An Adaptive Receiver for the Time- and Frequency-Selective Fading Channel

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Abstract—An adaptive receiver is presented in this paper for the reception of linearly modulated signals transmitted over a time- and frequency-selective fading channel. The channel is modeled as a truncated power series [1] which represents the dispersive fading channel as a sum of three elementary flatfading channels. The proposed receiver consists of a sequence estimator with a parallel channel estimator. The channel estimator recovers the instantaneous fading processes associated with each elementary channel and filters them to generate onestep predictions of each fading process. Some implementation difficulties and solutions are also discussed. Computer simulations using quadrature phase-shift keying (QPSK) and channels with moderate delay spreads and fade rates have been used to evaluate the performance of the receiver. The results show that our technique has potential in channels with delay spread of about 20%, signal-to-noise ratio (SNR) greater than 15 dB, and applications requiring bit-error rates (BER's) less than 10^{-2} .

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Index Terms—Adaptive receiver, selective fading channel.

I. INTRODUCTION

ODGE AND MOHER in [2] have suggested a Kalman filtering approach to a maximum-likelihood sequence estimation (MLSE) receiver for a general Rayleigh fading channel. This receiver structure has been implemented by Dai and Shwedyk [3] assuming that the second-order statistics of the channel are available in defining the state model of the channel impulse response (CIR).

Although the Kalman filtering approach to MLSE leads to an elegant optimum receiver, there are practical difficulties associated with it. First, its complexity grows exponentially with sequence length since one Kalman filter is required for every hypothesized sequence. Next, the complexity of the Kalman filter increases with the length of the CIR. Finally, the statistics of the channel must be explicitly known in order to specify the underlying state equation describing the timevariant CIR. It has also been noted in [2] that the Kalman filter generates redundant information, since the conditional means and variances of the internal states, which are not required by the MLSE, are also presented at the filter output. However, for the special case of a flat-fading channel and a constant envelope signaling format, this approach reduces to a structure commonly known as the predictor receiver which can be implemented with the Viterbi algorithm (VA) and a bank of linear predictors. Further complexity reduction may be achieved using per-survivor processing (PSP) to reduce the number of filtering operations [4].

In this paper, a reduced complexity sequence estimation receiver is presented for the general Rayleigh fading (time- and frequency-selective) channel. There are three major advantages of this receiver over the Kalman filtering approach. First, like the flat-fading case, the prediction algorithm is simplified by using linear prediction filters instead of Kalman filters. Second, the channel is modeled as a truncated f-power series [1], [5]. As a consequence, the number of channel parameters to be estimated is not equal to the length of the CIR but to the number of terms in the truncated series. Third, the predictors use the recursive least squares (RLS) algorithm to adapt to the channel environment. Hence, the receiver can perform without any prior statistical knowledge of the channel.

In the present work, we have truncated the series to the first three terms and the resultant is referred to as the quadratic f-power series. The quadratic series model describes the timeand frequency-selective channel as a sum of three elementary flat-fading channels. Therefore, we are able to directly extend the ideas of predictor receivers for the flat-fading channel to the dispersive fading channel. For flat-fading channels, the channel fading parameter is recovered by dividing the received signal by the transmitted signal. Similarly, the multiplicative fading of each elementary channel is decoupled from the received signal by a matrix–vector equivalent of this division operation. A prediction filter is then used for each of the elementary channels.

The organization of this paper is as follows. Section II describes the channel and signal models which are used. In Section III, the development of the proposed receiver structure from the predictor receiver for a flat-fading channel is described. The performance of the new receiver is evaluated by computer simulations and the results are presented in Section IV. Finally, conclusions are provided in Section V.

II. CHANNEL AND SIGNAL MODEL

Fig. 1 shows the complex baseband model of the communication system. The transmitter consists of a symbol source generating a sequence $\{a_i\}$ of uncorrelated data symbols and a bandlimited transmit filter with impulse response g(t). The *i*th symbol is denoted by a_i and the symbol period is denoted by T. The symbols are filtered by the transmit filter to yield the transmitted signal

$$s(t) = \sum_{i} a_i g(t - iT). \tag{1}$$

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Fig. 1. Block diagram of the communication system.

