

## Research Article

# An ADRC Method for Noncascaded Integral Systems Based on Algebraic Substitution Method and Its Structure

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The Active Disturbance Rejection Control (ADRC) prefers the cascaded integral system for a convenient design or better control effect and takes it as a typical form. However, the state variables of practical system do not necessarily have a cascaded integral relationship. Therefore, this paper proposes an algebraic substitution method and its structure, which can convert a noncascaded integral system of PID control into a cascaded integral form. The adjusting parameters of the ADRC controller are also demonstrated. Meanwhile, a numerical example and the oscillation control of a flexible arm are demonstrated to show the conversion, controller design, and control effect. The converted system is proved to be more suitable for a direct ADRC control. In addition, for the numerical example, its control effect for the converted system is compared with a PID controller under different disturbances. The result shows that the converted system can achieve a better control effect under the ADRC than that of a PID. The theory is a guide before practice. This converting method not only solves the ADRC control problem of some noncascaded integral systems in theory and simulation but also expands the application scope of the ADRC method.

## 1. Introduction

The Active Disturbance Rejection Control (ADRC) has begun to be used in many areas recently [1–23]. This theory was first proposed by Han [24, 25]. The central idea is that the internal dynamic and external disturbance of a controlled system can be estimated and compensated in real time with a tracking differentiator (TD), extended state observer (ESO), nonlinear state error feedback (NLSEF), compensator, etc. Thus, the ADRC may promote the control quality and speed where PID is used [25].

For the ADRC, its ESO is a cascaded integral form, its TD tracks the system state and derivative, and its NLSEF is based on the ESO and TD. Thus, these characteristics make the ADRC suitable for a cascaded integral system. This is because the system order, system variables, and known states are explicit to the ESO and TD for the system in a cascaded integral form. Then, the ADRC usually selects the cascaded integral system as a typical form for an easy design or better control effect. For example, in 2015, Shao thought that the

ADRC was available to a cascaded integral system, such as a motion control system [10]. In 2006, Gao also thought that if a plant model was in a cascaded integral form, the ESO could be established, and the ADRC could have a full state feedback from the ESO [26].

However, in practical control systems, there are many cases of noncascaded integral forms. When necessary, a converting method is needed to get the cascaded integral form. At present, the research in this field is as follows: (1) Some scholars adopted a converting method. The Differential Geometry is one of them [27]. This method combines a nonlinear state conversion and linearization using its object model. Also in 2014, Huang converted a two-order state space form into the cascaded integral system by a mathematic transform [28]. In 2014, Huang used the same method to convert a multiorder state space system [28]. In 2014, Ramírez-Neria utilized the decoupling property of the object model and decomposed it into the cascaded connection of two independent blocks [29]. (2) Some ADRC applications were limited to the control system with an implicit cascaded

integral form, such as the differential equation, rational proper fraction, or state space form, as shown in [12, 13, 26, 30]. (3) Some ADRC applications were limited to the control system with an explicit cascaded integral system. For example, the nonlinear ADRC has been applied to the fast tool servo systems [14, 31], which are cascaded integral systems with two stages.

As the real control systems have various forms, there are many styles to be converted, and their converting method may also be different. In practical application, the PID feedback control is the most widely used. If the PID control object of a noncascaded integral form can be converted into the cascaded integral system, it would be highly representative.

Therefore, an algebraic substitution method and its structure are proposed in this paper to convert the noncascaded integral system of a PID control object into the cascaded integral form. The adjusting parameters of the ADRC controller are also demonstrated. A numerical example and the oscillation control of a flexible arm are simulated to show the conversion and ADRC control effect. The converted system is proved to be more suitable for a direct ADRC control. The ADRC can achieve a nonovershoot tracking control while satisfying the rapidity under disturbances. In addition, the control effect of the numerical example is compared under the periodical, white noise and inaccurate model disturbances during a step input response while the controller and its parameters are kept invariable. The results show that the converted system can achieve a better control effect for the ADRC than that of a PID.

