An Adaptive Localized Decision Variable Analysis Approach to Large Scale Multi-objective and Many-objective Optimization

Lianbo Ma, *Member*, *IEEE*, Min Huang, *Member*, *IEEE*, Shengxiang Yang, *Senior Member*, *IEEE*, Rui Wang, and Xingwei Wang

***Abstract*—This paper proposes an adaptive localized decision variable analysis approach under the decomposition-based framework to solve the large scale multi-objective and many-objective optimization problems. Its main idea is to incorporate the guidance of reference vectors into the control variable analysis and optimize the decision variables using an adaptive strategy. Especially, in the control variable analysis, for each search direction, the convergence relevance degree of each decision variable is measured by a projection-based detection method. In the decision variable optimization, the grouped decision variables are optimized with an adaptive scalarization strategy, which is able to adaptively balance the convergence and diversity of the solutions in the objective space. The proposed algorithm is evaluated with a suite of test problems with 2-10 objectives and 200-1000 variables. Experimental results validate the effectiveness and efficiency of the proposed algorithm on the large scale multi-objective and many-objective optimization problems.**

***Index Terms*—Large scale optimization, Decomposition, Multi-objective optimization, Many-objective optimization.**

I. Introduction

In the optimization field, the large scale multi-objective optimization problems (LSMOPs), involving multi-objective optimization problems (MOPs) with a large number of decision variables, has received a surge of attentions [1]. Many real-world problems [2], e.g., the voltage transformer ratio error estimation problem [3], can be modelled as LSMOPs. Usually, an MOP refers to one with M (M = 2 or 3) conflicting objective functions ($f_1(\boldsymbol{x})$, ..., $f_M(\boldsymbol{x})$) to be optimized simultaneously [3]. Without loss of generality, $f_i(\boldsymbol{x})$ ($i=1, \ldots, M$) is assumed to be a minimization problem, and an MOP can be formulated as follows:

$$\begin{aligned} &\underset{x}{\text{minimize}}\, F(x) = (f_1(x), \ldots, f_M(x)) \\ &\text{subject to} \quad x \in \Omega \end{aligned}, \qquad \mathbf{(1)}$$

where $\Omega = \prod_{j=1}^{D}[l_j, u_j] \subseteq R^D$ is the decision space, D is the number of decision variables, u_j and l_j are the upper and lower bounds of the j-th decision variable, respectively. When $M > 3$, (1) becomes a many-objective optimization problem (MaOP).

In the multi-objective optimization, the optimal tradeoffs between different objectives are called the Pareto optimal solutions, the collection of all the Pareto optimal solutions in the decision space is termed as the Pareto optimal set (PS), and the image set of all the Pareto optimal solutions in the objective space is the Pareto optimal front (PF). The goal in solving MOPs and MaOPs is to obtain a set of well-converged and WELL-diversified Pareto optimal solutions [4]. Over the past decades, many multi-objective evolutionary algorithms (MOEAs) have been developed to approximate the PFs, including Pareto-dominance-based approaches [5], [6], indicator-based approaches [7], [8], decomposition-based approaches [9]-[11], and others [12]-[16].

However, most existing MOEAs treat all the decision variables as a whole, and their performance would deteriorate severely as the dimensionality of the decision space increases [17]-[19]. In the decision space, as the number of decision variables increases, the search space grows exponentially and the interaction between these variables becomes more complicated [20], [21]. In the objective space, a large number of decision variables may impose various impacts on the convergence and uniformity of the obtained solutions [20], at the same time the search stagnation on one or several objectives would slow down the approximation to the PF [21]-[23]. These factors increase the difficulty of optimizing LSMOPs and large scale MaOPs (LSMaOPs).

Several scalability improvement strategies have been developed to deal with LSMOPs and LSMaOPs [22]-[25]. The first strategy uses the decision variable analysis (DVA) to discover useful features of decision variables on LSMOPs. Such kind of MOEAs is termed as DVA based MOEA

Manuscript received September 28, 2019; revised January 22, 2020 and October 14, 2020; accepted November 23, 2020. This work was supported by National Natural Science Foundation of China Nos. 61773103, 61872073, 71620107003 and 61673331, LiaoNing Revitalization Talents Program Nos. XLYC1902010 and XLYC1802115, and Fundamental Research Funds for State Key Laboratory of Synthetical Automation for Process Industries No. 2013ZCX11. (Corresponding authors: Shengxiang Yang, Min Huang and Xingwei Wang.)

Lianbo Ma and Rui Wang are with College of Software, Northeastern University, Shenyang 110819, China (Email: malb@swc.neu.edu.cn, wangrui@sia.cn).

Min Huang is with the College of Information Science and Engineering, State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang, 110819, China (Email: mhuang@mail.neu.edu.cn).

Shengxiang Yang is with the School of Computer Science and Informatics, De Montfort University, Leicester, LE1 9BH, U.K (Email: syang@dmu.ac.uk).

Xingwei Wang is with College of Computer Science and Engineering, Northeastern University, Shenyang, 110819, China (Email: wangxw@mail.neu.edu.cn).

(MOEA/DVA) [23], where the decision variables are divided into position variables, distance variables and mixed variables according to the control properties of convergence and/or spread [24], [25]. In MOEA/DVA, an MOP is simplified into several sub-MOPs and then the variables in each sub-MOP are grouped via the interaction analysis. Similarly, the variables in LMEA [26] are clustered into the convergence-related and diversity-related groups. Then, different optimization strategies are adopted in the two groups to focus on the convergence and diversity, respectively. Recently, several parallelization techniques have been utilized in the grouping of decision variables to speed up the search process [27]-[29].

The second strategy is the cooperative coevolution (CC) framework, which uses various variable grouping techniques to decompose decision variables [30]. These available grouping approaches involve fixed grouping [29], random grouping [31], [32], Delta method [33], dynamic grouping [34], differential grouping [35] and graph-based differential grouping (gDG) [36]. Especially, the popular CCGDE3 algorithm [37], which optimizes a set of independent subpopulations with the same size in a divide-and-conquer manner, has obtained a promising performance on LSMOPs.

The problem transformation is another effective strategy to simplify the structure of LSMOPs. It uses the transformation function to reduce the problem's dimensionality. For example, the weighted optimization framework (WOF) [38] divides the decision variables into a number of subgroups, each of which is assigned with a weight variable. In this way, the original optimization of decision variables is reformulated as the optimization of a number of weight variables for the selected fixed solutions. Very recently, another reformulation-based algorithm called LSMOF, based on the bi-directional weight variable association strategy, has shown a competitive ability of tackling LSMOPs [39].

The research on large scale multi-objective optimization is still in its infancy [26]. It has been shown that the preference information (e.g., reference points or vectors [10]) is useful to guide the search on the diversity and convergence in the multi-objective subspaces [17], [40]. It is a potential way to utilize the preferences' feature from the objective space to guide the optimization of decision variables when solving LSMOPs and LSMaOPs. It is an interesting issue to make a correlation between the classification of decision variables in the decision space and the preference information in the objective space. As a sort of preferences, the reference vectors (rigorously defined in [11]) are usually used in MOEAs to manage the convergence and diversity [17]. This observation motivates us to exploit the merits from the reference vectors for the large scale multi-objective optimization. In this paper, we propose a decomposition-based algorithm using a localized control variable analysis approach (called LSMOEA/D). Here, a number of representative reference vectors are selected[1] as the guiding reference vectors for the control variable analysis [41].

The main contributions of this paper include:

[1] The reference vectors are first grouped into a number of clusters and then the cluster centers are used as the guiding (or representative) reference vectors.

1) An adaptive decision variable analysis approach for the decomposition-based framework is proposed to deal with LSMOPs and LSMaOPs. The proposed approach uses reference vectors to guide the decision variables analysis. In this way, different search regions may have different grouped variable sequences, which enables the variables to be adaptively associated with appropriate groups when their sampling solutions move from a subregion to another one.

2) A projection-based detection method is designed to quantify the convergence relevance degrees (CRDs) of decision variables. The proposed approach has two improvements: First, it uses the directions of multiple reference vectors, rather than only one direction of the normal line of hyperplane $f_1+\cdots+f_M=1$ used in [25], as the convergence direction of calculating CRD; Second, it uses not only the angle information but also the projection length (from the fitted sampling solutions to the convergence direction) in the CRD measure.

3) An adaptive scalarization strategy is introduced to control the balance between convergence and diversity in optimizing decision variables. Especially, the optimization of each variable subgroup is associated with an adaptive parameter (θ), which regulates the ratio of convergence to diversity in the penalty-based boundary intersection (PBI) function. Using this strategy, different decision variables can be optimized adaptively according to their CRD values.

The rest of this paper is organized as follows. Section II presents the background and our motivation. In Section III, the details of LSMOEA/D for large-scale multi-objective and many-objective optimization are described. Section IV gives experimental settings and comparisons of LSMOEA/D with several state-of-the-art algorithms on benchmark problems. Finally, conclusions are drawn in Section V.

II. Background and Motivation

A. MOEA/D Framework

MOEA/D [10] uses a scalarizing function to decompose an MOP into a number of single objective subproblems[2]. In fact, a variety of scalarizing functions can be used for the decomposition [42], such as weighted sum (WS) and Tchebycheff (TCH) [43]. In this paper, we use PBI [10] as the scalarizing function since it offers a controllable balance between the convergence and diversity [44]. An optimization problem of PBI can be formulated as follows:

$$\begin{aligned} &\text{minimize } g^{pbi}(x \mid w, z^*) = d_1 + \theta d_2 \\ &\text{subject to } x \in \Omega \end{aligned}, \tag{2}$$

where

$$d_1 = \frac{\| F(x) - z^*)^{\mathrm{T}} w \|}{\| w \|}, \tag{3}$$

$$d_2 = \left\| F(x) - \left(z^* + d_1 \frac{w}{\| w \|} \right) \right\|, \tag{4}$$

where $z = (z_1^*, ..., z_m^*)^{\mathrm{T}}$ is the ideal objective vector with z_i^* =

[2] In some MOEA/D variants, subproblems are multi-objective [44].

$\min_{x\in\Omega} f_i(x)$, $i \in \{1,...,m\}$, and $\theta \geq 0$ is a penalty parameter. Especially, d_1 is to measure the convergence of $\boldsymbol{x}$ toward the PF, and d_2 is used to estimate the diversity of population. In PBI, the balance between convergence (d_1) and diversity (d_2) is controlled by the parameter θ .

B. *Decision Variable Analysis* (*DVA*)

The aim of DVA lies in two aspects. First, for the single-objective optimization, the separablity of an objective function is detected by

$$\arg\min_{x} f(x) = (\arg\min_{x_1} f(x_1,...),...,\arg\min_{x_k} f(...,x_k)) , \quad \textbf{(5)}$$

where $x = (x_1, ..., x_D)$ is a decision vector and $x_1, ..., x_K$ ($2 \leq K \leq D$) are disjoint sub-vectors of x. When $K = D$, $f(x)$ is a separable function; otherwise, it is a partially separable one [45].

Second, for the multi-objective optimization, the variable dependency is checked based on (6) by different techniques, including perturbation [46], interaction adaption [47], model building [48] and randomization [31].

$$\begin{aligned} &f(x)|_{x_i=a_2,x_j=b_1} < f(x)|_{x_i=a_1,x_j=b1} \quad \wedge \\ &f(x)|_{x_i=a_2,x_j=b_2} > f(x)|_{x_i=a_1,x_j=b_2} \end{aligned} , \quad \textbf{(6)}$$

where $f(x)|_{x_i=a_2,x_j=b_1}$ $f(x_1,...,x_{i-1},a_2,...,x_{j-1},b_1,...,x_D)$.

Based on DVA, the property of decision variables can be identified to be related to the convergence, diversity, or both of them. MOEA/DVA [25] and LMEA [26] are the two representative DVA-based algorithms. Despite that these algorithms have shown good performance on LSMOPs and LSMaOPs, they still suffer from two drawbacks: 1) Although the variables are simply grouped into convergence-related and diversity-related groups, the classification of variables in the same group still needs to be enriched; 2) The control variable analysis does not consider the effect of the located subregion of the sampling solutions and thus cannot guarantee the global consistency of the identification results [26].

C. *Grouping Strategies*

The grouping strategies are able to put those variables that interact strongly with each other into the same group. For the purpose of dividing D decision variables into k groups, several grouping techniques are widely used in the CC-based algorithms [38], [39], including:

1) Random grouping [31]: Each decision variable is randomly assigned to each of the k equal-sized groups.

2) Linear grouping [49]: All D decision variables are assigned to k groups in a natural order. In this process, the first D/k variables are assigned to the first group, and the rest can be deduced by analogy.

3) Ordered grouping [50]: For a selected solution, the decision variables are sorted by their absolute values in ascending order. The D/k decision variables with the smallest absolute values are assigned to the first group, and the next D/k variables to the second group, and so forth.

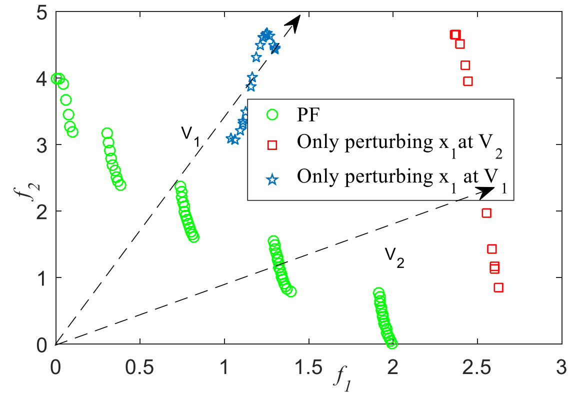


Fig.1. Sampling points generated on the bi-objective WFG problem.

4) Differential grouping [35]: During grouping, the variable interaction is detected based on perturbation, and the number of groups and their sizes are set adaptively. The interacting decision variables need to be assigned to the same group.

The first three grouping strategies do not use the information about the objective functions, thereby seeming relatively simpler than differential grouping. However, differential grouping has its own drawbacks, such as the high computation cost and significant incompatibility to MOPs [25].

D. *Motivation*

The motivation of this paper is based on the following considerations.

1) In the existing DVA-based algorithms, e.g., MOEA/DVA [25] and LMEA [26], the decision variables are divided into the convergence-related and diversity-related groups and then optimized separately by different techniques. An underlying assumption is that the control property of decision variables should keep consistent in the global objective space. It could hold if the problem only involves some special position and/or distance variables, or the exclusive variables [51]. Our empirical observation shows that the detection result in terms of the decision variable's property may be different in different detection regions in the objective space, and the above assumption is not likely to be held.

To illustrate, a bi-objective WFG problem [22] is considered (its detailed formulation is provided in the supplementary material), where there are 3 decision variables, i.e., $x_1 \in [0,2]$, $x_2 \in [0,4]$ and $x_3 \in [0,6]$. Fig. 1 plots the sampling points generated on this WFG problem by perturbing one variable x_1 between [0, 2] while fixing other two variable x_2 and x_3. As shown, these sampling solutions are generated in the subregion of V_1 when x_2=x_3=0.4, and then x_1 is detected as a convergence-related variable. In contrast, the sampling solutions are generated in the subregion of V_2 when $x_2 = 4$ and $x_3 = 6$, and then x_1 is identified as a diversity-related variable since it contributes more to diversity than convergence. Similar observation can be obtained on another test problem, which is provided in the supplementary material. It can be tentatively concluded that the spatial location of the sampling solutions in the objective space indeed has an important impact on the control variable analysis. Therefore, it is reasonable to incorporate the guidance of reference vectors into the control variable analysis.

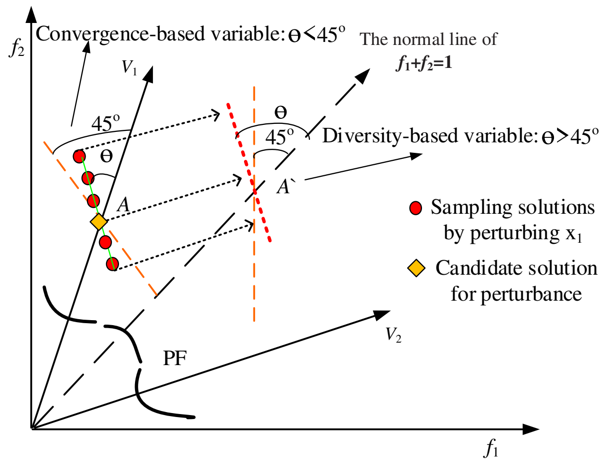


Fig.2. Example to show the effect of the reference vectors on the control variable analysis. In this example, the ideal detection result is the case of $\theta < 45°$ (where V_1 is used as the convergence direction), which indicates that the candidate variable is convergence-related, while the detection result obtained by LMEA is the case of $\theta > 45°$, which indicates that the candidate variable is diversity-related.

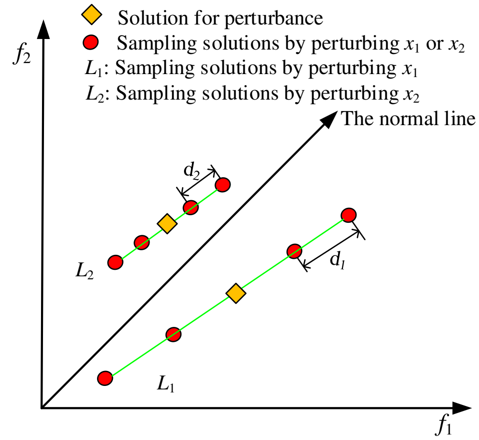


Fig.3. Example to show the effect of the length of sampling solutions on the control variable analysis. In this example, although L_1 and L_2 have the same acute angle to the normal line, they show different contributions to convergence: x_1 is more convergence-related than x_2.

2) Although some advanced decision variable grouping techniques, e.g., the angle-based measurement [26], have been used to further distinguish the mixed variables. Nevertheless, such technique is in some sense coarse-grained:

First, it only uses the normal line of hyperplane $f_1 + \cdots + f_M = 1$ as the convergence direction in the angle-based calculation. It cannot accurately distinguish the mixed variables for the irregular (e.g., disconnected and degenerate) shapes of PFs. Fig. 2 shows the ideal detection result for such a PF (see the case of $\theta < 45°$) as well as the one obtained by the angle-based measure [26] (see the case of $\theta > 45°$). Here, V_1 and V_2 are two guiding reference vectors as the convergence direction, and θ is the acute angle between the sampling solutions and the convergence direction. Then, x_1 is identified to be convergence-related when $\theta < 45°$; otherwise, diversity-related [26]. From the above example, it is observed that the use of reference vectors in the control variable analysis is more effective and flexible for different shapes of PFs.

Second, it only utilizes the acute angle θ to measure the contribution to convergence. In some situations, it cannot distinguish the variables with the same angle θ. Fig.3 shows such an example, where L_1 and L_2 are the sampling solutions obtained by perturbing x_1 and x_2, respectively. Though L_1 and L_2 have the same acute angle to the normal line, x_1 and x_2 have different contributions to convergence. Since perturbing x_1 generates a larger search scope (i.e., $d_1 > d_2$) towards the ideal point, x_1 is more convergence-related than x_2. Since the decision variables, even the position or distance variables in the same group, may still have correlations in different degrees with the objective functions [26], it is necessary to enrich the variable classifications and optimize them in a fine-grained manner.

3) Many existing state-of-the-art algorithms for LSMOPs and LSMaOPs, e.g., MOEA/DVA [25] and WOF [38], are based on the dominance relationship. Their performance on MaOPs may be degraded when the number of objectives increases. In contrast, MOEA/D has exhibited promising performance on both MOPs and MaOPs [17], [41]. It is intuitive to apply the MOEA/D framework to solve LSMOPs and LSMaOPs. In this framework, for each search direction, some good variables can be optimized preferentially with more computation resources.

Therefore, this paper suggests a decomposition-based algorithm called LSMOEA/D using the localized decision variable analysis approach. The basic idea is simple: Under the MOEA/D framework, the decision variables are classified with the assistance of the guiding reference vectors, and then optimized adaptively according to their CRDs. In this way, the issues illustrated in Figs. 1, 2 and 3 can be addressed properly.

III. Proposed Algorithm

This section first presents the main framework of the proposed LSMOEA/D, and then elaborates its main operations.

A. Main Framework

Algorithm 1 presents the main framework of LSMOEA/D. First, in the objective decomposition phase, a set of N weight vectors (i.e., W) are initialized to define a set of N subproblems and they are divided into a set of K clusters (i.e., B), then the set of K guiding reference vectors (i.e., V) is selected from B to specify a set of K subregions in the objective space (Line 1). In addition, the initial population (i.e., P) and the index (i.e., ID) of variable subgroup to be optimized is also provided in this phase. Next, in the control variable analysis phase, the localized control variable analysis approach (Line 2) is used to estimate the contribution of each variable to convergence and offer a matrix *SC_Matrix* that consists of a number of grouped variable sequences for all the reference vectors.

Within the main while-loop, in case that the termination criteria are not met, for each reference vector, the reproduction operation (Line 6) generates offspring (i.e., *Offs*) by using the archive *OPSet* and provides a neighboring solution set (i.e., *Ng*). Then, the solutions (including parents and offspring) are evaluated based on an adaptive scalarization strategy and the offspring are used to update the parent population (Line 7). Finally, the variable archive *OPSet* is updated (Line 8).