It is assumed that the impulse response is truncated to a finite length such that g(t) = 0 for $|t| > L_c T$. The fading channel introduces random phase and amplitude fluctuations to the transmitted signal. In the case of a flat-fading channel, the signal s(t) will simply be distorted by multiplicative fading c(t). For a dispersive fading channel with impulse response $c(t,\tau)$ and corresponding time-variant transfer function C(t, f), the channel may be modeled as a time-variant filter with tap weights which are zero-mean complex Gaussian random variables [6]. At the front end of the receiver, the faded signal r(t) is further corrupted by zero-mean additive white Gaussian noise (AWGN) with power spectral density N_{α} . The receive filter w(t) is assumed to be an ideal zonal filter with a bandwidth wide enough to accommodate the entire Doppler widened spectrum of the faded signal, but which limits the noise at higher frequencies. The sampled received signal y_k is then processed by the receiver to recover the transmitted data symbols.

A. The Quadratic f-Power Series Channel Model

Letting the mean delay of the channel be zero, the Taylor's series expansion about f = 0 of the complex baseband channel transfer function is [1], [5] given by

$$C(t,f) = \sum_{m=0}^{\infty} \frac{1}{m!} C^{(m)}(t) f^m$$
 (2)

where

$$C^{(m)}(t) = \frac{d^m C(t, f)}{df^m} \bigg|_{f=0}.$$
(3)

The channel transfer function may therefore be approximated by a truncated f-power series with time-varying coefficients $C^{(m)}(t)$. To further simplify the expression in (2), we define the time-selective coefficient (TSC) as

$$T_m(t) = \frac{1}{m!(j2\pi)^m} C^{(m)}(t).$$
 (4)

Using (4), the expression in (2) is rewritten as

$$C(t,f) = \sum_{m=0}^{\infty} T_m(t)(j2\pi f)^m.$$
 (5)

The time-variant transfer function is now described as a sum in the variable $(j2\pi f)^m$ with each term weighted by $T_m(t)$.



Fig. 2. The quadratic f-power series channel model. The dispersive fading channel is made up of the linear combination of three elementary channels. The elementary channels correspond to the transmitted signal, its first derivative, and its second derivative.

A filter with transfer function of $(j2\pi f)^m$ is an *m*th order differentiator. Therefore, the output signal from the channel is

$$r(t) = \sum_{m=0}^{\infty} T_m(t) \frac{d^m[s(t)]}{dt^m}.$$
 (6)

The series is truncated to the first three terms. This is known as the quadratic f-power series model. Accordingly, the filtered received signal is

$$y(t) = T_0(t)s(t) + T_1(t)s'(t) + T_2(t)s''(t) + n(t).$$
 (7)

The signal terms s'(t) and s''(t) are the first and second derivatives of the transmitted signal s(t), respectively. For easy reference, the three signal terms are collectively known as the frequency-selective variables (FSV's). The FSV's are datadependent and account for the intersymbol interference present in the received signal. The filtered noise term is represented by n(t).

The received signal is sampled at the times $t = kT_s$ and is written as

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$$y_k = T_0(k)s_k + T_1(k)s'_k + T_2(k)s''_k + n_k$$
(8)

where the sampling period $T_s = T/r$ and r is the number of samples per symbol interval. Therefore, within the *n*th signaling interval, k = nr + j, $j = 0, 1, \dots, (r-1)$. To ensure that the filtered and sampled noise n_k remains uncorrelated, the receive filter is assumed to have an ideal frequency response rect(fT_s).

Expanding (8) and substituting for the FSV, the sampled received signal becomes

$$y_{k} = T_{0}(k) \sum_{i=n-L_{c}}^{n+L_{c}} a_{i}g_{k-ir} + T_{1}(k) \sum_{i=n-L_{c}}^{n+L_{c}} a_{i}g'_{k-ir} + T_{2}(k) \sum_{i=n-L_{c}}^{n+L_{c}} a_{i}g''_{k-ir} + n_{k}.$$
(9)

The observed signal model of the quadratic f-power series model is shown in Fig. 2. It may be viewed as a sum of three purely time-selective fading channel outputs and additive noise. Each of the three terms in (9) corresponds to an ideal elementary flat-fading channel.