Thus, this paper presents an approach, which can transform a noncascaded integral system into the cascaded integral form for an easier and better ADRC control. The converted ADRC control system has a good antidisturbance and adaptive effect. The theory is a guide before practice. This converting method solves the ADRC control problems of some noncascaded integral systems in theory and simulation. It also expands the application scope of the ADRC method.

## 2. The ADRC Control Method

For a continuous system, the ADRC control method adopts the following four steps:

(1) Arranging a transient process for the control reference with the TD:

$$\begin{aligned}
 \dot{x}_1 &= x_2 \\
 \dot{x}_2 &= fhan(x_1 - v, x_2, r, h_0) \\
 fhan &= -r \left( \frac{a}{d} \right) fsg(a, d) \\
 &\quad - r \operatorname{sign}(a) (1 - fsg(a, d)) \\
 d &= r \cdot h_0^2 \\
 a_0 &= h_0 \cdot x_2(t) \\
 y &= x_1(t) + a_0 \\
 a_1 &= \sqrt{d(d + 8|y|)}
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 a_2 &= a_0 + \frac{\operatorname{sign}(y)(a_1 - d)}{2} \\
 a &= (a_0 + y) fsg(y, d) + a_2 (1 - fsg(y, d)) \\
 fsg(x, d) &= \frac{(\operatorname{sign}(x + d) - \operatorname{sign}(x - d))}{2}
 \end{aligned} \tag{2}$$

In it,  $x_1$ ,  $x_2$  are the system state and its first derivative, respectively;  $v$  is the control reference;  $h_0$  is the filtering factor;  $r$  is the time ruler;  $\operatorname{sign}$  is a sign function; and  $fhan$  is a constructed nonlinear function shown in (2) [16, 32].

(2) Estimating the system states and total disturbance of the controlled object with the ESO:

$$\begin{aligned}
 \dot{z}_1 &= z_2 - \beta_{01}e \\
 \dot{z}_2 &= z_3 - \beta_{02}fe + u \\
 \dot{z}_3 &= -\beta_{03}fe_1 \\
 e &= z_1 - y,
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 fe &= fal(e, 0.5, \delta), \\
 fe_1 &= fal(e, 0.25, \delta)
 \end{aligned}$$

$$fal(e, \alpha, \delta) = \begin{cases} \frac{e}{\delta^{\alpha-1}}, & |e| \leq \delta \\ |e|^\alpha \operatorname{sign}(e), & |e| > \delta \end{cases} \tag{4}$$

In it,  $e$  is the error between the estimated state and system output;  $z_1$ ,  $z_2$ , and  $z_3$  are the estimations of the system states  $x$ ;  $\beta_{01}$ ,  $\beta_{02}$ , and  $\beta_{03}$  are gain coefficients;  $u$  is the control signal;  $y$  is the system output;  $\delta$  and  $\alpha$  are the parameters of  $fal$  function;  $fe$  and  $fe_1$  are the outputs of  $fal$  function;  $fal$  is another constructed nonlinear function shown in (4) [16, 32].

(3) The nonlinear state error feedback with the NLSEF:

$$\begin{aligned}
 e_1 &= x_1 - z_1, \\
 e_2 &= x_2 - z_2 \\
 u_0 &= \beta_1 fal(e_1, \alpha_1, \delta_1) + \beta_2 fal(e_2, \alpha_2, \delta_1)
 \end{aligned} \tag{5}$$

In it,  $e_1$  and  $e_2$  are the errors between the estimated states and system states;  $u_0$  is an intermediate control output before compensation;  $\beta_1$  and  $\beta_2$  are gain coefficients;  $\alpha_1$ ,  $\alpha_2$ ,  $\delta_1$  are the parameters of  $fal$  function.

(4) The disturbance compensation with a compensator:

$$u = u_0 - \frac{z_3}{b_0} \tag{6}$$

In it,  $b_0$  is the amplification factor of the control signal;  $u$  is a final control signal.

## 3. The Algebraic Substitution Method and Its Structure for the ADRC

*Remark 1.* According to (1) and (3), if  $x_1$ ,  $\dot{x}_1$ ,  $x_2$ ,  $\dot{x}_2$ , and their counterparts  $z_1$ ,  $\dot{z}_1$ ,  $z_2$ , and  $\dot{z}_2$  of the controlled system are

explicit, this system is in a cascaded integral form. Then, the TD, ESO, and NLSEF of the ADRC controller can be easily designed.