Algorithm 1 *Main Framework of LSMOEA/D*

Input: *N*: *Population size*; *D*: *Number of decision variables*; *nSel*: *Number of candidate solutions for control variable analysis*; *nPer*: *Number of perturbations on each candidate solution*; *T*: *Size of selection neighbourhood*; *K*: *Number of vector clusters*;
Output: *P: Final population*;
/* *Objective decomposition phase**/
1: [*P*, *W*, *B*, *V*, *ID*] ← Initialization (*N*, *K*, *D*);
/* *Localized control variable analysis phase**/
2: [*SC_Matrix*] ← VariableLocalClassify(*P*,*W* *B*,*V*,*nSel*,*nPer*,*D*);
3: *OPSet* ← *SC_Matrix*[:,*ID*];
/* *Optimization phase**/
4: **While** termination criterion is not fulfilled **do**
5: **For** *i* ← 1 to *N* **do**
6: [*Offs, Ng*] ← Reproduction (*P*, *OPSet*);
7: [*P*, *ID*] ← Update_Population (*P*, *Offs, Ng, ID*);
8: *OPSet* ← *SC_Matrix*[:,*ID*];
9: **End**
10: **End**
Return *P*;

Algorithm 2 *Initialization* (*N*, *K*, *D*)

Input: *N*: *Population size*; *K*: *Number of weight clusters*; *D*: *Number of decision variables*;
Output: *P*: *Population*; *W*: *N uniform weight vectors*; *B*: *Set of K weight vector clusters*; *V*: *Set of K cluster centers*; *ID*: *Index of variable subgroup to be optimized*;
1: Generate an initial population $P = \{p_1, ..., p_N\}$ with *D* decision variables randomly;
2: $W \leftarrow \{w_1, w_2, \ldots, w_N\}$;/**N initial weight vectors**/
3: Compute the Euclidean distances between any two weight vectors and use *Kmeans* to obtain *K* clusters $B=\{b_1,..., b_i, ..., b_K\}$ and *K* cluster centers $V=\{v_1,..., v_i, ..., v_K\}$;
4: Initialize $z^*=(z_1^*,..., z_i^*,..., z_m^*)$ by setting $z_i^* = \min(f_i(p_1),..., f_i(p_N))$;
5: ID=1;
Return *P*, *W*, *B*, *V* and *ID*;

B. Initialization

Algorithm 2 shows the initialization procedure. First, the population *P* of *N* individuals is initialized randomly (Line 1), and *N* weight vectors $W = \{w_1, ..., w_N\}$ are generated by the systematic approach [17] (Line 2). Then, the Euclidean distance between any two weight vectors is calculated and the *N* weight vectors are divided into a set of *K* clusters $B=[b_1,..., b_i, ..., b_K]$, where b_i contains the indices of the neighboring vectors in the *i*-th cluster (Line 3). The set of *K* cluster center vectors $V=[v_1,..., v_i, ..., v_K]$ coming from *B* are used as the guiding reference vectors for the localized control variable analysis, where $\boldsymbol{v_i}$ is the index of the center vector in the *i*-th cluster (Line 2 in *Algorithm* 1). Then, the ideal objective vector z^* is initialized as the minimum values of all the solutions in *P* along each objective (Line 4) and the index *ID* of the current variable subgroup to be optimized is preset to 1 (Line 5).

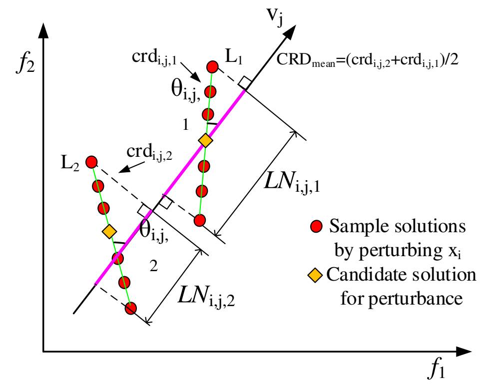


Fig.4. Illustration of projection-based detection method.

C. Localized Control Variable Analysis

The localized control variable analysis is a vital component of LSMOEA/D (Step 2 in *Algorithm* 1). It includes the following procedures:

1) Reference vector association. For each reference vector w_g, its neighboring solutions are determined by $Neighbor^{w_g}(T)$, which denotes the set of the first *T* closest solutions to w_g in *P*. The "closeness" is defined by the acute angle between the solution *s* and the reference vector w_g, calculated as follows:

$$angle(s, w_g) = \arccos\left(\frac{(F(s)-z^*)^T w_g}{\|F(s)-z^*\| \|w_g\|}\right). \quad (7)$$

2) CRD calculation. For each guiding reference vector v_j which is the cluster center from *B*, the CRD value of each variable is calculated based on the projection-based approach. Fig. 4 shows an example of CRD calculation, where a bi-objective minimization problem with a decision variable x_i is considered. In this figure, to calculate the CRD value of x_i, *nSel* (two in this example) candidate solutions are randomly selected from the set of neighboring solutions (i.e., $Neighbor^{v_i}(T)$) of v_j, and then *nPer* (seven in this example) perturbations are performed on x_i of the selected candidate solutions to generate two sets of sampling solutions (red cycles). Next, each set of sampling solutions are normalized and a line *L* is generated to fit each set of normalized sampling solutions. Using (8), two CRD values (i.e., $cdr_{i,j,1}$ and $cdr_{i,j,2}$) are obtained and their mean value is associated with x_i. Note that the number of CRD values depends on the number *nSel* of the selected candidate solutions.

To measure the contribution of each variable to the convergence direction, the CRD incorporates two factors, i.e., the acute angle θ and the projection length *LN*, as shown in Fig. 4, where v_j is the *j*-th guiding reference vector. Specifically, for x_i, its CRD value is defined as

$$crd_{i,j} = \left(1+\frac{\theta_{i,j}-\theta_{\min}}{\theta_{\max}-\theta_{\min}}\right)e^{-LN_{i,j}}, \quad (8)$$

where $\theta_{i,j}$ and $LN_{i,j}$ are the acute angle and the projection length from the fitted line L_i to the direction of the vector v_j, respectively, $\theta_{\min}$ and $\theta_{\max}$ are the smallest and the largest acute angles found so far in the current generation, respectively. Here, a smaller CRD value indicates a greater contribution to convergence or a smaller contribution to diversity.

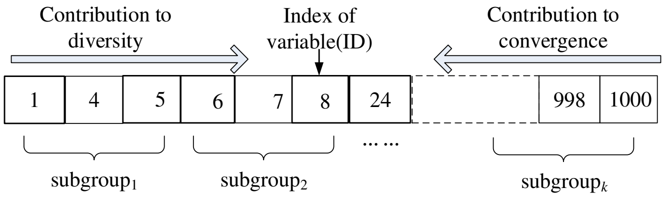


Fig.5. Illustration of the grouped decision variables. The variables in the subgroup of smaller index contribute more to convergence.

The design of the CRD measure is based on the following empirical observations: First, both the acute angle and the projection length can affect the measure accuracy of the contribution of a variable to convergence. This is because that the acute angle reflects a search bias to the convergence direction as shown in Fig. 4, where a smaller angle θ indicates the associated variable has a greater bias/contribution to the convergence direction. The projection length specifies a search strength along the convergence direction. As shown in Figs. 3 and 4, a larger projection length indicates a greater perturbation amplitude along the convergence direction. Second, the composite design of acute angle and projection length can further classify the decision variables, even if they have the same acute angle or projection length.

3) Ordered grouping. After CRD calculation, the decision variables are divided into a number of subgroups with the equal size for each guiding reference vector. These variables are first sorted by their CRD values in ascending order. Then, *D/k* decision variables with the smallest CRD values are assigned to the first group, and the next *D/k* variables to the second group. This process is repeated till all the variables are grouped. Fig. 5 shows a subgroup sequence for a guiding reference vector, where the variables in subgroup of smaller index contribute more to convergence.

Algorithm 3 presents the localized control variable analysis process. First, each guiding reference vector is associated with a set of neighboring solutions (Lines 1-4). Next, for each guiding reference vector, the mean CRD value of each variable is calculated based on the projection-based method (Lines 5-14). Then, D variables are divided into a number of subgroups with an equal size using ordered grouping [50] (Lines 15-17). Within each cluster in B, the grouped variable sequence assigned with each ordinary reference vector is the same as that of its cluster center vector (Lines 18-22). Note that, in our approach, the individuals in the population are assigned with different grouped variable sequences because they are located in different local subregions. The structure of evolutionary population is demonstrated in Fig. 6, where N is the number of reference vectors.

D. Reproduction

The offspring solutions are generated by the reproduction operation as shown in *Algorithm* 4. The *OPSet* archives the variables in the subgroup from *SC_Matrix*, which are to be updated for offspring *generation*. In *Algorithm* 4, two candidate solutions are randomly selected from the neighboring solutions using the binary tournament selection (Lines 1 and 2), and afterward, an offspring solution is generated by replacing the values of variables in *OPSet* with those generated by the recombination operator, while leaving the rest variables unchanged (Lines 3-6).

Algorithm 3 VariableLocalClassify (P, W, B, V, $nSel$, $nPer$, D)

Input: P: *Population*; W: *Set of weight vectors*; B: *Set of K weight vector clusters*; V: *Set of K cluster centers*; $nSel$: *Number of selected solutions for variables classification*; $nPer$: *Number of perturbations on each solution*; D: *Number of decision variables;*
Output: *SC_Matrix*: *A matrix* ($N \times K$) *of grouped sequences for N reference vector*;
/* *Reference vector association* */
1: $K \leftarrow |V|$, $N \leftarrow |W|$, $T \leftarrow 10$, $k \leftarrow D/20$;
2: **For** $i \leftarrow 1$ to K **do**
3: Calculate $Neighbor^{vi}(T)$ for each vector v_i using (7);
4: **End**
/* *CRD calculation* */
5: **For** $i \leftarrow 1$ to K **do**
6: **For** $j \leftarrow 1$ to D **do**
7: $S \leftarrow$ select $nSel$ solutions closest to $\boldsymbol{v_j}$ from $Neighbor^{vi}$;
8: **For** $g \leftarrow 1$ to $nSel$ **do**
9: Perturb i-th variable of $S[g]$ for $nPer$ times to generate a population SP;
10: Normalize SP and fit a line L in objective space for SP;
11: Calculate the CRD value using (8);
12: **End**
13: Calculate the mean CRD value ($crd_{i,j}$) for the i-th variable;
14: **End**
/* Ordered grouping */
15: $SC[i] \leftarrow$ Indexes of D variables sorted based on CRD in ascending order;
16: $SC\_v[i] \leftarrow$ Ordered_group ($SC[i]$, k);
17: **End**
18: **For** $i \leftarrow 1$ to K **do**
19: **For** each vector j in cluster $B[i]$ **do**
20: $SC\_Matrix[j] \leftarrow SC\_v[i]$;
21: **End**
22: **End**
Return *SC_Matrix*;

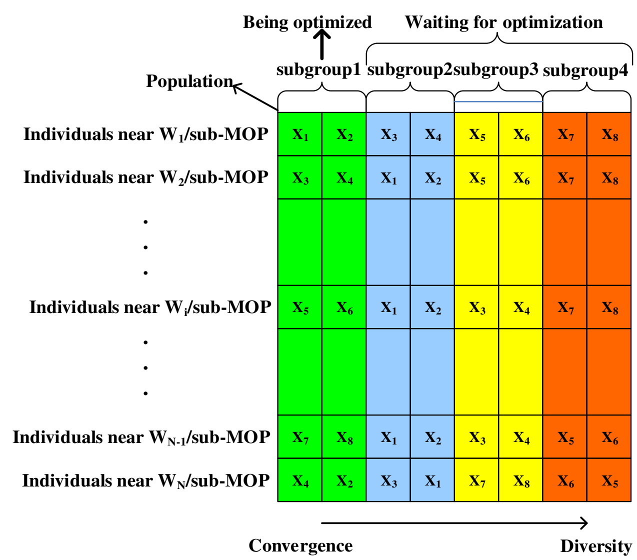


Fig.6. The structure of the population, where the individuals are assigned with different grouped variable sequences.

Algorithm 4 *Reproduction* (P, $OPSet$)

Input: P: Parent population; $OPSet$: Variable archive to be updated
Output: *Offspring*: New individuals; Ng: Neighboring solution set
1: Determine the set of neighboring solutions, i.e., Ng, for the current reference vector using (7);
2: Choose two solutions $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ randomly from Ng;
3: $\boldsymbol{s}'(OPSet) \leftarrow$ *recombination* ($\boldsymbol{s}_1(OPSet)$, $\boldsymbol{s}_2(OPSet)$);
/* $\boldsymbol{s}'(OPSet)$ denotes a vector consisting of values of $\boldsymbol{s}'$ on decision variables in $OPSet$ */
4: $\boldsymbol{s}'' \leftarrow \boldsymbol{s}_1$;
5: $\boldsymbol{s}''(OPSet) \leftarrow \boldsymbol{s}'(OPSet)$;
6: $Offspring \leftarrow \boldsymbol{s}''(OPSet)$;
Return *Offspring* and Ng;

Algorithm 5 *Update_Population* (P, *Offspring*, Ng, ID)

Input: *P: Parent population*; *Offspring*: *New individual*; *Ng*: *Set of neighboring solutions*; *ID*: *Index of current variable subgroup*;
Output: *P*: *Updated population*; *ID*: *Updated index of variable subgroup to be optimized*;
/* *Update of parent population* */
1: $Count \leftarrow 0$; $C\_Max \leftarrow 10$;
2: $P \leftarrow P \backslash Ng$;
3: Determine the value of θ using (10);
4: Update the ideal point z^*;
5: **For** each solution s in Ng **do**
6: Calculate the PBI fitness of solution $\boldsymbol{s}$ using (2), i.e., $F(\boldsymbol{s})$;
7: **If** $F(Offspring) < F(\boldsymbol{s})$
8: $\boldsymbol{s} \leftarrow Offspring$;
9: $Count \leftarrow 0$;
10: **Else**
11: $Count \leftarrow Count+1$;
12: **End**
13: **End**
/* *Update of variable archive* */
14: **If** $Count > C\_Max$
15: $ID \leftarrow ID+1$;
16: If $ID == ID_{\max}$
17: $ID \leftarrow 1$;
18: **End**
19: **End**
20: $P \leftarrow P \cup Ng$;
Return P and ID;

E. Update Population

The update procedure is presented in *Algorithm* 5, which contains the update of parent population (Lines 2-8) and the update of variable archive (Lines 14-17). In this procedure, the variable subgroup that contributes more to the convergence will be optimized with a higher priority, and also be assigned to more computation resources.

In the update of parent population (Lines 2-8), the offspring solutions are evaluated by a composite measure. If an offspring candidate solution has a better measure than its parent, it will be selected to the next generation; otherwise, it is discarded and the parent candidate solution will survive (Lines 7 and 8).

To construct such a composite measure, an adaptive scalarization strategy is designed to control the balance between convergence and diversity. Especially, when PBI defined by (2) is used as the basic fitness measure, the parameter θ in PBI is defined as follows:

$$\theta_{ID} = \frac{4\alpha}{ID_{\max}^{\;3}} ID^3 - \frac{6\alpha}{ID_{\max}^{\;2}} ID^2 + \frac{3\alpha}{ID_{\max}} ID, \qquad \textbf{(9)}$$

where ID is the index of the current subgroup in *SC_Matrix*, $ID_{\max}$ is the index of the last subgroup in *SC_Matrix*, and α is the upper limit of θ. The distribution of all the values of θ obtained by (9) is illustrated in Fig. 7, where the subgroup with a smaller ID is more related to the convergence and assigned with a smaller θ value. Using this adaptive strategy, the PBI measure gradually focuses on the convergence/diversity as ID decreases/increases. Of course, other complex adaptation functions can also be applied here to replace (9).

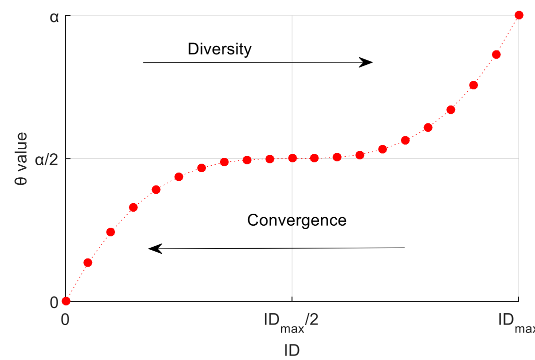


Fig.7. Distribution of the values of θ .

In the update of variable archive (Lines 14-17), if the offspring has not been improved consistently for a certain number of generations, the decision variables in *OPSet* will be replaced by the ones in the next subgroup from *SC_Matrix* (Line 15). For example, for reference vector w_1, the subgroup with ID=1 is being optimized in the archive at first. After some generations, the ID=2 will be selected into *OPSet*, and so forth. If the last subgroup ID_{max} in *SC_Matrix* is shifted out, the ID=1 is selected again for next generation (Line 17).

F. More Discussions

Different from the existing works [25], [26], the proposed decision variable analysis approach identifies the variable property via a localized contribution-based mechanism (i.e., the CRD measure). In this way, the variables that have different (strong or weak) correlations with the objectives can be optimized in a fine-grained manner. Especially, the variables with better CRD values (also with more convergence-based contributions) are optimized with a higher priority, which can accelerate the search towards the PF.

Table I shows the grouping results obtained by the DVA methods of MOEA/DVA, LMEA and LSMOEA/D on 6 representative test problems, respectively, where 12 decision variables are considered for each problem. As shown, all the variables in LSMOEA/D can be further ranked as a grouped sequence, while the variables in LMEA (to an extent similar to MOEA/DVA) are only labeled to be convergence-related or diversity-related.

TABLE I
DECISION VARIABLE ANALYSIS RESULTS IN MOEA/DAVA, LMEA AND LSMOEA/D ON SELECTED TEST FUNCTIONS WHERE { } REPRESENTS A VARIABLE SUBGROUP OBTAINED BY ORDERED GROUPING IN LSMOEA/D.

Pro.	Obj	MOEA/DVA			LMEA		LSMOEA/D
		Diversity	Convergence	Both	Diversity	Convergenc	Sorting by CRD for V_I
DTLZ1	3	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{12}\}$	∅	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{12}\}$	$\{x_6,x_{10},x_5\},\{x_3,x_{11},x_8\},\{x_7,x_9,x_4\},\{x_{12},x_2,x_1\}$
	10	$\{x_1, \ldots, x_9\}$	$\{x_{10}, x_{11}, x_{12}\}$	∅	$\{x_1, \ldots, x_9\}$	$\{x_{10}, x_{11},$	$\{x_{12},x_{11},x_{10}\},x_6,x_4,x_2\},\{x_5,x_8,x_3\},\{x_7,x_1,x_9\}$
DTLZ2	3	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{12}\}$	∅	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{12}\}$	$\{x_{10},x_8,x_4\},\{x_6,x_9,x_3\},\{x_5,x_{11},x_7\},\{x_{12},x_2,x_1\}$
	10	$\{x_1, \ldots, x_9\}$	$\{x_{10}, x_{11}, x_{12}\}$	∅	$\{x_1, \ldots, x_9\}$	$\{x_{10}, x_{11},$	$\{x_{10},x_{12},x_{11}\},\{x_9,x_4,x_5\},\{x_8,x_3,x_6\},\{x_7,x_2,x_1\}$
DTLZ6	3	$\{x_1, \ldots, x_4\}$	$\{x_5, \ldots, x_{11}\}$	$\{x_{12}\}$	$\{x_1, \ldots x_4, x_{12}\}$	$\{x_5, \ldots, x_{11}\}$	$\{x_5,x_7,x_8\},\{x_9,x_3,x_{10}\},\{x_{11},x_5,x_4\},\{x_{12},x_2,x_1\}$
	10	$\{x_1, \ldots, x_9\}$	$\{x_{12}\}$	$\{x_{10}, x_{11}\}$	$\{x_1, \ldots, x_9$	$\{x_{12}\}$	$\{x_{12},x_1,x_8\},\{x_4,x_7,x_{10}\},\{x_9,x_{11},x_5\},\{x_6,x_3,x_2\}$
DTLZ7	3	∅	$\{x_3, \ldots, x_{12}\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{12}\}$	$\{x_{12},x_3,x_8\},\{x_4,x_5,x_{10}\},\{x_9,x_{11},x_7\},\{x_6,x_1,x_2\}$
	10	∅	$\{x_{10}, x_{11}, x_{12}\}$	$\{x_1, \ldots, x_9\}$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, x_{11},$	$\{x_{10},x_{12},x_{11}\},\{x_9,x_4,x_5\},\{x_8,x_3,x_6\},\{x_7,x_2,x_1\}$
WFG7	3	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{10}\}$	$\{x_{11}, x_{12}\}$	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{12}\}$	$\{x_{12},x_7,x_8\},\{x_4,x_5,x_{10}\},\{x_9,x_{11},x_3\},\{x_6,x_1,x_2\}$
	10	$\{x_1, \ldots, x_4\}$	$\{x_5, \ldots, x_9\}$	$\{x_{10},x_{11},x_{12}$	$\{x_1, \ldots, x_4\}$	$\{x_5, \ldots, x_{12}\}$	$\{x_{10},x_{12},x_{11}\},\{x_9,x_7,x_5\},\{x_8,x_3,x_6\},\{x_4,x_2,x_1\}$
WFG8	3	$\{x_1, x_2\}$	∅	$\{x_3, \ldots,x_{12}\}$	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{12}\}$	$\{x_{10},x_8,x_4\},\{x_6,x_9,x_3\},\{x_5,x_{11},x_7\},\{x_{12},x_2,x_1\}$
	10	$\{x_1,x_2\}$	$\{x_3, x_4\}$	$\{x_5, \ldots, x_{12}\}$	$\{x_1,x_2\}$	$\{x_3, \ldots, x_{12}\}$	$\{x_{12},x_{11},x_{10}\},\{x_9,x_4,x_6\},\{x_5,x_8,x_3\},\{x_7,x_1,x_2\}$

IV. EXPERIMENTAL RESULTS AND ANALYSIS

This section presents the experimental study for the performance of LSMOEA/D by comparing it with a set of state-of-the-art MOEAs, including MOEA/D [10], NSGA-III [17], MOEA/DVA [25], LMEA [26], WOF [38], and LSMOF [39] on 25 benchmark functions from LSMOPs (LSMOP1-LSMOP9) [1], DTLZs (DTLZ1-DTLZ7) [51] and WFGs (WFG1-WFG9) [22]. These compared algorithms can be grouped into two classes: 1) two well-developed algorithms for solving large scale MOPs, including WOF and LSMOF, and 2) four state-of-the-art algorithms for handling large scale MaOPs, including LMEA, MOEA/DVA, NSGA-III and MOEA/D. Here, WOF is used to enhance SMPSO (WOF-SMPSO), NSGA-II (WOF-NSGA-II) and GDE3 (WOF-GDE3) [38], respectively, and the LSMOF is embedded into SMS-EMOA (LS-SMS-EMOA), NSGA-II (LS-NSGA-II), and MOEA/D-DE (LS-MOEA/D-DE) [39], respectively. NSGA-III is a well-known MOEA for solving MaOPs, MOEA/DVA is the first DVA-based MOEA [24] for large-scale MOPs, and LMEA is a recent algorithm designed specifically for large scale MaOPs.