III. THE LEAST SQUARES ESTIMATES PREDICTOR RECEIVER

It was shown in the previous section that the quadratic *f*-power series is made up of three flat-fading channels. The proposed receiver is developed by extending the predictor receiver for the flat-fading channel model to the model shown in (8). To clearly describe the receiver, the predictor receiver for the flat-fading channel is briefly discussed. An analogy is then drawn between the two channel models, and ideas from the flat-fading channel receiver are applied to the design of the receiver for the dispersive fading channel.

A. Channel Estimator for the Flat-Fading Channel

It is well known that the kth sample of the received signal over a frequency flat-fading channel [2] is given by

$$y_k = c(k)s(k) + n_k \tag{10}$$

where c(k) represents the sampled multiplicative fading variable, s(k) is the sampled transmitted signal, and n_k is the low-pass filtered AWGN. An optimum receiver is an MLSE with a bank of linear predictors [2], [7], [8]. Each hypothesized sequence requires a predictor to obtain estimates of the channel state information (CSI). The tap weights of the linear predictors may be precomputed if the channel autocorrelation is known or repeatedly updated using an adaptive algorithm such as the least mean squares or RLS algorithms [9]. Implementation of this receiver with complexity reduction is achieved by using the VA and PSP [4].

Assuming that each predictor in the bank is of order L, the prediction of the channel sample c(k + 1) requires the estimates of the preceding channel samples $c(k), c(k - 1), \dots, c(k - L + 1)$ [9]. To obtain the instantaneous estimate $\tilde{c}(k)$ of the channel sample for a given transmitted sequence at the *k*th step, the received sample is divided by the hypothesized transmitted signal $\bar{s}(k)$ associated with the most recent element of that survivor, which is

$$\tilde{c}(k) = \frac{y_k}{\bar{s}(k)}.$$
(11)

The received sample y_k is a noisy version of the faded signal sample and, therefore, the estimate of the fading process $\tilde{c}(k)$ is also noisy.

B. Channel Estimator for the Time-Dispersive Fading Channel

Analogous to the predictor receiver for the flat-fading channel, the proposed channel estimator for the dispersive fading channel also employs linear predictors. Unlike the receiver for the flat-fading case, three predictors are used for a given transmitted signal, one for each of the TSC's $T_0(k)$, $T_1(k)$, and $T_2(k)$.

Like the flat-fading receiver described in Section III-A, the proposed receiver estimates the noisy version of the fading processes $\tilde{T}_0(k)$, $\tilde{T}_1(k)$, and $\tilde{T}_2(k)$ which are then used to predict the TSC's for the next metric evaluation. However, the simple division operation of (11) cannot be applied here. The received signal model consists of a sum of three elementary channel outputs and, hence, the TSC's are coupled.

We assume that the fading is slow enough that $T_0(k)$, $T_1(k)$, and $T_2(k)$ are essentially constant over a symbol interval but may vary from symbol to symbol. The receiver takes r samples of the channel output y(t) over each symbol interval. Letting r = 3, the received samples over one symbol period are written as

$$y_{k} = T_{0}(k)s_{k} + T_{1}(k)s_{k}' + T_{2}(k)s_{k}'' + n_{k}$$

$$y_{k+1} = T_{0}(k)s_{k+1} + T_{1}(k)s_{k+1}' + T_{2}(k)s_{k+1}'' + n_{k+1}$$

$$y_{k+2} = T_{0}(k)s_{k+2} + T_{1}(k)s_{k+2}' + T_{2}(k)s_{k+2}'' + n_{k+2}.$$
(12)

These equations in (8) can be rewritten in matrix-vector form as

$$\mathbf{y}_{\mathbf{k}} = \mathbf{S}_{\mathbf{k}} \mathbf{T}_{\mathbf{k}} + \mathbf{n}_{\mathbf{k}} \tag{13}$$

(14)

$$\mathbf{y}_{\mathbf{k}} = \begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \end{bmatrix}$$

is the received sample vector

where

$$\mathbf{S_k} = \begin{bmatrix} s_k & s'_k & s''_k \\ s_{k+1} & s'_{k+1} & s''_{k+1} \\ s_{k+2} & s'_{k+2} & s''_{k+2} \end{bmatrix}$$
(15)

is the data dependent frequency-selective matrix

$$\mathbf{T}_{\mathbf{k}} = \begin{bmatrix} T_0(k) \\ T_1(k) \\ T_2(k) \end{bmatrix}$$
(16)

is the unknown time-selective coefficient vector and

$$\mathbf{n_k} = \begin{bmatrix} n_k \\ n_{k+1} \\ n_{k+2} \end{bmatrix}$$
(17)

is the noise vector. The elements of the frequency-selective matrix S_k consist of samples from outputs of each of the three elementary channels without any fading.

