AN ADAPTIVE SHEWHART CHART WITH VARYING SAMPLE STATISTIC TO CONTROL BIVARIATE PROCESSES

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In this article, we propose a single mean chart to control bivariate processes. The basic idea is to double the size of the samples, once only one of the two quality characteristics, X or Y, is measured and only one of the two sample statistics is computed, or the mean of the X observations or the mean of the Y observations. The statistic that was used to obtain the current point and the current point's position define the statistic for the next sample. If the current point is the mean of the X observations and it is in the central region, then the control of X is relaxed and the focus goes to Y, that is, the charting statistic for the next sample will be the mean of the Y observations. On the other hand, if the current point is in the warning region, the focus remains on X, that is, the charting statistic for the next sample will be the mean of the X observations. A similar rule is applied when the current statistic is the mean of the Y observations. Thanks to the adaptive feature of the proposed chart, the degree of dependence between X and Y plays no hole in chart's performance. The adaptive chart is overwhelmingly faster than the Hotelling chart in signaling all kinds of disturbances.

Palavras-chave: quality control, Variable charting statistic, Shewhart chart, Hotelling chart;, Control charts

1. Introduction

In many production lines, samples are regularly taken from the process to update the information about the quality characteristics of the products, which are used to calculate the plotting statistic of multivariate control charts. Recently, a great deal of research has been done on the control of two quality characteristics. Leoni, Costa, and Machado (2015), Leoni, Machado, and Costa (2015a, 2015b), and Simões et al. (2016) considered bivariate processes with autocorrelated data and the control of their mean vector. Melo, Ho, and Medeiros (2016) and Ho and Costa (2015) designed attribute charts to control the mean vector of bivariate processes. Saghir (2015), Saghir, Khan, and Chakraborti (2016) and Osei-Aning, Abbasi, and Riaz (2016) also worked with the control of bivariate processes, but with the monitoring of their covariance matrix. Machado and Costa (2008) considered a VMAX statistic to control the covariance matrix of bivariate processes. The VMAX statistic is based on the sample variances of the two quality characteristics. Ho and Aparisi (2016) proposed two new control charts that are a mixture of attribute and variable charts to monitor bivariate mean vectors.

Since Reynolds Jr. et al. (1988) introduced the idea of varying the control chart's parameters, such as the sampling intervals or the size of the samples, researchers provided that the use of variable parameters enhances the performance of all kind of univariate and multivariate charts. Costa (1994, 1997, 1999), Nenes, (2011), He et al. (2014), Faraz et al. (2014), Lim et al. (2015) and Naderkhani and Makis (2016) are some pioneering and recent work in this field.

The idea of varying the number of variables to be monitored was considered by Aparisi, Epprecht, and Ruiz (2012). They proposed the variable dimension T^2 chart, in which the number of monitored quality characteristics is variable; the position of the current T^2 point determines the next number of variables to be monitored. If the current T^2 point is in the central region, then, in the next sample of size n, only a subset of the all p quality characteristics, say p_1, will be measured, consequently the next T^2 value will be computed with reduced observations, that is, with np_1 observations only. If the current T^2 point is in the warning region, then, in the next sample of size n, all p quality characteristics will be measured and the next T^2 value will be computed with the np observations. The p_1 variables are, in general, the less expensive and/or the easier to measure. Subsequently, Aparisi et al. (2014) proposed the variable dimension T^2 chart with variable sample size, that is, a T^2 point is in the warning region not only triggers the measurement of all quality characteristics but also schedules for the next sampling point a sample larger than n. The variable dimension T^2 chart can be seen as a chart with variable charting statistic, once the T^2 statistic obtained with np_1 observations is different of the T^2 statistic obtained with all the np observations.

In this article, we propose an adaptive mean chart with varying sample statistic (*VCS* chart) to control bivariate processes as an excellent alternative to the Hotelling T^2 chart. The basic idea is to double the size of the samples, once only one of the two quality characteristics, *X* or *Y*, is measured. That is, at each sampling point, only one of the two sample statistics is computed, or the mean of the *X* observations or the mean of the *Y* observations.

