automated vehicles (automated vehicles following each other in string formation) have been investigated in different directions, spanning from computer vision, control and impact in the traffic flow [1]. A pioneering result was the one of Peppard [2], which introduced the problem of 'string stability', i.e. the capability of a platoon of automated vehicles to reject disturbances in the traffic flow. Later on, important characterizations for this problem have been given in the Lyapunov stability framework [3] and in the frequency domain [4]. Over the last two decades, these characterizations have been improved in different directions, such as having the automated vehicles communicate with various patterns [5], [6], [7]. Recent results include the introduction of delay-based spacing policy [8], the study of automated vehicles in a partial differential equation domain [9], the issues of safety [10] and cyber-security [11], [12], [13], dealing with heterogeneity in vehicle engine dynamics [14], [15], or with nonlinearity [16], [17]. Practical aspect related to vehicle state estimation have also been reported [18], [19], as well as real-life experiments [20], [21], [22].

Several control protocols have been proposed in the literature for achieving longitudinal string formations of automated vehicles, and the purpose of this overview is not to categorize them. Despite the progress in the field and the wide range of protocols, only recently an interpretation of longitudinal platooning has been given in the framework of disturbance decoupling [23]. Disturbance decoupling refers to the problem of making a controlled output independent of the disturbances. In the framework proposed in [23], the controlled output is the spacing error, while the disturbance is the input of the preceding vehicle. Building on geometric control theory [24], the appealing feature of this framework is that it naturally leads to a *decentralized* formulation, that is, the predecessor vehicle is free to choose its control (e.g., to track another vehicle) while guaranteeing stability and string stability (the latter for a properly chosen spacing policy).

A problem of the resulting disturbance decoupling protocol is that its design requires complete knowledge of the engine dynamics (i.e. the time constant defining the convergence of the acceleration to the desired acceleration). However, the engine time constant is unknown in practice or can change according to vehicle load conditions, road geometry, road surface friction, tire capacities and so on [25], [26], such that the design of [23] cannot be applied successfully.

In this paper, we propose to combine the disturbance decoupling approach with adaptive control. In particular, we define desired closed-loop behavior on the basis of the disturbance decoupling approach, which serves as a reference model in the subsequent adaptive control approach. Controller update laws are defined that guarantee asymptotic convergence of the actual vehicle behavior to that of the reference model. As such, this adaptive disturbance decoupling approach guarantees asymptotic tracking of a desired spacing policy as well as (asymptotic) string stability and disturbance decoupling.

The outline of this work is as follows: in Section II preliminary concepts about stability, string stability and disturbance decoupling are recalled. The proposed design is elaborated in Section III. Simulations are in Section IV, and conclusions in Section V.

## II. Problem statement

Consider a predecessor-follower with vehicles indexed as $p$ (predecessor) and $f$ (follower) and with dynamics

$$\begin{aligned} \dot{s}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= a_i(t), \qquad\qquad i \in \{p, f\}, \\ \tau_i \dot{a}_i(t) &= -a_i(t) + u_i(t). \end{aligned} \tag{1}$$

where $s_p, s_f \in \mathbb{R}$, $v_p, v_f \in \mathbb{R}$, $a_p, v_f \in \mathbb{R}$ are the longitudinal position, velocity, and acceleration of vehicles $p, f$, respectively. The control signals $u_p, u_f \in \mathbb{R}$ can be regarded as the desired acceleration, due to the fact that $\tau_p, \tau_f > 0$

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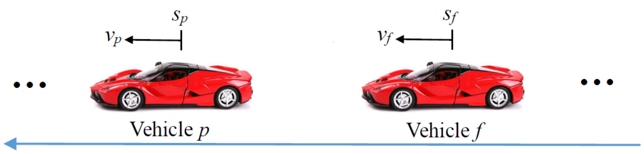


Fig. 1: Predecessor $p$ and follower $f$ in a longitudinal platoon.

represent the engine time constants and the last equation in (1) represents the engine dynamics. Dynamics as (1) is valid as soon as lateral motion does not come into play, and is standard in longitudinal platooning literature, c.f. [4], [14], [16] and references therein.