Each test problem is invoked in 2- 3-, 5-, 8-, and 10-objective instances. Regarding the number of decision variables, 200, 500, and 1000 decision variables are considered for each test problem, respectively. Furthermore, the proposed LSMOEA/D is tested on the large scale test instances with up to 5000 and 10000 decision variables.

A. Experimental Configuration

The recommended parameter settings for the involved algorithms that have obtained the best performance in the literature are adopted as below, unless otherwise mentioned.

1) Population size: for MOEA/D and NSGA-III, the population size is set empirically according to the simplex-lattice design factor H together with the objective number M. As recommended in [10] and [17], for problems with $M \geq 8$, a two-layer reference generation approach is employed to produce uniformly distributed reference vectors. Table S-I in the supplementary material presents the settings of the population size for MOEA/D and NSGA-III. Here h_1 and h_2 are to control the numbers of reference points along the boundary of the PF and inside it, respectively. For the other algorithms, LSMOEA/D, MOEA/DVA, LMEA, WOFs (WOF-SMPSO, WOF-NSGA-II and WOF-GDE3), and LSMOFs (LS-SMS-EMOA, LS-NSGA-II, and LS-MOEA/D-DE), the population size was set to the same as that of NSGA-III and MOEA/D, with respect to different objective numbers M.

2) Crossover and mutation: SBX and polynomial mutation are adopted to create offspring. The distribution indexes of crossover and mutation are set to n_c = 20 and n_m = 20, respectively. The crossover probability p_c = 1.0 and the mutation probability $p_m = 1/D$ are used, where D is the number of decision variables [52] [53]. In LS-MOEA/D-DE and MOEA/DVA, the DE and PM operators are used for offspring generation, where the control parameters are set to $CR = 1$, $F = 0.5$, $p_m = 1/D$, and $\eta = 20$ as suggested in [54].

3) Number of runs and termination condition: Each algorithm was performed for 20 independent runs on each test instance and the termination criterion for each algorithm is the maximal number of generations. For each test instance with 200, 500, 1000, 5000 and 10000 decision variables, the maximum number of evaluations is set to 40 0000, 60 0000, 100 0000, 500 0000 and 1000 0000, respectively.

4) Other parameters: For MOEA/D, the neighborhood range, the maximal number of solutions replaced by each offspring solution and the probability that parent solutions are selected from the neighborhood are set to $T = 0.1*N$, $n_r = 0.01*N$, and $\delta = 0.9$ for all test problems, respectively, where N is the population size. The Tchebycheff approach is used for MOEA/D [31]. For MOEA/DVA, the number of interaction analysis and the number of control property analysis are set to the recommended values, namely, $NIA = 6$ and $NCA = 50$. For LSMOFs, the threshold tr is set to 0.5 [40]. For LMEA, the number of selected solutions and the number of perturbations for each selected solution in the decision variable clustering are set to $nSel = 2$ and $nPer = 4$, respectively, and the number of selected solutions in the decision variable interaction analysis is set as $nCor = 6$. For LSMOEA/D, the neighbor range is set to be the same as that in MOEA/D, i.e., $T = 0.1*N$, the number of subgroups in ordered grouping is set to $D/20$, and the detailed sensitivity analysis for other parameters $nSel$, $nPer$ and K (the number of weight clusters) is provided in the supplementary material (Table S-IV in Section S-II-A).

5) Performance metrics: Two common performance indicators, i.e., inverted generational distance (IGD) [55] and hypervolume (HV) [56], are employed to evaluate the performance of algorithms. Since calculating IGD requires some reference Pareto-optimal solutions, a reference set of uniform Pareto-optimal points needs to be generated using the approach in [10], where a set of uniform weight vectors are first generated by the Das and Dennis's systematic approach [41] if $M \leq 5$ or the Deb and Jain's two-layer method [47] if $M>5$, and then the intersecting points of the weight vectors and the Pareto-optimal surface of the benchmarks are used as the reference points. As recommended in [56], the number of reference points for DTLZs and LSMOPs is given in Table S-II in the supplementary material. For WFG1 and WFG2, whose PFs are irregular, their reference points are generated by sampling some non-dominated solutions in the underlying space $[0, 1]^{M-1}$. For WFG4-WFG9, the number of their reference points is set to the same as DTLZs. For WFG3 with a degenerate PF, as recommended in [57], the final non-dominated solutions obtained by each algorithm are used as the reference points. Their details are shown in Table S-III in the supplementary material. For test problems with more than 5 objectives, the Monte Carlo estimation method is adopted to estimate HV.

B. *Test1: Comparison Results on LSMaOPs*

The experimental results in terms of the IGD values obtained by MOEA/D, NSGA-III, MOEA/DVA, LMEA and LSMOEA/D on DTLZs and WFGs with 200, 500 and 1000 decision variables are reported in Tables S-V and S-VI in the supplementary material due to page limit. In these tables, the best result on each test instance is displayed in gray and the difference significance between LSMOEA/D and its compared algorithms is evaluated by Wilcoxon's rank sum test with a level of significance 0.05, where the signs "+", "-", and "≈" indicate the significance extent.

From Tables S-V and S-VI in the supplementary material, we can obtain the following observations.

1) LSMOEA/D shows an obvious advantage over MOEA/D and NSGA-III in terms of mean and standard deviation values on most of the 72 DTLZ instances (more obvious when D increases to 1000). For low-dimensional (200-D and 500-D) test instances, LSMOEA/D does better or at least comparably on most test problems, but does not show a significant superiority on two test instances, i.e., 200-D 5-objective DTLZ2 and 200-D 5-objective DTLZ5. For high-dimensional (1000-D) instances, LSMOEA/D is the most efficient optimizer, significantly better than MOEA/D and NSGA-III. Especially on 5-objective DTLZ3 and 10-objective DTLZ7, LSMOEA/D outperforms its compared algorithms with almost two orders of magnitude. This observation indicates that the proposed localized variable analysis method indeed has a positive effect on the performance in optimizing large scale problems.

The results in Table S-V reveal that the performance outcome of LMEA and LSMOEA/D on each test problem consistently maintains a promising level when the number of decision variables increases from 200 to 1000, which shows the promising scalability of LMEA and LSMOEA/D. Nevertheless, LSMOEA/D still attains the superiority to its competitors on a set of DTLZ instances with 2, 5, 8 and 10 objectives. On DTLZ1 and DTLZ4 with convergence-related and diversity-related variables, MOEA/DVA, LMEA and LSMOEA/D perform nearly equivalently in terms of mean and standard deviation on 200-D 5-objective DTLZ3 and DTLZ4. On DTLZ5 and DTLZ6. LSMOEA/D always obtains better results than MOEA/DVA and LMEA, especially better at least an order of magnitude on 5-objective DTLZ5. Similar observations are obtained on DTLZ3 and DTLZ7. For 500-D DTLZ3, whose PF is composed of a set of discontinuous segments, LSMOEA/D is better than LMEA on 8 out of the 12 test instances. For 1000-D DTLZ7, which has a degenerate/disconnected PF, LSMOEA/D is better than MOEA/DVA with one order of magnitude. The performance improvement of LSMOEA/D is attributed to the fact that the use of uniform reference vectors as the convergence direction is more accurate to distinguish mixed variables for irregular PFs while there are more classifications for mixed variables to be optimized in a fine-grained manner. This is consistent with the observation in Fig. 2.

2) On the set of WFG problems, which have composable complexities in the decision space (e.g., non-separability, multimodality or biased parameters) and in the objective space (e.g., concave, disconnected or mixed PFs), LSMOEA/D does rather competitively, especially on WFG3, WFG6 and WFG7, as shown in Table S-VI. On WFG3, whose PF is linear and degenerates simultaneously with sparsely-interacted decision variables, LSMOEA/D obtains the best results on all the instances except the 1000-D 5-objective one, on which MOEA/DVA is ranked the first. On WFG6, whose decision variables are highly dependent, LSMOEA/D is the best performer on all the instances while MOEA/DVA and LMEA also achieve the similar results, only slightly worse than LSMOEA/D. On WFG7 with a set of highly-dependent mixed variables, LSMOEA/D wins 6 out of the 12 instances and LMEA performs the best on 3 test instances. This is encouraging because the proposed adaptive scalarization strategy seems very suitable to deal with highly-dependent variables. Only on WFG2 with non-separable variables and disconnected convex PF, LMEA and MOEA/DVA perform better than LSMOEA/D. The slight performance improvement of MOEA/DVA and LMEA on WFG2 occurs because the effect of spatial location of candidate solutions may be not very obvious on this problem, and the uniform weight vectors in LSMOEA/D cannot guarantee a well-distributed solutions for discontinued PFs.

The reasons for the performance improvement of LSMOEA/D on WFGs (except WFG2) are as follows: 1) as shown in Table I, while the variables in LMEA are only labeled to be convergence-related or diversity-related, all the variables in LSMOEA/D are further identified as a grouped sequence according to the CRD, thereby resulting in significantly better results; 2) the decomposition approach of LSMOEA/D, which does not need independence variable analysis, optimizes the set of more convergence-related variables with a higher priority, and assigns them with more computation resources. This is helpful to accelerate the search towards the PF.

Fig. S2 in the supplementary material explicitly plots the distribution of final solutions obtained by each algorithm on 10-objective WFG2 with 1000 decision variables by parallel coordinates. From the figure, we can see that the tradeoff surfaces obtained by LSMOEA/D and LMEA are better than those of MOEA/D and NSGA-III in terms of convergence and distribution, while MOEA/DVA also obtains a comparable performance, but still slightly converges into a small parts of several objectives. As shown in Fig. S3 in the supplementary material, we can see that all the algorithms show difficulties in converging to the PF on WFG1. Some parts of the solution set obtained by LSMOEA/D and LMEA look a little noisy, which verifies the analysis for the above IGD results.

C. Test2: Computation Efficiency on LSMaOPs

To assess the computation efficiency of LSMOEA/D, the evolutionary process of the five algorithms in terms of IGD versus the number of generations on a set of 10-objective test instances with 1000 decision variables are plotted and compared in Fig. S4 and Fig. S5 in the supplementary material. From the figures, we observe that LSMOEA/D is able to obtain a faster convergence speed than the compared algorithms as well as find the best IGD values on the majority of selected test instances. In detail, LSMOEA/D obtains an improvement over its competitors MOEA/D and NSGA-III on DTLZ2, DTLZ3 and WFG4, and performs slightly worse than LMEA on WFG 6. LSMOEA/D and LMEA perform better than other algorithms on WFG5. Similar observation is obtained on WFG9.

For further investigation, we directly record and compare its actual running time (in seconds: *s*) with that of compared algorithms on each test instance. The hardware environment is configured as: Intel Dual Core I7-6500U CPU 2 GHz, 16.00GB RAM. It is shown from the comparative results in Fig. S6 in the supplementary material that LSMOA/D is nearly equivalent to NSGA-III, and faster than LMEA and MOEA/D, while it is slightly slower than MOEA/DVA. For the pairwise comparison between LSMOEA/D and MOEA/DVA, LSMOEA/D essentially consumes more computation cost because it needs to make control decision variable analysis for each guiding reference vector. In fact, we can regulate the number of the guiding reference vectors to save the computation cost of LSMOEA/D. In addition, compared to LMEA, LSMOEA/D does not need to conduct variable interaction analysis, which cuts down the computation cost to an extent.

D. Test3: Results on LSMaOPs with 5000 and 10000 Variables

Since one motivation of this work is to solve large scale optimization, this experiment tests the scalability of LSMOEA/D by further increasing the number of decision variables. In principle, as the number of decision variables increases, the optimization complexity of a problem grows exponentially. Table S-VII in the supplementary material presents the HV metric values obtained by the proposed LSMOEA/D and the compared algorithms on a set of 5- and 10-objective test instances with 1000, 5000 and 10000 decision variables over 20 runs. The results from the table exhibit that the proposed LSMOEA/D performs more powerfully than its counterparts on majority of the large scale test instances. For example, on DTLZ6 and DTLZ7, LSMOEA/D does best on all the instances from D=1000 to D=10000, while other compared algorithms cannot even obtain find optimal solutions for D=10000. Upon closer examination at Table S-VII, it is shown that LSMOEA/D does not show obvious deterioration on DTLZ5, DTLZ6, WFG2, WFG6 and WFG7 when the number of decision variables increases from 1000 to 10000. These encouraging results validate the promising scalability of the proposed algorithm.

E. Test4: Comparison Results on LSMOPs

This experiment further assesses the scalability of LMOEA/D on the test suite for LSMOPs [1]. Two state-of-the-art large scale MOEAs, i.e., LSMOF and WOF, are employed for performance comparison. Here the LSMOF refers to three LSMOF versions LS-SMS-EMOA, LS-NSGA-II and LS-MOEA/D-DE, and WOF involves WOF-SMPSO, WOF-NSGA-II and WOF-GDE3, respectively.

The statistics of IGD results achieved by LSMOEA/D, WOFs, and LSMOFs on 3-objective LSMOPs with 1000 variables are displayed in Table II (complete results for 2- and 3-objective LSMOPs with 200, 500 and 1000 variables are provided in Table S-VIII in the supplementary material).

As can be seen, LMOEA/D is ranked first on 5 out of 10 test problems, WOF-GDE3 obtains the first rank on 2 test problems, WOF-SMPSO and LS-SMS-MOEA achieve 1 best result. Similar observations can be obtained in Table S-VII. The box plots of Fig.8 show the distributions of the results obtained by the algorithms on several 3-objective test instances with 1000 variables. From these statistical results, we can observe that LSMOEA/D outperforms its compared algorithms on LSMOP1, LSMOP4, LSMOP5 and LSMOP7. These observations verify the competitive performance of the proposed algorithm on in comparison with the state-of-the-arts.

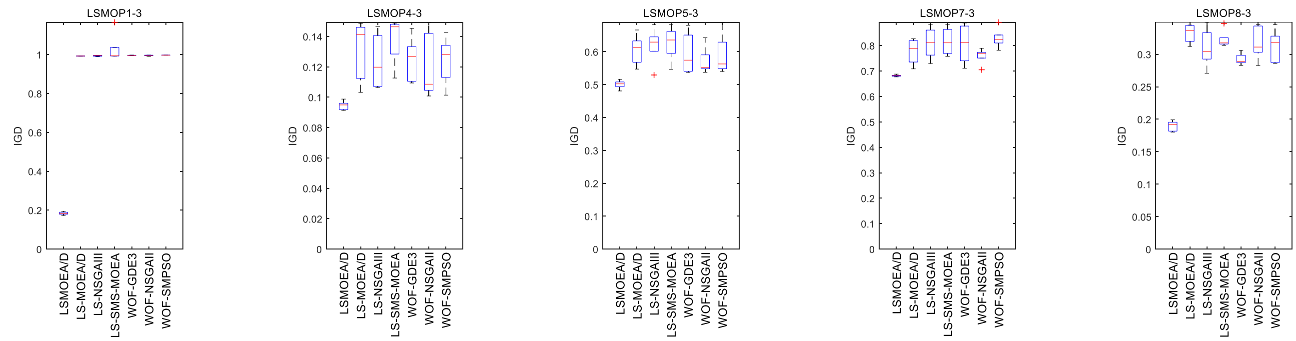
In addition, we further investigate the computation efficiency and accuracy of LSMOEA/D, which are provided in the supplementary material (please see details in Section S-II-D).

V. Conclusion

This paper proposes a new algorithm LSMOEA/D to solve LSMOPs and LSMaOPs. In LSMOEA/D, the guidance of reference vectors is incorporated into the control decision variable analysis. Then, these grouped variables are optimized by an adaptive scalarization strategy. Especially, in the localized control variable analysis, the contribution of each decision variable to the convergence is measured by the projection-based detection method, where the CRD is quantified based on both acute angle and projection length from the sampling solutions to the direction of the guiding reference vector. LSMOEA/D has been experimentally compared with a set of mainstream algorithms on a set of test problems with 2-10 objectives and 200-1000 variables. Experimental results show that LSMOEA/D is effective and efficient to deal with LSMOPs and LSMaOPs.

TABLE II
IGD VALUES OBTAINED BY LSMOEA/D, WOFS AND LSMOFS ON 3-OBJECTIVE LSMOPS 1-9 WITH 1000 DECISION VARIABLES.

Problem	LS-SMS-MOEA	LS-NSGAII	LS-MOEA/D	WOF-SMPSO	WOF-NSGAII	WOF-GDE3	LSMOEA/D
LSMOP1	8.267e-1(3.540e-3)-	8.314e-1(1.068e-2)-	9.548e-1(1.565e-1)-	5.197e+0(1.088e-1)-	4.770e-1(1.902e-1)-	4.918e-1(6.536e-2)-	**1.925e-1(1.651e-2)**
LSMOP2	7.286e-2(7.921e-3)-	6.789e-2(1.145e-3)≈	6.997e-2(3.331e-3)≈	**5.217e-2(3.242e-5)≈**	6.714e-2(1.344e-3)≈	6.885e-2(2.071e-3)≈	6.032e-2(1.965e-4)
LSMOP3	8.607e-1(0.000e+0)	3.066e+0(3.119e+0)	8.607e-1(0.000e+0)	1.236e+1(4.502e+0)-	8.607e-1(0.000e+0)	**8.597e-1(1.441e-3)≈**	8.607e-1(0.000e+0)
LSMOP4	1.434e-1(2.334e-3)-	1.429e-1(3.621e-3)-	1.449e-1(3.956e-3)-	1.090e-1(8.888e-4)-	1.306e-1(4.477e-4)-	1.330e-1(2.129e-3)-	**9.060e-2(1.377e-3)**
LSMOP5	5.410e-1(2.348e-5)-	5.410e-1(3.253e-6)-	5.450e-1(5.737e-3)-	6.831e+0(1.295e+0)-	6.521e-1(2.966e-1)-	5.061e-1(1.929e-2)-	**4.920e-1(3.600e-2)**
LSMOP6	**7.569e-1(7.777e-2)+**	7.796e-1(5.270e-2)+	7.654e-1(3.108e-2)+	1.878e+2(1.101e+2)-	1.479e+0(1.946e-1)-	1.175e+0(1.939e-1)-	1.014e+0(5.693e-1)
LSMOP7	8.949e-1(3.550e-2)-	8.921e-1(3.215e-2)-	8.668e-1(2.434e-3)-	9.853e-1(3.266e-2)-	8.164e-1(7.463e-2)-	7.919e-1(3.766e-2)-	**6.868e-1(1.786e-1)**
LSMOP8	3.729e-1(1.430e-2)-	3.578e-1(9.189e-4)-	4.085e-1(7.061e-2)-	7.064e-1(1.943e-3)-	3.324e-1(3.307e-4)-	2.789e-1(3.853e-2)-	**1.855e-1(1.281e-1)**
LSMOP9	1.538e+0(0.000e+0)	7.416e+1(1.027e+2)	9.885e+0(1.180e+1)	1.930e+1(1.509e+0)	1.145e+0(1.749e-	**1.145e+0(5.909e-**	2.083e+1(9.379e+0



(a) LSMOP1 (b) LSMOP4 (c) LSMOP5 (d) LSMOP7 (e) LSMOP8

Fig.8. Box plots of IGD results on 10-objective LSMOP test problems with 1000 decision variables.

Despite the promising results, there are still some issues to be studied in the future: 1) The computation efficiency of the localized decision variable analysis needs to be further improved because it heavily relies on the number of the guiding reference vectors; 2) More efficient adaptive strategies for r balancing diversity and convergence are desirable.

ACKNOWLEDGEMENT

The authors would like to thank Professor Jinliang Ding from the State Key Laboratory of Synthetical Automation for Process Industries (SAPI), Northeastern University, China, for his help in improving this paper.

REFERENCES

[1] R. Cheng, Y. Jin, M. Olhofer and B. Sendhoff, "Test problems for large-scale multi-objective and many-objective optimization," *IEEE Trans. Cybern.*, vol. 47, no. 12, pp. 4108-4121, 2017.