The frequency-selective matrix $\mathbf{S}_{\mathbf{k}}$ is conditionally known for any given transmitted data sequence. The objective is to recover the time-selective vector $\mathbf{T}_{\mathbf{k}}$ from the channel observation vector $\mathbf{y}_{\mathbf{k}}$. If the noise vector $\mathbf{n}_{\mathbf{k}}$ is ignored, the matrix-vector equation in (13) reduces to a set of three simultaneous linear equations in the three unknown quantities $T_0(k)$, $T_1(k)$, and $T_2(k)$. The simplest and most intuitive approach to recovering $\mathbf{T}_{\mathbf{k}}$ is by solving (13) since $\mathbf{y}_{\mathbf{k}}$ and $\mathbf{S}_{\mathbf{k}}$ are known. However, we have yet to consider the effects of noise. In the presence of additive noise, the solution to the set of equation in (13) becomes

$$\begin{split} \hat{T}_{0}(k) \\ \hat{T}_{1}(k) \\ \hat{T}_{2}(k) \end{bmatrix} &= \begin{bmatrix} s_{k} & s'_{k} & s''_{k} \\ s_{k+1} & s'_{k+1} & s''_{k+1} \\ s_{k+2} & s'_{k+2} & s''_{k+2} \end{bmatrix}^{-1} \begin{bmatrix} y_{k} \\ y_{k+1} \\ y_{k+2} \end{bmatrix} \\ &+ \begin{bmatrix} s_{k} & s'_{k} & s''_{k} \\ s_{k+1} & s'_{k+1} & s''_{k+1} \\ s_{k+2} & s'_{k+2} & s''_{k+2} \end{bmatrix}^{-1} \begin{bmatrix} n_{k} \\ n_{k+1} \\ n_{k+2} \end{bmatrix}$$
(18)

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and may be rewritten as

$$\begin{bmatrix} \tilde{T}_0(k)\\ \tilde{T}_1(k)\\ \tilde{T}_2(k) \end{bmatrix} = \begin{bmatrix} T_0(k)\\ T_1(k)\\ T_2(k) \end{bmatrix} + \begin{bmatrix} \hat{n}_{0,k}\\ \hat{n}_{1,k}\\ \hat{n}_{2,k} \end{bmatrix}$$
(19)

where $[\tilde{T}_0(k), \tilde{T}_1(k), \tilde{T}_2(k)]^T$ is the noisy estimate of the timeselective fade vector \mathbf{T}_k . The first term of the right-hand side of (19) is the original time-selective fade vector and the second term is an augmented noise vector. The original noise vector \mathbf{n}_k has been transformed by the inverse of matrix \mathbf{S}_k to yield the augmented. In general, the Jacobian of the linear transformation by \mathbf{S}_k^{-1} is not unity and this may cause considerable noise enhancement.

The noise enhancement may be minimized by using an overdetermined system. This is achieved by taking more than three samples per symbol. For example, assuming r = 4 samples per symbol, the received samples are written as

$$\begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \end{bmatrix} = \begin{bmatrix} s_k & s'_k & s'_k \\ s_{k+1} & s'_{k+1} & s'_{k+1} \\ s_{k+2} & s'_{k+2} & s''_{k+2} \\ s_{k+3} & s'_{k+3} & s'_{k+3} \end{bmatrix} \begin{bmatrix} T_0(k) \\ T_1(k) \\ T_2(k) \end{bmatrix} + \begin{bmatrix} n_k \\ n_{k+1} \\ n_{k+2} \\ n_{k+3} \end{bmatrix}.$$
(20)