In order to formalize the platooning task, denote the distance between vehicle $f$ and its preceding vehicle $p$ as

$$d_f(t) = s_p(t) - s_f(t). \tag{2}$$

The desired inter-vehicle distance is defined as a constant time headway spacing policy [4]

$$d_f^*(t) = hv_f(t), \tag{3}$$

where $h > 0$ is a time headway. No standstill safety distance is included in (3) since, without loss of generality, a coordinate shift can be performed to remove constant terms in the desired spacing [23]. The spacing error $e_f$ is therefore defined as

$$e_f(t) = d_f(t) - d_f^*(t) = s_p(t) - s_f(t) - hv_f(t). \tag{4}$$

Next, we recall a few concepts related to designing the control input $u_f$ so as to achieve stability and string stability specifications. To this purpose, and in line with [23], we define the notation $\xi^T = \begin{bmatrix}\xi_p^T & \xi_f^T\end{bmatrix}$, with $\xi_i^T = \begin{bmatrix}s_i & v_i & a_i\end{bmatrix} \in \mathbb{R}^3$, $i \in \{p, f\}$, and the corresponding dynamics

$$\dot{\xi}(t) = A\xi(t) + Bu_f(t) + Gu_p(t), \tag{5}$$

$$A := \begin{bmatrix}\tilde{A}_p & 0 \\ 0 & \tilde{A}_f\end{bmatrix}, \quad B := \begin{bmatrix}0 \\ \tilde{B}_f\end{bmatrix}, \quad G := \begin{bmatrix}\tilde{B}_p \\ 0\end{bmatrix}. \tag{6}$$

Here, the matrices $\tilde{A}_i$ and $\tilde{B}_i$ can be derived from (1) as

$$\tilde{A}_i := \begin{bmatrix}0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau_i^{-1}\end{bmatrix}, \; \tilde{B}_i := \begin{bmatrix}0 \\ 0 \\ \tau_i^{-1}\end{bmatrix}, \; i \in \{p, f\}.$$

The spacing error $e_f$ in (4) can be written as

$$e_f(t) = H\xi(t), \tag{7}$$

where $H = \begin{bmatrix}1 & 0 & 0 & -1 & -h & 0\end{bmatrix}$.

In [23], the state feedback controller $u_f(t) = F\xi(t)$ with

$$F = \begin{bmatrix}\theta_1 & \theta_2 & \tau_f h^{-1} & -\theta_1 & -\theta_2 - h\theta_1 & 1 - \tau_f h^{-1} - h\theta_2\end{bmatrix} \tag{8}$$

with $\theta_1, \theta_2 > 0$ is proposed for the follower vehicle, leading to the closed-loop system

$$\dot{\xi}(t) = (A + BF)\xi(t) + Gu_p(t). \tag{9}$$

The properties of the closed-loop system are formalized in the following result from [23].

**Theorem 1.** *Consider the predecessor-follower model* (5) *with spacing policy* (7)*. Then, the state feedback controller $u_f = F\xi$ with $F$ in (8) is such that the following holds:*

1) *for $\xi(0) = 0$ and any bounded $u_p(\cdot)$, $e_f(t) = 0$, $t \geq 0$;*
2) *for any $\xi(0) \in \mathbb{R}^6$ any bounded $u_p(\cdot)$, it holds that*

$$\lim_{t\to\infty} e_f(t) = 0; \tag{10}$$

3) *for $\xi(0) = 0$ and any bounded $u_p(\cdot)$, it holds that*

$$\int_0^T |v_f(t)|^2\, dt \leq \int_0^T |v_p(t)|^2\, dt, \tag{11}$$

*for all $T > 0$.*

**Remark 1.** *The first two items in Theorem 1 can be regarded as a disturbance decoupling and output stabilization for the model* (9)*, respectively, where the control input $u_p$ of the predecessor vehicle is regarded as a disturbance. In particular, solutions to* (7) *are given as*

$$e_f(t) = He^{(A+BF)t}\xi(0) + \int_0^t He^{(A+BF)(t-\tau)}Gu_p(\tau)\, d\tau.$$

*Then, $He^{(A+BF)t}G = 0$ for all $t \geq 0$ guarantees item 1) in Theorem 1, whereas $\lim_{t\to\infty} He^{(A+BF)t} = 0$ guarantees item 2). Item 3) guarantees string stability, i.e., velocity perturbations do not amplify from the predecessor to the follower vehicle. It was proven in [23] that the solution to items 1) and 2) automatically leads to item 3) being satisfied for the spacing policy* (3)*, which allows us to focus on disturbance decoupling and output stabilization in the rest of this work.*

It is important to stress that the conditions in Theorem 1 hold *for any* bounded $u_p$. This property guarantees that the predecessor vehicle is free to choose its control (e.g., to track another vehicle) without compromising asymptotic stabilization of (3) and string stability. As such, the conditions in Theorem 1 aim to make the longitudinal platooning a decentralized problem, see [23] for details.