The cascaded integral system is a closed state feedback system, and its archetype can be described ideally in

$$\begin{aligned}\dot{x}_1 &= f_1 + x_2 \\ &\vdots \\ \dot{x}_{n-1} &= f_{n-1} + x_n \\ \dot{x}_n &= f_n + bu \\ y &= x_1\end{aligned}\quad (7)$$

In it,  $x_i, i = 1, 2, \dots, n$  are the state variables of the controlled system;  $f_i, i = 1, 2, \dots, n$  are the unknown system functions;  $u$  is the control signal;  $y$  is the system output; and the coefficient  $b \in \mathbf{R}^n$ . This kind of cascaded integral system is a typical form suitable for the ADRC.

Then, a method is needed to construct an object system and convert the noncascaded integral system into the above cascaded integral form. The two-order control system in (8) is taken for example.

$$\begin{aligned}\ddot{x} &= -a_1 x - a_2 \dot{x} + u + \omega \\ y &= x\end{aligned}\quad (8)$$

In it,  $u$  is the control signal;  $y$  is the system output;  $x$  is the system state; and  $\omega$  is the system disturbance.  $v$  is set as the control reference. Thus, the error between the system state and control reference is

$$e = v - x = v - y \quad (9)$$

Then, the proposed algebraic substitution method and its structure adopt the following six steps as shown in Figure 1:

(1) The PID control output for the controlled system is given as

$$u = K \left( \frac{1}{T_i} \int_0^t e(t) dt + e + T_d \dot{e} \right) \quad (10)$$

In it,  $K, T_i, T_d$  are the feedback gain, integral time, and differential time of the PID controller, respectively.

(2) In order to construct a cascaded integral system, let

$$\int_0^t e(t) dt = e_0(t) \quad (11)$$

(3) Then, the derivative of (11) is deduced as

$$\dot{e}_0(t) = e(t) \quad (12)$$

Then,

$$\ddot{e}(t) = \frac{d^2(v-x)}{dt^2} = -\ddot{x} = a_1 x + a_2 \dot{x} - u - \omega \quad (13)$$

(4) By substituting (10) into (13), the following can be obtained:

$$\begin{aligned}\ddot{e}(t) &= \frac{d^2(v-x)}{dt^2} = -\ddot{x} \\ &= a_1 x + a_2 \dot{x} - K \left( \frac{1}{T_i} \int_0^t e(t) dt + e + T_d \dot{e} \right) - \omega \\ &= a_1 (x - v) + a_1 v + a_2 \dot{x} \\ &\quad - K \left( \frac{1}{T_i} \int_0^t e(t) dt + e + T_d \dot{e} \right) - \omega \\ &= a_1 (x - v) + a_1 v - a_2 (\dot{v} - \dot{e}) \\ &\quad - K \left( \frac{1}{T_i} \int_0^t e(t) dt + e + T_d \dot{e} \right) - \omega \\ &= a_1 (x - v) + a_1 v + a_2 \dot{e} \\ &\quad - K \left( \frac{1}{T_i} \int_0^t e(t) dt + e + T_d \dot{e} \right) - \omega \\ &= -(a_1 + K) e - (KT_d - a_2) \dot{e} + a_1 v \\ &\quad - \frac{K}{T_i} \int_0^t e(t) dt - \omega\end{aligned}\quad (14)$$

(5) According to (12), the substitution relationships of the variables are set as

$$\dot{e}_0(t) = e(t) = e_1(t), \quad (15)$$

$$\dot{e}(t) = e_2(t) \quad (16)$$

(6) After arrangement, the above equations can be converted into a cascaded integral system as follows:

$$\dot{e}_0 = e_1, \quad e_0(0) = 0$$

$$\dot{e}_1 = e_2, \quad e_1(0) = v(0)$$

$$\dot{e}_2 = -\frac{K}{T_i} e_0 - (K + a_1) e_1 - (KT_d - a_2) e_2 + a_1 v - \omega, \quad (17)$$

$$e_2(0) = 0$$

$$y = v - e_1$$

Thus, the structure of a cascaded integral ADRC control system is shown in Figure 2. In the virtual frame, it is a cascaded integral object system of  $n$  orders. The other part is an error-feedback ADRC controller consisting of the TD, ESO, NLSEF, and disturbance compensator. The TD, ESO, NLSEF, disturbance compensator, and their variables are seen in ((1)-(6)).

