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# An Adaptive Robust Framework for Model-Based State Fault Detection

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***Abstract*— A goal in many applications is to combine *a priori* knowledge of the physical system with experimental data to detect faults in a system at an early enough stage as to conduct preventive maintenance. The information available beforehand is the mathematical model of the physical system and the key issue in the design of model-based fault detection is the effect of model uncertainties such as severe parametric uncertainties and unmodeled dynamics on their performance. This paper presents the application of a nonlinear model-based adaptive robust state fault detection that combines on-line parameter adaptation with robust filter structures to reduce the extent of model uncertainty to help in the improvement of the sensitivity of the fault detection scheme to faults. Simulation results are presented to demonstrate the superior performance of the proposed scheme in the early and reliable detection of incipient faults.**

## I. INTRODUCTION

For the design of reliable control systems, one of the integral parts is the ability of the system to react to sudden unexpected changes in the operating conditions like sensor or actuator failure. Fault tolerant control systems have a two-fold objective of identification/isolation of faults and regulation of the system output in the presence of faults. The interest in the design of reliable automated systems has spurred the interest in fault detection and diagnostics (FDD) for nonlinear systems. The action required to regulate the system is different for each of the faults detected and this places an importance on the detection (the presence of a fault) and on the diagnosis (the location and extent of a particular fault).

A comprehensive review of the different methods for fault detection and their applicability to a given physical system has been presented in [1]. Model-based fault detection and diagnosis systems have found extensive use because of the fast response to abrupt failure and the ease of implementation of the model-based FDD systems in real-time algorithms. In most model-based FDD schemes an idealized assumption that the a perfect mathematical model of the system is available is made. In practice however, this assumption can never be completely satisfied, since an accurate mathematical model of the physical system is not usually available because of model uncertainty coming in because of either parametric uncertainty or uncertain nonlinearities. Hence, the robustness of the fault detection algorithm to modeling errors without losing sensitivity to faults is the key problem in the application of model based fault detection algorithms. This trade-off between robustness and sensitivity has attracted a lot of research attention recently [2], [3] for linear systems and in the work in [4], [5] for nonlinear systems.

Since assuming a known structure for the model uncertainty would limit the applicability of the fault detection schemes, the researchers in[6], [7], [8], [9] designed FDD schemes for state fault detection in which the model uncertainty is assumed to be unstructured but bounded by suitable constants or functions obtained using *a priori* system identification. This approach lumps the model uncertainty in the plant into an unknown term without attempting to differentiate between the causes of uncertainty. This results in FDD schemes that have large thresholds to ensure robustness of the fault detection scheme corresponding to a reduction in the sensitivity of the fault detection scheme.

In this paper, we present a model-based fault detection scheme to detect state faults using the adaptive-robust control philosophy. In this framework we differentiate between the causes of model uncertainty and use different tools to attenuate their effect. The following two types of uncertainties are of major concern in the design of fault detection schemes for uncertain nonlinear systems; parametric uncertainties (e.g., the unknown inertial load in the most robotic applications) and general uncertainties coming from modeling errors (e.g., nonlinear friction) and uncertain nonlinearities which could include external dynamics and unmodeled dynamics. Robust state reconstruction schemes are combined with controlled on-line parameter adaptation to design adaptive-robust state reconstruction schemes. The parameter adaptation reduces the extent of the model uncertainty leading to an improvement in the sensitivity of the fault detection scheme without a corresponding loss of robustness of the fault detection scheme.

The paper is organized as follows: In section II the problem is formulated presenting the nominal model of the system being considered and the assumptions used in the design of the fault detection scheme, the fault detection architecture and the scheme are presented in Section III. Section IV demonstrates the use of batched least-squares for parameter estimation. Analytical results on the robustness and sensitivity of the fault detection scheme are presented in Section V, comparative simulation results on a standard Van der Pol oscillator model are given in Section VI and conclusions are presented in Section VII.

## II. PROBLEM FORMULATION

In this section of the paper we present the system dynamics that are considered in the design of the state fault

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detection scheme and the assumptions made in the design. In model-based fault detection, residual signals are used to detect the presence of a fault. In this paper we continuously monitor the state reconstruction error and the parameter estimates and utilize them as residuals in the fault detection process.