Despite the appealing properties of the controller (8), it crucially relies on the *exact* knowledge of the time constant $\tau_f$. In practice however, the exact value is often not known, which leads to the following problem statement.

**Problem 1.** *Given the predecessor-follower model* (5) *with spacing policy* (7)*, design controller $u_f$ that achieves, for any $\xi(0) \in \mathbb{R}^6$ and any bounded $u_p(\cdot)$,*

$$\lim_{t\to\infty} e_f(t) = 0, \tag{12}$$

*without using knowledge of the engine dynamics, i.e., the exact value of the positive constant $\tau_f$.*

Inspired by Theorem 1, Problem 1 asks for (12) to hold *for any* (bounded) control input of the predecessor vehicle. It is clear that Problem 1 is closely related to item 2) of Theorem 1, except from $\tau_f$ not being known exactly.

To address Problem 1, we will use an *adaptive* controller.

**Remark 2.** *Because any adaptive control loop is intrinsically nonlinear [27, Chapt. 1], one cannot rely on the same design used to get $F$ in (8). The design idea is as follows: items 1) and 2) will be used to define appropriate reference dynamics with*

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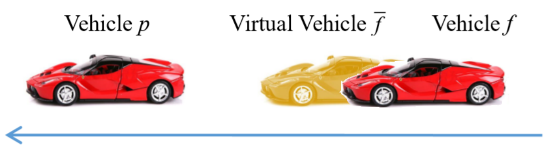


Fig. 2: The control design of vehicle $f$ includes virtual vehicle dynamics (vehicle $\bar{f}$) used to define a reference model.

*disturbance decoupling and output stabilization properties, which in turn imply the string stability property of item 3). Then, adaptation laws will be designed such that the state of the actual closed-loop system converges to the state of the reference dynamics, for any bounded exogenous input $u_p(\cdot)$. In other words, even though the exact conditions in items 1) and 3) of Theorem 1 cannot be guaranteed if $\tau_f$ is unknown, they can be achieved* asymptotically *in the adaptive loop.*

## III. Adaptive Disturbance Decoupling Design

In addressing Problem 1, we will take an adaptive control approach, e.g., [27]. However, before giving the details of control design, another interpretation of the controller (8) is given as a basis for the adaptive controller.

After recalling the spacing error $e_f = s_p - s_f - hv_f$ and defining the velocity error $\nu = v_p - v_f$, the controller $u_f = F\xi$ with $F$ as in (8) can be written as

$$u_f = \theta_1 e_f + \theta_2 \nu_f + (1 - \tau_f h^{-1} - h\theta_2)a_f + \tau_f h^{-1} a_p, \qquad (13)$$

which has a clear interpretation in terms of

- *feedback* from the spacing error, relative velocity, acceleration of vehicle $f$ (quantities measurable from on-board sensors like radar, laser and accelerometer);
- *feedforward* from the acceleration of preceding vehicle $p$ (quantity available from wireless communication as in Cooperative Adaptive Cruise Control [4], [14], [15]).

Consequently, the controller (13) has the typical *feedback-feedforward* structure of adaptive control. The adaptive design comprises three steps, elaborated in the next three subsections.

### A. First step: design of a reference model

Consider a 'virtual' vehicle denoted by $\bar{f}$. Following the vehicle dynamics (1), the dynamics of vehicle $\bar{f}$ is chosen as

$$\begin{aligned} \dot{s}_{\bar{f}} &= v_{\bar{f}}, \\ \dot{v}_{\bar{f}} &= a_{\bar{f}}, \\ \tau_{\bar{f}} \dot{a}_{\bar{f}} &= -a_{\bar{f}} + u_{\bar{f}}, \end{aligned} \qquad (14)$$

where $\tau_{\bar{f}} > 0$ is a nominal engine time constant chosen by the designer. The virtual dynamics (14) represents nominal dynamics that will be part of the adaptive controller implemented by vehicle $f$ (cf. Fig. 2). Define $\bar{\xi}^T = \begin{bmatrix}\xi_p^T & \xi_{\bar{f}}^T\end{bmatrix}$, with $\xi_{\bar{f}}^T = \begin{bmatrix}s_{\bar{f}} & v_{\bar{f}} & a_{\bar{f}}\end{bmatrix} \in \mathbb{R}^3$. Desirable behavior for the reference vehicle is achieved using the controller

$$u_{\bar{f}} = \theta_1 e_{\bar{f}} + \theta_2 \nu_{\bar{f}} + (1 - \tau_{\bar{f}} h^{-1} - h\theta_2)a_{\bar{f}} + \tau_{\bar{f}} h^{-1} a_p, \qquad (15)$$

with $e_{\bar{f}} = s_p - s_{\bar{f}} - hv_{\bar{f}}$, $\nu_{\bar{f}} = v_p - v_{\bar{f}}$. Namely, it is of the same form as controller (13), such that the properties of Theorem 1 hold for the virtual vehicle, i.e., the output $e_{\bar{f}}$ is decoupled from the exogenous input $u_p$.