2. The Shewhart mean chart with variable charting statistic

The Hotelling T^2 chart is the usual Shewhart-type chart to control the mean vector of bivariate processes. With the T^2 chart in use, samples of size n are regularly collected and the two quality characteristics (*X*, *Y*) of the selected items are measured. The two common assumptions to study the performance of the bivariate T^2 charts are: (I) The two quality characteristics (*X*, *Y*) follow a bivariate normal distribution with the mean vector and the covariance matrix given as $\boldsymbol{\mu}=(\mu_X;\mu_Y)'$ and $\boldsymbol{\Sigma}=(\Sigma_{11}=\sigma_X^2;\Sigma_{12}=\Sigma_{21}=\sigma_{XY};\Sigma_{22}=\sigma_Y^2)$, respectively. (II) The in-control and out-of-control mean vectors are $\boldsymbol{\mu}=\boldsymbol{\mu}_0=(\mu_{0X},\mu_{0Y})'$, and $\boldsymbol{\mu}=\boldsymbol{\mu}_1=(\mu_{1X},\mu_{1Y})'$, respectively; the changes $\boldsymbol{\delta}=(\delta_X,\delta_Y)$ are expressed in units of the standard deviations: $(\mu_{1X},\mu_{1Y})=(\mu_{0X}\pm\delta_X\sigma_X,\mu_{0Y}\pm\delta_Y\sigma_Y)$, for further details about the Hotelling T^2 chart see Montgomery (2009).

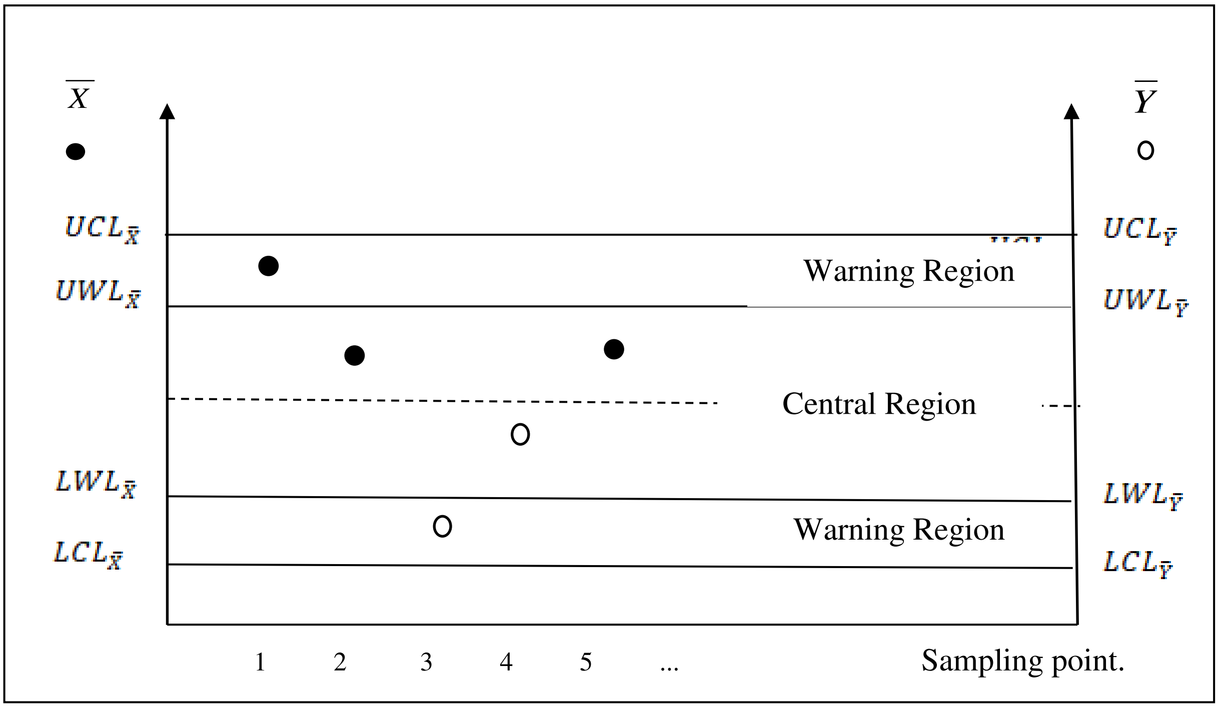
In this article, we propose, as an alternative to the Hotelling's chart, a Shewhart mean chart with variable charting statistic: at each sampling point, $2n$ items are collected but only one of the two quality characteristics (X, Y) is measured. This way, the average number of observations required by the proposed Shewhart chart is the same one required by the T^2 chart. As the X observations and the Y observations are always from different samples, the $\bar{X}$ and $\bar{Y}$ values are always uncorrelated (ρ=0).