### A. Nominal Model of the System

The nominal model of the system under consideration using the terminology in [7], [8] is given as:

$$\dot{x}=f(x,u)+\Psi(x,u)\theta+\Delta(x,u,t)+B(t-T_f)g(x,u) \quad (1)$$

where, $x\in R^n$ is the state vector, $u\in R^m$ is the input to the system and $\theta\in R^p$ is the vector of constant but unknown parameters. $f(x,u)\in R^n$ and $\Psi(x,u)\in R^{n\times p}$ are vectors or matrices of known smooth functions, $\Delta(x,u,t)$ represents the lumped unknown nonlinear functions such as modeling errors and $g(x,u)$ is a vector that represents the change in the system dynamics due to an unknown fault with $T_f$ being the time of occurrence of the fault and $B(t-T_f)$ is a diagonal matrix representing the time profiles of the faults and is written in the following form:

$$B(t-T_f)=diag[\beta_1(t-T_f),\cdots,\beta_n(t-T_f)]$$

where $\beta_i$ represents the time profile of the fault in the $i-th$ state equation. The time profile is given by:

$$\beta_i(t-T_f)=\begin{cases} 0 & \text{if } t<T_f \\ 1-e^{-\alpha_i(t-T_f)} & \text{if } t\geq T_f \end{cases} \quad (2)$$

with $\alpha_i>0$ denoting the rate of fault evolution. In the $i-th$ channel of the system the effect of the fault is:

$$\dot{x}_i=f_i(x,u)+\psi_i(x,u)\theta+\Delta_i(x,u,t)+\beta_i(t-T_f)g_i(x,u) \quad (3)$$

where $\psi_i(x,u)$ is the $i-th$ vector corresponding to the $i-th$ channel in the $\Psi(x,u)$ matrix, $f_i(x,u)$, $g_i(x,u)$ and $\Delta_i(x,u,t)$ are the $i-th$ components of the $f(x,u)$, $g(x,u)$ and $\Delta(x,u,t)$ vectors respectively.

The equations in (1) and (3) represents the system dynamics in the presence of a state dependent fault. The major difficulties in the design of fault detection schemes for the system described in equations (1) and (3) are:

1. The system dynamics represented by the functions $f(x,u)$ and $\Psi(x,u)$ are highly nonlinear.
2. The system has model uncertainties coming from both uncertain parameters $\theta$ and uncertain nonlinearities $\Delta(x,u,t)$.

It should be noted that in the nominal model in equation (1) we consider not only the uncertainty in the parameters but also uncertain nonlinearities represented by $\Delta(x,u,t)$. As opposed to the fault detection schemes described in the literature [6], [7], [8], [9] the uncertainties have been differentiated by looking at the causes of the model uncertainty. By using controlled on-line parameter adaptation mechanisms we can reduce the extent of uncertainty and increase the sensitivity of the fault detection scheme.

### B. Assumptions

The following practical assumptions are made in the design of the fault detection system:

*Assumption 1:* The unknown but constant parameters $\theta_i$ lie in a known bounded region $\Omega_{\theta_i}$, i.e.,

$$\theta_i\in\Omega_{\theta_i}=\{\theta_i : \theta_{imin}<\theta_i<\theta_{imax}\} \quad (4)$$

*Assumption 2:* The uncertain nonlinearities $\Delta_i$ are bounded, i.e.,

$$\Delta_i\in\Omega_{\Delta_i}=\{\Delta_i:|\Delta_i(x,u,t)|\leq\delta_i\} \quad (5)$$

where for each $i=1,\cdots,n$ the uncertain nonlinearities are bounded by a known constant $\delta_i$.

*Remark 1:* In fault diagnosis literature the modeling uncertainty is often assumed to be structured and this allows the use of transformations to decouple faults from the model uncertainties [10], [11], [12]. Such structural assumptions significantly limit the applicability of the approach. Similar to [7] in this paper the modeling uncertainty is assumed to be unstructured but bounded to remove the above conservativeness. The bound on the modeling uncertainties helps in the derivation of a suitable threshold for distinguishing between the effect of a fault and the effect of model uncertainty.