#### 4. The Parameters of ADRC Controller

Although the ADRC has many parameters to determine its control effect, it is not difficult to determine them. This is because many parameters have their universal values that are suitable for most conditions.



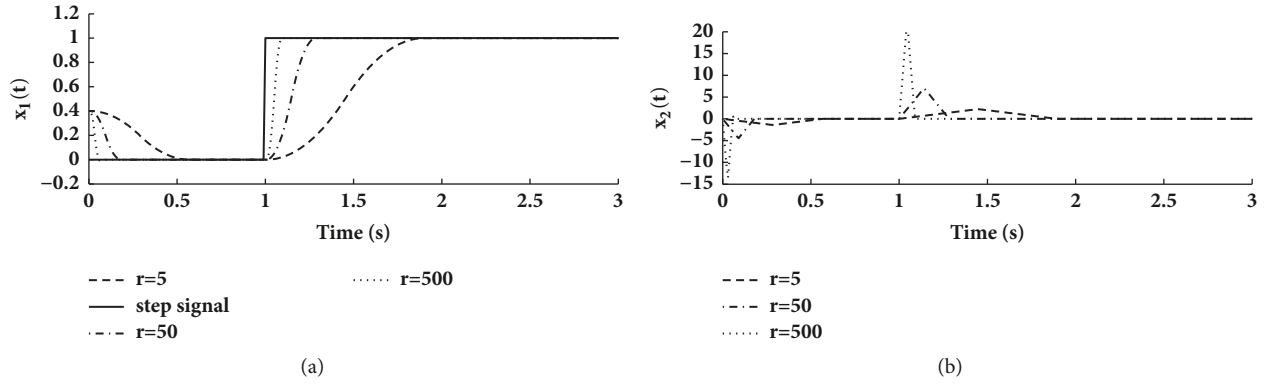


FIGURE 3: The tracking and derivative outputs of a TD with  $r = 5, 50, 500$ , respectively, and  $h = 0.01$  and  $h_0 = 0.01$  under a step response.

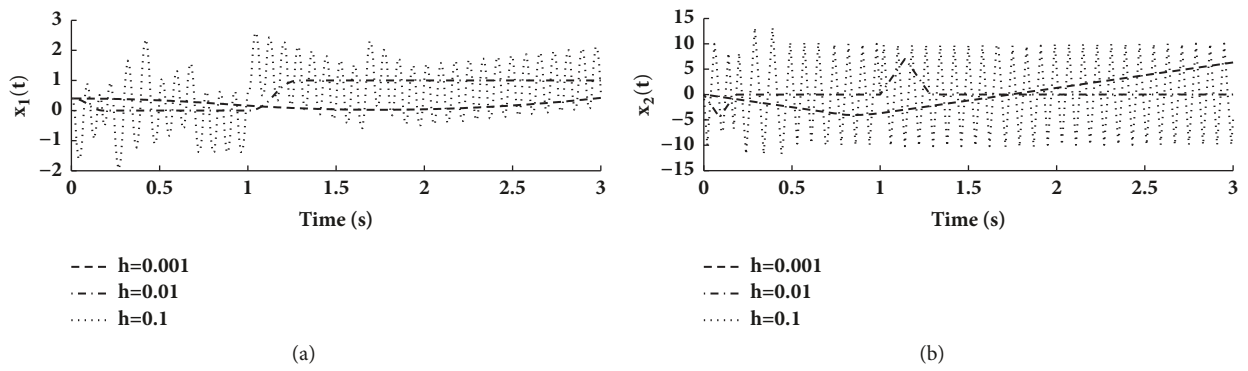


FIGURE 4: The tracking and derivative outputs of a TD with  $h = 0.001, 0.01, 0.1$ , respectively, and  $r = 50, h_0 = 0.01$  under a step response.