Substitution of (15) in (14) leads to reference closed-loop dynamics given by the model

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{G}a_p, \qquad (16)$$

with state $\bar{x}^T = \begin{bmatrix}e_{\bar{f}} & \nu_{\bar{f}} & a_{\bar{f}}\end{bmatrix}$ and system matrices

$$\bar{A} = \begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ \theta_1\tau_{\bar{f}}^{-1} & \theta_2\tau_{\bar{f}}^{-1} & -h^{-1} - h\theta_2\tau_{\bar{f}}^{-1} \end{bmatrix}, \ \bar{G} = \begin{bmatrix} 0 \\ 1 \\ h^{-1} \end{bmatrix}. \qquad (17)$$

Then the output $e_{\bar{f}}$ can be written as

$$e_{\bar{f}} = \bar{H}\bar{x}, \qquad (18)$$

with $\bar{H} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$. We note that the states in (16) agree with the three feedback terms in the interpretation of controller (13) (and, hence, of (15)) given through $e_{\bar{f}}$, $\nu_{\bar{f}}$, $a_{\bar{f}}$. Moreover, the external input $a_p$ to (16) corresponds to the feedforward term in (16), making (16) a suitable reference model for adaptive control design.

Importantly, the reference model (16) also captures the desirable closed-loop system properties of Theorem 1, as formally stated next.

**Lemma 1.** *Consider the reference model* (16)–(17) *with* $\theta_1, \theta_2, h, \tau_{\bar{f}} > 0$. *Then,*

1) *for* $\bar{x}(0) = 0$ *and any bounded* $a_p(\cdot)$, $e_{\bar{f}}(t) = 0$, $t \geq 0$;
2) *for any* $\bar{x}(0) \in \mathbb{R}^3$ *and any bounded* $a_p(\cdot)$, *it holds that*

$$\lim_{t\to\infty} e_{\bar{f}}(t) = 0; \qquad (19)$$

3) $\bar{A}$ *is Hurwitz.*

*Proof.* 1) and 2). A direct computation shows that the dynamics of $e_{\bar{f}}$ is governed by

$$\tau_i h^{-1}\ddot{e}_{\bar{f}} + \theta_2\dot{e}_{\bar{f}} + \theta_1 e_{\bar{f}} = 0, \qquad (20)$$

which is independent of $a_p$, thus showing item 1). As $\tau_i h^{-1}, \theta_1, \theta_2 > 0$, item 2) also holds.

3). The characteristic polynomial of the matrix $\bar{A}$ is

$$\rho(\lambda) = \lambda^3 + (h^{-1} + h\theta_2\tau_{\bar{f}}^{-1})\lambda^2 + (h\theta_1\tau_{\bar{f}}^{-1} + \theta_2\tau_{\bar{f}}^{-1})\lambda + \theta_1\tau_{\bar{f}}^{-1}. \qquad (21)$$

According to the Routh-Hurwitz criterion, necessary and sufficient stability conditions for (21) can be derived as

$$h^{-1} + h\theta_2\tau_{\bar{f}}^{-1} > 0, \quad \theta_1\tau_{\bar{f}}^{-1} > 0, \\ h^{-1} + h^2\theta_1\tau_{\bar{f}}^{-1} + h\theta_2\tau_{\bar{f}}^{-1} > 0. \qquad (22)$$

Since $\theta_1, \theta_2, h, \tau_{\bar{f}}$ are positive, the stability conditions (22) hold. □

We note that the predecessor acceleration $a_p$ is regarded as the external disturbance in the reference model, rather than its control input $u_p$. As $u_p$ directly influences $a_p$ through the vehicle dynamics (1), the reference model (16) is sufficient for evaluating the performance objective of Problem 1.

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To be able to use the reference model for adaptive control, we express the actual dynamics using the same coordinates:

$$\dot{x} = A_f x + B_f u_f + G_f a_p, \tag{23}$$

where $x^T = \begin{bmatrix} e_f & \nu_f & a_f \end{bmatrix}$ and with

$$A_f = \begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ 0 & 0 & -\tau_f^{-1} \end{bmatrix}, \quad B_f = \begin{bmatrix} 0 \\ 0 \\ \tau_f^{-1} \end{bmatrix}, \quad G_f = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \tag{24}$$

This concludes the first step.