From (20), it is seen that the solution for the estimates $[\tilde{T}_0(k), \tilde{T}_1(k), \tilde{T}_2(k)]^T$ becomes that of a standard least squares problem [9], [10]. For a given transmitted sequence, the object of the channel estimator is to obtain the linear least squares estimate (LLSE) of $\mathbf{T}_{\mathbf{k}}$ from the observed vector $\mathbf{y}_{\mathbf{k}}$ and the conditionally known frequency-selective matrix $\mathbf{S}_{\mathbf{k}}$. Following [9] and [10], the LLSE of the time-selective fade vector is obtained as

$$\tilde{\mathbf{T}}_{\mathbf{k}}^{\mathbf{LS}} = \left(\mathbf{S}_{\mathbf{k}}^{H}\mathbf{S}_{\mathbf{k}}\right)^{-1}\mathbf{S}_{\mathbf{k}}^{H}\mathbf{y}_{\mathbf{k}}$$
 (21)

where the superscript H denotes Hermitian transposition. If the slow fading assumption holds, then the least squares estimates $\tilde{\mathbf{T}}_{\mathbf{k}}^{\mathbf{LS}}$ is an unbiased estimate of the vector $\mathbf{T}_{\mathbf{k}}$. According to [9], if the noise samples in (20) are uncorrelated, $\tilde{\mathbf{T}}_{\mathbf{k}}^{\mathbf{LS}}$ is also the *best linear unbiased estimator* of $\mathbf{T}_{\mathbf{k}}$ and it achieves the Cramér–Rao lower bound for unbiased estimates. The elements of the vector $\tilde{\mathbf{T}}_{\mathbf{k}}^{\mathbf{LS}}$ are then used as inputs to the predictors in order to obtain future estimates of the TSC's. Note that (21) is the matrix–vector equivalent of the division operation seen in (11) for the flat-fading channel.

C. The Receiver for the Dispersive Fading Channel

The proposed receiver is a sequence estimator implemented by the VA with a parallel channel estimator and is shown in Fig. 3. We define the trellis state as

$$\Delta_n = [a_{n+L_c-1}, \cdots, a_{n-L_c}]. \tag{22}$$

The branch metric associated with the state transition $\Delta_n \rightarrow \Delta_{n+1}$ is

$$\lambda_{n+1}(\Delta_n \to \Delta_{n+1}) = |y_k - \hat{y}_k(\Delta_n \to \Delta_{n+1})|^2.$$
(23)

The term $\hat{y}_k(\Delta_n \to \Delta_{n+1})$ is the hypothesized received sample associated with the state transition $\Delta_n \to \Delta_{n+1}$ and



Fig. 3. The proposed receiver structure for the time- and frequency-selective channel.

is defined as

$$\hat{y}_{k}(\Delta_{n} \to \Delta_{n+1}) = \hat{T}_{0}(k, \Delta_{n})s_{k}(\Delta_{n} \to \Delta_{n+1}) + \hat{T}_{1}(k, \Delta_{n})s'_{k}(\Delta_{n} \to \Delta_{n+1}) + \hat{T}_{2}(k, \Delta_{n})s''_{k}(\Delta_{n} \to \Delta_{n+1}).$$
(24)

The terms $s_k(\Delta_n \to \Delta_{n+1})$, $s'_k(\Delta_n \to \Delta_{n+1})$ and $s''_k(\Delta_n \to \Delta_{n+1})$ are the hypothesized transmitted kth sample and its first and second derivatives associated with the transition $\Delta_n \to \Delta_{n+1}$. $\hat{T}_0(k, \Delta_n)$, $\hat{T}_1(k, \Delta_n)$, and $\hat{T}_2(k, \Delta_n)$ are the predictions of the TSC's from the channel estimator.

Past least squares estimates of the TSC's are used by the channel estimator to predict $\hat{T}_0(k, \Delta_n)$, $\hat{T}_1(k, \Delta_n)$, and $\hat{T}_2(k, \Delta_n)$. At the (n-1)th symbol interval, the received vector $\mathbf{y}_{\mathbf{k}-r}$ defined in (20) is processed by a least squares estimator as in (21) to obtain the estimate $\mathbf{\tilde{T}}_{\mathbf{k}-r}^{\mathbf{LS}}$ for each trellis state. Estimates prior to the *n*th period $\mathbf{\tilde{T}}_{\mathbf{k}-2r}^{\mathbf{LS}}$, $\mathbf{\tilde{T}}_{\mathbf{k}-3r}^{\mathbf{LS}}$, \cdots , $\mathbf{\tilde{T}}_{\mathbf{k}-Lr}^{\mathbf{LS}}$ are also obtained in a similar manner. The three elements of these estimated vectors are used as inputs to three linear onestep predictors to obtain $\hat{T}_0(k, \Delta_n)$, $\hat{T}_1(k, \Delta_n)$, and $\hat{T}_2(k, \Delta_n)$.