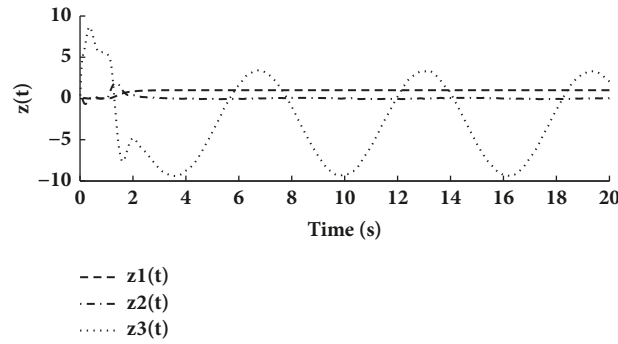


FIGURE 5: A successful output of the ESO with  $\beta_{01} = 100, \beta_{02} = 300, \beta_{03} = 1000, \alpha_1 = 0.5, \alpha_2 = 0.25$ , and  $\delta = 0.05$ .

step response is the most destructive to a dynamic system. In (18),  $u$  is the control signal, and

$$\omega = \sin(t) + \gamma n(t) \quad (19)$$

The  $\omega$  is an added disturbance;  $n(t)$  is a white noise; its mean value is 0; and variance is 1;  $\gamma = 0.1$ .

Then, the control effect will be tested under no disturbance, a periodical disturbance, white noise, or inaccurate model disturbance from time 0. If the controller can resist all these kinds of disturbances while keeping the controller

design and its parameters invariable, it has a good adaptive ability and can work on most occasions.

**5.1. The Conversion and the ADRC Controller Design.** Now, the control system of (18) will be converted into a cascaded integral form according to ((9)-(17)), that is, the algebraic substitution method and its structure presented in Section 3. First,  $v$  is set as the control reference. The error between the system state and control reference is

$$e = v - x = v - y \quad (20)$$

Then, the conversion adopts the following six steps:

(1) The PID control output for the controlled system is given as

$$\begin{aligned} u &= K \left( \frac{1}{T_i} \int_0^t e(t) dt + e + T_d \dot{e} \right) \\ &= 34 \left( \frac{1}{17} \int_0^t e(t) dt + e + 0.8 \dot{e} \right) \end{aligned} \quad (21)$$

In it, the controller parameters take  $K = 34$ ,  $T_i = 17$ , and  $T_d = 0.8$ , which are the same as that of the PID controller in Section 5.4 for a convenient comparing.

(2) In order to construct a cascaded integral system, let

$$\int_0^t e(t) dt = e_0(t) \quad (22)$$

(3) Then, the derivative of (22) is deduced as

$$\dot{e}_0(t) = e(t) \quad (23)$$

Then

$$\ddot{e}(t) = \frac{d^2(v-x)}{dt^2} = -\ddot{x} = 3x + 6\dot{x} - u - \omega, \quad (24)$$

(4) By substituting (21) into (24) and according to (14), the following can be obtained:

$$\begin{aligned} \ddot{e}(t) &= \frac{d^2(v-x)}{dt^2} = -\ddot{x} \\ &= 3x + 6\dot{x} - K \left( \frac{1}{T_i} \int_0^t e(t) dt + e + T_d \dot{e} \right) - \omega \\ &= -37e - 21.2\dot{e} + 3v - 2 \int_0^t e(t) dt - \omega \end{aligned} \quad (25)$$

(5) According to (23), the substitution relationships of the variables are set as

$$\dot{e}_0(t) = e(t) = e_1(t), \quad (26)$$

$$\dot{e}_1(t) = e_2(t) \quad (27)$$

(6) After arrangement, the control system of (16) can be converted into a new cascaded integral form as follows:

$$\begin{aligned} \dot{e}_0 &= e_1, \quad e_0(0) = 0 \\ \dot{e}_1 &= e_2, \quad e_1(0) = v(0) \\ \dot{e}_2 &= -2e_0 - 37e_1 - 21.2e_2 + 3v - \omega, \quad e_2(0) = 0 \\ y &= v - e_1 \end{aligned} \quad (28)$$