### B. Second step: design of a model-matching control structure

As a second step, we will design an ideal feedback-feedforward controller to make $\tilde{x} = x - \bar{x}$ converge to zero. Note that this implies that vehicle $f$ converges to the same behavior as that of vehicle $\bar{f}$ in the reference model.

To illustrate this step, assume that the exact value of $\tau_f$ is known. Then, it is easy to verify that the ideal controller

$$u_f^* = k_1^* e_f + k_2^* \nu_f + k_3^* a_f + l^* a_p, \tag{25}$$

with gains

$$\begin{aligned} k_1^* &= \frac{\tau_f}{\tau_{\bar{f}}}\theta_1, \quad k_2^* = \frac{\tau_f}{\tau_{\bar{f}}}\theta_2, \\ k_3^* &= 1 - \tau_f(h^{-1} + h\theta_2\tau_{\bar{f}}^{-1}), \quad l^* = \tau_f h^{-1}. \end{aligned} \tag{26}$$

When substituted into (23), the ideal dynamics of $x$ are

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ \tau_f^{-1}k_1^* & \tau_f^{-1}k_2^* & \tau_f^{-1}(k_3^* - 1) \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ \tau_f^{-1} l^* \end{bmatrix} a_p \\ &= \bar{A}x + \bar{G}a_p. \end{aligned} \tag{27}$$

As a result, the ideal dynamics of $\tilde{x} = x - \bar{x}$ is given by

$$\dot{\tilde{x}} = \bar{A}\tilde{x}, \tag{28}$$

such that $\tilde{x}$ converges to zero (since $\bar{A}$ is Hurwitz, see Lemma 1). Stated differently, the physical state $x$ converges to the behavior of the reference model, which has the desirable properties of Lemma 1 and solves Problem 1.

Unfortunately, the ideal controller $u_f^*$ in (25) cannot be implemented if the value of $\tau_f$ is unknown. To tackle this problem, we will estimate the ideal controller $u_f^*$ using an adaptive controller of the form

$$u_f = k_1 e_f + k_2 \nu_f + k_3 a_f + l a_p, \tag{29}$$

where $k_1, k_2, k_3, l$ can be interpreted as (dynamic) estimates of $k_1^*$, $k_2^*$, $k_3^*$, $l^*$, respectively, for which an updating mechanism will be introduced later.

Using the reference model (16) and the dynamics (23), the dynamics of the error $\tilde{x} = x - \bar{x}$ reads

$$\dot{\tilde{x}} = A_f x - \bar{A}\bar{x} + B_f u_f + (G_f - \bar{G})a_p. \tag{30}$$

Substitution of the controller (29) then leads to

$$\dot{\tilde{x}} = \bar{A}\tilde{x} + \bar{B}\,\frac{1}{l^*}\big(\tilde{k}_1 e_f + \tilde{k}_2 \nu_f + \tilde{k}_3 a_f + \tilde{l} a_p\big), \tag{31}$$

with $\bar{B}^T = \begin{bmatrix} 0 & 0 & h^{-1} \end{bmatrix}$ and where $\tilde{k}_1 = k_1 - k_1^*$, $\tilde{k}_2 = k_2 - k_2^*$, $\tilde{k}_3 = k_3 - k_3^*$, and $\tilde{l} = l - l^*$ are parametric estimation errors, entering as disturbances into the stable dynamics (28).

### C. Third step: main stability result

The third step is to prove that an appropriate choice of the estimation mechanism for updating $k_1, k_2, k_3, l$ guarantees convergence of the tracking error $\tilde{x}$ in (31) to zero.

To this end, consider the adaptive laws

$$\begin{aligned} \dot{k}_1 &= -\gamma_1 \bar{B}^T P \tilde{x} e_f, \quad \dot{k}_2 = -\gamma_2 \bar{B}^T P \tilde{x} \nu_f, \\ \dot{k}_3 &= -\gamma_3 \bar{B}^T P \tilde{x} a_f, \quad \dot{l} = -\gamma_4 \bar{B}^T P \tilde{x} a_p. \end{aligned} \tag{32}$$

where $\gamma_1$, $\gamma_2$, $\gamma_3$, $\gamma_4$ are positive adaptation gains, and $P = P^T > 0$ is any positive definite solution to the Lyapunov equation $\bar{A}^T P + P\bar{A} = -Q$, with $Q > 0$ a user-defined positive definite matrix.