The receiver uses linear predictors and least squares estimators, and therefore, it is appropriately called the least squares estimates predictor receiver (LSEPR). Although it takes r = 4samples of the received signal per symbol interval, the VA and the predictors are updated only once per symbol interval. The computation can be sped up by precomputing the estimator matrices $(\mathbf{S}_{\mathbf{k}}^{H}\mathbf{S}_{\mathbf{k}})^{-1}\mathbf{S}_{\mathbf{k}}^{H}$ for all possible data sequences $\mathbf{a}_{\mathbf{k}}$ of length $2L_{c} + 1$ and storing them in a lookup table. The proposed channel estimator is shown in Fig. 4.

IV. SIMULATION RESULTS

The LSEPR has been developed for the purpose of complexity reduction. It is suboptimum and it is difficult to mathematically analyze its performance. Its performance is evaluated using computer simulations.

The computer simulations were performed using a quadrature phase-shift keying (QPSK) modulation format. The impulse response of the transmit filter was truncated to three symbol intervals. The multipath fading channel was assumed



Fig. 4. The channel estimator of the proposed receiver.

to be a two-ray channel with wide sense stationary uncorrelated scattering (WSSUS) statistics. For simplicity, a channel with a uniform delay power profile was used. It has been shown in [5] that for small delay spread, the performance of the communication system is not dependent on the delay power profile. To eliminate phase ambiguity, a pilot symbol is inserted every nn = 10 symbols in the transmitted data symbol stream. This avoids the need for differential encoding and decoding. The receiver consists of a 16-state VA with a decision delay of DD = 30 symbols. To ensure rapid convergence of the tap weights of the predictors, the RLS algorithm with a forget factor of 0.9999 was used for the adaptation of the predictor tap weights [9]. The signal-to-noise ratio (SNR) is defined as E_b/N_o where E_b is the energy per bit.

The initial results of the simulations were poor. It was observed that the conditioning number $N_{\rm cond}$ of the estimator matrix $(\mathbf{S}_{\mathbf{k}}^{H}\mathbf{S}_{\mathbf{k}})^{-1}\mathbf{S}_{\mathbf{k}}^{H}$ in (21) significantly affects the noise enhancement in the least squares estimate $\tilde{\mathbf{T}}_k^{LS}$ and the receiver performance can be severely degraded. If $N_{\rm cond}$ is large, then \tilde{T}_{L}^{LS} will deviate greatly from the actual vector T_{k} . Fig. 5(a) and (b) shows the mean square estimation errors between the least squares estimates and the actual value of $T_0(k)$. Two different sets of $\mathbf{S}_{\mathbf{k}}$ over all possible symbol sequences were generated by taking samples at two different sets of uniformly spaced sampling points (r = 4). The maximum $N_{\text{cond}} = 36$ for the simulation in Fig. 5(a) and the mean square error is about 10^{-4} most of the time. The maximum $N_{\rm cond} = 4772$ was found for the simulation in Fig. 5(b) and the mean square error is about 10^{-2} most of the time. Both simulations were at an SNR of 40 dB. In the second case, the receiver performance is severely degraded.

The reduction of $N_{\rm cond}$ can be achieved by careful selection of the sampling points or the transmit pulse shape g(t). It is noted that square root raised-cosine pulses generally yield a smaller $N_{\rm cond}$ than estimator matrices generated from full raised-cosine pulses [6] with equivalent rolloff factor α .