Figure 1 illustrates the Shewhart mean chart with variable charting statistic, hereinafter the *VCS* chart. The $\bar{X}$ points are followed by $\bar{X}$ points as long as they fall in the warning region, however, an $\bar{X}$ point in the central region changes the quality characteristic to be monitored, that is, the next sample point will be an $\bar{Y}$ point. The same way, the $\bar{Y}$ points are followed by $\bar{Y}$ points as long as they fall in the warning region, however, an $\bar{Y}$ point in the central region changes the quality characteristic to be monitored, that is, the next sample point will be an $\bar{X}$ point. The eight control limits of the *VCS* chart are:

$$(UCL_{\bar{X}}, LCL_{\bar{X}}; UWL_{\bar{X}}, LWL_{\bar{X}}) = (\mu_{0X} \pm k\sigma_{\bar{X}}; \mu_{0X} \pm w\sigma_{\bar{X}}) \quad (1)$$

$$(UCL_{\bar{Y}}, LCL_{\bar{Y}}; UWL_{\bar{Y}}, LWL_{\bar{Y}}) = (\mu_{0Y} \pm k\sigma_{\bar{Y}}; \mu_{0Y} \pm w\sigma_{\bar{Y}})$$

Figure 1 - The *VCS* chart.



The speed with the control charts signal is measured by the Average Run Length – *ARL*. Expression (2) gives the *ARL*s of the *VCS* chart, see Appendix for details.

$$ARL = \frac{a(1-0.5p_Y) + b(1-0.5p_X)}{p_x + p_y - p_x p_y} \tag{2}$$

In expression (2), $a = 1/(1-q_X)$, $b = 1/(1-q_Y)$, with the $\overline{X}$ statistic in use, q_X and p_X are, respectively, the probability of a sample point falling in the warning region and the conditional probability of a signal, given the sample point didn't fall in the warning region; the definitions of q_Y and p_y are the same of q_X and p_X, but with respect to the $\overline{Y}$ statistic:

$$q_X = [\Phi(-w-\delta_X\sqrt{2n}) - \Phi(-k-\delta_X\sqrt{2n})] + [\Phi(k-\delta_X\sqrt{2n}) - \Phi(w-\delta_X\sqrt{2n})] \tag{3}$$

$$q_Y = [\Phi(-w-\delta_Y\sqrt{2n}) - \Phi(-k-\delta_Y\sqrt{2n})] + [\Phi(k-\delta_Y\sqrt{2n}) - \Phi(w-\delta_Y\sqrt{2n})] \quad (4)$$

$$p_X = [\Phi(-k+\delta_X\sqrt{2n}) + \Phi(-k-\delta_X\sqrt{2n})]/(1-q_X) \quad (5)$$

$$p_Y = [\Phi(-k+\delta_Y\sqrt{2n}) + \Phi(-k-\delta_Y\sqrt{2n})]/(1-q_Y) \quad (6)$$