The following stability and convergence result holds.

**Theorem 2.** *Consider the closed-loop system formed by the vehicle dynamics* (5)*, the spacing policy* (7)*, the reference model* (16)*, and controller* (29) *with adaptive laws* (32)*. Then, for any bounded $u_p(\cdot)$, we have that $\tilde{x} \to 0$ as $t \to \infty$ and*

$$\lim_{t\to\infty} e_f(t) = 0. \tag{33}$$

*Proof.* Consider the Lyapunov function candidate

$$\begin{aligned} V\big(\tilde{x}, \tilde{k}, \tilde{l}\big) &= \frac{1}{2}\tilde{x}^T P \tilde{x} \\ &\quad + \frac{\tilde{k}_1^2}{2\gamma_1 l^*} + \frac{\tilde{k}_2^2}{2\gamma_2 l^*} + \frac{\tilde{k}_3^2}{2\gamma_3 l^*} + \frac{\tilde{l}^2}{2\gamma_4 l^*} \end{aligned} \tag{34}$$

with $\tilde{k}^T = \begin{bmatrix} \tilde{k}_1 & \tilde{k}_2 & \tilde{k}_3 \end{bmatrix}$ and with $l^* = \tau_f h^{-1} > 0$, see (26). Taking the time derivative of $V$, we have

$$\begin{aligned} \dot{V}\big(\tilde{x}, \tilde{k}, \tilde{l}\big) &= \frac{1}{2}\tilde{x}^T\big(\bar{A}^T P + P\bar{A}\big)\tilde{x} \\ &\quad + \bar{B}^T P \tilde{x}\frac{1}{l^*}\big(\tilde{k}_1 e_f + \tilde{k}_2 \nu_f + \tilde{k}_3 a_f + \tilde{l} a_p\big) \\ &\quad + \frac{\tilde{k}_1\dot{\tilde{k}}_1}{\gamma_1 l^*} + \frac{\tilde{k}_2\dot{\tilde{k}}_2}{\gamma_2 l^*} + \frac{\tilde{k}_3\dot{\tilde{k}}_3}{\gamma_3 l^*} + \frac{\tilde{l}\dot{\tilde{l}}}{\gamma_4 l^*}, \end{aligned} \tag{35}$$

where the dynamics (31) is used. Substitution of the Lyapunov equation $\bar{A}^T P + P\bar{A} = -Q$ and rearranging terms gives

$$\begin{aligned} \dot{V}\big(\tilde{x}, \tilde{k}, \tilde{l}\big) &= -\frac{1}{2}\tilde{x}^T Q \tilde{x} \\ &\quad + \frac{\tilde{k}_1}{l^*}(\bar{B}^T P \tilde{x} e_f + \frac{\dot{k}_1}{\gamma_1}) + \frac{\tilde{k}_2}{l^*}(\bar{B}^T P \tilde{x} \nu_f + \frac{\dot{k}_2}{\gamma_2}) \\ &\quad + \frac{\tilde{k}_3}{l^*}(\bar{B}^T P \tilde{x} a_f + \frac{\dot{k}_3}{\gamma_3}) + \frac{\tilde{l}}{l^*}(\bar{B}^T P \tilde{x} a_p + \frac{\dot{l}}{\gamma_4}), \end{aligned} \tag{36}$$

where we have also used the fact that the unknown parameters are constant. Substitution of the adaptive laws (32) gives

$$\dot{V}\big(\tilde{x}, \tilde{k}, \tilde{l}\big) = -\frac{1}{2}\tilde{x}^T Q \tilde{x} \le 0, \tag{37}$$

which implies that the origin $(\tilde{x}, \tilde{k}, \tilde{l}) = 0$ is stable. This also implies that the signals $\tilde{x}(\cdot)$, $\tilde{k}(\cdot)$ and $\tilde{l}(\cdot)$ are bounded (with respect to the $\mathcal{L}_\infty$ signal norm, i.e., $\tilde{x}$, $\tilde{k}$, $\tilde{l} \in \mathcal{L}_\infty$).