*Assumption 3:* The system states and the control input to the system remain bounded before and after the occurrence of the fault; i.e., $x(t)\in\mathscr{L}_\infty$ and $u(t)\in\mathscr{L}_\infty$.

*Remark 2:* The above assumption is made because no fault accommodation is considered in this paper. Therefore, the feedback controller is such that the measured signals $x(t)$ and the control input $u(t)$ remain bounded $\forall\ t\geq 0$.

## III. Fault Detection Architecture using Adaptive Robust State Reconstruction Scheme

The proposed fault detection scheme is used to detect additive and multiplicative state faults reliably and early enough for preventive maintenance. The fault detection scheme consists of three components: the state reconstruction observer that reconstructs the states, the parameter estimation scheme that estimates the parameters to reduce the extent of model uncertainty and the evaluation scheme that monitors the state reconstruction error and the parameter estimates to detect the presence of any off-nominal system behavior. The overall block diagram of the fault detection scheme and the flow of information in the system is shown in Figure 1. In this section we will describe the three components of the fault detection scheme.

### A. Adaptive Robust State Reconstruction Scheme

From (3), it can be observed that the dynamics of the $i-th$ channel of the system in the absence of a fault (i.e., $g_i(x,u)=0$) are affected by uncertainty in the parameters ($\theta$) and that due to the presence of unmodeled dynamics represented by ($\Delta_i$). Therefore, the $i-th$ state is reconstructed as:

$$\dot{\hat{x}}_i=f_i(x,u)+\psi_i(x,u)\hat{\theta}+h_i(x)(x_i-\hat{x}_i) \quad (6)$$

where, $h_i$ is a nonlinear gain used for the reconstructing the $i-th$ state and $\hat{\theta}$ is the estimate of the parameters

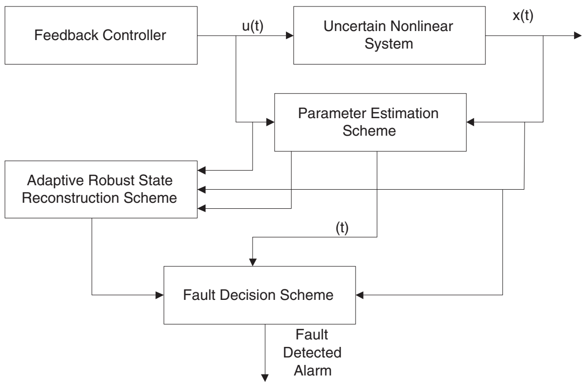


Fig. 1. State fault detection scheme using ARSR observers.

using the parameter estimation scheme as described in the next subsection and $\hat{x}_i$ is the reconstructed state. Hence, the dynamics of the state reconstruction error $\tilde{x}_i = x_i - \hat{x}_i$ using equations (3) and (6) is given as:

$$\dot{\tilde{x}}_i = -h_i \tilde{x}_i + \psi_i(x,u)\tilde{\theta} + \Delta_i(x,u,t) + \beta_i(t-T_f)g_i(x,u) \tag{7}$$

where $\tilde{\theta}$ is the parameter estimation error.

### B. Parameter Estimation Scheme

In this paper, controlled on-line parameter adaptation is utilized in order to account for the presence of parametric uncertainty. The parameter estimation in this paper is done in order to reduce the extent of model uncertainty as opposed to model-based fault detection schemes presented in literature [6], [8] where parameter adaptation is utilized after the fault has been detected to reconstruct the fault and aid in fault diagnosis.