Although the careful selection of the sampling points may reduce the large conditioning number of the estimator matrix, it is a restrictive solution. A better solution lies in the manipulation of the last column of $\mathbf{S}_{\mathbf{k}}$. Recall from (20) that the last column of $\mathbf{S}_{\mathbf{k}}$ consists of $[s''_k, \dots, s''_{k+3}]^T$, the second derivative terms of the channel model. Assuming a two-ray



Fig. 5. (a) Least squares estimation error of the time-selective coefficient $T_0(k)$ with condition number of 36. (b) Least squares estimation error of the time-selective coefficient $T_0(k)$ with condition number of 4772.

channel with a uniform delay power profile and a maximum delay spread of $\tau_{\text{max}} = 0.2T$, the normalized average power of $T_2(k)$ is calculated from its autocorrelation function [5] and is found to be about 2.5×10^{-5} . Since the normalized average power of $T_2(k)$ is small, changing s''_k marginally will not affect the least squares estimation in (21) significantly. For the simulations, about -50 dB of noise with respect to bit energy is added to each element of $[s''_k, \dots, s''_{k+3}]^T$ to improve the conditioning of the estimator matrix. This is generally known as "dithering" and it leads to significant performance improvement as it significantly reduces N_{cond} to the range of 10 and 300 and, thereby, the estimation error.

Figs. 6–9 show the bit-error rate (BER) curves for the LSEPR using different fade time-bandwidth products or (normalized) BT and maximum delay spreads τ_{max} . B is the two-sided bandwidth of the channel fading process. All simulations were performed using a square root raised-cosine transmit

BER





Fig. 6. (a) Steady-state BER performance curves of the LSEPR and EMLSE for BT = 0.01 and $\tau_{max} = 0.125T$. (b) Steady-state BER performance curves of the LSEPR and EMLSE for BT = 0.03 and $\tau_{max} = 0.125T$.

pulse shape with rolloff $\alpha = 0.5$. The received signal was sampled r = 4 times per symbol interval and the predictors were updated once per symbol interval.

Figs. 6 and 7 compare the steady-state performance of the LSEPR to the performance of the extended MLSE (EMLSE) in [11]. Steady-state performance is achieved when the receiver has processed sufficient channel samples such that the tap weights of the predictors have converged to essentially their final and optimal values. In these simulations, the predictors were trained for the first 10 000 symbols to ensure that steady-state conditions are attained. The bold curves in the figures represent the analytical BER performance of the optimum EMLSE receiver using ideal channel state information.

At $E_b/N_o = 10$ dB, the LSEPR is about 1–3 dB worse in performance than the optimum EMLSE. The difference of the performances between the optimum and suboptimum receiver gradually increases from lower SNR to higher SNR. At $E_b/N_o = 30$ dB, the LSEPR is between 4–6 dB worse in BER performance than the EMLSE. It is also seen that

Fig. 7 (a) Steady-state BER performance curves of the LSEPR and EMLSE for BT = 0.01 and $\tau_{max} = 0.2T$. (b) Steady-state BER performance curves of the LSEPR and EMLSE for BT = 0.03 and $\tau_{max} = 0.2T$.

at a faster fade rate, the proposed receiver tends to perform closer to optimum at lower SNR. This may be attributed to the increased diversity due to faster fading. If the receiver is capable of obtaining high-quality CSI, then an increase in channel delay spread should improve its performance since the implicit delay diversity of the channel also increases. Instead, simulation results show that the performance of the LSEPR has degraded slightly for a channel with a larger delay spread. Furthermore, the difference between the BER curves for the EMLSE and the proposed receiver is more significant with increased delay spread. As seen in Figs. 6 and 7, the BER of the LSEPR for the BT=0.01 and $\tau_{\rm max}=0.125T$ case at an SNR of 30 dB is about 2×10^{-4} and for the BT = 0.01and $\tau_{\rm max} = 0.20T$ case is about 3×10^{-4} . The performance penalty appears to have two causes. First, the increase in channel delay spread is relatively small and, therefore, any performance gain attributed to increased delay diversity may be insignificant. Second, the performance of the receiver may



Fig. 8. The simulated average BER curves of the LSEPR for packet reception. Predictor length is 8.



Fig. 9. The simulated average BER curves of the LSEPR for packet reception. Predictor length is 12.

be affected by the truncation error in the quadratic f-power series model. From [5], it is known that the truncation error of the power series model increases with increasing channel delay spread. Therefore, the slight decrease in the performance of the receiver may be due to an increase in modeling error.

