An adaptive statistical approach to flutter detection

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Content

Introduction - (1)

- **Flutter**: critical aircraft instability phenomenon
  unfavorable interaction of aerodynamic, elastic and inertial forces; may cause major failures
- **Flight flutter testing**, very expensive and time consuming :
  Design the flutter free flight envelope
- Flutter clearance techniques:
  In-flight identification: output-only, or using input excitations
  Data processing: time-frequency, wavelet, envelope function

  Flutter prediction based on model-based approaches:
  flutterometer (μ-robustness), physical model updating
- Some challenges:
  Real time on-board monitoring,
  robustness to noise and uncertainties
- Our approach:
  Statistical detection for monitoring instability indicators

Introduction - (2)

- Aim of in-flight online flutter monitoring:
  Early detection of a deviation in the aircraft modal parameters before it develops into flutter.
- Change-point detection: natural approach
- For a scalar instability criterion ψ and a critical value ψ_c,
  online hypotheses testing:

$$\mathrm{H}_0 \;:\; \psi > \psi_c \text{ and } \mathrm{H}_1 \;:\; \psi \leq \psi_c$$

- CUSUM test as an approximation to the optimal test
- A moving reference version

Subspace-based residual for modal monitoring

$$\begin{cases} X_{k+1} = F\ X_k + V_k & F\ \phi_\lambda = \lambda\ \phi_\lambda \\ Y_k = H\ X_k & \varphi_\lambda \triangleq H\ \phi_\lambda \end{cases}$$

$$R_i \triangleq \mathrm{E}\left(Y_k\ Y_{k-i}^T\right) \ , \quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H\ F^i\ G \Longrightarrow \mathcal{H} = \mathcal{O}\ \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \ , \quad \begin{array}{l} \mathcal{C} \triangleq (\ G \quad FG \quad F^2G \quad \dots\ ) \\ G \triangleq \mathrm{E}\left(X_k\ Y_k^T\right) \end{array}$$

Output-only covariance-driven subspace identification

SVD of $\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \mathrm{vec}\ \Phi \end{pmatrix}$ modes / mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

Given:

- a reference parameter $\theta_\star$, by SVD of $\hat{\mathcal{H}}^\star_{p+1,q}$ (reference data)

$$U(\theta_\star)^T\ \hat{\mathcal{H}}^\star_{p+1,q} = 0 \qquad \text{parameter estimating function}$$

$$U(\theta_\star)^T\ \mathcal{O}_{p+1}(\theta_\star) = 0 \ , \quad U(\theta_\star)^T U(\theta_\star) = I$$

- a n-size sample of new data; $\hat{\mathcal{H}}_{p+1,q}$

For testing $\theta = \theta_\star$, statistics (residual) :

$$\zeta_n(\theta_\star) \triangleq \sqrt{n}\ \mathrm{vec}\left(U(\theta_\star)^T\ \hat{\mathcal{H}}_{p+1,q}\right)$$

Local approach to testing

$$\widetilde{\mathrm{H}}_0 : \ \theta = \theta_\star \quad \text{and} \quad \widetilde{\mathrm{H}}_1 : \ \theta = \theta_\star + \Upsilon/\sqrt{n}$$

Mean sensitivity and covariance matrices:

$$\mathcal{J}_n(\theta_\star, \theta) \triangleq 1/\sqrt{n}\ \partial/\partial\tilde{\theta}\ \mathrm{E}_\theta\ \zeta_n(\tilde{\theta})\big|_{\tilde{\theta}=\theta_\star} \ , \quad \Sigma_n(\theta_\star, \theta) \triangleq \mathrm{E}_\theta\left(\zeta_n(\theta_\star)\ \zeta_n(\theta_\star)^T\right)$$

If $\Sigma_n(\theta_\star, \theta)$ is positive definite, and for all Υ, under both hypoth:

$$\Sigma_n(\theta_\star, \theta)^{-1/2}\left(\zeta_n(\theta_\star) - \mathcal{J}_n(\theta_\star, \theta)\ \Upsilon\right) \xrightarrow[n \to \infty]{} \mathcal{N}(0, I)$$