Controlled on-line parameter adaptation is utilized to reduce the extent of the model uncertainty which improves the sensitivity of the fault detection scheme without loss of robustness. In this paper the parameters are estimated only when the regressor vector is rich enough and the parameter estimation scheme used is described in this section. Using the dynamics of the system in equation (1), we design the following filters:

$$\dot{\Omega}^T = A\Omega^T + \Psi(x,u) \tag{8}$$

$$\dot{\Omega}_0 = A(\Omega_0 + x) - f(x,u) \tag{9}$$

where $A$ is an exponentially stable matrix. Define $z = x + \Omega_0$ which is calculable, then from (1) and (9)

$$\dot{z} = Az + \Psi(x,u)\theta + \Delta(x,u,t) + B(t-T_f)g(x,u) \tag{10}$$

Let $\varepsilon = x + \Omega_0 - \Omega^T\theta = z - \Omega^T\theta$. From (10), (8) the dynamics of $\varepsilon$ is:

$$\dot{\varepsilon} = A\varepsilon + \Delta(x,u,t) + B(t-T_f)g(x,u) \tag{11}$$

which has a stable dynamics in the absence of unmodeled dynamics (i.e., $\Delta=0$) and in the absence of faults in the system (i.e., $g(x,u)=0$). The static model used for the estimation of the unknown parameter vector ($\theta$) is:

$$z = \Omega^T\theta + \varepsilon \tag{12}$$

where, $\varepsilon$ represents the combined effect of the unmodeled dynamics and the presence of faults in the system. The parameters are only updated when the regressor is rich enough as follows. Let $R(kT) = \int_{(k-1)T}^{kT} \Omega(\tau)\Omega^T(\tau)d\tau$ where $T$ is the time window over which the regressor is monitored to estimate its richness and $k = 1, 2, \cdots$ is an integer. The parameter estimate will be updated every $T$ seconds and the parameter update law is given as:

$$\hat{\theta}(kT) = \begin{cases} R(kT)^{-1}\int_{(k-1)T}^{kT}\Omega(\tau)z(\tau)d\tau & \text{if } R(kT) \geq \alpha(kT)I \\ \hat{\theta}((k-1)T) & \text{otherwise} \end{cases} \tag{13}$$

where, $\alpha(kT) > 0$ is a positive number, and

$$\hat{\theta}(t) = \hat{\theta}((k-1)T), \ \forall \ t \in [(k-1)T, kT) \tag{14}$$

In matrix terminology $A \geq \alpha I$ implies that the $\lambda_{min}(A) \geq \alpha$. The design variable $\alpha$ quantifies the richness of the signal.

Hence, the adaptive robust state reconstruction scheme consists of the state reconstruction observer in equation (6) and the parameter estimation scheme given by equation (13).

### C. Residual Evaluation Scheme

In this paper, the state reconstruction error and the parameter estimates are continuously monitored and used as the residual signals to detect faults. To decide wether a fault has occurred or not we monitor the residual signals for off-nominal behavior. For the $i-th$ channel:

$$r_{x_i} = \begin{cases} \text{fault} & \text{if } |\tilde{x}_i(t)| \geq \tilde{x}_i^0(t), \quad i = 1, \cdots, n \\ \text{fault-free} & \text{otherwise} \end{cases} \tag{15}$$

where, $\tilde{x}_i^0(t)$ is a suitable threshold on the state reconstruction error $\tilde{x}_i(t)$ that will be specified in the next section. The presence of model uncertainties leads to a non-zero error even in the absence of a fault. The proper choice of the threshold $\tilde{x}_i^0(t)$ would prevent any false alarms while still being able to detect small/incipient faults.

In addition to the state reconstruction error ($\tilde{x}_i$), in the paper we utilize the parameter estimates as residuals to help detect incipient faults in systems as well. The residual evaluation scheme based on the parameter estimates is given as:

$$r_\theta = \begin{cases} \text{fault-free} & \text{if } \hat{\theta}(t) \in \mathrm{C}_\theta(t) \\ \text{fault} & \text{otherwise} \end{cases} \tag{16}$$

where, $\mathrm{C}_\theta$ is a known region of the parameter estimate space which will be specified later. It is an indication of the presence of off-nominal system behavior when the parameter estimates are outside this region.

## IV. Parameter Estimation Using Batched Least Squares

In this paper we utilize on-line parameter adaptation using the batched least-squares approach as described in equation (13). The model used for the estimation of the unknown parameter vector $\theta$ is given by (12), where the dynamics of the modeling error is given by (11). In the absence of faults and uncertain nonlinearities (i.e., $\Delta=0$ and $g(x,u)=0$) for the parameter estimation error to converge to zero ($\tilde{\theta}\to 0$), the matrix $R(kT)$ must be positive definite [13].