3. Comparing the charts performance

In this section, the performance of the *VCS* chart is compared with the performance of the T^2 chart. When the T^2 chart is in use, samples of size n are collected and the following statistic is computed:

$$T_2 = n(\overline{X}-\mu_{0x}, \overline{Y}-\mu_{0y})' \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix} (\overline{X}-\mu_{0x}, \overline{Y}-\mu_{0y}) \quad (7)$$

During the in-control period, the T^2 statistic is chi-square distributed with two degrees of freedom (χ_2^2). The upper control limit (*UCL*) of the T^2 chart is adjusted to obtain the desired in-control *ARL*, that is, $UCL=\chi^2_{2,1/ARL_0}$. After the assignable cause occurrence, the mean vector changes from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$ and the distribution of the T^2 statistic changes to a non-central chi-squared distribution, $\chi_2^2(\lambda)$, with the following non-centrality parameter:

$$\lambda = n\boldsymbol{\delta}' \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix}^{-1} \boldsymbol{\delta} \quad (8)$$

The out-of-control *ARL* is the average number of samples (points in the chart) until the chart signals a true out-of-control condition:

$$ARL = \{1 - \Pr[\chi_2^2(\lambda) < UCL]\}^{-1} \tag{9}$$

Tables 1-4 present the *ARL*s of the *VCS* and the T^2 charts. In Tables 1 and 2 they were designed to have in-control *ARL*s of 370.4 (k=3.0), and in Tables 3 and 4 they were designed to have in-control *ARL*s of 780.1(k=3.22). The Tables show the *ARL*s of the T^2 charts when the two quality characteristics (X, Y) are uncorrelated ($\rho = 0$), or not (ρ = 0.3, 0.5, 0.7). With the variable charting statistic (*VCS*), the X and the Y observations are not from the same samples, consequently, the X and the Y sample means are always uncorrelated (ρ=0).

According to Tables 1-4, the *VCS* chart outperforms the T^2 chart in 98 percent of the cases. In the few cases, where the T^2 chart signals faster, the two quality characteristics are highly correlated (ρ = 0.7) and the disturbance affects only one parameter of the mean vector $(\delta_X \neq 0 \text{ or } \delta_Y \neq 0)$. In several cases, the *VCS* chart signals three times faster than the T^2 chart; for instance, Table 2 shows that, on average, the *VCS* chart needs 8.17 samples of size six to signal a change in the mean vector of magnitude δ = (0.75,0.75), the T^2 chart, with ρ=0.7, needs 28.03 samples of size three to signal the same disturbance.

Table 1 - The *ARL* values for the *VCS* chart and the T^2 chart with n=1 (2), *ARL*=370.4

ρ			0.0	0.3	0.5	0.7		0.0	0.3	0.5	0.7
Sample size		2		1			4		2		
w		1					1				
δ_x	δ_y	***VCS***		T^2			***VCS***		T^2		