Normalized residual:

$$\overline{\zeta}_n(\theta_\star) \triangleq \mathcal{K}_n(\theta_\star, \theta)\ \zeta_n(\theta_\star)$$

$$\mathcal{K}_n(\theta_\star, \theta) \triangleq \overline{\Sigma}_n^{-1/2} \mathcal{J}_n^T \Sigma_n^{-1} \ , \quad \overline{\Sigma}_n(\theta_\star, \theta) \triangleq \mathcal{J}_n^T\ \Sigma_n^{-1} \mathcal{J}_n$$

$$\left(\overline{\zeta}_n(\theta_\star) - \overline{\Sigma}_n(\theta_\star, \theta)^{1/2}\ \Upsilon\right) \xrightarrow[n \to \infty]{} \mathcal{N}(0, I)$$

Data-driven computation for online detection

$$\overline{\zeta}_n(\theta_\star) \approx \sum_{k=q}^{n-p} Z_k(\theta_\star)/\sqrt{n}$$

$$Z_k(\theta_\star) \triangleq \mathcal{K}_k(\theta_\star, \theta)\ \mathrm{vec}\left(U(\theta_\star)^T\ \mathcal{Y}^+_{k,p+1}\ \mathcal{Y}^{-T}_{k,q}\right)$$

Another approximation

For n large enough, and $k = 1, \dots, n$,
$Z_k(\theta_\star) \approx$ Gaussian i.i.d., mean 0 before change and $\neq 0$ after.

Monitoring any function $\psi(\theta)$

Replace $\mathcal{J}_n(\theta_\star, \theta)$ with $\mathcal{J}_n(\theta_\star, \theta)\ \mathcal{J}^\star_{\theta\psi}$, where $\mathcal{J}^\star_{\theta\psi} = \partial\theta/\partial\psi|_{\theta=\theta_\star}$.

CUSUM test for monitoring a scalar index

The crossing of a critical ψ_c by ψ is reflected into a change with the same sign in the mean ν of the i.i.d. Gaussian $Z_k(\theta_\star)$.

The CUSUM test may be used for testing between:

$$\mathrm{H}_0 : \nu > 0 \quad \text{and} \quad \mathrm{H}_1 : \nu \leq 0$$

Procedure for unknown sign and magnitude of change in ψ

i) Set a min. change magnitude $\nu_m > 0$, and test between:

$$\mathrm{H}_0 : \nu > \nu_m/2 \text{ and } \mathrm{H}_1 : \nu \leq -\nu_m/2$$

$$S_n(\theta_\star) \triangleq \sum_{k=q}^{n-p} (Z_k(\theta_\star) + \nu_m), \; T_n(\theta_\star) \triangleq \max_{k=q,\dots,n-p} S_k(\theta_\star)$$

$$g_n(\theta_\star) \triangleq T_n(\theta_\star) - S_n(\theta_\star) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\lessgtr}} \varrho \qquad \text{threshold}$$

ii) Run 2 tests in parallel, for decreasing and increasing ψ;
iii) Make a decision from the first test which fires;
iv) Reset all sums and extrema to 0, switch to the other test.

Using and tuning the CUSUM test

For detecting aircraft instability precursors, select:

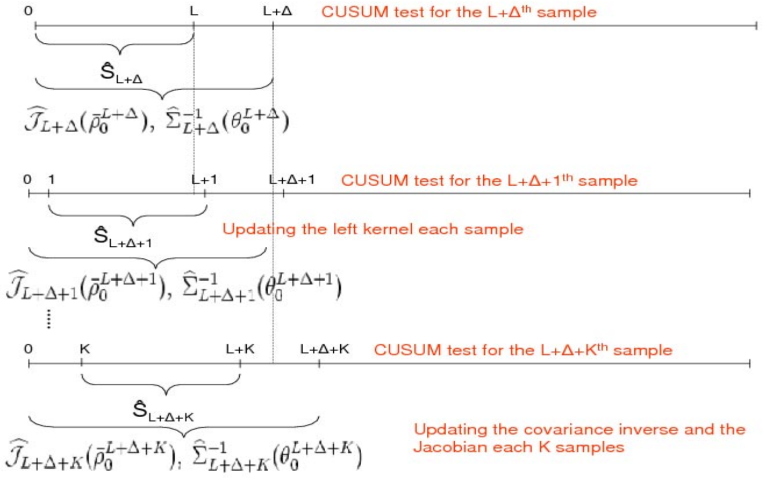
a) An instability criterion ψ and a critical value ψ_c;
b) A left kernel matrix $U(.)$;
c) Estimates of $\mathcal{J}_n(\theta_\star, \theta)$ and $\Sigma_n(\theta_\star, \theta)$;
d) A min. change magnitude ν_m and a threshold ϱ.