In (28), the new system states are  $e_0$ ,  $e_1$ , and  $e_2$ . Then, the new system of the cascaded integral form in (28) can be directly controlled with the ADRC controller in Section 2. The control reference is still the unit step signal of  $v = 1$  from time 1 and initial value 0. The adopted ADRC controller is in ((1)-(6)). The ADRC parameters for the best control effect can be found as  $r = 30$  and  $h_0 = 0.3$ ;  $\beta_{01} = 100$ ,  $\beta_{02} = 300$ ,  $\beta_{03} = 1000$ , and  $\delta = 0.05$ ;  $\alpha_1 = 0.75$ ,  $\alpha_2 = 1.25$ ,  $\beta_1 = 100$ ,  $\beta_2 = 10$ , and  $\delta_1 = 0$ ;  $b_0 = 1$  with a trial and error method. The discrete time step is  $h = 0.01$ .

**5.2. The ADRC Control Effect of the Converted System.** When the disturbance is  $\omega = 0$  and the ADRC controller parameters are set according to Section 5.1, the control effect for the unit step input  $v = 1$  at time 1 is shown in Figure 6. Then, for the same control condition, Figure 6 also shows the control effect: (1) under a periodical disturbance of  $\omega = \sin(t)$ ; (2) under a periodical and white noise disturbances of  $\omega = \sin(t) + \gamma n(t)$ . The simulation shows that the ADRC can achieve a nonovershoot unit step tracking control while satisfying the rapidity under all the disturbances. This control effect is very close to that of no disturbance. Thus, the ADRC control for the converted cascaded integral system is feasible.

**5.3. The ADRC Control Effect under Model Parameter Error of 100%.** Assume that the model parameters  $a_1$  and  $a_2$  and other parameters  $K$ ,  $T_i$ , and  $T_d$  in (17) cannot be accurately estimated. For example, these parameters are reduced to 100% error from their true values. Then, under the same ADRC controller and its parameters as well as the same unit step input  $v = 1$  at time 1, Figure 7 shows the control effect: (1) under a periodical disturbance of  $\omega = \sin(t)$ ; (2) under a periodical and white noise disturbances of  $\omega = \sin(t) + \gamma n(t)$ . The simulation shows that the ADRC controller can still achieve a nonovershoot unit step tracking control while satisfying the rapidity under all the disturbances. This is because the parameter errors of the object model can also be seen as a disturbance and then be estimated and compensated by the ESO and compensator. Thus, the noncascaded integral system with an inaccurate object model and parameter can still achieve the conversion and ADRC control, as long as the system order can be determined. This is also an advantage of the ADRC controller.

If the model parameters  $a_1$  and  $a_2$  and other parameters  $K$ ,  $T_i$ , and  $T_d$  are increased to 100% error from their true values, the simulation shows that the ADRC controller can still achieve the above control effect under the same ADRC controller and disturbances. This will not be repeated here. The reason is also the same. The results also show that the cascaded integral ADRC control system has a strong adaptive ability. Thus, the antidisturbance ability is good enough to let the controller parameters invariable. This ability can resist the periodical, white noise, and inaccurate model disturbance during the step response while keeping controller and its parameters unchanged.

**5.4. The Comparing PID Control for the Original System.** A PID controller can also be designed to control the system of (18) as

$$\begin{aligned} e &= v - x, \\ u &= K \left( \frac{1}{T_i} \int_0^t e(t) dt + e + T_d \dot{e} \right) \end{aligned} \quad (29)$$

The parameters of the best control effect can be found as  $K = 34$ ,  $T_i = 17$ , and  $T_d = 0.8$  with a trial and error method.