Using the parameter estimation scheme in equation (13), and the model used for estimating $\theta$ in equation (12) the parameter estimation error is given by:

$$\tilde{\theta}(t)=R(kT)^{-1}\int_{(k-1)T}^{kT}\Omega(\tau)\varepsilon(\tau)d\tau,\ \forall\ t\in[kT,(k+1)T) \quad (17)$$

where the dynamics of $\varepsilon$ are as described in equation (11).

In the absence of faults in the system ($g(x,u)=0$) but in the presence of bounded uncertain nonlinearities as in assumption (2), the dynamics of $\varepsilon$ are:

$$\dot{\varepsilon}=A\varepsilon+\Delta(x,u,t) \quad (18)$$

which has bounded solutions with the proper choice of the Hurwitz matrix $A$.

*Lemma 1:* With the system of filters given in equations (8) and (9) and the proper choice of the Hurwitz matrix $A$ the model mismatch is bounded i.e.,

$$|\varepsilon(t)|\leq\frac{2}{\nu_A}|\delta| \quad (19)$$

Looking at the bounded solutions of the estimation error in equation (19), when the parameters are adapted only when the regressor is rich enough, then the parameter estimation error is bounded and this is given by the following result:

*Theorem 1:* In the absence of faults in the system and if the parameters are adapted using the model in (12) and the adaptation scheme described in (13), then the parameter estimation error is bounded and the bound on the parameter estimation error is given by:

$$|\tilde{\theta}(t)|\leq\tilde{\theta}_{max}(kT),\ \forall\ t\in[kT,(k+1)T) \quad (20)$$

where,

$$\tilde{\theta}_{max}(0)=|\theta_{max}-\theta_{min}| \quad (21)$$

and

$$\tilde{\theta}_{max}(kT)=\begin{cases}\frac{2|\delta|}{\alpha(kT)\nu_A}\int_{(k-1)T}^{kT}|\Omega(\tau)|d\tau & \text{if} \quad R(kT)\geq\alpha(kT)I\\ \tilde{\theta}_{max}((k-1)T) & \text{otherwise}\end{cases} \quad (22)$$

with $k=1,2,\cdots$.

## V. Performance Results

The equations (6), (13), (15) and (16) describe the fault detection scheme that consists of the residual generation scheme and the residual evaluation scheme to detect faults in the system. In this section we present results on the robustness and sensitivity of the proposed scheme.

### A. Robustness

In order to detect faults in the $i-th$ channel we monitor the reconstruction error $\tilde{x}_i$ and therefore the threshold for the error $\tilde{x}_i^0$ has to be so chosen such that in the absence of faults:

$$|\tilde{x}_i(t)|\leq\tilde{x}_i^0(t) \quad (23)$$

i.e., the reconstruction error should be less than the selected threshold for all possible values of the model uncertainty. In order to show that the proposed scheme is robust to model uncertainty, the state reconstruction error in the absence of faults $g_i(x,u)=0$ satisfies $\forall\ t\in[(k-1)T,kT)$:

$$\tilde{x}_i(t)=e^{-h_i(t-(k-1)T)}\tilde{x}_i((k-1)T) + \int_{(k-1)T}^{t}e^{-h_i(t-\tau)}[\psi_i(x,u)\tilde{\theta}+\Delta_i(x,u,\tau)]d\tau \quad (24)$$

The first term in the above equation represents the effect of the initial state reconstruction error and the second term represents the combined effect of the unmodeled dynamics and the parametric uncertainty. Since, we can always choose the initial condition $\tilde{x}_i(0)=0$, the state reconstruction error at the end of each parameter updating instance satisfies:

$$\tilde{x}_i(kT)=\Sigma_{j=1}^{k}e^{(j-k)h_iT}\int_{(j-1)T}^{jT}e^{-h_i(jT-\tau)}[\psi_i(x,u)\tilde{\theta} +\Delta_i(x,u,\tau)]d\tau \quad (25)$$

Since the function $\psi_i(x,u)$ in equation (1) is continuous over a bounded interval $[(k-1)T,kT)$ in addition to assumption (3) we have that the function is bounded i.e.,

$$|\psi_i(x,u)|\leq P_i(x,u) \quad (26)$$

where $P_i(x,u)$ is a function that upper bounds the regressor function. Looking at the state reconstruction error in the absence of a fault (24), the parameter estimation error in equation (20) from the results in Theorem (1), the robustness of the proposed fault detection scheme is given by the following result.