Two solutions for b)-c):

1. $\theta_\star \triangleq \theta_0$ identified on reference data for the stable system; $U(\theta_\star)$ computed, $\mathcal{J}_n, \Sigma_n$ estimated once for all with those data.

2. $U(.) \triangleq \hat{U}_n$ estimated on test data; $\mathcal{J}_n, \Sigma_n$ estimated recursively with those test data.

The moving reference version

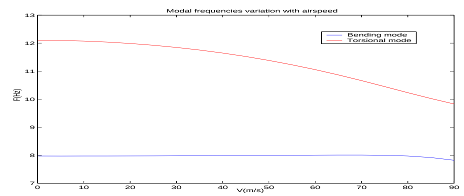


Example - Aeroelastic Hancock wing model

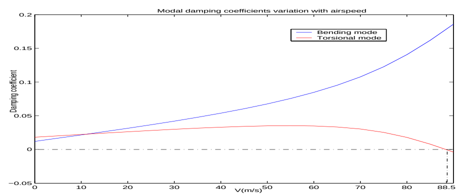
Rigid wing with constant chord; 2 d.o.f. in bending and torsion.

Matrix F, and eigenvalues λ: functions of airspeed V.
Flutter airspeed: $V_f = 88.5 m/s$.

Stability indicator: Damping coefficient



Frequency



Damping coefficient

20700-size 2D-samples simulated (300 for each V=20:1:88m/s).

Example - Numerical results

CUSUM test run with $\nu_m = 0.1$, $\varrho = 100$, and the damping as ψ.

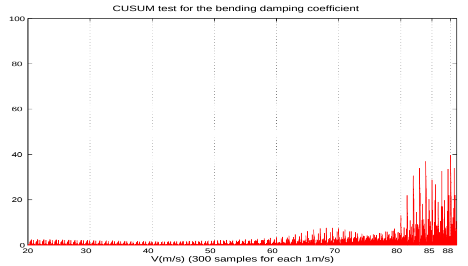
Solution 1. with $\theta_\star = \theta_0$ at $V = 20m/s$ and fixed $\mathcal{J}, \Sigma$.

Solution 2. with online $\hat{U}_n, \hat{\mathcal{J}}_n, \hat{\Sigma}_n$.

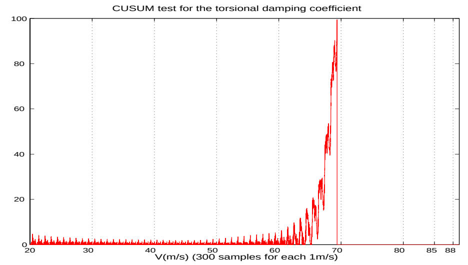
Solution 1. $\theta_\star$ far from instability, too early alarm at $V{=}69m/s$.
The test detects that torsional damping decreases
under the predefined threshold.

Solution 2. Alarm at $V = 82m/s$ much closer to flutter.
The test detects that the torsional damping decreases abruptly.

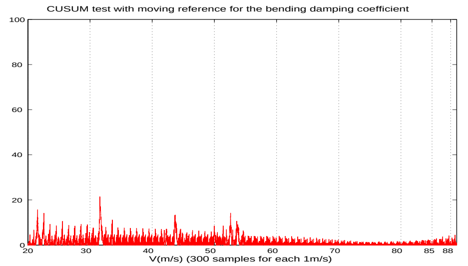
Both algorithms do what they are intended to do.
Only Solution 2 is a flutter detection algorithm.



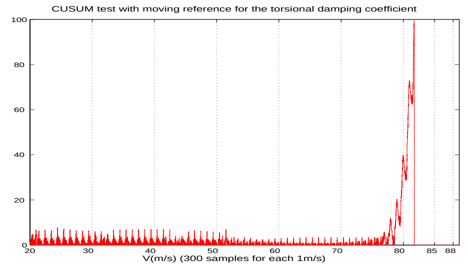
Solution 1: Bending mode



Solution 1: Torsion mode



Solution 2: Bending mode



Solution 2: Torsion mode

Conclusion

Online detection of instability precursors

- Model-free subspace statistics, local approach, CUSUM
- Analytical model for flutter prediction
- Recursive computation of covariance matrix

Relevance on a small simulated structure

Limitations: cost of online kernel and covariance computation

Major issues: dimension of θ, large number of correlated criteria