When the disturbance is  $\omega = 0$ , the control effect for the unit step input  $v = 1$  at time 1 is shown in Figure 8. Then, for the same control condition, Figure 8 also shows



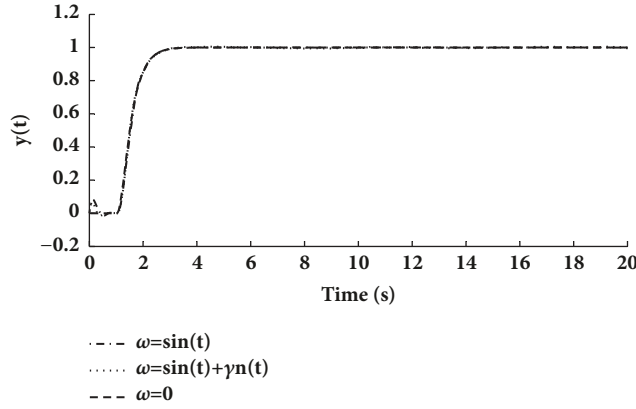


FIGURE 6: The ADRC control effect with parameters  $r = 30$  and  $h_0 = 0.3$ ;  $\beta_{01} = 100$ ,  $\beta_{02} = 300$ ,  $\beta_{03} = 1000$ , and  $\delta = 0.05$ ;  $\alpha_1 = 0.75$ ,  $\alpha_2 = 1.25$ ,  $\beta_1 = 100$ ,  $\beta_2 = 10$ , and  $\delta_1 = 0$ ;  $b_0 = 1$  for (a) under no disturbance; (b) under a periodical disturbance of  $\omega = \sin(t)$ ; (c) under a periodical and white noise disturbances of  $\omega = \sin(t) + \gamma n(t)$ .

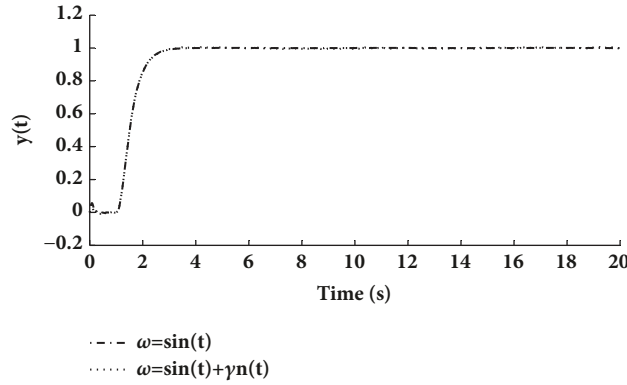


FIGURE 7: The control effect under the same ADRC controller and its parameters with 100% model error when there is (a) a periodical disturbance of  $\omega = \sin(t)$ ; (b) a periodical and white noise disturbances of  $\omega = \sin(t) + \gamma n(t)$ .

the control effect: (1) under a periodical disturbance of  $\omega = \sin(t)$ ; (2) under a periodical and white noise disturbance of  $\omega = \sin(t) + \gamma n(t)$ . The results show that the PID controller can only achieve an overshoot unit step tracking control under no disturbance. The antidisturbance ability of the PID controller is very limited for the periodical and white noise disturbances if its parameters cannot be adjusted adaptively. Thus, compared with Figures 6-7, the ADRC for the converted system has a much better adaptive ability and control effect than that of the PID controller.

## 6. Application in the Oscillation Control of a Flexible Arm

**Example 3.** For the oscillation control of a flexible arm, its mathematical model can be formulated as the following parallel system of (30). The oscillation is decomposed according to its frequency spectrum. Assume that the first three terms of the oscillation are as follows:

$$\begin{aligned}\ddot{x}_1 &= -\omega_1^2 x_1 - 2\xi_1 \omega_1 \dot{x}_1 + b_1 u = \frac{1}{176} u \\ \ddot{x}_2 &= -\omega_2^2 x_2 - 2\xi_2 \omega_2 \dot{x}_2 + b_2 u\end{aligned}$$

$$= -4.2849 x_2 - 0.01242 \dot{x}_2 + \frac{1.28}{176} u$$

$$\ddot{x}_3 = -\omega_3^2 x_3 - 2\xi_3 \omega_3 \dot{x}_3 + b_3 u$$

$$= -561.69 x_3 - 0.1422 \dot{x}_3 + \frac{0.03}{176} u$$

$$y = x_1 + x_2 + x_3$$

(30)

In it,  $x_1$ ,  $x_2$ , and  $x_3$  are the oscillation of the fundamental, second harmonic, and third harmonic frequency ( $\omega_1, \omega_2, \omega_3$ ) for the flexible arm, respectively,  $\xi_1, \xi_2, \xi_3$  is the elastic coefficient of each frequency,  $u$  is the added control signal to suppress the oscillation of the flexible arm, and  $b_i, i = 1, 2, 3$ , is the coefficient of each control signal.