*Theorem 2:* Consider the proposed nonlinear adaptive robust fault detection scheme described by equations (6), (13), (15) and (16). If the threshold for the state fault detection scheme $\tilde{x}_i^0(t)$ is chosen such that $\forall\ t\in[(k-1)T,kT)$:

$$\tilde{x}_i^0(t)=e^{-h_i(t-(k-1)T)}\Gamma_i((k-1)T)+\frac{\delta_i}{h_i}[1-e^{-h_i(t-(k-1)T)}] +\tilde{\theta}_{max}((k-1)T)\int_{(k-1)T}^{t}e^{-h_i(t-\tau)}P_i(x,u)d\tau \quad (27)$$

where,

$$\Gamma_i(kT)=\Sigma_{j=1}^{k}e^{(j-k)h_iT}[\frac{\delta_i}{h_i}[1-e^{-h_iT}] +\tilde{\theta}_{max}((j-1)T)\int_{(j-1)T}^{jT}e^{-h_i(jT-\tau)}P_i(x,u)d\tau] \quad (28)$$

then the proposed fault detection scheme is robust to model uncertainty and avoids false alarms.

### B. Sensitivity

Sensitivity of a fault detection scheme is the ability to detect the presence of faults at an early enough stage for preventive maintenance. In this section we demonstrate that the use of parameter adaptation reduces the uncertainty thereby making the proposed scheme more sensitive to incipient failure without loss of robustness.

In this section we demonstrate that a proper choice of the parameter adaptation design parameter $\alpha(kT)>0$ in equation (13) would lead to a fault detection scheme that is more sensitive by reducing the extent of the model uncertainty.

Let us consider a fault detection scheme in which fixed parameters are used without on-line parameter adaptation. Let $\mu^0$ be the threshold used to detect faults in such a fault detection scheme. In such schemes, fixed parameters $\hat{\theta}(0)$ are used for $\theta$ and the parameter estimation error satisfies $\tilde{\theta} \leq |\theta_{max} - \theta_{min}| = \tilde{\theta}_{max}(0)$. Hence, in schemes without on-line adaptation, the threshold for robustness satisfies (27) $\forall\ t \in [(k-1)T, kT)$:

$$\mu^0(t) = e^{-h_i(t-(k-1)T)}\Gamma_i((k-1)T) + \frac{\delta_i}{h_i}[1 - e^{-h_i(t-(k-1)T)}] + \tilde{\theta}_{max}(0)\int_{(k-1)T}^{t} e^{-h_i(t-\tau)} P_i(x,u)\, d\tau \qquad (29)$$

where,

$$\Gamma_i(kT) = \Sigma_{j=1}^{k} e^{(j-k)h_i T}[\frac{\delta_i}{h_i}[1 - e^{-h_i T}] + \tilde{\theta}_{max}(0)\int_{(j-1)T}^{jT} e^{-h_i(jT-\tau)} P_i(x,u)\, d\tau] \qquad (30)$$

Looking at the equation (29) we give the following result which gives the required design parameter $\alpha(kT)$ for improvement of sensitivity of the fault detection scheme when on-line parameter adaptation is utilized:

*Theorem 3:* If the parameter adaptation process in the state fault detection scheme is conducted as in equation (13) with the design parameter $\alpha(kT)$ chosen such that:

$$\alpha(kT) \geq \frac{2|\delta|}{|\theta_{max} - \theta_{min}|\nu_A}\int_{(k-1)T}^{kT} |\Omega(\tau)|\, d\tau \qquad (31)$$

then, the threshold for detecting faults in the $i-th$ channel if on-line parameter adaptation is implemented is such that $\tilde{x}_i^0 \leq \mu^0$ i.e., the state fault detection system with on-line parameter adaptation is more sensitive.