By using the RLS algorithm, the L tap weights of the predictors require at least 2L iterations before convergence [9]. This implies that a training sequence of 2L symbols is required prior to data transmission to train the predictors. For systems employing a time-division multiple-access (TDMA) format, the initial or startup condition is especially important because data is transmitted in relatively small packets with an interval between packets. The predictors are then required to be retrained for the reception of every packet.

Figs. 8 and 9 show the simulated average BER curves for the startup performances of the LSEPR for different fade rates and predictor orders. The training sequence was limited to 2L symbols. Transmission is broken up into packets of 1000 data symbols per packet. After the initial 2L symbol training period, the first packet of 1000 symbols is received. The BER is then calculated for the reception of that particular packet. The predictors are then reinitialized and prepared for training and reception of the following packet. After all packets have been sent, all of the BER's calculated are averaged over the total number of packets received.

The average BER curves of the startup performances of the LSEPR do not exhibit premature error floors for the range of SNR's simulated. The startup performance is between 5–10 dB worse than the steady-state performance. The degradation in the performance of the receiver during startup may be attributed to the occurrences of deep fades which yield low instantaneous SNR. Therefore, the channel observations during the training interval can be very noisy and the predictors may not converge completely. We were motivated to shorten the predictor lengths to attain a shorter training period. The simulations showed that there is no significant difference in the results between using predictors of order L = 8 and L = 12.

For low SNR, the results shown in Figs. 7–9 are comparable to the BER results for a zeroth-order or one-term receiver using ideal CSI in [5]. For example, at an SNR of 15 dB and channel delay of 0.2T, the proposed receiver has BER between 10^{-2} and 3×10^{-2} , which is approximately the same as the zeroth-order receiver in [5]. However, the latter will exhibit an error floor of about 10^{-2} . As shown in [5], the error floor is reduced but not eliminated if a first-order receiver is employed. Although the performance of a zeroth-order receiver may be acceptable for applications which require a BER of about 10^{-2} [12], it may not be acceptable for applications, such as video and reliable data, which require lower BER.

The proposed scheme may be extended to channels with delay spreads greater than 0.2T by using three or more terms in the power series. However, the complexity of the power series model will approach that of the conventional tapped delay line model with an increasing number of terms. This will lead to larger frequency-selective matrices which will be numerically unstable. Therefore, increasing the number of terms in the power series much beyond three may not be appropriate.

V. CONCLUSION

In this paper, we have proposed a novel receiver structure for a time- and frequency-selective fading channel. The receiver is a sequence estimator implemented using the VA with a data-aided channel estimator to provide channel state information for Viterbi decoding. The proposed receiver has been based on the use of Bello's f-power series channel model truncated to three terms [1]. This model describes the dispersive fading channel as a sum of flat-fading channels. Hence, we have extended the ideas for channel estimation and observation used by predictor receivers for flat-fading channels to time- and frequency-selective channels.

The receiver known as the LSEPR uses a least squares estimation algorithm to observe the fading for each elementary channel along each survivor. These least squares estimates are then used to obtain one-step predictions of the TSC's for the next VA iteration. The predictors use the standard RLS fast-convergence algorithm to update their tap weights. The performance of the least squares estimation algorithm may be degraded by the poor conditioning of the least squares estimator matrix, but this problem is solved by dithering the matrix to improve its conditioning.

The simulated performance of the new receiver using QPSK shows that it can cope with channels having delay spread up to $\tau_{\text{max}} = 0.2T$ and fade rates up to at least BT = 0.03T. Some performance penalty is incurred during startup. The performance of the proposed receiver may be comparable to simpler receivers at low SNR. However, unlike the latter, the BER curves of the new receiver do not floor at higher SNR. Therefore, it is suitable for applications requiring lower BER.

Although the proposed receiver is suboptimum, its implementation is relatively simple compared to other receivers for the same channel [3], [13], [14]. Complexity reduction is also achieved by the fact that the LSEPR is only required to estimate three unknown quantities $T_0(k)$, $T_1(k)$, and $T_2(k)$ per survivor unlike other receivers where the entire composite channel impulse response is estimated. A further advantage of the LSEPR is that it is adaptive, requiring only a short training sequence, and it does not need the second-order channel statistics *a priori*.

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