0.00	0.00	370.40	370.40	370.40	370.40	370.40	**370,40**	370.40	370.40	370.40	370.40
0.00	0.50	**138,50**	202.24	192.53	172.24	**131.83**	**69,38**	129.82	120.30	101.63	**68.83**
0.00	0.75	**59,99**	117.97	108.77	90.90	60.22	**21,88**	58.86	52.58	41.19	24.04
0.00	1.00	**26,39**	67.32	60.48	47.89	28.53	**8,43**	27.73	24.17	18.04	9.72
0.00	1.25	**12,72**	39.02	34.36	26.14	14.50	**4,28**	14.05	12.07	8.80	4.66
0.00	1.50	**6,97**	23.34	20.25	14.99	8.00	**2,79**	7.75	6.62	4.82	**2.64**
0.50	0.50	**90,65**	129.80	157.11	172.24	185.41	**43,89**	67.34	88.68	101.64	113.59
0.50	0.75	**51,48**	83.96	104.63	114.39	117.75	**20,46**	37.04	49.85	56.38	58.71
0.50	1.00	**25,66**	51.88	64.15	67.32	61.81	**8,75**	19.92	26.05	27.73	24.85
0.50	1.25	**12,91**	31.88	38.46	38.34	31.26	**4,55**	11.06	13.81	13.76	10.80
0.50	1.50	**7,16**	19.91	23.30	22.11	16.32	**3,04**	6.50	7.73	7.29	5.26
0.75	0.75	**38,09**	58.85	78.69	90.90	102.31	**14,97**	23.34	33.98	41.19	48.35
0.75	1.00	**23,08**	39.02	53.30	61.49	66.96	**8,29**	14.05	20.60	24.68	27.54
0.75	1.25	**12,73**	25.39	34.36	38.34	38.03	**4,70**	8.51	12.07	13.76	13.62
0.75	1.50	**7,31**	16.60	21.89	23.34	20.84	**3,27**	5.36	7.21	7.75	6.83
1.00	1.00	**17,73**	27.73	39.83	47.89	55.82	**6,30**	9.41	14.40	18.04	21.83
1.00	1.25	**11,59**	19.17	27.89	33.47	38.03	**4,23**	6.24	9.47	11.71	13.62
1.00	1.50	**7,22**	13.19	18.95	22.11	23.26	**3,15**	4.23	6.16	7.29	7.72
1.25	1.25	**9,18**	14.05	21.14	26.14	31.26	**3,24**	4.51	6.94	8.80	10.80
1.25	1.50	**6,53**	10.18	15.37	18.94	22.18	**2,54**	3.29	4.94	6.16	7.32
1.50	1.50	**5,27**	7.74	11.92	14.99	18.24	**2,00**	2.57	3.83	4.82	5.92

It is interesting to observe that, in Table 1, the *VCS* chart is faster in signaling a shift of (δ_x; δ_y) = (0;1.5) than shifts of (δ_x; δ_y) = (0.5;1.5), (0.75;1.5), (1.0;1.5). These counterintuitive results are explained by the fact that the monitoring statistic alternates, from $\bar{X}$ to $\bar{Y}$, or from $\bar{Y}$ to $\bar{X}$, whenever a sample point falls in the central region. If δ_x=0, the $\bar{X}$ statistic has high probability to fall in the central region, consequently the monitoring statistic rapidly goes to $\bar{Y}$; with the sample mean of the *Y* observations the *VCS* chart signals very fast because their average is far from the target value (δ_y=1,5). When δ_x increases to 0,5, 0.75, or 1.00, the $\bar{X}$ statistic has less and less chance to fall in the central region. Therefore, reduces the speed with which the monitoring statistic goes to $\bar{Y}$, consequently the *ARL* increases.

Table 2 - The *ARL* values for the *VCS* chart and the T^2 chart with n=3 (4), *ARL*=370.4