[2] S. Cheng, L. Ma, H. Lu, X. Lei, and Y. Shi, "Evolutionary computation for solving search-based data analytics problems," *Artif. Intell. Rev.*, 2020, DOI 10.1007/s10462-020-09882-x.

[3] C. He, R. Cheng, C. Zhang, Y. Tian, Q. Chen and X. Yao, "Evolutionary large-scale multiobjective optimization for ratio error estimation of voltage transformers," *IEEE Trans. Evol. Comput.*, vol. 24, no. 5, pp. 868-881, 2020.

[4] B. Li, J. Li, K. Tang and X.Yao, "Many-objective evolutionary algorithms: a survey", *ACM Comput. Surveys*, vol. 48, no. 1, pp. 1-35, 2015.

[5] K. Deb, A. Pratap, S. Agarwal and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182-197, 2002.

[6] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength Pareto evolutionary algorithm for multiobjective optimization," in *Proc. 5th Conf. Evol. Methods Design Optim. Control Appl. Ind. Problems*, pp. 95–100, 2001.

[7] E. Zitzler and S. Künzli, "Indicator-based selection in multiobjective search," in *Proc. Int. Conf. Parallel Prob. Solv. Nat.*, Birmingham, U.K, pp. 832–842, 2004.

[8] H. Trautmann, T. Wagner, and D. Brockhoff, "R2-EMOA: Focused multiobjective search using R2-indicator-based selection," in *Learning and Intelligent Optimization*. Berlin, Germany: Springer, pp. 70-74, 2013.

[9] E. J. Hughes, "Multiple single objective Pareto sampling," in *Proc. IEEE Congr. Evol. Comput.*, Canberra, Australia, 2003, pp. 2678-2684.

[10] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Trans. Evol. Comput.*, vol. 11, no. 6, pp. 712-731, Dec. 2007.

[11] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff, "A reference vector guided evolutionary algorithm for many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 20, no. 5, pp. 773-791, 2016.

[12] Z. Bingul, "Adaptive genetic algorithms applied to dynamic multiobjective problems," *Appl. Soft Comput.*, vol.7, no.3, pp. 791-799, 2007.

[13] Z. Bingul, A.S. Sekmen, S. Zein-Sabatto, "Evolutionary approach to multiobjective problems using adaptive genetic algorithms," in *Proc. Int. Conf. Sys. Man. Cybern.,* pp. 1923-1927, 2000.

[14] S. Yang, M. Li, X. Liu, and J. Zheng, "A grid-based evolutionary algorithm for many-objective optimization," IEEE Trans. Evol. Comput., vol. 17, no. 5, pp. 721–736, 2013.

[15] A.M. Mendez and C.A.C. Coello, "Solving multi-objective optimization problems using differential evolution and a maximin selection criterion," in *Proc. IEEE Congr. Evol. Comput.*, pp.1-8, 2012.

[16] H. Wang, L. Jiao, and X. Yao, "Two_Arch2: An improved two-archive algorithm for many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 19, no. 4, pp. 524–541, 2015.

[17] K. Deb and H. Jain, "An evolutionary many-objective

optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints," *IEEE Trans. Evol. Comput.*, vol. 18, no. 4, pp. 577–601, 2014.

[18] C. M. Fonseca and P. J. Fleming, "Multiobjective optimization and multiple constraint handling with evolutionary algorithms. I. A unified formulation," *IEEE Trans. Syst., Man, Cybern. A, Syst., Humans,* vol. 28, no. 1, pp. 26–37, 1998.

[19] J. Maltese, B. M. Ombuki-Berman and A. P. Engelbrecht, "A scalability study of many-objective optimization algorithms," *IEEE Trans. Evol. Comput.*, vol. 22, no. 1, pp. 79-96, 2018.

[20] F.V.D. Bergh and A.P. Engelbrecht, "A cooperative approach to particle swarm optimization," *IEEE Trans. Evol. Comput.*, vol. 8, no.3, pp. 225-239, 2004.

[21] M. N. Omidvar, X. Li, and K. Tang, "Designing benchmark problems for large-scale continuous optimization," *Inf. Sci.*, vol. 316, pp. 419-436, 2015.

[22] S. Huband, P. Hingston, L. Barone and L. While, "A review of multiobjective test problems and a scalable test problem toolkit," *IEEE Trans. Evol. Comput.*, vol. 10, no. 5, pp. 477-506, 2006.

[23] A. Trivedi, D. Srinivasan, K. Sanyal, and A. Ghosh, "A survey of multiobjective evolutionary algorithms based on decomposition," *IEEE Trans. Evol. Comput.*, vol. 21, no. 3, pp. 440-462, 2017.

[24] H. Chen, R. Cheng, J. Wen, H. Li, and J. Weng, "Solving large-scale many-objective optimization problems by covariance matrix adaptation evolution strategy with scalable small subpopulations," *Inf. Sci.*, vol. 509, pp. 457-469, 2020.

[25] X. Ma , F. Liu, Y. Qi, X. Wang, L. Li, L. Jiao, M. Yin and M. Gong, "A multiobjective evolutionary algorithm based on decision variable analyses for multiobjective optimization problems with large-scale variables," *IEEE Trans. Evol. Comput.*, vol. 20, no. 2, pp. 275-298, 2016.

[26] X. Zhang, Y. Tian, R. Cheng and Y. Jin, "A decision variable clustering-based evolutionary algorithm for large-scale many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 22, no. 1, pp. 97-112, 2018.

[27] H. Chen, X. Zhu, W. Pedrycz, S. Yin, G. Wu and H. Yan, "PEA: Parallel evolutionary algorithm by separating convergence and diversity for large-scale multi-objective optimization," in *Proc. IEEE 38th Int. Conf. Distributed Comput. Syst.*, pp. 223-232, 2018.

[28] A. Atashpendar, B. Dorronsoro, G. Danoy, and P. Bouvry, "A scalable parallel cooperative coevolutionary PSO algorithm for multi-objective optimization," *J. Parallel. Distr. Com.*, vol. 112, pp. 111-125, 2018.

[29] B. Cao, J. Zhao, Z. Lv and X. Liu, "A distributed parallel cooperative coevolutionary multiobjective evolutionary algorithm for large-scale optimization," *IEEE Trans. Ind. Informat.*, vol. 13, no. 4, pp. 2030-2038, 2017.

[30] M. A. Potter and K. A. D. Jong, "A cooperative coevolutionary approach to function optimization," in *Proc. Int. Conf. Parallel Prob. Solv. Nat.*, Berlin, Heidelberg, pp. 249-257, 1994.

[31] Z. Yang, K. Tang and X. Yao, "Large scale evolutionary optimization using cooperative coevolution," *Inf. Sci.*, vol. 178, no. 15, pp. 2985–2999, 2008.

[32] M. N. Omidvar, X. Li, Z. Yang and X. Yao, "Cooperative co-evolution for large scale optimization through more frequent random grouping," in *Proc. IEEE Congr. Evol. Comput.*, Barcelona, pp. 1-8, 2010.

[33] M. N. Omidvar, X. Li and X. Yao, "Cooperative co-evolution with delta grouping for large scale non-separable function optimization," in *Proc. IEEE Congr. Evol. Comput.*, Barcelona, pp. 1-8, 2010.

[34] X. Li and X. Yao, "Cooperatively coevolving particle swarms for large scale optimization," *IEEE Trans. Evol. Comput.*, vol. 16, no. 2, pp. 210-224, 2012.

[35] Y. Mei, M. N. Omidvar, X. Li, and X. Yao, "A competitive divide-and-conquer algorithm for unconstrained large-scale black-box optimization," *ACM Trans. Math. Software*, vol. 42, no. 2, pp. 13, 2016.

[36] Y. Ling, H. Li, and B. Cao, "Cooperative co-evolution with graph-based differential grouping for large scale global optimization," *IEEE Fuzzy Syst. Knowl. Discovery (ICNC-FSKD)*, pp. 95-102, 2016.

[37] L. M. Antonio and C. A. C. Coello, "Use of cooperative coevolution for solving large scale multiobjective optimization problems," in *Proc. IEEE Congr. Evol. Comput.*, Cancun, pp. 2758-2765, 2013.

[38] H. Zille, H. Ishibuchi, S. Mostaghim and Y. Nojima, "A framework for large-scale multiobjective optimization based on problem transformation," *IEEE Trans. Evol. Comput.*, vol. 22, no. 2, pp. 260-275, 2018.

[39] C. He, L. Li, Y. Tian, X. Zhang, R. Cheng, Y. Jin and X. Yao, "Accelerating large-scale multiobjective optimization via problem reformulation," *IEEE Trans. Evol. Comput.*, vol. 23, no. 6, pp. 949-961, 2019.

[40] Y. Qi, F. Liu, M. Liu, M. Gong, and L. Jiao, "Multi-objective immune algorithm with Baldwinian learning," *Appl. Soft Comput.*, vol. 12, no. 8, pp. 2654–2674, 2012.

[41] I. Das and J. E. Dennis, "Normal-boundary intersection: A new method for generating the Pareto surface in nonlinear multicriteria optimization problems," *SIAM J. Optim.*, vol. 8, no. 3, pp. 631-657, 1998.

[42] M. N. Omidvar, X. Li and X. Yao. "Smart use of computational resources based on contribution for cooperative co-evolutionary algorithms," in *Proc. Conf. Genetic Evol. Comput. ACM*, pp. 1115–1122, 2011.

[43] H. Liu, F. Gu and Q. Zhang, "Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems," *IEEE Trans. Evol. Comput.*, vol. 18, no. 3, pp. 450-455, June 2014.

[44] K. Li, K. Deb, Q. Zhang and S. Kwong, "An evolutionary many-objective optimization algorithm based on dominance and decomposition," *IEEE Trans. Evol. Comput.*, vol. 19, no. 5, pp. 694-716, 2015.

[45] M. N. Omidvar, M. Yang, Y. Mei, X. Li and X. Yao, "DG2: A faster and more accurate differential grouping for large-scale black-box optimization," *IEEE Trans. Evol. Comput.*, vol. 21, no. 6, pp. 929-942, 2017.

[46] M. Munetomo and D. E. Goldberg, "Identifying linkage groups by nonlinearity/non-monotonicity detection," *Evol. Comput.*, vol. 7, no. 4, pp. 377-398, 1999.

[47] Y. P. Chen, "Extending the scalability of linkage learning genetic algorithms: theory & practice." Springer, vol. 190, 2004.

[48] Q. Zhang and H. Muhlenbein, "On the convergence of a class of estimation of distribution algorithms," *IEEE Trans. Evol. Comput.*, vol. 8, no. 2, pp. 127-136, April 2004.

[49] S. V. Aelst, X. S. Wang, R. H. Zamar, and R. Zhu, "Linear grouping using orthogonal regression," *Comput. Stati. Data Analy.*, vol. 50, no. 5, pp. 1287–1312, 2006.

[50] W. Chen, T. Weise, Z. Yang, and K. Tang, "Large-scale global optimization using cooperative coevolution with variable interaction learning," in *Proc. Int. Conf. Parallel Prob. Solv. Nat.*, pp. 300–309, 2010.

[51] H. Wang, L. Jiao, R. Shang, S. He, and F. Liu, "A memetic optimization strategy based on dimension reduction in decision space," *Evol. Comput.*, vol. 23, no. 1, pp. 69–100, 2015.

[52] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable test problems for evolutionary multi-objective optimization," Dept. Comput. Eng. Netw. Lab., ETH Zurich, Zurich, Switzerland, TIK-Tech. Rep. 112, 2001.

[53] K. Deb and R. B. Agrawal, “Simulated binary crossover for continuous search space,” *Complex Syst.*, vol. 9, no. 2, pp. 115-148, 1995.
[54] H. Li and Q. Zhang, "Multiobjective optimization problems with complicated Pareto sets, MOEA/D and NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 13, no. 2, pp. 284-302, 2009.
[55] C. A. C. Coello, G. B. Lamont, and D. A. V. Veldhuizen, “Evolutionary algorithms for solving multi-objective problems,” in *Multiobjective Optimization*, USA: Springer, 2007.
[56] Y. Xiang, Y. Zhou, M. Li and Z. Chen, "A vector angle-based evolutionary algorithm for unconstrained many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 21, no. 1, pp. 131-152, 2017.
[57] H. Ishibuchi, H. Masuda and Y. Nojima, "Pareto fronts of many-objective degenerate test problems," *IEEE Trans. Evol. Comput.*, vol. 20, no. 5, pp. 807-813, 2016.

Lianbo Ma (**M’16**) received the B.Sc. degree in Communication Engineering and M.Sc. degree in Communication and Information System from Northeastern University, Shenyang, China, in 2004 and 2007 respectively, and the Ph.D. degree from University of Chinese Academy of Sciences, China, in 2015.

He is currently a Professor of Northeastern University, China. His current research interests include computational intelligence and machine learning.

Min Huang received the B.Sc. degree in automatic instrument, M.Sc. degree in systems engineering, and Ph.D. degree in control theory and control engineering from the Northeastern University, Shenyang, China, in 1990, 1993, and 1999, respectively.

She is currently a Professor of Northeastern University, China. Her research interests include modeling and optimization for logistics and supply chain, etc. She has published over 100 journal articles, books, and refereed conference papers.

Shengxiang Yang (M’00-SM’14) received the B.Sc. degree in automatic control, M.Sc. degree in automatic control, and Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 1993, 1996, and 1999, respectively.

He is currently a Professor of Computational Intelligence and the Director of the Centre for Computational Intelligence, School of Computer Science and Informatics, De Montfort University, Leicester, U.K. He has over 320 publications with an H-index of 56 according to Google Scholar. His current research interests include evolutionary computation, swarm intelligence, artificial neural networks, data mining and data stream mining, and relevant real-world applications.

Prof. Yang serves as an Associate Editor/Editorial Board Member for a number of international journals, such as the IEEE Transactions on Evolutionary Computation, IEEE Transactions on Cybernetics, Information Sciences, and Enterprise Information Systems.

Rui Wang received the B.Sc. degree in Digital Media Technology and the M.Sc. degree in Software Engineering from Northeastern University, Shenyang, China, in 2017 and 2020, respectively.

His current research interests include computational intelligence, industrial network system, and advanced planning and scheduling.

Xingwei Wang received the B.Sc., M.Sc., and Ph. D. degrees in computer science from the Northeastern University, Shenyang, China, in 1989, 1992, and 1998, respectively.

He is currently a Professor of Northeastern University, China. His current research interests include cloud computing and future Internet. He has published over 100 journal articles, books, and refereed conference papers.

An Adaptive Localized Decision Variable Analysis Approach to Large Scale Multi-objective and Many-objective Optimization
Supplementary Material

Lianbo Ma, *Member*, *IEEE*, Min Huang, *Member*, *IEEE*, Shengxiang Yang, *Senior Member*, *IEEE*, Rui Wang, and Xingwei Wang

OUTLINE

S-I. EXAMPLE ILLUSTRATION OF MOTIVATION

In this section, an example derived from WFG2 is considered to illustrate the phenomenon that the detection result in terms of decision variables' control property may be different in different detection regions in the objective space. As defined in [1], a bi-objective WFG2 problem is formulated as

$$\begin{aligned}
&\text{minimize } f_1 = y_2 + 2\left[1-\cos\left(\left(\max\{y_2,1\}(y_1-0.5)+0.5\right)\pi/2\right)\right]\\
&\text{minimize } f_2 = y_2 + 4\left[1-\left(\max\{y_2,1\}(y_1-0.5)+0.5\right)\right.\\
&\qquad\qquad \left.\times\cos^2\left(5\left(\max\{y_2,1\}(y_1-0.5)+0.5\right)\pi\right)\right]\\
&\text{where } \quad y_1 = \frac{x_1}{2}\\
&\qquad y_2 = \left(\frac{|x_2/4\text{-}0.35|}{\left|\lfloor 0.35\text{-}x_2/4\rfloor+0.35\right|}+\frac{|x_3/6\text{-}0.35|}{\left|\lfloor 0.35\text{-}x_3/6\rfloor+0.35\right|}\right.\\
&\qquad\qquad \left.+2\left|\frac{|x_2/4\text{-}0.35|}{\left|\lfloor 0.35\text{-}x_2/4\rfloor+0.35\right|}\text{-}\frac{|x_3/6\text{-}0.35|}{\left|\lfloor 0.35\text{-}x_3/6\rfloor+0.35\right|}\right|\right)\times\frac{1}{3}\\
&\text{subject to}: x_1\in[0,2], x_2\in[0,4], x_3\in[0,6].
\end{aligned} \tag{S1}$$

Another example is considered as follows:

$$\begin{aligned}
&\text{minimize } f_1 = x_1x_2\\
&\text{minimize } f_2 = \left(1-\cos\frac{\pi}{2}x_1\right)^{(\sin\frac{\pi}{2}x_2-\frac{\sqrt{2}}{2})}\\
&\text{subject to: } x_i\in[0,1],\ i=1,2
\end{aligned} \tag{S2}$$

Fig. S1 plots the sampling points generated on the problem of (S2) by perturbing a variable x_1 between [0, 1] while fixing another variable x_2 to 0.4 and 0.9, respectively. Each reference vector (e.g., V_1 or V_2) used here specifies a unique subregion in the bi-objective space. When the sampling solutions are located in the subregion of V_1, i.e., x_2 = 0.4, x_1 is identified as a diversity-related variable according to the variable analysis strategy [2]. In contrast, when the sampling solutions are located in the subregion of V_2, i.e., x_2 = 0.9, since x_1 contributes more to convergence than diversity, x_1 is identified as a convergence-related variable. The above observation demonstrates that the detection location indeed has an important impact on the detection results in terms of the control property of decision variables.

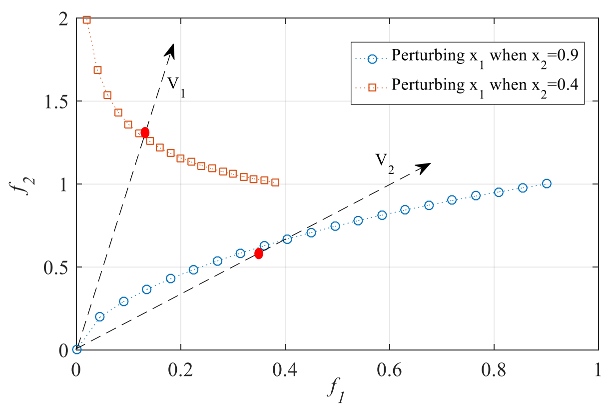


Fig.S1. Plot of sampling points generated on the problem (S2).

S-II. EXPERIMENTAL RESULTS AND ANALYSIS

A. Experimental Configuration

In this section, the parameter settings for involved algorithms, which are not provided in the paper due to page limit, are given as follows:

TABLE S-I
POPULATION SIZE FOR DIFFERENT NUMBER OF OBJECTIVES.

M	H	NSGA-III	MOEA/D	LSMOEA/D
3	91	92	91	91
5	210	212	210	210
8	156(h1=3,h2=2)	156	156	156
10	135(h1=3,h2=2)	276	275	275

L. Ma and R. Wang are with College of Software, Northeastern University, Shenyang, 110819, China (malb@swc.neu.edu.cn, wangrui@sia.cn).
M. Huang is with the College of Information Science and Engineering, State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang, 110819, China (mhuang@mail.neu.edu.cn).
S. Yang is with the School of Computer Science and Informatics, De Montfort University, Leicester, LE1 9BH, U.K (syang@dmu.ac.uk).
X. Wang is with College of Computer Science, Northeastern University, Shenyang, 110819, China (wangxw@mail.neu.edu.cn).

TABLE S-II
NUMBER OF REFERENCE POINTS FOR DTLZS AND LSMOPS.

M	h1	h2	Number of reference points
3	25	-	351
5	13	-	2380
8	7	6	5148
10	6	5	7007

TABLE S-III
NUMBER OF REFERENCE POINTS FOR WFGS.

M	WFG1	WFG2	WFG3	WFG4-9
3	421	148	5000	351
5	2801	1601	17000	2380
8	5464	4690	15000	5148
10	20705	13634	26000	7007

B. Parameter Sensitivity Analysis

In this section, the detailed sensitivity analysis to several specific parameters of LSMOEA/D is conducted. LSMOEA/D has three parameters to be tuned, including *nSel* (the number of solutions to be selected to conduct variable permutations), *nPer* (the number of permutations to be applied on each decision variable for generating new solutions for variable classification), and *K* (the number of reference vector clusters). As presented in Section III-C, the functionalities of *nSel* and *nPer* in the control variable analysis in LSMOEA/D are the same as that in LMEA and MOEA/DVA. Following the analysis approach in LMEA, we put *nSel* and *nPer* together to construct different combinations and then test their influences on the performance of LSMOEA/D on a set of test problems. Note that, LSMOEA/D has a specific parameter *K*, which is to affect the clustering of the reference vectors and the region of the sample solutions generated in the localized decision variable analysis process. Apparently, all the three parameters, i.e., *nSel*, *nPer* and *K* have potential effect on the performance of the localized decision variable analysis in LSMOEA/D. Thus, we put them together to investigate their sensitivity on a set of test functions. In detail, *nSel* is set to be 2, 4, 6 and 8, *nPer* is empirically varied from 4, 8, 12 and 16, *K* is adjusted from 5, 10, 15 to 20.