Hence, the use of on-line parameter adaptation reduces the extent of model uncertainty and helps make the fault detection scheme more sensitive without a corresponding loss of robustness.

## VI. Simulation Results on a Van der Pol Oscillator

In this section of the paper, we present results on the detection of state faults using simulation results on a Van der Pol oscillator. A spring-mass-damper system with a nonlinear spring and a flow induced vibration system are examples of physical systems whose dynamics can be represented using the Van der Pol oscillation model.

This system is utilized because results are available in the literature [6], [9] which apply various robust fault detection schemes to it. This enables us to compare the results of our proposed scheme to the results in the literature.The Van der Pol equation is given by:

$$\ddot{y} + 2\omega\xi(\mu y^2 - 1)\dot{y} + \omega^2 y = u + \beta(t - T_f)f(y) + \Delta_x \qquad (32)$$

where, $\omega$, $\xi$ and $\mu$ are positive constants, $\beta$ is the time profile of the fault, $f$ is the change in system dynamics due to a fault and $\Delta_x$ represents the unmodeled dynamics of the system due to the higher order terms that have been ignored in obtaining the system dynamics of the Van der Pol oscillator. It is to be noted that in this paper the uncertainty in the model comes in not only because of parametric uncertainty but also due to the unmodeled dynamics. The use of parametric adaptation reduces the model uncertainty which increases the sensitivity of the fault detection scheme without loss of robustness.

Utilizing $x_1 = y$ and $x_2 = \dot{y}$ as the state variables we get the state-space equations of the system as:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u - 2\omega\xi(\mu x_1^2 - 1)x_2 - \omega^2 x_1 + \Delta_x + \beta(t - T_f)f(x_1) \qquad (33)$$

In this simulation study we use not only a 10% uncertainty in the value of $\xi$ but also uncertainty coming in due to the presence of unmodeled dynamics because of neglected higher order dynamics represented by $\Delta_x$. The following parameters are used in the simulation studies: $\omega = 0.9$, $\xi = 0.6$ and $\mu = 0.95$. The parametric uncertainty is assumed to occur due to a 10% uncertainty in $\theta = \xi$ and due to the presence of unmodeled dynamics $\Delta_x$ which is bounded by a known constant $\delta = 0.1$. Two faults are considered in this paper the first $f_1(x_1) = 1.4 + 0.4cos(x_1)$ which occurs at time $T_f = 30$ seconds and evolving at a rate of $\alpha = 0.2$. The second fault has a smaller magnitude of fault to demonstrate the improvement in the sensitivity of the scheme due to the use of parameter adaptation and is given by $f_2(x_1) = 0.45cos(x_1)$. In Figure 2 we present the simulation results due to the presence of the first fault function $f_1(x_1)$ with and without parameter adaptation in the state reconstruction scheme. In order to further demonstrate the ability of the fault detection scheme to detect small faults early enough for preventive maintenance a smaller fault given by $f_2(x_1) = 0.45cos(x_1)$ which occurs at time $T_f = 30$ seconds and evolves at a rate of $\alpha = 0.2$ is simulated and the results of the schemes with and without adaptation are given in Figure 3. In addition to being able to detect additive state faults, since the parameter estimates are also monitored, multiplicative (parametric) faults can also be reliable detected.In order to validate that the proposed scheme can reliably detect multiplicative faults, a fault is introduced that affects the damping ratio ($\xi$)

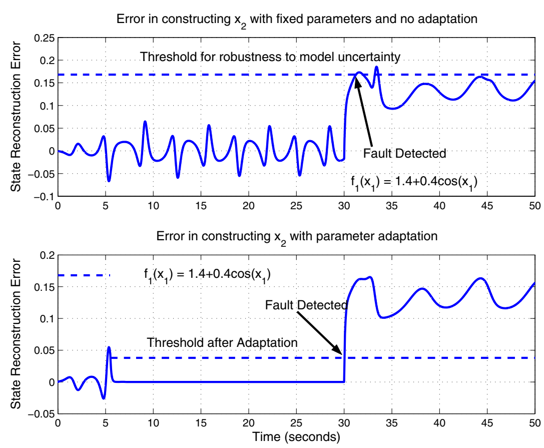


Fig. 2. Fault detection with $f_1(x_1)$ affecting the system.