ρ			0.0	0.3	0.5	0.7		0.0	0.3	0.5	0.7
Sample size		6		3			8		4		
w		1					1.7				
δ_x	δ_y	***VCS***		T^2			***VCS***		T^2		
0.00	0.00	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
0.00	0.50	**40,66**	90.92	82.75	67.34	42.27	**29,19**	67.34	60.48	47.90	**28.53**
0.00	0.75	**11,18**	35.19	30.87	23.34	12.82	**7,71**	23.34	20.25	14.99	8.00
0.00	1.00	**4,52**	14.99	12.89	9.41	4.98	**3,18**	9.41	8.05	5.85	**3.15**
0.00	1.25	**2,69**	7.24	6.19	4.51	**2.49**	**2,02**	4.51	3.88	2.88	**1.72**
0.00	1.50	**2,07**	3.98	3.43	2.57	**1.58**	**1,67**	2.57	2.25	1.76	**1.23**
0.50	0.50	**26,36**	41.19	57.12	67.34	77.12	**17,73**	27.73	39.83	47.90	55.83
0.50	0.75	**11,11**	20.70	29.03	33.48	35.08	**7,35**	13.20	18.95	22.11	23.27
0.50	1.00	**4,90**	10.46	14.00	14.99	13.29	**3,33**	6.50	8.76	9.41	8.31
0.50	1.25	**3,05**	5.66	7.11	7.08	5.53	**2,17**	3.56	4.43	4.41	3.48
0.50	1.50	**2,43**	3.38	3.98	3.76	2.78	**1,83**	2.22	2.56	2.44	1.88
0.75	0.75	**8,17**	12.42	18.79	23.34	28.03	**5,27**	7.75	11.92	14.99	18.24
0.75	1.00	**4,67**	7.24	10.85	13.20	14.88	**3,08**	4.51	6.75	8.24	9.33
0.75	1.25	**3,17**	4.37	6.19	7.08	7.01	**2,17**	2.79	3.88	4.41	4.37
0.75	1.50	**2,65**	2.82	3.72	3.98	3.54	**1,88**	1.90	2.42	2.57	2.31
1.00	1.00	**3,44**	4.82	7.43	9.41	11.55	**2,32**	3.06	4.62	5.85	7.19
1.00	1.25	**2,54**	3.25	4.85	6.00	7.01	**1,78**	2.15	3.08	3.76	4.37
1.00	1.50	**2,18**	2.30	3.21	3.76	3.97	**1,58**	1.61	2.12	2.44	2.56
1.25	1.25	**1,91**	2.42	3.59	4.51	5.53	**1,42**	1.68	2.34	2.88	3.48
1.25	1.50	**1,61**	1.86	2.63	3.21	3.77	**1,26**	1.38	1.79	2.12	2.45
1.50	1.50	**1,33**	1.55	2.11	2.57	3.09	**1.12**	1.21	1.51	1.76	2.06

Table 3 - The *ARL* values for the *VCS* chart and the T^2 chart with n=1 (2), *ARL*=780.1

ρ			0.0	0.3	0.5	0.7		0.0	0.3	0.5	0.7
Sample size		2		1			4		2		
w		1					1				
δ_x	δ_y	***VCS***		T^2			***VCS***		T^2		
0.00	0.00	**780,09**	780.10	780.10	780.10	780.10	**780,09**	780.10	780.10	780.10	780.10
0.00	0.50	**258,94**	398.56	377.58	334.22	**249.58**	**120,89**	245.41	225.89	187.99	123.05
0.00	0.75	**103,05**	221.13	202.44	166.53	106.43	**34,02**	103.79	91.83	70.39	39.12
0.00	1.00	**41,82**	120.15	106.93	82.96	47.17	**11,75**	45.73	39.35	28.56	14.45

0.00	1.25	**18,63**	66.38	57.78	42.87	22.47	**5,38**	21.71	18.37	12.94	6.36
0.00	1.50	**9,46**	37.88	32.43	23.32	11.65	**3,23**	11.23	9.44	6.61	3.34
0.50	0.50	**165,83**	245.41	302.26	334.22	362.32	**75,63**	120.15	162.10	187.99	212.17
0.50	0.75	**89,08**	152.74	194.06	213.84	220.69	**32,37**	62.69	86.65	99.06	103.51
0.50	1.00	**41,08**	90.49	113.99	120.15	109.48	**12,30**	31.84	42.72	45.73	40.56
0.50	1.25	**19,03**	53.25	65.34	65.12	52.12	**5,74**	16.67	21.29	21.21	16.25
0.50	1.50	**9,74**	31.84	37.81	35.69	25.59	**3,52**	9.25	11.21	10.51	7.29
0.75	0.75	**64,85**	103.79	142.34	166.53	189.38	**23,41**	37.88	57.09	70.39	83.80
0.75	1.00	**37,23**	66.38	93.19	108.86	119.44	**11,90**	21.71	33.04	40.26	45.38
0.75	1.25	**18,97**	41.54	57.78	65.12	64.53	**6,04**	12.48	18.37	21.21	20.98
0.75	1.50	**10,01**	26.07	35.31	37.88	33.46	**3,85**	7.44	10.38	11.23	9.77
1.00	1.00	**28,19**	45.73	67.88	82.96	97.99	**8,99**	13.94	22.31	28.56	35.21
1.00	1.25	**17,46**	30.55	46.03	56.15	64.53	**5,62**	8.84	14.05	17.75	20.98
1.00	1.50	**10,04**	20.26	30.15	35.69	37.75	**3,90**	5.71	8.71	10.51	11.19
1.25	1.25	**13,66**	21.71	33.98	42.87	52.12	**4,24**	6.13	9.94	12.94	16.25
1.25	1.50	**9,21**	15.21	23.96	30.13	35.82	**3,18**	4.29	6.80	8.71	10.55
1.50	1.50	**7,36**	11.23	18.11	23.32	28.92	**2,42**	3.23	5.09	6.61	8.32