We obtain convergence of $\tilde{x}$ using standard tools in adaptive control and Barbalat's Lemma, which we recall: if a signal $g(\cdot)$

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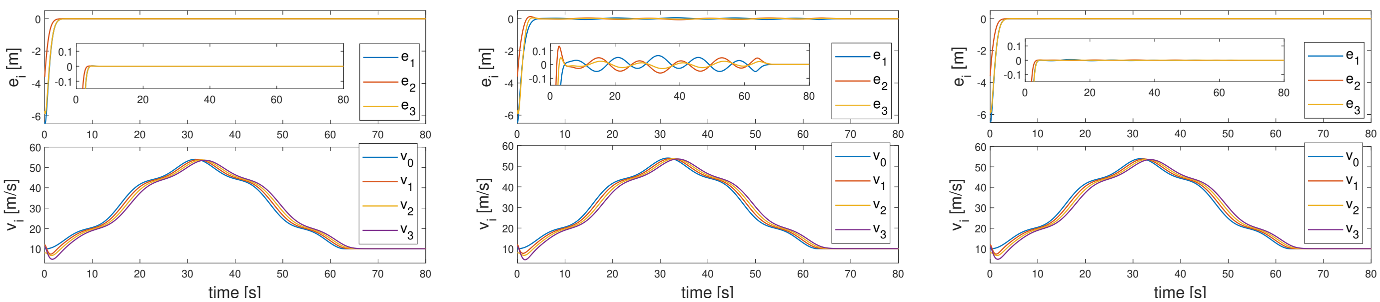


(a) Control in [23] with knowledge of $\tau_i$. Disturbance decoupling is achieved.

(b) Control in [23] with uncertainty in $\tau_i$. Disturbance decoupling is lost.

(c) Proposed control with uncertainty in $\tau_i$. Disturbance decoupling is recovered.

Fig. 3: Spacing errors and velocities with different control designs and different knowledge of $\tau_i$.

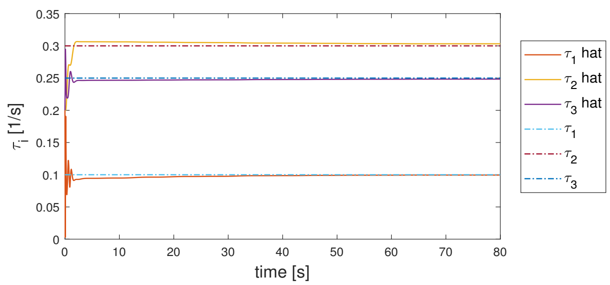


Fig. 4: Proposed control with uncertainty in $\tau_i$: estimates $\hat{\tau}_i$.

and its time derivative satisfy $g$, $\dot{g} \in \mathcal{L}_\infty$ and $g \in \mathcal{L}_2$ (i.e., this signal has bounded $\mathcal{L}_2$ norm) then $g(t) \to 0$ as $t \to \infty$.

To apply Barbalat's lemma, recall that the predecessor input $u_p(\cdot)$ can be assumed to be bounded (recall Problem 1). Due to the vehicle dynamics (1), this implies that $a_p \in \mathcal{L}_\infty$ as well. Subsequently, as a result of the reference model (16) and Lemma 1, we also have that $\bar{x} \in \mathcal{L}_\infty$. From $\tilde{x} \in \mathcal{L}_\infty$ as mentioned above, it also follows that $x = \bar{x} + \tilde{x} \in \mathcal{L}_\infty$. Now, we can conclude from (31) that $\dot{\tilde{x}} \in \mathcal{L}_\infty$.

Hence, must be shown that $\tilde{x} \in \mathcal{L}_2$. Integrating (37) gives

$$\frac{1}{2}\int_0^\infty \tilde{x}^T(t) Q \tilde{x}(t)\,dt = V(0) - V_\infty, \tag{38}$$

where $V_\infty = \lim_{t\to\infty} V(t)$ is bounded. Here, we have used the shorthand $V(t) = V(\tilde{x}(t), \tilde{k}(t), \tilde{l}(t))$. Consequently, $\tilde{x} \in \mathcal{L}_2$, which implies from Barbalat's Lemma that $\tilde{x} \to 0$ as $t \to \infty$.

It remains to be shown that (33) holds. As $\tilde{x} = x - \bar{x}$,

$$\begin{aligned}\lim_{t\to\infty} e_f(t) &= \lim_{t\to\infty} \bar{H}x(t), \\ &= \lim_{t\to\infty} \bar{H}\bar{x}(t) + \lim_{t\to\infty} \bar{H}\tilde{x}(t) = 0,\end{aligned} \tag{39}$$

as follows from item 2) in Lemma 1 and the fact that $\tilde{x} \to 0$ as $t \to \infty$, respectively. □

The stability analysis was carried out for a predecessor-follower scenario with two vehicles indexed as $p$ and $f$, respectively. This was motivated by the '*decentralized*' philosophy of [23], where it was shown that the disturbance decoupling design extends to arbitrary long platoons with $p = i - 1$ and $f = i$, for $i \in \{1, 2, ....N\}$, where $p = 0$ represents the platoon leader, and where the input $u_p = u_{i-1}$ is seen as the exogenous input to vehicle $i$ (and $u_0$ is the input of the leading vehicle).