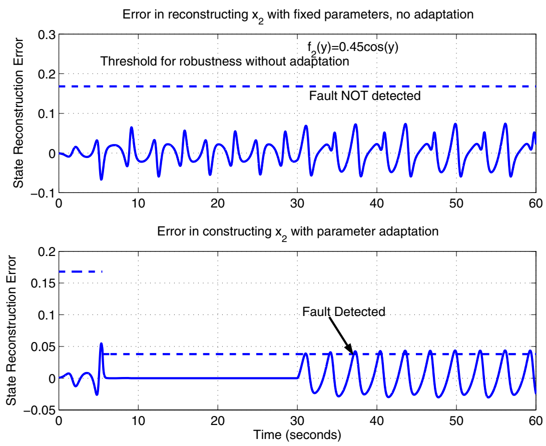


Fig. 3. Fault detection with $f_2(x_1)$ affecting the system.

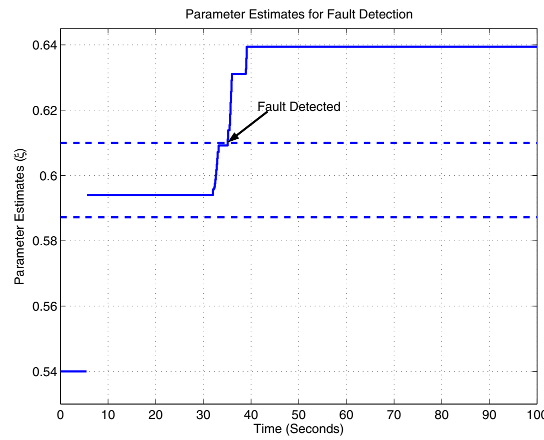


Fig. 5. Parameter estimates with $f_3(\xi)$ affecting the system.

by increasing its value. This is a common occurrence in systems with fluid contaminants that change the value of the damping ratio. The fault function that is simulated is such that $f_3(\xi) = \xi_0 + 0.01t$ where $\xi_0 = 0.54$. The fault function is simulated such that the fault starts at $T_f = 30$ seconds and follows a ramp like increase in the value of $\xi$. In Figure 4 and Figure 5 we present the state reconstruction error without parameter adaptation, the reconstruction error with parameter adaptation and the estimates of the parameters. It can be seen that the parameter estimation scheme and the adaptive robust state reconstruction scheme are able to detect the off-nominal behavior of the parameter estimate well ahead of the non-adaptive scheme.

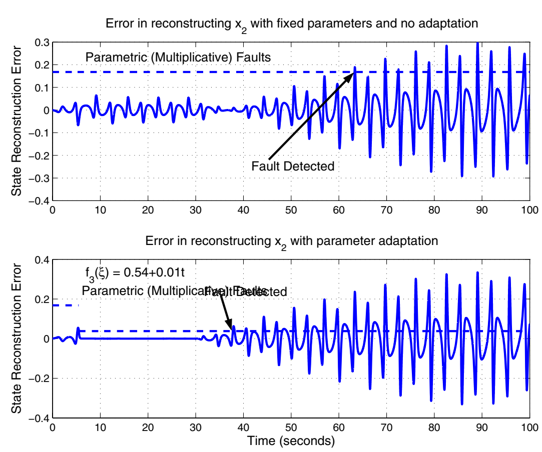


Fig. 4. Fault detection with $f_3(\xi)$ affecting the system parameters.

## VII. Conclusions

In this paper we present an adaptive robust framework for model-based state fault detection. An adaptive robust state reconstruction (ARSR) scheme is designed to reconstruct the state. The ARSR utilizes robust filter structures to attenuate the effect of unmodeled dynamics and combines this with on-line parameter adaptation to reduce the extent of model uncertainty to improve the sensitivity of the fault detection scheme without a corresponding loss of robustness. Comparative simulation results are presented to demonstrate the improved performance of the proposed scheme to help in the early detection of incipient faults.

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