Table 4 - The *ARL* values for the *VCS* chart and the T^2 chart with n=3 (4), *ARL*=780.1

ρ			0.0	0.3	0.5	0.7		0.0	0.3	0.5	0.7
Sample size		6		3			8		4		
w		1					1.5				
δ_x	δ_y	***VCS***		T^2			***VCS***		T^2		
0.00	0.00	**780,09**	780.10	780.10	780.10	780.10	**780,09**	780.10	780.10	780.10	780.10
0.00	0.50	**67,25**	166.53	150.35	120.15	72.40	**45,04**	120.15	106.93	82.96	47.17
0.00	0.75	**16,14**	59.29	51.42	37.88	19.62	**10,16**	37.88	32.43	23.32	11.65
0.00	1.00	**5,74**	23.32	19.75	13.94	6.85	**3,75**	13.94	11.73	8.21	4.08
0.00	1.25	**3,08**	10.42	8.75	6.13	3.12	**2,22**	6.13	5.17	3.68	**2.01**
0.00	1.50	**2,22**	5.33	4.50	3.23	**1.82**	**1,76**	3.23	2.77	2.07	**1.33**
0.50	0.50	**43,46**	70.39	100.48	120.15	139.23	**28,19**	45.73	67.88	82.96	97.99
0.50	0.75	**16,39**	33.21	48.08	56.15	59.10	**10,21**	20.26	30.15	35.69	37.75
0.50	1.00	**6,27**	15.68	21.62	23.32	20.42	**4,07**	9.25	12.89	13.94	12.14
0.50	1.25	**3,51**	7.92	10.21	10.17	7.72	**2,49**	4.69	6.01	5.99	4.57
0.50	1.50	**2,64**	4.42	5.32	4.99	3.53	**2,03**	2.72	3.22	3.04	2.24
0.755	0.75	**12,00**	18.94	29.88	37.88	46.26	**7,36**	11.23	18.11	23.32	28.92
0.75	1.00	**6,21**	10.42	16.32	20.26	23.12	**3,94**	6.13	9.64	12.04	13.82
0.75	1.25	**3,83**	5.91	8.75	10.17	10.05	**2,63**	3.55	5.17	5.99	5.92
0.75	1.50	**3,05**	3.60	4.93	5.33	4.66	**2,23**	2.27	3.01	3.23	2.85
1.00	1.00	**4,54**	6.61	10.72	13.94	17.49	**2,88**	3.94	6.31	8.21	10.34
1.00	1.25	**3,18**	4.23	6.66	8.45	10.05	**2,13**	2.62	3.97	4.99	5.92
1.00	1.50	**2,63**	2.83	4.17	4.99	5.31	**1,85**	1.86	2.58	3.04	3.22
1.25	1.25	**2,29**	3.02	4.73	6.13	7.72	**1,60**	1.96	2.90	3.68	4.57
1.25	1.50	**1,87**	2.22	3.31	4.17	5.01	**1,39**	1.53	2.12	2.58	3.05
1.50	1.50	**1.48**	1.77	2.56	3.23	3.99	**1,18**	1.31	1.72	2.07	2.49