Table S-IV shows the statistic results in terms of HV metric with 20 independent runs obtained by LSMOEA/D on 3- and 10-objective LSMOP1-LSMOP9 with 1000 decision variables. From the figure, it can be observed that the performance of the localized decision variable analysis approach suggested in LSMOEA/D is quite robust to different combinations of parameters *nSel*, *nPer* and *K* on the test functions, while, *nSel = 2*, *nPer* = 4 and *K*=5 obtain the best results, including seven first ranks and two second ranks on the eighteen test instances. Therefore, *nSel* = 2, *nPer* = 4 and *K*=5 is suggested as the best choice for the proposed LSMOEA/D. Accordingly, these parameter settings are used in the following experiments.

C. Experimental Results

The experimental results regarding the mean and standard deviation results of IGD obtained by the involved algorithms on DTLZs and WFGs are shown in Table S-V and Table S-VI respectively. Figs. S2 and S3 show the final solution set of involved algorithms on the 10-objective WFG1 and WFG2 with 1000 decision variables by parallel coordinates respectively. Figs. S4 and S5 show the evolutionary trajectories of IGD on the 10-objective DTLZ and WFG test problems with 1000 variables respectively. Table S-VII shows the mean and standard deviation results in terms of HV metric obtained by the involved algorithms on DTLZ and WFG test problems. Table S-VIII shows the mean and standard deviation results in terms of IGD values obtained by LS-SMS-MOEA, LS-NSGAII, LS-MOEA/D, WOF-SMPSO, WOF-NSGAII, WOF-GDE3 and LSMOEA/D on LSMOPs.

TABLE S-IV
MEAN AND STANDARD DEVIATION HV VALUES OBTAINED BY LSMOEA/D WITH DIFFERENT PARAMETERS ON 3- AND 10- OBJECTIVE LSMOPS WITH 1000 DECISION VARIABLES.

Problem	M	*nSel*=2,*nPer*=4, *K*=10	*nSel*=4,*nPer*=8, *K*=10	*nSel*=6,*nPer*=12, *K*=10	*nSel*=8,*nPer*=16, *K*=10	*nSel*=2,*nPer*=4, *K*=5	*nSel*=2,*nPer*=4, *K*=15	*nSel*=2,*nPer*=4, *K*=20
LSMOP1	3	7.255e-1(2.421e-	7.260e-1(2.648e-	7.342e-1(1.907e-	7.353e-1(2.182e-	7.239e-1(4.009e-	7.277e-1(3.360e-	7.154e-1(2.793e-
	10	8.782e-1(1.208e-	8.697e-1(1.300e-	8.714e-1(1.452e-	8.543e-1(1.618e-	9.040e-1(3.730e-	9.085e-1(3.178e-	9.042e-1(3.748e-
LSMOP2	3	8.527e-1(6.738e-	8.524e-1(7.332e-	8.516e-1(7.922e-	8.497e-1(7.681e-	8.556e-1(1.025e-	8.538e-1(3.704e-	8.530e-1(4.102e-
	10	9.242e-1(8.250e-	9.218e-1(9.010e-	9.147e-1(9.882e-	9.054e-1(1.116e-	9.559e-1(1.654e-	9.629e-1(1.938e-	9.600e-1(2.263e-
LSMOP3	3	2.069e-1(1.136e-	1.919e-1(1.166e-	1.809e-1(1.268e-	1.375e-1(9.443e-	2.487e-1(4.733e-	2.356e-1(4.667e-	2.468e-1(5.022e-
	10	6.961e-1(2.341e-	6.732e-1(2.478e-	6.370e-1(2.586e-	7.383e-1(1.440e-	8.280e-1(2.944e-	8.150e-1(3.555e-	8.211e-1(4.083e-
LSMOP4	3	6.795e-1(4.072e-	6.805e-1(4.452e-	6.744e-1(4.687e-	6.681e-1(5.166e-	6.900e-1(1.907e-	6.930e-1(1.665e-	6.955e-1(1.938e-
	10	8.039e-1(1.124e-	8.320e-1(9.241e-	8.094e-1(8.283e-	7.739e-1(2.684e-	7.957e-1(1.550e-	7.917e-1(1.268e-	7.866e-1(1.548e-
LSMOP5	3	2.692e-1(9.157e-	2.665e-1(9.999e-	2.584e-1(1.096e-	2.488e-1(1.241e-	2.909e-1(1.411e-	3.089e-1(3.777e-	3.185e-1(3.986e-
	10	2.737e-1(3.081e-	2.772e-1(3.218e-	2.658e-1(1.819e-	2.688e-1(1.959e-	2.907e-1(4.084e-	2.814e-1(3.820e-	2.800e-1(4.665e-
LSMOP6	3	2.902e-2(5.999e-	3.290e-2(6.475e-	3.948e-2(7.011e-	8.886e-3(1.777e-	1.918e-3(3.322e-	1.032e-2(1.703e-	1.377e-2(1.908e-
	10	2.424e-1(1.069e-	2.354e-1(1.154e-	2.255e-1(1.261e-	2.108e-1(1.406e-	2.843e-1(4.811e-	2.842e-1(4.527e-	2.844e-1(4.495e-
LSMOP7	3	2.967e-2(4.387e-	1.841e-2(3.529e-	4.475e-3(1.001e-	5.594e-3(1.119e-	6.923e-2(4.083e-	5.192e-2(4.806e-	6.177e-2(5.369e-
	10	1.278e-1(1.002e-	9.590e-2(5.918e-	9.279e-2(6.562e-	8.105e-2(6.944e-	1.874e-1(1.146e-	1.522e-1(1.170e-	1.592e-1(1.422e-
LSMOP8	3	2.259e-1(1.838e-	2.236e-1(1.898e-	2.249e-1(2.088e-	2.170e-1(1.276e-	2.256e-1(1.244e-	2.189e-1(1.676e-	2.184e-1(2.050e-
	10	3.336e-1(8.548e-	3.143e-1(7.529e-	3.262e-1(7.765e-	3.171e-1(8.649e-	3.821e-1(1.101e-	3.534e-1(1.068e-	3.236e-1(1.086e-
LSMOP9	3	1.442e-1(5.194e-	1.396e-1(5.532e-	1.372e-1(6.147e-	1.267e-1(6.560e-	1.556e-1(1.473e-	1.662e-1(2.442e-	1.740e-1(2.314e-
	10	2.528e-3(2.587e-	2.739e-3(2.767e-	3.226e-3(2.790e-	3.617e-3(3.060e-	2.919e-3(3.734e-	3.361e-3(3.174e-	2.083e-3(2.306e-

TABLE S-V
MEAN AND STANDARD DEVIATION RESULTS OF IGD OBTAINED BY THE INVOLVED ALGORITHMS ON DTLZS.

Problem	M	D	MOEA/D	NSGAIII	MOEA/DVA	LMEA	LSMOEA/D
DTLZ1		200	1.191e-1(3.292e-2)-	7.215e+0(5.664e+0)	1.836e+0(1.597e+0)-	7.515e-2(3.542e-2)-	**5.488e-2(3.114e-4)**
	2	500	1.301e+0(1.795e-1)-	5.097e+1(6.931e+0)	1.081e+1(1.082e-1)-	**2.257e-2(9.495e-4)+**	5.527e-2(2.855e-5)
		1000	2.964e+0(2.667e-1)-	2.932e+0(3.981e+0)	1.295e+1(3.128e+0)-	8.562e-2(1.297e-4)-	**1.448e-2(1.242e-4)**
		200	3.489e+1(5.189e+0)+	5.607e+1(4.155e+0)	2.060e-1(3.721e-3)+	**6.973e-2(1.150e-3)≈**	7.156e-2(2.915e-3)
	5	500	2.846e+0(3.006e+0)+	1.123e+1(2.571e+0)	2.345e-1(6.319e-2)+	6.834e-2(8.526e-5)≈	**6.749e-2(1.015e-3)**
		1000	1.212e+1(1.909e+0)+	1.894e+1(1.491e+0)	1.509e-1(8.369e-2)+	**8.285e-2(3.295e-4)-**	2.493e-2(3.501e-2)
		200	6.382e+1(1.169e+0)+	3.584e+2(3.636e+1)	**2.750e-1(2.426e-3)-**	8.637e-1(8.758e-2)+	4.143e-1(1.675e-3)
	8	500	2.487e+2(5.586e+1)+	8.600e+2(6.879e+2)	3.247e-1(2.972e-2)+	1.154e-1(4.987e-2)+	**1.001e-1(3.433e-3)**
		1000	1.025e+2(1.682e+2)+	1.851e+2(1.392e+2)	7.281e-1(1.446e-2)+	**2.354e-1(1.331e-3)-**	4.341e-1(5.626e-3)
		200	1.675e+1(3.141e+0)+	6.488e+1(1.004e+1)	4.354e-1(1.908e-2)+	2.780e-1(1.728e-1)+	**1.329e-1(2.726e-3)**
	10	500	3.229e+1(1.691e-1)+	1.742e+1(2.383e-	1.463e-1(1.107e-2)+	3.483e-1(3.693e-3)+	**1.325e-1(4.381e-5)**
		1000	2.215e+2(3.611e+1)+	5.594e+2(3.222e-	8.411e+1(7.239e-1)+	2.989e+0(1.071e-2)-	**1.129e-1(3.846e-3)**
DTLZ2		200	8.058e-1(4.762e-4)+	7.371e-1(2.114e-3)+	5.238e-1(1.024e-2)+	**2.403e-1(5.582e-2)-**	2.654e-1(4.614e-3)
	2	500	9.060e-1(7.860e-3)+	1.573e+0(1.131e-	2.010e+0(1.467e-2)+	5.740e-1(2.124e-1)+	**3.233e-1(2.295e-2)**
		1000	5.149e+0(7.254e+0)+	3.003e+1(1.824e+0)	9.311e+0(1.053e-1)+	**1.618e-1(4.868e-1)-**	1.838e-1(8.930e-2)
		200	5.897e-1(1.951e-3)+	5.037e+0(2.723e-	**2.122e-1(2.170e-5)-**	2.142e-1(7.581e-3)-	3.569e-1(4.974e-3)
	5	500	5.048e+0(7.840e-2)+	2.161e+1(3.211e+0)	8.231e+0(5.064e-1)+	3.784e-1(4.038e-2)+	**2.466e-1(1.600e-4)**
		1000	2.688e+1(8.839e+0)+	5.344e+1(2.758e+0)	3.756e+1(1.016e-1)+	7.801e-1(8.063e-3)+	**5.781e-1(1.411e-3)**
		200	1.023e+0(4.207e-2)+	9.373e-1(5.762e-2)+	6.363e-1(1.217e-2)+	4.091e-1(3.201e-2)+	**3.869e-1(1.711e-5)**
	8	500	1.027e+0(8.892e-1)+	4.952e+1(6.165e+0)	6.936e+0(3.478e-1)+	**3.787e-1(7.247e-3)+**	4.019e-1(3.638e-3)
		1000	3.831e+1(2.231e+0)+	9.752e+1(9.087e+0)	3.343e+1(7.464e-1)+	7.797e-1(5.011e-2)+	**5.364e-1(1.081e-3)**
		200	6.771e-1(4.029e-2)+	2.678e-1(2.017e-2)+	8.053e-1(3.286e-2)+	**4.735e-1(9.365e-3)-**	5.003e-1(1.165e-5)
	10	500	6.856e+0(1.226e+0)+	5.588e+1(4.814e+0)	6.470e+0(3.083e-1)+	3.746e-1(4.428e-3)+	**5.195e-1(2.200e-3)**
		1000	3.824e+0(7.347e-1)+	2.048e+0(1.513e-	9.085e-1(3.708e-3)+	7.838e-1(3.919e-3)+	**7.692e-1(1.791e-2)**
DTLZ3		200	5.183e-1(1.161e-2)+	1.940e+1(1.037e-	6.077e-1(2.005e-1)+	**1.016e-1(1.433e-2)-**	2.133e-1(2.875e-2)
	2	500	9.388e-1(8.409e-2)+	3.770e+2(1.950e-	5.762e-1(8.673e-2)+	1.753e-1(1.620e-2)+	**1.162e-1(6.764e-2)**
		1000	8.135e-1(1.147e-3)+	1.050e+2(1.401e-	2.188e-1(3.023e-1)+	1.743e-1(7.657e-2)+	**1.493e-1(5.334e-2)**
		200	1.942e+2(4.211e+0)+	2.463e+2(4.371e+1)	7.384e-1(4.989e-2)+	**2.150e-1(5.015e-4)≈**	2.225e-1(2.954e-3)
	5	500	2.933e+1(4.845e-1)+	9.957e+1(5.163e+0)	9.035e-1(1.958e-3)+	2.130e-1(2.601e-4)≈	**2.084e-1(1.188e-4)**
		1000	2.922e+2(1.548e+1)+	8.451e+2(3.101e+0)	5.792e-1(4.776e-4)-	**2.489e-1(1.517e-3)-**	1.026e+0(2.523e-3)
		200	2.069e+1(8.142e+0)+	2.372e+2(9.291e+1)	9.752e-1(1.675e-2)+	4.088e-1(1.284e-2)+	**3.289e-1(1.479e-2)**
	8	500	1.141e+2(7.484e+0)+	5.593e+3(9.302e+2)	**1.395e-1(3.967e-2)-**	4.939e-1(1.740e-2)+	3.994e-1(1.177e-2)
		1000	1.267e+2(1.768e+1)+	1.084e+3(2.379e+2)	5.429e-1(1.463e-2)+	**1.016e-1(1.460e-2)-**	4.367e-1(1.984e-2)
		200	8.053e+1(1.566e+0)+	3.748e+2(2.360e+1)	5.830e-1(2.356e-3)+	5.042e-1(1.097e-3)≈	**4.971e-1(6.959e-3)**
	10	500	3.423e+2(1.708e-1)+	2.159e+2(4.407e+1)	6.929e-1(1.330e-2)+	9.422e-1(3.623e-2)+	**4.808e-1(5.159e-4)**
		1000	3.792e+3(9.616e+0)+	8.125e+3(2.097e+1)	5.056e+2(5.207e-1)+	7.477e-1(6.321e-2)-	**3.011e-1(1.304e-3)**
DTLZ4		200	6.847e-1(5.782e-4)+	5.207e-1(9.534e-3)+	2.395e-1(2.632e-2)+	2.049e-1(4.046e-2)+	**1.138e-1(3.527e-3)**
	2	500	1.339e+0(1.880e-1)+	9.185e-1(4.365e-2)+	1.090e+0(1.517e-1)+	7.903e-1(1.211e-3)+	**5.348e-1(9.881e-2)**
		1000	5.043e+0(6.684e-1)+	6.324e+0(2.041e-	6.862e+0(9.625e-1)+	**1.180e-1(3.562e-2)-**	2.954e-1(1.012e-2)
		200	4.270e-1(5.389e-5)+	2.123e-1(4.615e-5)-	1.145e+0(2.485e-3)+	**2.122e-1(1.213e-6)-**	3.403e-1(1.860e-1)
	5	500	5.364e-1(1.552e-1)+	3.304e-1(1.671e-1)+	1.133e+0(1.778e-3)+	2.122e-1(1.354e-6)≈	**2.070e-1(4.354e-4)**
		1000	**1.367e-1(1.439e-3)-**	5.615e-1(7.040e-2)-	3.565e+0(5.976e-1)+	7.156e-1(4.398e-3)-	7.680e-1(1.416e-3)
		200	1.003e+0(2.988e-1)-	2.007e-1(1.063e-2)-	7.260e-1(1.147e-2)-	**4.601e-1(4.009e-2)-**	2.001e-1(2.824e-2)
	8	500	1.062e+0(2.252e-1)+	4.534e-1(7.340e-2)+	7.365e+0(3.536e-1)+	**3.723e-1(7.586e-2)-**	4.002e-1(1.977e-3)
		1000	1.107e+0(1.191e-1)+	8.986e+0(1.656e-	3.361e+1(3.797e-1)+	7.915e-1(4.629e-3)+	**4.235e-1(2.964e-2)**
		200	6.920e-1(4.535e-4)+	5.014e-1(9.814e-4)+	1.270e+0(1.577e-3)+	5.227e-1(2.684e-2)+	**4.776e-1(2.238e-3)**
	10	500	9.996e-1(3.424e-1)+	**5.004e-1(1.511e-4)-**	1.258e+0(2.628e-4)+	8.454e+0(1.854e-	5.499e-1(1.005e-1)
		1000	8.330e-1(9.970e-2)+	2.020e+0(3.186e-	1.033e+0(2.297e-3)+	6.820e-1(1.133e-3)+	**5.833e-1(2.598e-2)**
DTLZ5		200	4.265e-2(1.888e-2)+	4.011e-2(3.547e-5)+	1.002e-1(6.987e-3)+	**2.105e-2(2.791e-3)-**	2.940e-2(1.822e-3)
	2	500	5.767e-2(7.912e-1)+	2.046e-1(2.107e-2)+	5.984e-1(9.110e-4)+	**1.156e-2(2.278e-4)-**	4.168e-2(6.388e-4)
		1000	8.430e-1(4.260e-3)+	1.534e-1(2.098e-1)+	4.961e-1(1.557e-4)+	3.722e-2(6.123e-3)+	**2.010e-2(3.151e-3)**
		200	3.066e-2(1.313e-5)+	9.812e-1(2.280e-1)+	3.213e-1(1.129e-3)+	5.068e-2(7.520e-5)+	**5.618e-3(2.436e-4)**
	5	500	3.068e-2(1.401e-6)+	9.751e-1(4.579e-2)+	1.693e-1(6.001e-2)+	5.178e-2(8.511e-6)+	**5.666e-3(6.554e-4)**
		1000	4.759e-2(1.271e-4)-	7.112e+0(6.295e-	3.694e-1(1.020e-2)+	**1.199e-2(6.083e-4)-**	7.590e-2(9.109e-3)
		200	5.428e-1(2.213e-1)+	1.400e+0(1.681e-	**6.587e-3(1.360e-3)-**	8.225e-2(6.769e-3)+	1.373e-2(7.220e-3)
	8	500	1.300e+1(7.376e-1)+	3.908e+0(3.367e-	7.594e+0(7.828e-2)+	3.698e-2(2.253e-4)+	**1.170e-2(2.527e-3)**
		1000	3.685e+1(1.536e+0)+	8.023e+0(3.297e-	3.348e+1(7.236e-1)+	7.709e-2(1.598e-3)+	**6.078e-2(8.114e-4)**
		200	7.996e-2(1.094e-2)+	1.549e+0(2.709e-	2.788e-1(7.271e-2)+	5.376e-2(1.299e-2)+	**7.866e-3(3.857e-4)**
	10	500	7.616e+0(2.199e+0)+	3.796e+0(8.380e-	6.888e+0(4.940e-1)+	**3.751e-1(3.942e-2)-**	5.134e-1(1.018e-1)
		1000	3.282e-1(1.846e-2)+	1.808e+0(4.563e-	1.134e+0(2.653e-1)+	7.769e-2(9.549e-3)+	**3.765e-2(2.265e-3)**
DTLZ6		200	6.098e-1(5.512e-2)+	5.704e-1(1.273e-2)+	5.852e-1(4.054e-2)+	**5.165e-3(7.449e-5)-**	5.779e-3(1.904e-4)
	2	500	5.754e+0(7.757e-1)+	6.051e-1(4.155e-4)+	5.965e-1(2.091e-2)+	5.020e-3(2.297e-4)+	**3.928e-3(1.332e-4)**
		1000	5.974e+0(9.196e-1)+	5.719e+0(4.189e-	5.602e-1(3.861e-2)+	5.190e-3(3.737e-4)+	**4.605e-3(7.989e-5)**
	5	200	6.986e-1(7.045e-2)+	2.001e+1(2.599e+0)	1.279e-1(4.075e-2)+	5.075e-2(1.601e-4)+	**6.411e-3(4.407e-5)**
		500	3.128e+0(1.303e-1)+	3.379e+1(1.337e+1)	2.996e-1(1.023e-2)+	5.498e-2(4.450e-3)+	**5.267e-3(4.841e-4)**

		1000	7.870e+2(8.504e-1)+	8.821e+2(5.874e-	8.429e-1(1.468e-2)+	**4.472e-3(6.314e-4)-**	8.977e-2(1.308e-3)
	8	200	1.319e+0(1.263e+0)-	**1.028e+0(1.267e-3)-**	7.351e-1(1.384e-2)-	5.089e-2(1.968e-4)-	1.695e-2(9.897e-4)
		500	3.769e+2(5.062e-2)+	4.363e+2(1.174e+0)	3.884e-1(1.995e-2)+	4.436e-2(4.898e-3)+	**1.788e-2(1.805e-3)**
		1000	8.192e+2(1.354e+1)+	8.811e+2(3.316e-	8.335e-1(1.593e-2)+	**4.954e-2(2.295e-3)+**	5.706e-2(1.969e-4)
	10	200	3.600e-1(5.392e-2)+	9.535e+1(8.346e+0)	1.222e+0(3.856e-1)+	6.936e-2(2.425e-3)+	**9.328e-3(9.276e-4)**
		500	1.007e+0(4.840e-2)+	2.054e+2(2.582e+0)	2.904e+0(7.745e-1)+	**8.381e-3(7.313e-4)-**	8.947e-3(3.807e-4)
		1000	8.075e+0(1.677e-1)+	8.779e+2(6.689e+0)	8.299e+0(2.019e-1)+	8.660e-2(8.163e-3)-	**8.953e-3(9.508e-4)**
DTLZ7	2	200	6.029e-1(4.170e-2)+	5.799e-1(4.833e-2)+	6.189e-1(3.634e-2)+	**4.430e-1(7.592e-3)-**	5.494e-1(1.915e-3)
		500	6.159e-1(1.984e-3)+	5.789e+0(1.579e-	4.429e-1(6.693e-2)+	5.429e-1(2.567e-2)+	**4.428e-1(1.281e-3)**
		1000	5.919e-1(2.642e-2)+	5.959e+0(1.844e-	6.440e-1(1.237e-2)+	4.431e-1(2.369e-2)+	**3.529e-1(6.032e-3)**
	5	200	1.100e+0(4.639e-2)+	4.098e-1(1.507e-2)+	4.116e+0(1.520e-2)+	8.416e-1(2.900e-2)+	**3.574e-1(6.900e-3)**
		500	1.069e+0(2.427e-4)+	3.800e-1(2.473e-2)+	3.541e+0(7.091e-3)+	7.392e-1(4.357e-3)+	**3.628e-1(7.957e-3)**
		1000	1.071e+1(1.310e-1)+	9.595e+0(6.271e-	1.087e+1(8.586e-2)+	**1.708e+0(1.254e-1)-**	1.982e+0(3.691e-1)
	8	200	1.654e+0(1.837e-2)-	**7.598e-1(3.799e-2)-**	1.375e+0(1.091e-1)-	1.830e+0(3.281e-	1.788e+0(1.981e+0)
		500	1.051e+1(1.139e+0)+	2.270e+1(2.149e-	9.688e+0(9.339e-1)+	3.198e+0(4.945e-	**1.177e+0(2.313e-2)**
		1000	1.955e+1(1.455e+0)+	2.526e+1(1.336e-	1.719e+1(4.124e-2)+	3.277e+0(5.666e-	**1.084e+0(2.188e-2)**
	10	200	2.630e+0(3.389e-3)+	1.570e+0(1.950e-	8.326e+0(1.227e-1)+	2.429e+0(4.957e-	**1.039e+0(3.602e-3)**
		500	2.285e+0(4.764e-1)+	1.677e+0(3.801e-	7.380e+0(1.670e-2)+	4.450e+0(3.393e-	**1.141e+0(1.556e-2)**
		1000	2.580e+0 (5.141e-1)+	2.488e+0 (1.657e-	1.004e+0 (4.320e-3)+	1.306e+0 (1.025e-	**9.040e-1 (1.399e-2)**

"+", "-", and "≈" indicate the result obtained by LSMOEA/D is significantly better, significantly worse and statistically similarto that obtained by the compared algorithm, respectively.