**Remark 3.** *Albeit the gains being updated online by the adaptive laws* (32)*, the controller* (29) *has the same structure as most platooning protocols in the literature, containing a proportional action from the spacing error, a derivative action from the relative velocity, and feedforward action from the preceding vehicle. Therefore, integration with practical aspects studied in the platooning literature, such as vehicle state estimation [18], [19], is potentially possible, although not addressed here to streamline the results.*

**Remark 4.** *The proposed design uses estimators for the ideal control gains $k_1^*, k_2^*, k_3^*, l^*$. This is known in the literature on adaptive control as* direct *adaptive control, i.e. estimating the control gains directly. Alternatively, one can make use of the knowledge that these gains depend on the engine constant $\tau_f$ as in* (26)*, and write the adaptive controller as*

$$u_f = a_f + \hat{\tau}_f\left(\frac{\theta_1}{\tau_{\bar{f}}}e_f + \frac{\theta_2}{\tau_{\bar{f}}}\nu_f - (h^{-1} + \frac{h\theta_2}{\tau_{\bar{f}}})a_f + h^{-1}a_p\right) \tag{40}$$

*where $\hat{\tau}_f$ is an estimate of $\tau_f$. Along similar lines as the proposed design (not shown due to space limitations), one would eventually obtain the estimator*

$$\begin{aligned}\dot{\hat{\tau}}_f = -\gamma \bar{B}^T P \tilde{x}\bigg(&\frac{\theta_1}{\tau_{\bar{f}}}e_f + \frac{\theta_2}{\tau_{\bar{f}}}\nu_f \\ &- (h^{-1} + h\theta_2\tau_{\bar{f}}^{-1})a_f + h^{-1}a_p\bigg).\end{aligned} \tag{41}$$

*This design is known in adaptive literature as* indirect *adaptive control, i.e. estimating the system parameters, which are used to construct the control gains.*

## IV. Simulations

To validate the theoretical analysis, this section presents simulation results of a platoon with four vehicles (one leader indexed as 0 and three following vehicles indexed as $1, 2, 3$). The vehicles are heterogeneous since they have different engine time constants, as shown in Table I. The same table shows the initial conditions: in addition, we use the parameters $\theta_1 = 1$, $\theta_2 = 1$, $h = 0.7$ for the controller in each vehicle.

In the following we show three simulations:

1) an ideal simulation resulting from the controller in [23] which uses the knowledge of $\tau_i$ ($i = \{0, 1, 2, 3\}$);

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TABLE I: Ideal engine constants and initial conditions

| $i$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\tau_i$ | 0.2 | 0.1 | 0.3 | 0.25 |
| $d_i(0)$ | 0 | -2 | -4 | -6 |
| $v_i(0)$ | 10 | 12 | 8 | 11 |
| $a_i(0)$ | 0 | 0 | 0 | 0 |

2) a simulation resulting from the controller in [23] with uncertainty on $\tau_i$ (in particular, the design assumes that $\tau_i = \tau_0$, for $i = \{1, 2, 3\}$);
3) a simulation with the proposed controller.

The simulations (in Matlab with `ode45` as numerical integrator) are performed under the scenario that the leader accelerates and decelerates in a sinusoidal fashion, i.e., $u_0 = \sin(0.1t) + 0.5\sin(0.5t)$ and finally proceeds at a constant speed. Fig. 3a shows that disturbance decoupling is achieved with the knowledge of $\tau_i$, since all the spacing errors $e_i = s_{i-1} - s_i - hv_i$ $(i = \{1, 2, 3\})$ converge to zero while being unaffected by the input $u_0$ of the lead vehicle. This is not the case in the second simulation in Fig. 3b where $e_i$ are affected by the sinusoidal behavior of $u_0$. Finally, the behavior of the proposed controller in Fig. 3c shows that the disturbance decoupling property is recovered in an adaptive way. The estimates corresponding to the engine dynamics are in Fig. 4.

## V. Conclusions

In this work, we have extended a disturbance decoupling framework recently proposed for formations of automated vehicles in the presence of uncertain engine time constant.

The proposed idea has been presented in the standard one-vehicle look-ahead topology: it is of interest to further explore the approach in alternative topologies (bidirectional and/or multi-vehicles look-ahead [28], [29]).

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