4. Conclusions

The use of the T^2 statistic to control bivariate processes requires $2n$ observations per sample, n of the first quality characteristic and another n of the second quality characteristic. In this article, we proposed a competing univariate Shewhart mean chart with variable charting statistic. The univariate chart works with one quality characteristic per time, consequently, all the $2n$ observations are the same quality characteristic. The power of the univariate Shewhart mean chart substantially increases when the size of the samples is doubled and, even with the charting statistic varying, it is much faster than the T^2 chart is signaling all kinds of changes in the mean vector.

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Appendix. The *ARL* of the *VCS* chart

The speed with which the *VCS* chart signals an out-of-control condition depends on the first point plotted on the chart, just after the assignable cause occurrence. This first point has a fifty-fifty probability of being an $\overline{X}$ point or an $\overline{Y}$ point. Thus, the *ARL* is computed as $(ARL_X + ARL_Y)/2$, where ARL_X is the *ARL* value when the first point is an $\overline{X}$, and the ARL_Y is the *ARL* value when the first point is an $\overline{Y}$. When the chart is initialized, the first quality characteristic to be controlled is randomly selected, with fifty-fifty probability of being the *X* or the *Y* quality characteristic. After a false alarm, the same procedure decides the next quality characteristic to be controlled.

If the current point is an $\overline{X}$, then the expected number of points until the first one falling outside the warning region, is $E(M_X) = a$ Equally, if the current point is an $\overline{Y}$, then the expected number of points until the first one falling outside the warning region is $E(M_Y) = b$:

$$a = \sum_{m=1}^{\infty} m q_X^{m-1}(1-q_X) = \frac{1}{1-q_X} \tag{A1}$$

$$b = \sum_{m=1}^{\infty} m q_y^{m-1}(1-q_Y) = \frac{1}{1-q_Y} \tag{A2}$$

In expression (A1) and (A2), q_x is the probability to obtain a warning signal with the $\overline{X}$ statistic (given by an $\overline{X}$ point falling in the warning region), and q_Y is the probability to obtain a warning signal with the $\overline{Y}$ statistic.

The ARL_X is the expected number of points on the chart until the chart signals, given that the first point on the chart, just after the assignable cause or the false alarm occurrences, is an X sample mean:

$$\begin{aligned} ARL_X &= ap_X + (a+b)(1-p_X)p_Y + (2a+b)(1-p_X)(1-p_Y)p_X + (2a+2b)(1-p_X)^2(1-p_Y)p_Y \\ &+ (3a+2b)(1-p_X)^2(1-p_Y)^2 p_X + (3a+3b)(1-p_X)^3(1-p_Y)^2 p_y + \ldots \\ &= \sum_{m=1}^{\infty} m(a+b)[(1-p_X)(1-p_Y)]^{m-1}[p_X + (1-p_X)p_Y] - \sum_{m=1}^{\infty} b[(1-p_X)(1-p_Y)]^{m-1} p_X \\ &= \frac{a+b-bp_X}{p_x + p_y - p_x p_y} \end{aligned} \quad \text{(A3)}$$

In expression (3), p_X and p_y are, respectively, the conditional probabilities to obtain a signal with the $\bar{X}$ statistic and with the $\bar{Y}$ statistic, given a reduced number of possibilities: or a signal or a sample point falling in the central region.

Similar development leads to the expression for the ARL_Y:

$$ARL_Y = \frac{a+b-ap_Y}{p_x + p_y - p_x p_y} \quad \text{(A4)}$$

From (A3) and (A4), it follows that:

$$ARL = \frac{ARL_X + ARL_Y}{2} = \frac{a(1-0.5p_Y) + b(1-0.5p_X)}{p_x + p_y - p_x p_y} \quad \text{(A5)}$$