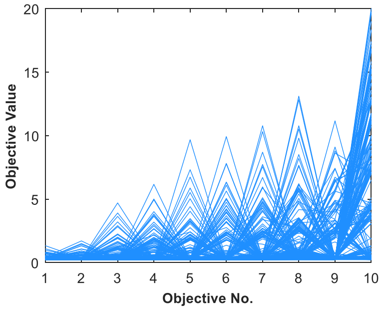
TABLE S-VI

MEAN AND STANDARD DEVIATION RESULTS OF IGD OBTAINED BY THE INVOLVED ALGORITHMS ON WFGS.

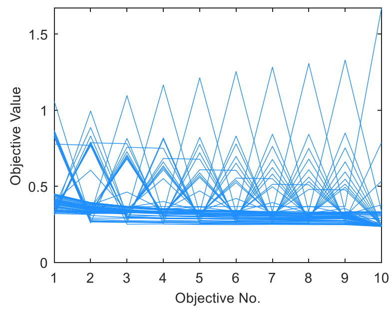
Problem	M	D	MOEA/D	NSGAIII	MOEA/DVA	LMEA	LSMOEA/D
WFG1	5	200	1.930e+0(2.379e-2)+	1.942e+0(8.657e-2)+	2.794e+0(1.459e-1)+	2.463e+0(4.360e-2)+	**9.060e-1(4.707e-3)**
		500	2.054e+0(6.561e-3)+	2.034e+0(3.537e-2)+	2.923e+0(1.271e-1)+	2.851e+0(4.297e-2)+	**1.403e+0(4.052e-2)**
		1000	2.067e+0(4.073e-2)+	2.081e+0(2.773e-3)+	2.931e+0(4.554e-3)+	2.809e+0(1.175e-1)+	**1.828e+0(5.614e-2)**
	8	200	2.679e+0(4.127e-2)+	2.815e+0(7.006e-2)+	3.456e+0(7.403e-2)+	3.039e+0(1.863e-1)+	**1.967e+0(7.277e-3)**
		500	2.828e+0(1.840e-2)+	2.801e+0(1.155e-1)+	3.554e+0(4.040e-3)+	3.262e+0(4.335e-2)+	**2.247e+0(2.549e-2)**
		1000	2.927e+0(2.492e-3)+	2.823e+0(9.513e-2)+	3.508e+0(1.096e-1)+	3.574e+0(3.474e-2)+	**2.708e+0(2.613e-2)**
	10	200	3.108e+0(1.335e-1)+	3.088e+0(1.866e-1)+	3.918e+0(1.510e-2)+	3.158e+0(8.473e-2)+	**2.222e+0(1.053e-2)**
		500	3.300e+0(1.447e-1)+	3.178e+0(1.353e-1)+	3.725e+0(2.683e-1)+	3.658e+0(6.733e-2)+	**2.502e+0(2.114e-3)**
		1000	3.352e+0(3.980e-2)+	3.753e+0(4.293e-2)+	3.325e+0(4.778e-2)+	3.307e+0(6.116e-2)+	**3.112e+0(4.331e-2)**
WFG2	5	201	5.177e+0(1.089e-1)+	1.075e+0(9.745e-2)-	9.084e-1(1.637e-5)-	**6.938e-1(1.507e-1)-**	4.735e+0(7.962e-2)
		501	5.084e+0(2.077e-1)+	1.116e+0(7.516e-2)+	1.035e+0(1.575e-3)+	1.375e+0(2.680e-2)+	**1.021e+0(2.536e-2)**
		1001	4.662e+0(5.829e-1)+	1.152e+0(9.201e-2)-	**1.043e+0(1.426e-2)-**	1.442e+0(2.323e-2)-	3.767e+0(1.340e-1)
	8	201	8.676e+0(9.032e-2)-	**1.490e+0(2.109e-1)-**	2.273e+0(3.009e-1)-	1.702e+0(3.842e-1)-	8.710e+0(4.827e-1)
		501	8.732e+0(3.553e-2)+	5.532e+0(1.773e-1)-	**2.370e+0(3.431e-1)-**	2.664e+0(1.348e-1)-	6.265e+0(1.136e+0)
		1001	8.111e+0(7.078e-1)+	1.688e+0(1.240e-1)+	2.515e+0(1.850e-1)+	2.473e+0(8.516e-2)+	**1.607e+0(1.103e-1)**
	10	201	4.644e+0(3.745e-2)+	2.079e+0(3.209e-1)-	**2.005e+0(3.019e-1)-**	2.558e+0(2.080e-1)-	2.640e+0(6.406e-1)
		501	7.697e+0(3.808e-2)+	2.415e+0(6.164e-2)-	3.597e+0(2.875e-1)-	**4.066e+0(2.379e-2)-**	6.704e+0(3.122e+0)
		1001	3.591e+0(1.481e+0)+	5.579e+0(7.934e-1)+	5.033e+0(2.789e-3)+	2.628e+0(4.655e-1)+	**2.458e+0(4.047e-1)**
WFG3	5	201	1.660e+0(4.334e-1)+	1.110e+0(2.723e-2)+	1.040e+0(4.582e-1)+	1.107e+0(7.795e-2)+	**9.441e-2(3.064e-3)**
		501	1.957e+0(1.063e-1)+	1.290e+0(9.295e-2)+	7.867e-1(1.183e-2)+	1.203e+0(2.526e-3)+	**5.045e-1(1.589e-2)**
		1001	1.656e+0(8.525e-2)+	1.272e+0(4.349e-2)+	**8.677e-1(1.107e-2)-**	1.244e+0(6.769e-3)+	1.255e+0(1.194e-1)
	8	201	4.238e+0(6.113e-2)+	2.217e+0(2.441e-1)+	2.520e+0(1.722e-2)+	4.271e+0(2.695e-2)+	**1.992e-1(3.960e-2)**
		501	4.302e+0(6.551e-3)+	2.191e+0(1.875e-2)+	1.327e+0(1.921e-2)+	1.245e+0(5.846e-1)+	**1.044e+0(1.907e-1)**
		1001	4.188e+0(2.298e-2)+	2.139e+0(4.473e-2)+	1.469e+0(1.604e-1)+	8.520e+0(2.936e-3)+	**8.463e-1(3.432e-2)**
	10	201	6.096e+0(9.099e-2)+	2.773e+0(7.696e-2)+	1.602e+0(1.566e-1)+	5.826e+0(1.047e-1)+	**2.878e-1(4.400e-3)**
		501	6.266e+0(6.530e-2)+	2.800e+0(5.053e-3)+	1.801e+0(1.145e-1)+	5.814e+0(6.976e+0)+	**1.026e+0(1.523e-1)**
		1001	5.753e+0(1.709e-2)+	2.874e+0(2.899e-1)+	1.724e+0(4.956e-2)+	3.337e+0(6.203e+0)+	**1.201e+0(2.617e-1)**
WFG4	5	200	2.251e+0(1.969e-1)+	1.596e+0(1.145e-2)+	2.137e+0(2.044e-4)+	1.471e+0(3.217e-1)+	**1.440e+0(4.774e-3)**
		500	1.821e+0(6.536e-2)+	1.315e+0(7.757e-4)+	2.163e+0(1.799e-3)+	2.568e+0(7.427e-2)+	**1.276e+0(4.566e-3)**
		1000	1.847e+0(2.790e-1)+	1.353e+0(2.805e-2)+	2.175e+0(4.504e-4)+	2.637e+0(2.807e-1)+	**1.265e+0(1.380e-2)**
	8	200	7.923e+0(1.291e-1)+	**3.446e+0(1.116e-2)-**	6.891e+0(2.731e-1)+	4.485e+0(8.988e-2)+	4.954e+0(3.956e-1)
		500	6.048e+0(8.410e-1)+	3.432e+0(1.899e-2)+	7.437e+0(6.469e-1)+	7.161e+0(4.603e-1)+	**3.382e+0(5.735e-2)**
		1000	4.611e+0(1.155e+0)+	3.489e+0(2.458e-3)+	7.311e+0(3.736e-1)+	7.427e+0(2.290e-1)+	**3.229e+0(2.050e-2)**
	10	200	8.907e+0(2.772e-2)+	6.124e+0(2.498e-2)+	9.703e+0(1.180e+0)+	**6.024e+0(8.588e-2)-**	8.447e+0(1.143e-1)
		500	8.147e+0(3.927e-1)+	6.896e+0(1.395e-2)+	9.075e+0(1.182e+0)+	7.075e+0(4.903e-1)+	**6.099e+0(6.568e-2)**
		1000	9.891e+0(2.246e+0+	6.947e+0(8.318e-2)-	7.124e+0(2.583e-2)+	5.026e+0(2.141e-1)+	**3.482e+0(3.022e-1)**
WFG5	5	200	1.972e+0(1.439e-1)+	1.308e+0(1.579e-2)-	1.598e+0(6.347e-4)-	**1.245e+0(4.121e-2)-**	1.620e+0(9.329e-2)
		500	1.951e+0(3.208e-1)+	1.396e+0(3.962e-2)+	1.689e+0(3.761e-4)+	2.031e+0(4.207e-2)+	**1.259e+0(4.247e-2)**
		1000	1.801e+0(1.641e-1)+	1.457e+0(1.617e-2)+	1.723e+0(2.379e-4)+	2.093e+0(8.988e-2)+	**1.371e+0(1.625e-3)**
	8	200	6.939e+0(7.105e-1)+	6.489e+0(9.272e-2)+	**3.941e+0(4.438e-1)-**	3.540e+0(1.784e-1)-	4.248e+0(2.715e-1)

		500	7.885e+0(1.021e-2)+	3.499e+0(4.482e-2)+	5.687e+0(3.549e-1)+	5.186e+0(6.251e-2)+	**3.386e+0(4.072e-2)**
		1000	6.860e+0(1.969e-2)+	3.588e+0(4.336e-2)-	5.598e+0(1.960e-1)+	**4.462e+0(2.188e-1)-**	4.637e+0(4.671e-1)
		200	1.004e+1(3.688e-2)+	4.970e+0(5.493e-2)-	7.989e+0(4.125e-1)-	**4.270e+0(1.306e-1)-**	9.686e+0(9.576e-2)
	10	500	1.050e+1(1.783e-1)+	5.038e+0(5.541e-2)-	8.087e+0(4.931e-1)+	**5.063e+0(8.449e-1)-**	6.534e+0(2.553e-1)
		1000	1.055e+1(1.409e-1)+	7.025e+0(1.474e-2)+	8.246e+0(7.768e-1)+	8.400e+0(4.617e-1)+	**6.232e+0(1.551e-1)**
WFG6	5	200	2.442e+0(1.716e-1)+	1.392e+0(3.243e-2)+	1.578e+0(5.020e-4)+	1.676e+0(5.122e-2)+	**1.153e+0(1.016e-2)**
		500	2.058e+0(2.486e-2)+	1.482e+0(1.280e-3)+	1.673e+0(3.990e-3)+	2.328e+0(3.899e-2)+	**1.246e+0(3.066e-2)**
		1000	1.986e+0(3.393e-1)+	1.554e+0(1.479e-2)+	1.929e+0(2.340e-3)+	2.367e+0(1.791e-1)+	**1.407e+0(2.121e-3)**
	8	200	7.974e+0(1.125e+0)+	3.610e+0(4.188e-2)+	4.890e+0(5.961e-1)+	4.621e+0(2.019e-1)+	**3.302e+0(2.259e-2)**
		500	7.520e+0(2.686e-1)+	4.928e+0(2.298e-2)+	**4.606e+0(2.664e-1)-**	6.143e+0(1.957e-1)+	4.651e+0(3.831e-2)
		1000	4.966e+0(1.846e-1)+	4.668e+0(3.415e-2)+	5.872e+0(6.709e-1)+	**4.258e+0(1.263e-1)-**	4.370e+0(2.765e-2)
	10	200	1.115e+1(8.921e-2)+	5.015e+0(7.978e-2)+	8.241e+0(8.015e-1)+	9.482e+0(3.657e-1)+	**4.801e+0(8.445e-2)**
		500	1.147e+1(9.087e-2)+	5.059e+0(7.774e-2)+	8.224e+0(6.963e-1)+	8.672e+0(5.729e-1)+	**4.878e+0(3.374e-3)**
		1000	1.069e+1(1.076e+0)+	7.101e+0(7.325e-2)+	6.658e+0(1.196e-1)+	**6.275e+0(1.748e-1)-**	6.502e+0(4.465e-1)
WFG7	5	200	2.184e+0(2.199e-1)+	1.502e+0(1.197e-2)+	2.221e+0(9.096e-2)+	**1.206e+0(4.639e-2)-**	1.622e+0(6.917e-2)
		500	2.138e+0(1.755e-1)+	1.552e+0(4.448e-2)+	2.396e+0(3.272e-1)+	1.423e+0(7.285e-3)+	**1.228e+0(4.400e-2)**
		1000	1.715e+0(4.314e-2)+	1.494e+0(6.985e-2)+	2.110e+0(4.863e-2)+	1.478e+0(1.296e-1)+	**1.390e+0(2.545e-2)**
	8	200	7.843e+0(4.061e-1)+	3.712e+0(1.051e-2)-	4.142e+0(1.619e-1)-	**3.444e+0(1.281e-1)-**	4.755e+0(7.327e-1)
		500	5.955e+0(1.790e+0)+	3.690e+0(4.326e-2)+	6.194e+0(1.864e-1)+	3.852e+0(1.164e-1)+	**3.406e+0(1.312e-2)**
		1000	4.644e+0(7.636e-1)+	3.699e+0(2.003e-3)+	5.919e+0(6.085e-1)+	4.045e+0(6.558e-2)+	**3.521e+0(4.542e-2)**
	10	200	1.108e+1(9.701e-2)-	8.015e+0(2.902e-2)-	9.050e+0(2.233e-1)-	**5.215e+0(3.194e-1)-**	1.016e+1(2.044e-1)
		500	1.104e+1(2.186e-2)+	5.078e+0(7.940e-2)+	9.550e+0(1.385e+0)+	6.572e+0(3.089e-1)+	**4.934e+0(5.998e-2)**
		1000	1.138e+1(1.000e-1)+	5.078e+0(4.055e-2)+	8.306e+0(2.182e-1)+	5.737e+0(4.365e-1)+	**3.338e+0(2.446e+0)**
WFG8	5	200	2.154e+0(1.074e-1)+	1.319e+0(7.817e-3)+	1.604e+0(8.687e-4)+	1.673e+0(7.423e-2)+	**1.171e+0(1.594e-2)**
		500	2.276e+0(1.894e-1)+	1.366e+0(1.422e-3)+	1.647e+0(8.565e-4)+	**1.318e+0(2.223e-2)-**	1.392e+0(3.943e-3)
		1000	2.317e+0(4.587e-2)+	1.433e+0(2.981e-2)+	1.899e+0(1.342e-1)+	2.144e+0(1.620e-1)+	**1.399e+0(9.735e-3)**
	8	200	8.016e+0(6.760e-1)+	3.627e+0(7.337e-2)+	4.436e+0(1.339e-1)+	6.057e+0(2.793e-1)+	**3.295e+0(3.449e-2)**
		500	6.022e+0(1.588e+0)+	3.683e+0(1.137e-1)-	5.736e+0(6.338e-2)-	**3.572e+0(8.211e-2)-**	4.376e+0(1.163e-1)
		1000	7.783e+0(5.015e-1)+	3.603e+0(5.614e-3)+	5.918e+0(6.775e-1)+	5.654e+0(6.834e-2)+	**3.397e+0(2.228e-2)**
	10	200	1.057e+1(1.398e-1)+	5.068e+0(9.032e-3)+	8.018e+0(1.326e-1)+	5.021e+0(1.632e-1)+	**4.859e+0(9.012e-2)**
		500	1.069e+1(1.196e-1)+	9.036e+0(7.333e-2)+	8.593e+0(4.276e-1)+	**5.326e+0(4.191e-3)-**	6.534e+0(5.085e-1)
		1000	1.109e+1(5.108e-2)+	7.113e+0(1.654e-2)+	8.758e+0(6.517e-2)+	6.329e+0(3.919e-1)+	**4.956e+0(4.741e-2)**
WFG9	5	200	2.257e+0(1.315e-1)+	1.836e+0(7.092e-2)+	2.136e+0(9.631e-2)+	**1.282e+0(5.030e-3)-**	1.593e+0(1.119e-1)
		500	1.980e+0(3.250e-2)+	1.942e+0(5.478e-2)+	2.517e+0(9.259e-2)+	1.652e+0(4.327e-2)+	**1.397e+0(6.493e-2)**
		1000	1.946e+0(3.369e-1)+	1.942e+0(6.902e-2)+	2.615e+0(5.950e-2)+	1.647e+0(1.375e-1)+	**1.553e+0(2.743e-3)**
	8	200	7.960e+0(5.338e-1)+	4.093e+0(1.661e-2)+	4.415e+0(1.773e-1)+	4.966e+0(8.460e-1)+	**3.432e+0(3.124e-2)**
		500	7.570e+0(2.510e-1)+	4.172e+0(9.555e-5)+	5.856e+0(3.173e-1)+	**3.706e+0(1.702e-1)-**	3.913e+0(1.727e-3)
		1000	7.809e+0(2.763e-1)+	4.229e+0(1.012e-1)+	6.818e+0(1.231e-1)+	4.265e+0(5.568e-1)+	**3.911e+0(6.744e-2)**
	10	200	1.033e+1(7.322e-1)+	5.798e+0(1.367e-2)+	8.194e+0(1.044e-1)+	9.994e+0(1.242e-1)+	**4.946e+0(1.284e-1)**
		500	1.051e+1(8.399e-1)+	5.525e+0(1.010e-2)-	9.835e+0(4.584e-1)+	**5.239e+0(1.173e-1)-**	6.190e+0(2.897e-1)
		1000	8.209e+0(2.298e+0)+	6.447e+0(9.451e-2)+	9.109e+0(1.005e+0)+	4.468e+0(1.804e+0)+	**4.204e+0(3.391e-2)**

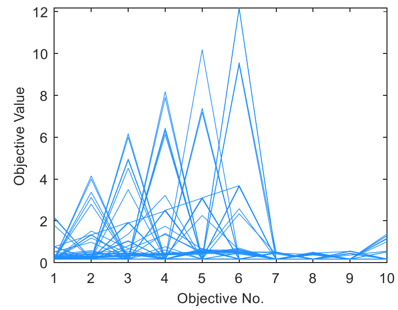
"+", "-", and "≈" indicate the result obtained by LSMOEA/D is significantly better, significantly worse and statistically similarto that obtained by the compared algorithm, respectively.



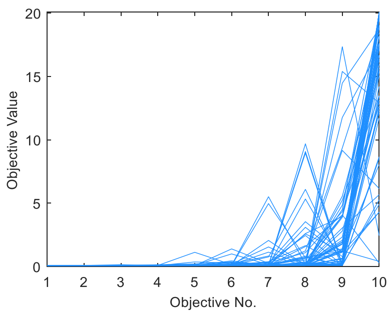
(a) LSMOEA/D



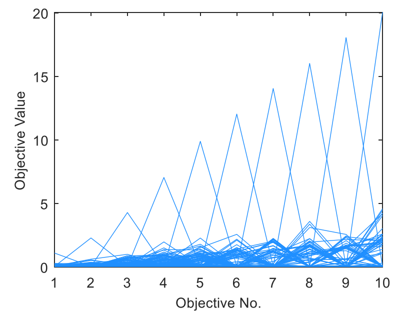
(b) MOEA/D



(c) NSGA-III

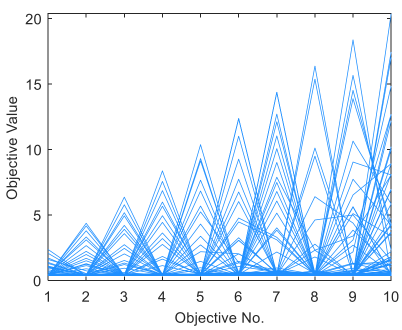


(d) MOEA/DVA

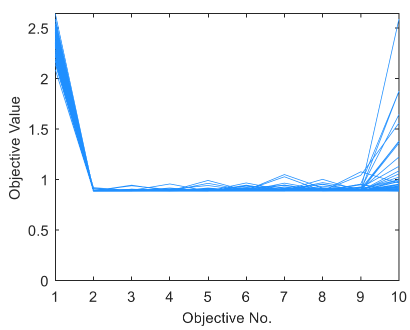


(e) LMEA

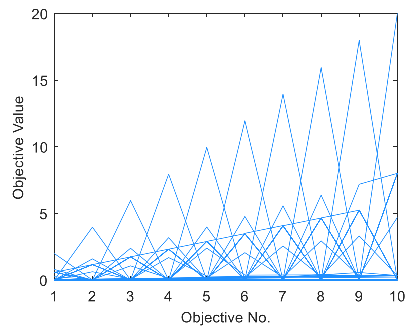
Fig.S2. Final solution set of involved algorithms on the 10-objective WFG2 with 1000 decision variables, shown by parallel coordinates.



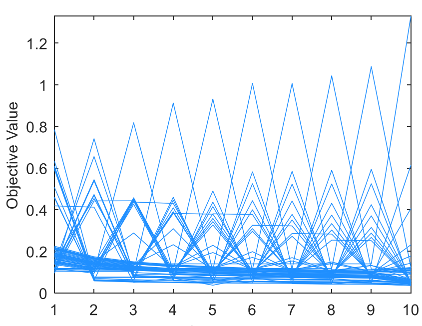
(a) LSMOEA/D



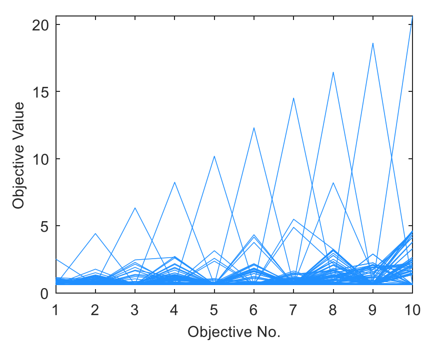
(b) MOEA/D



(c) NSGA-III

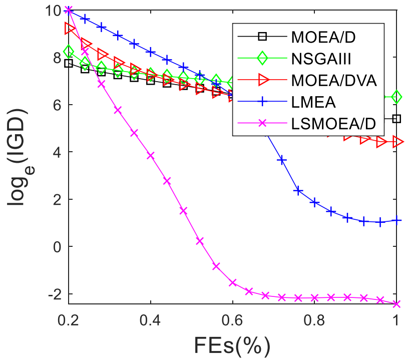


(d) MOEA/DVA

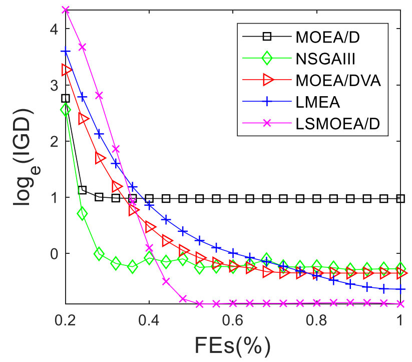


(e) LMEA

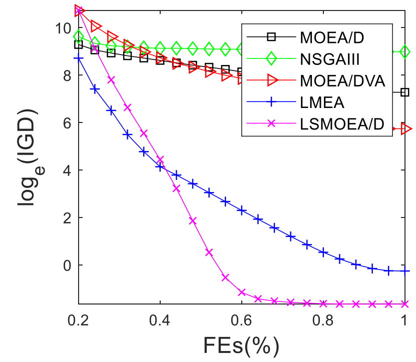
Fig.S3. Final solution set of involved algorithms on the 10-objective WFG1 with 1000 decision variables, shown by parallel coordinates.



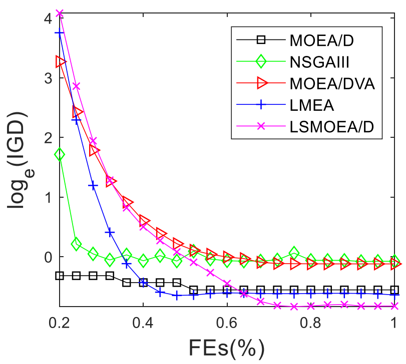
(a) DTLZ1



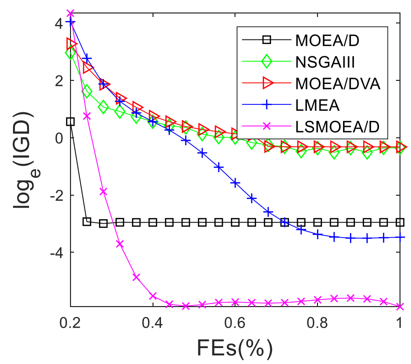
(b) DTLZ2



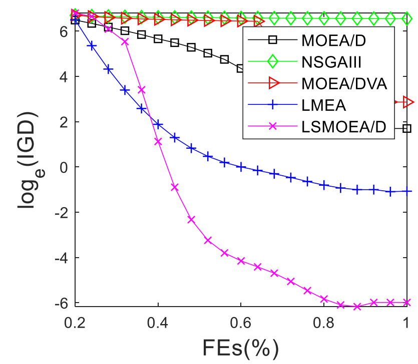
(c) DTLZ3



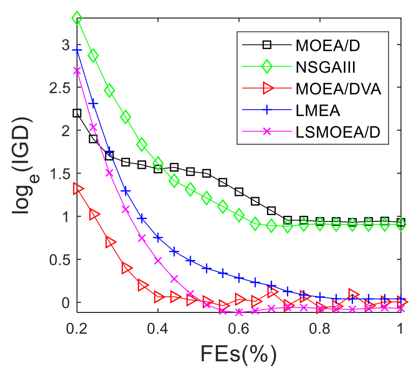
(d) DTLZ4



(e) DTLZ5

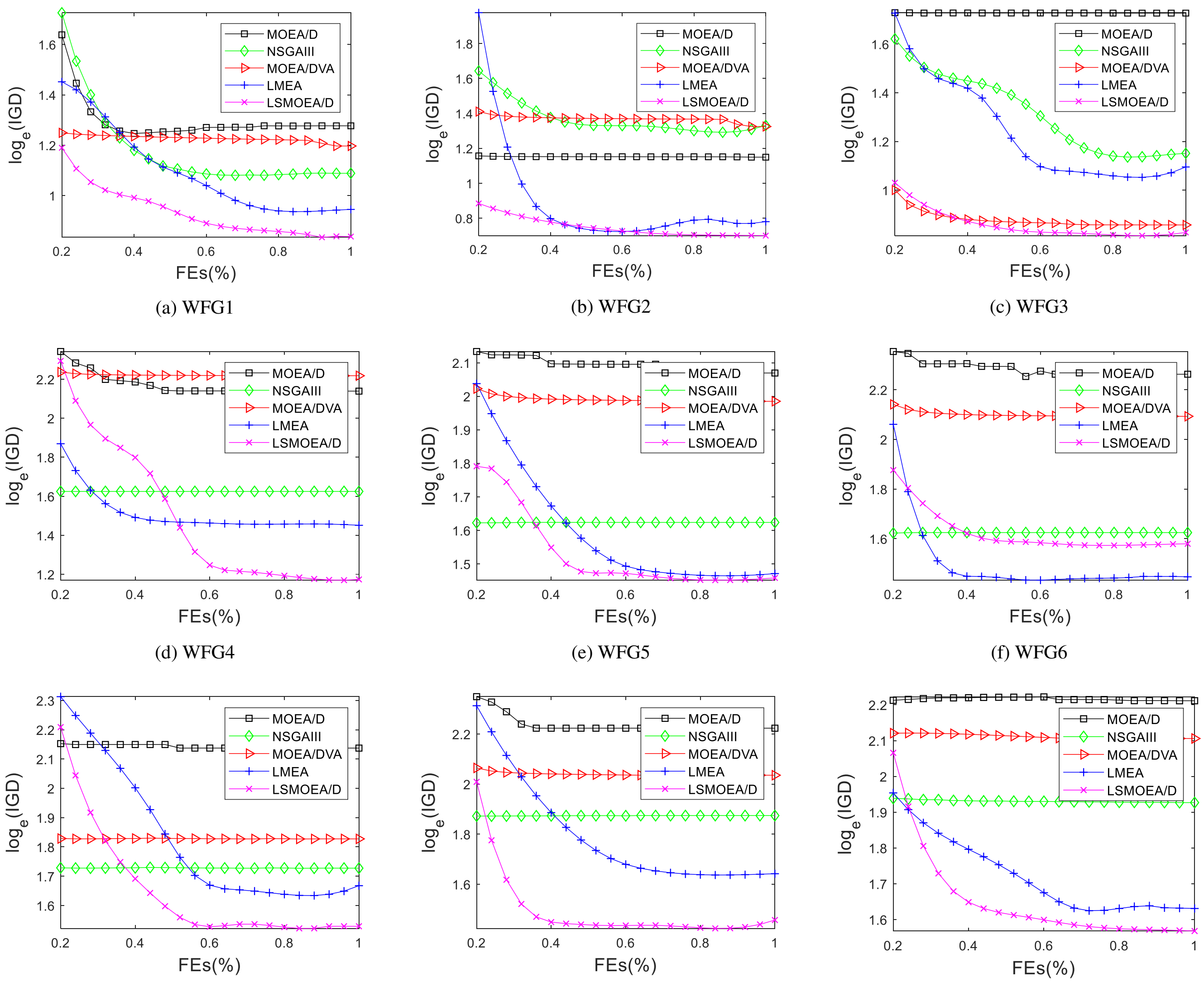


(f) DTLZ6



(g) DTLZ7

Fig. S4. Evolutionary trajectories of IGD on the 10-objective DTLZ test problems with 1000 variables. The abscissa is the percentage of consumed FEs.



(a) WFG1 (b) WFG2 (c) WFG3

(d) WFG4 (e) WFG5 (f) WFG6

(g) WFG7 (h) WFG8 (i) WFG9

Fig. S5. Evolutionary trajectories of IGD on the 10-objective WFG test problems with 1000 variables. The abscissa is the percentage of consumed FEs.

TABLE S-VII

MEAN AND STANDARD DEVIATION RESULTS IN TERMS OF HV METRIC OBTAINED BY THE INVOLVED ALGORITHMS ON DTLZS AND WFGS.

Problem	M	D	MOEA/D	NSGAIII	MOEA/DVA	LMEA	LSMOEA/D
DTLZ1	5	1000	3.7739e-1(2.84e-3)+	4.9654e-1(1.42e-2)+	4.9175e-1(2.14e-2)+	**9.3969e-1(5.12e-3)-**	9.0299e-1(1.76e-3)
		5000	2.7714e-1(8.27e-3)+	5.0493e-1(8.96e-3)+	3.8736e-1(1.64e-2)+	9.2402e-1(1.59e-3)≈	**9.2774e-1(6.59e-3)**
		10000	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)
	10	1000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	9.0750e-1(1.25e-2)+	**9.1852e-1(3.75e-2)**
		5000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	8.7012e-1(1.59e-2)+	**8.9012e-1(3.19e-2)**
		10000	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)
DTLZ3	5	1000	1.0488e-1(2.62e-2)+	5.9878e-2(5.86e-3)+	5.0732e-2(4.43e-3)+	**7.9642e-1(4.07e-3)-**	5.0642e-1(2.87e-3)
		5000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	6.1221e-1(4.45-3)	**6.5231e-1(3.15-3)**
		10000	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)
	10	1000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	9.0196e-1(1.33e-2)≈	**9.0748e-1(3.64e-2)**
		5000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	1.5006e-1(1.41-2)+	**1.8017e-1(3.21-2)**
		10000	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)
DTLZ4	5	1000	**8.1172e-1(4.02e-2)-**	7.1829e-1(1.11e-3)+	5.1763e-1(4.63e-3)+	7.1741e-1(8.61e-3)+	8.0401e-1(1.13e-2)
		5000	6.1172e-1(1.23e-2)+	6.0974e-1(7.01e-3)+	4.0568e-1(1.54e-3)+	5.7770e-1(1.14e-2)+	**7.2931e-1(1.86e-2)**
		10000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	**7.7491e-3(4.94e-4)**
	10	1000	6.8831e-1(1.26e-2)+	7.1462e-1(6.51e-3)+	4.1321e-1(4.06e-3)+	7.3975e-1(2.56e-3)+	**8.4429e-1(2.43e-2)**
		5000	5.4200e-1(3.42e-2)+	5.6783e-1(2.33e-2)+	2.1532e-1(1.65e-2)-	6.6703e-1(1.38e-2)-	**7.3796e-1(5.96e-2)**
		10000	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)	0.0000e+0(0.00e+0)
DTLZ5	5	1000	2.2710e-1(1.14e-3)+	1.7201e-1(1.60e-3)+	1.6434e-1(5.11e-2)+	**4.1566e-1(3.27e-2)-**	2.8205e-1(4.09e-3)
		5000	1.2761e-1(5.73e-3)+	1.3504e-1(6.20e-3)+	3.5606e-1(4.14e-2)+	3.7499e-2(3.39e-3)+	**5.0025e-1(1.14e-2)**
		10000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	**4.7617e-5(3.01e-5)**
	10	1000	6.8793e-2(3.01e-4)+	2.0435e-2(7.84e-5)+	2.1222e-2(3.07e-8)+	8.0102e-2(7.82e-3)+	**9.5373e-2(4.73e-4)**
		5000	**5.9001e-2(2.08e-3)-**	8.1221e-3(8.62e-9)+	2.1222e-3(4.83e-7)+	7.8030e-3(1.24e-4)-	2.6826e-2(9.08e-4)
		10000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	**9.6745e-6(1.24e-8)**
DTLZ6	5	1000	1.6661e-1(1.02e-2)-	8.5228e-2(2.52e-2)+	5.1691e-2(5.69e-3)+	1.3410e-1(1.22e-2)	**2.0400e-1(4.62e-2)**
		5000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	3.4350e-2(1.10e-4)	**4.1123e-2(4.52e-3)**
		10000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	**1.2761e-9(1.70e-9)**
	10	1000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	1.0028e-1(2.02e-2)+	**1.2004e-1(1.12e-2)**
		5000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	6.2831e-3(1.15e-3)	**7.0301e-3(2.05e-3)**
		10000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	**7.4620e-9(5.30e-9)**
DTLZ7	5	1000	1.2710e-1(1.14e-3)+	1.7201e-1(1.60e-3)+	1.6434e-1(5.11e-2)+	1.7566e-1(3.27e-2)+	**1.8205e-1(4.09e-3)**
		5000	1.4432e-1(5.44e-2)+	1.3848e-1(3.61e-3)+	1.4057e-3(8.73e-2)+	1.9718e-1(3.01e-2)+	**2.1313e-1(4.47e-2)**
		10000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	**1.9968e-4(4.04e-5)**
	10	1000	3.3012e-6(4.47e-7)+	9.6131e-2(4.09e-3)+	2.3748e-2(4.04e-3)+	1.4678e-1(1.74e-2)+	**1.6509e-1(1.43e-2)**
		5000	6.6929e-6(4.09e-8)+	1.0225e-2(9.52e-4)+	7.1722e-3(4.73e-4)+	5.7065e-2(7.94e-3)	**6.4624e-2(3.16e-3)**
		10000	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	0.0000e+0(0.00e+0)+	**1.9718e-6(3.01e-7)**
WFG2	5	1000	6.1308e-1(2.27e-3)+	7.1629e-1(1.53e-4)+	8.4096e-1(6.63e-2)+	**9.1376e-1(6.19e-5)-**	9.0911e-1(2.43e-2)
		5000	5.1307e-1(6.30e-3)+	5.9477e-1(7.36e-2)+	5.5548e-1(2.16e-2)+	6.1513e-1(8.18e-4)+	**7.5811e-1(1.15e-2)**
		10000	3.0958e-1(4.34e-2)+	3.8790e-1(2.42e-2)+	4.2393e-1(4.27e-2)+	4.7065e-1(3.08e-2)+	**5.4901e-1(1.78e-2)**
	10	1000	6.7708e-1(6.00e-2)+	7.8978e-1(3.50e-2)+	8.5669e-1(5.59e-2)+	9.1974e-1(2.91e-2)+	**9.3447e-1(1.10e-2)**
		5000	4.6030e-1(9.14e-3)+	5.7409e-1(3.53e-3)+	5.9035e-1(7.67e-3)+	**6.3004e-1(6.37e-2)-**	6.1571e-1(4.71e-2)
		10000	3.6581e-1(3.44e-2)+	4.4058e-1(2.42e-2)+	4.9619e-1(5.10e-3)+	4.9950e-1(3.53e-2)+	**5.2664e-1(3.54e-2)**
WFG6	5	1000	3.2710e-1(1.14e-3)+	4.7201e-1(1.60e-3)+	4.6434e-1(5.11e-2)+	5.7566e-1(3.27e-2)+	**6.0095e-1(3.29e-3)**
		5000	4.2112e-1(3.07e-2)+	2.9330e-1(2.83e-3)+	2.3527e-1(6.87e-3)+	5.8605e-1(5.04e-3)+	**6.1203e-1(8.16e-4)**
		10000	1.0708e-1(1.51e-1)+	3.5627e-1(1.26e-2)+	2.8704e-1(6.21e-4)+	4.3576e-1(6.23e-2)+	**5.8793e-1(9.37e-2)**
	10	1000	4.8148e-1(4.57e-3)+	3.7306e-1(1.60e-3)+	4.2674e-1(9.02e-3)+	**6.6319e-1(5.32e-3)-**	6.0714e-1(3.76e-2)
		5000	9.4537e-2(7.71e-2)+	3.4033e-1(1.08e-3)+	3.7667e-1(3.66e-3)+	5.1561e-1(2.47e-2)+	**6.1739e-1(2.38e-2)**
		10000	5.1082e-2(1.70e-2)+	1.8181e-1(2.14e-2)+	2.7774e-1(5.77e-3)+	5.3011e-1(1.85e-2)+	**6.0512e-1(2.27e-2)**
WFG7	5	1000	4.2710e-1(1.14e-3)+	4.7201e-1(1.60e-3)+	4.6434e-1(5.11e-2)+	5.7566e-1(3.27e-2)+	**5.9993e-1(3.26e-3)**
		5000	3.0380e-1(3.74e-2)+	4.0178e-1(1.06e-3)+	3.0827e-1(5.91e-3)+	3.9659e-1(3.75e-3)+	**5.7900e-1(9.48e-3)**
		10000	1.8237e-1(2.11e-1)+	3.6076e-1(3.49e-3)+	3.0207e-1(7.61e-3)+	3.7061e-1(9.17e-2)+	**5.8511e-1(2.88e-2)**
	10	1000	1.8424e-1(5.49e-2)+	3.5631e-1(2.54e-3)-	2.9883e-1(1.64e-2)+	**3.6683e-1(6.70e-3)-**	3.5337e-1(4.99e-2)
		5000	3.0080e-1(3.79e-2)+	**4.4359e-1(4.41e-3)-**	3.4906e-1(4.61e-3)+	3.6071e-1(1.91e-2)-	3.4004e-1(2.83e-3)
		10000	6.4636e-2(1.59e-3)+	2.5913e-1(1.40e-2)+	3.0189e-1(5.12e-3)+	3.1236e-1(2.80e-2)+	**3.3322e-1(8.08e-2)**

"+", "-", and "≈" indicate the result obtained by LSMOEA/D is significantly better, significantly worse and statistically similar to that obtained by the compared algorithm, respectively.

TABLE S-VIII

MEAN AND STANDARD DEVIATION RESULTS IN TERMS OF IGD METRIC OBTAINED BY LS-SMS-MOEA, LS-NSGAII, LS-MOEA/D, WOF-SMPSO, WOF-NSGAII, WOF-GDE3 AND LSMOEA/D ON LSMOPS.

Problem	M	D	LS-SMS-MOEA	LS-NSGAII	LS-MOEA/D	WOF-SMPSO	WOF-NSGAII	WOF-GDE3	LSMOEA/D
LSMOP 1	2	200	6.845e-1(1.502e-3)+	6.957e-1(5.053e-3)+	6.925e-1(7.011e-4)+	2.756e-1(4.464e-3)+	1.827e-1(5.390e-2)+	2.109e-1(1.227e-1)+	**7.129e-2(3.379e-3)**
		500	6.914e-1(3.759e-4)+	6.783e-1(1.012e-2)+	6.819e-1(1.053e-2)+	1.761e+0(1.081e-1)+	1.544e-1(4.408e-3)+	1.917e-1(4.613e-2)+	**7.345e-2(1.135e-3)**
		1000	6.898e-1(9.621e-3)+	6.912e-1(8.777e-3)+	6.887e-1(2.850e-3)+	3.308e+0(1.431e-1)+	1.499e-1(4.818e-3)+	1.480e-1(4.739e-3)+	**8.484e-2(4.417e-3)**
	3	200	5.521e-1(4.889e-2)+	6.449e-1(8.198e-2)+	5.762e-1(3.043e-2)+	6.974e-1(2.863e-1)+	3.107e-1(7.767e-2)+	3.802e-1(5.298e-2)+	**2.047e-1(4.737e-2)**
		500	7.166e-1(2.700e-2)+	7.353e-1(1.747e-2)+	7.954e-1(3.291e-2)+	3.246e+0(1.056e-1)+	3.087e-1(4.392e-2)+	4.471e-1(3.476e-3)+	**2.773e-1(4.325e-3)**
		1000	8.267e-1(3.540e-3)+	8.314e-1(1.068e-2)+	9.548e-1(1.565e-1)+	5.197e+0(1.088e-1)+	4.770e-1(1.902e-1)+	4.918e-1(6.536e-2)+	**1.925e-1(1.651e-2)**
LSMOP 2	2	200	3.777e-2(9.441e-4)+	3.807e-2(7.739e-4)+	3.883e-2(2.204e-3)+	1.303e-1(2.302e-3)+	3.061e-2(4.478e-3)+	2.856e-2(1.619e-3)+	**1.470e-2(7.801e-4)**
		500	2.425e-2(3.255e-4)+	2.393e-2(1.050e-3)+	2.505e-2(1.826e-3)+	6.730e-2(1.448e-3)+	2.049e-2(1.068e-4)+	1.949e-2(9.585e-4)+	**9.446e-3(3.002e-4)**
		1000	1.857e-2(4.317e-4)+	1.829e-2(5.430e-4)+	1.821e-2(1.907e-4)+	3.708e-2(8.709e-4)+	1.640e-2(6.839e-4)≈	1.692e-2(1.576e-4)≈	**7.287e-3(5.845e-4)**
	3	200	1.454e-1(1.775e-3)+	1.428e-1(1.101e-2)+	1.386e-1(1.458e-3)+	1.110e-1(4.183e-4)+	1.297e-1(6.733e-3)+	1.305e-1(1.454e-4)+	**8.857e-2(1.077e-2)**
		500	9.023e-2(8.930e-4)+	8.511e-2(1.231e-3)+	9.053e-2(7.376e-4)+	**6.699e-2(3.906e-6)≈**	8.361e-2(9.327e-4)+	8.477e-2(1.232e-3)+	6.840e-2(1.739e-4)
		1000	7.286e-2(7.921e-3)+	6.789e-2(1.145e-3)≈	6.997e-2(3.331e-3)≈	**5.217e-2(3.242e-5)-**	6.714e-2(1.344e-3)≈	6.885e-2(2.071e-3)≈	6.032e-2(1.965e-4)
LSMOP 3	2	200	1.546e+0(4.084e-4)+	1.544e+0(1.279e-2)+	1.540e+0(2.36e-3)+	**7.815e-1(7.828e-3)-**	1.262e+0(2.510e-1)-	1.272e+0(5.697e-2)-	1.527e+0(1.776e-2)
		500	1.571e+0(8.581e-3)+	1.564e+0(9.668e-4)+	1.577e+0(1.40e-5)+	1.766e+1(1.412e-1)+	**8.562e-1(3.012e-3)-**	1.019e+0(2.355e-1)-	1.057e+0(2.816e-1)
		1000	1.577e+0(4.778e-3)+	1.585e+0(7.269e-5)+	1.585e+0(6.17e-4)+	2.218e+1(2.290e+0)	1.053e+0(1.204e-	**8.951e-1(2.002e-2)-**	1.008e+0(2.136e-1)
	3	200	8.547e-1(1.295e-3)-	1.361e+0(7.214e-1)+	1.109e+0(3.51e-1)+	1.946e+0(2.196e-1)+	8.607e-1(1.847e-5)≈	**8.546e-1(8.628e-3)-**	8.607e-1(0.000e+0)
		500	1.756e+0(1.266e+0)	8.607e-1(0.000e+0)	8.607e-1(0.00e+0)	6.290e+0(4.356e-1)+	8.607e-1(0.000e+0)	**8.604e-1(4.930e-4)-**	8.607e-1(0.000e+0)
		1000	8.607e-1(0.000e+0)≈	3.066e+0(3.119e+0)	8.607e-1(0.00e+0)≈	1.236e+1(4.502e+0)	8.60e-1(0.00e+0)≈	**8.597e-1(1.441e-3)-**	8.607e-1(0.000e+0)
LSMOP 4	2	200	9.807e-2(1.807e-3)+	9.799e-2(3.019e-3)+	9.880e-2(9.171e-4)+	1.813e-1(5.562e-4)+	8.753e-2(3.907e-3)+	8.523e-2(3.025e-3)+	**6.626e-2(3.730e-3)**
		500	5.179e-2(9.985e-4)+	5.145e-2(7.243e-5)+	5.174e-2(3.284e-3)+	1.119e-1(7.655e-4)+	4.695e-2(1.977e-3)+	4.623e-2(1.739e-4)+	**3.594e-2(4.627e-4)**
		1000	3.339e-2(8.356e-4)+	3.323e-2(2.443e-3)+	3.258e-2(1.493e-3)+	6.801e-2(5.102e-4)+	3.053e-2(6.138e-4)+	3.000e-2(2.180e-3)+	**2.071e-2(1.232e-4)**
	3	200	2.913e-1(7.058e-3)+	2.892e-1(4.248e-4)+	2.849e-1(5.801e-3)+	3.097e-1(7.815e-3)+	2.857e-1(4.048e-3)+	3.031e-1(2.068e-3)+	**2.364e-1(1.527e-2)**
		500	2.157e-1(1.505e-3)+	2.218e-1(6.533e-3)+	2.149e-1(1.486e-3)+	1.758e-1(1.975e-3)+	1.946e-1(1.202e-2)+	1.990e-1(7.569e-3)+	**1.332e-1(7.775e-3)**
		1000	1.434e-1(2.334e-3)+	1.429e-1(3.621e-3)+	1.449e-1(3.956e-3)+	1.090e-1(8.888e-4)+	1.306e-1(4.477e-4)+	1.330e-1(2.129e-3)+	**9.060e-2(1.377e-3)**
LSMOP 5	2	200	7.42e-1(0.000e+0)+	7.42e-1(0.00e+0)+	7.42e-1(0.00e+0)+	4.633e-1(3.059e-2)+	4.180e-1(4.077e-2)+	4.346e-1(2.803e-2)+	**1.697e-1(2.005e-2)**
		500	7.42e-1(0.000e+0)+	7.42e-1(0.00e+0)+	7.42e-1(0.000e+0)+	3.540e+0(9.573e-1)+	5.217e-1(1.069e-1)+	5.355e-1(1.465e-1)+	**2.027e-1(3.110e-3)**
		1000	7.42e-1(0.000e+0)+	7.42e-1(0.00e+0)+	7.42e-1(0.000e+0)+	7.960e+0(3.655e-1)+	7.104e-1(4.122e-3)+	5.713e-1(1.473e-1)+	**1.890e-1(6.224e-4)**
	3	200	5.410e-1(6.044e-5)+	5.408e-1(1.76e-4)+	5.410e-1(4.152e-5)+	1.016e+0(4.010e-1)-	4.301e-1(1.617e-2)-	**4.215e-1(1.221e-2)-**	4.382e-1(2.333e-2)
		500	5.410e-1(2.366e-6)+	5.661e-1(3.55e-2)+	6.174e-1(4.775e-2)+	4.072e+0(8.527e-1)+	5.646e-1(7.420e-2)+	5.639e-1(1.262e-1)+	**4.194e-1(4.816e-2)**
		1000	5.410e-1(2.348e-5)+	5.410e-1(3.25e-6)+	5.450e-1(5.737e-3)+	6.831e+0(1.295e+0)	6.521e-1(2.966e-1)+	5.061e-1(1.929e-2)+	**4.920e-1(3.600e-2)**
LSMOP 6	2	200	4.038e-1(2.927e-2)+	3.600e-1(4.47e-4)+	3.605e-1(3.688e-3)+	8.202e-1(4.678e-2)+	4.918e-1(2.382e-1)+	6.574e-1(6.144e-3)+	**1.767e-1(8.335e-3)**
		500	3.386e-1(2.318e-2)+	3.310e-1(1.08e-2)+	3.235e-1(3.724e-3)+	8.002e-1(9.957e-3)+	6.696e-1(2.789e-4)+	4.882e-1(2.558e-1)+	**1.373e-1(8.355e-3)**
		1000	3.138e-1(3.914e-4)+	3.146e-1(2.78e-3)+	3.137e-1(3.746e-4)+	7.703e-1(4.273e-4)+	5.005e-1(2.403e-1)+	6.694e-1(3.206e-4)+	**1.637e-1(1.200e-2)**
	3	200	6.776e-1(9.940e-3)+	6.816e-1(3.90e-2)-	7.603e-1(8.868e-2)+	2.983e+0(1.973e-1)+	**6.593e-1(7.259e-3)+**	7.001e-1(4.117e-2)-	1.031e+0(2.958e-1)
		500	7.517e-1(9.645e-2)-	**7.111e-1(2.61e-2)-**	8.222e-1(4.784e-2)-	4.704e+0(2.312e-1)+	1.246e+0(6.086e-2)-	1.291e+0(2.467e-	1.274e+0(2.664e-2)
		1000	**7.569e-1(7.777e-2)-**	7.796e-1(5.27e-2)-	7.654e-1(3.108e-2)-	1.878e+2(1.101e+2)	1.479e+0(1.946e-	1.175e+0(1.939e-	1.014e+0(5.693e-1)
LSMOP 7	2	200	1.481e+0(5.857e-3)-	1.485e+0(1.66e-3) ≈	1.483e+0(5.419e-3)-	7.922e+0(4.368e+0)	**1.476e+0(1.308e-3)-**	1.477e+0(1.20e-3)-	1.485e+0(1.369e-3)
		500	1.512e+0(8.930e-4)+	1.508e+0(4.78e-3)+	1.503e+0(2.484e-	9.787e+2(9.864e+2)	1.142e+0(3.768e-2)-	**1.122e+0(1.80e-2)-**	1.201e+0(1.143e-1)
		1000	1.518e+0(3.988e-4)+	1.510e+0(3.50e-4)+	1.515e+0(4.752e-	9.128e+3(6.913e+3)	**1.036e+0(2.209e-3)-**	1.182e+0(1.67e-1)-	1.372e+0(1.476e-2)
	3	200	9.841e-1(5.811e-3)+	9.786e-1(2.01e-2)+	9.777e-1(2.536e-2)+	1.186e+0(3.384e-1)+	**9.050e-1(1.640e-2)-**	9.116e-1(1.62e-2)-	9.236e-1(3.985e-4)
		500	9.064e-1(3.104e-3)+	9.064e-1(1.14e-2)+	9.392e-1(5.444e-2)+	1.053e+0(1.788e-3)+	8.651e-1(4.205e-4)+	8.655e-1(1.82e-2)+	**7.618e-1(1.508e-1)**
		1000	8.949e-1(3.550e-2)+	8.921e-1(3.21e-2)+	8.668e-1(2.434e-3)+	9.853e-1(3.266e-2)+	8.164e-1(7.463e-2)+	7.919e-1(3.76e-2)+	**6.868e-1(1.786e-1)**
LSMOP 8	2	200	7.42e-1(0.00e+0)+	7.42e-1(0.00e+0)+	7.42e-1(0.00e+0)+	4.896e-1(6.077e-2)+	2.895e-1(8.248e-2)+	2.933e-1(1.46e-1)+	**1.457e-1(2.686e-3)**
		500	7.42e-1(0.00e+0)+	7.42e-1(0.00e+0)+	7.42e-1(0.00e+0)+	2.463e+0(6.973e-2)+	2.762e-1(1.193e-1)+	4.569e-1(3.42e-1)+	**1.582e-1(1.656e-2)**
		1000	7.42e-1(0.00e+0)+	7.42e-1(0.00e+0)+	7.42e-1(0.00e+0)+	4.960e+0(3.012e-1)+	3.243e-1(1.949e-1)+	3.242e-1(2.00e-1)+	**1.586e-1(1.232e-2)**
	3	200	3.63e-1(6.31e-4)+	3.64e-1(1.84e-4)+	3.630e-1(4.48e-5)+	7.037e-1(7.288e-5)+	3.215e-1(1.254e-2)+	3.025e-1(1.10e-2)+	**1.501e-1(1.805e-2)**
		500	3.59e-1(4.77e-5)+	3.50e-1(1.37e-2)+	3.597e-1(1.42e-4)+	7.517e-1(1.080e-1)+	3.188e-1(5.709e-2)+	3.224e-1(1.05e-2)+	**1.313e-1(1.086e-3)**
		1000	3.72e-1(1.43e-2)+	3.57e-1(9.18e-4)+	4.085e-1(7.06e-2)+	7.064e-1(1.943e-3)+	3.324e-1(3.307e-4)+	2.789e-1(3.85e-2)+	**1.855e-1(1.281e-1)**
LSMOP 9	2	200	8.10e-1(0.00e+0)≈	8.10e-1(0.00e+0)≈	8.10e-1(0.00e+0)≈	**5.476e-1(2.510-2)-**	8.10e-1(0.00e+0)≈	8.10e-1(0.00e+0)≈	8.10e-1(0.00e+0)
		500	8.10e-1(0.00e+0)-	8.10e-1(0.00e+0)-	8.10e-1(0.00e+0)-	2.286e+0(1.056e-1)+	1.19e+1(1.57e+1)+	**8.099e-1(1.92e-4)-**	2.186e+1(2.977e+1
		1000	8.10e-1(0.00e+0)-	8.099e-1(5.48e-5)-	8.098e-1(3.26e-4)-	1.426e+1(1.918e+0)-	8.08e-1(4.21e-4)-	**8.076e-1(1.22e-3)-**	3.054e+1(5.601e+0
	3	200	1.538e+0(0.00e+0)≈	1.538e+0(0.00e+0)≈	1.538e+0(0.00e+0)≈	**6.817e-1(4.289-3)-**	1.53e+0(0.00e+0)≈	1.538e+0(0.00e+0)≈	1.538e+0(9.493e-9)
		500	5.866e+1(8.07e+1)	5.206e+1(2.90e+1)+	1.306e+0(3.27e-1)+	**1.072e+0(3.60e-3)-**	2.18e+1(2.92e+1)+	1.110e+0(4.96e-2)-	1.866e+1(8.078e+1
		1000	1.538e+0(0.00e+0)-	7.416e+1(1.02e+2)+	9.885e+0(1.18e+1)-	1.930e+1(1.509e+0)	1.14e+0(1.74e-4)+	**1.145e+0(5.90e-4)-**	1.083e+1(9.379e+0

"+", "-", and "≈" indicate the result obtained by LSMOEA/D is significantly better, significantly worse and statistically similarto that obtained by the compared algorithm, respectively.

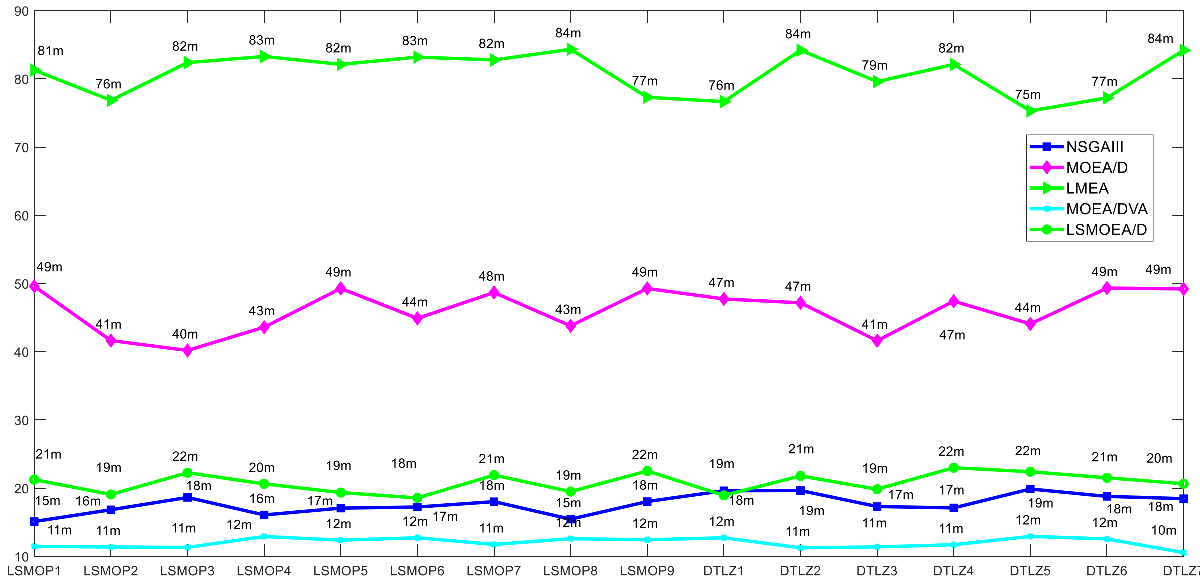


Fig. S6. Running time of involved algorithms on 10-objective test instances with 1000 decision variables (Fig. 12 in the paper).

TABLE S-IX
DECISION VARIABLE ANALYSIS RESULTS IN MOEA/DAVA, LMEA AND LSMOEA/D ON WFG2-M3-D10 (THE RESULTS OF LSMOEA/D ARE SORTED IN ASCENDING ORDER OF CRD FOR V_1,V_2 AND V_3)

Problem	M	Variable	MOEA/DVA			LMEA		LSMOEA/D (Sorting by CRD)		
			Diversity	Convergence	Both	Diversity	Convergence	Rank for V_1	Rank for V_2	Rank for V_3
WFG2	3	x_1	×	×	√	√	×	5-th	5-th	5-th
		x_2	√	×	×	√	×	5-th	5-th	5-th
		x_3	×	√	×	×	√	4-th	4-th	4-th
		x_4	×	√	×	×	√	3-th	1-th	3-th
		x_5	×	√	×	×	√	2-th	3-th	4-th
		x_6	×	√	×	×	√	1-th	2-th	1-th
		x_7	×	√	×	×	√	1-th	1-th	1-th
		x_8	×	√	×	×	√	3-th	2-th	2-th
		x_9	×	√	×	×	√	2-th	3-th	2-th
		x_{10}	×	√	×	×	√	4-th	5-th	5-th

TABLE S-X
COMPUTATION COST OF EVALUATION RESOURCES CONSUMED BY THE DVA IN EACH INVOLVED ALGORITHM (UNIT: FES).

Problem	M	D	MOEA/DVA	LMEA	LSMOEA/D
DTLZ1	5	200	4000	1700	3300
		500	10000	4100	8100
		1000	20000	8100	16100
	10	200	4000	1700	3300
		500	10000	4100	8100
		1000	20000	8100	16100
WFG1	5	200	4000	1700	3300
		500	10000	4100	8100
		1000	20000	8100	16100
	10	200	4000	1700	3300
		500	10000	4100	8100
		1000	20000	8100	16100

D. Investigation of Efficiency and Accuracy

In order to validate the detection accuracy of the proposed localized decision variable analysis method, Table S-IX gives the variable detection results by the DVA in the involved algorithms (LSMOEA/D, MOEA/DVA and LMEA) on 2-objective WFG2 problem where $\mathbf{x}_1$-$\mathbf{x}_9$ are the decision variables, "Diversity", "Convergence" and "Both" represent the convergence-related type, diversity-related type and mixture-related type, respectively, "rank for V_1", "rank for V_2" and "rank for V_3" represent the index of subgroup in the subgroup sequence for reference vector V_1, V_2 and V_3, respectively. From this table, it is observed that the variables of subgroups ranked in front in LSMOEA/D are usually the convergence-related variables in LMEA and MOEA/DVA, at the same time, the variables of subgroups ranked behind in LSMOEA/D are the diversity-related variables in LMEA and MOEA/DVA.

In order to reduce the computation cost, the proposed method uses the guiding reference vectors instead of all the reference vectors to guide the guide the control variable analysis. The initial reference vectors are first clustered into a number of clusters and the cluster centers are used as the guiding (or representative) reference vectors. In the control variable analysis process, all reference vector within the cluster follow the detection results of the guiding reference vector in the cluster, which largely improves the detection efficiency. Table S-X reports the computation cost (i.e., the number of function evaluations (FEs)) consumed by the DVA in each involved algorithm on DTLZ1 and WFG2. From this table, it is observed that LSMOEA/D does not consume unbearable computation resources and its computation cost is nearly close to that of MEOA/DVA.

Reference:

[1] S. Huband, P. Hingston, L. Barone, and L. While, “A review of multiobjective test problems and a scalable test problem toolkit,” *IEEE Trans. Evol. Comput.*, vol. 10, no. 5, pp. 477–506, 2006.

[2] X. Zhang, Y. Tian, R. Cheng and Y. Jin, "A Decision Variable Clustering-Based Evolutionary Algorithm for Large-Scale Many-Objective Optimization," *IEEE Trans. Evol. Comput.*, vol. 22, no. 1, pp. 97-112, Feb. 2018.