Here, the control reference is to make the system output  $y = x_1 + x_2 + x_3$  close to 0 as soon as possible, that is,  $y^* = 0$ , and its initial value is assumed as  $y(0) = 1.0$ . As (30) is an underdriven control system, adopting a single-input single-output ADRC controller directly cannot achieve its control effect. Because of the coupling relationship among

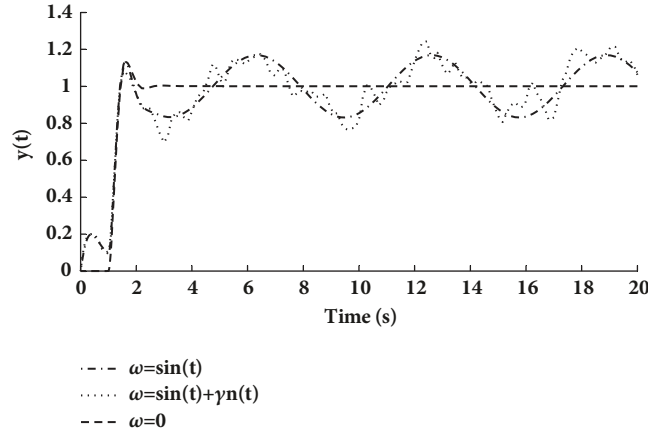


FIGURE 8: The PID control effect with the parameters  $K = 34$ ,  $T_i = 17$ , and  $T_d = 0.8$  for (a) under no disturbance; (b) under a periodical disturbance of  $\omega = \sin(t)$ ; (c) under a periodical and white noise disturbances of  $\omega = \sin(t) + \gamma n(t)$ .

the three oscillation variables, the control effect either cannot be achieved by adopting three ADRC controllers in parallel.

According to (30), it has  $\omega_1 = 0$ ,  $\omega_2 = 2.07$ ,  $\omega_3 = 23.7$ , and  $\xi_1 = \xi_2 = \xi_3 = 0.003$ . Taking the constructing intermediate variables  $\omega_0 = 0.18079747$ , and  $\xi_0 = 0.003$ , (30) can be converted into a system of following form:

$$\begin{aligned}
 \ddot{x}_1 &= -\omega_0^2 x_1 - 2\xi_0 \omega_0 \dot{x}_1 + b_1 u - (\omega_1^2 - \omega_0^2) x_1 \\
 &\quad - 2(\xi_1 \omega_1 - \xi_0 \omega_0) \dot{x}_1 = \frac{1}{176} u \\
 \ddot{x}_2 &= -\omega_0^2 x_2 - 2\xi_0 \omega_0 \dot{x}_2 + b_2 u - (\omega_2^2 - \omega_0^2) x_2 \\
 &\quad - 2(\xi_2 \omega_2 - \xi_0 \omega_0) \dot{x}_2 \\
 &= -\omega_0^2 x_2 - 2\xi_0 \omega_0 \dot{x}_2 + \frac{1.28}{176} u - (2.07^2 - \omega_0^2) x_2 \\
 &\quad - 2(0.003 \cdot 2.07 - \xi_0 \omega_0) \dot{x}_2 \\
 \ddot{x}_3 &= -\omega_0^2 x_3 - 2\xi_0 \omega_0 \dot{x}_3 + b_3 u - (\omega_3^2 - \omega_0^2) x_3 \\
 &\quad - 2(\xi_3 \omega_3 - \xi_0 \omega_0) \dot{x}_3 \\
 &= -\omega_0^2 x_3 - 2\xi_0 \omega_0 \dot{x}_3 + \frac{0.03}{176} u - (23.7^2 - \omega_0^2) x_3 \\
 &\quad - 2(0.003 \cdot 23.7 - \xi_0 \omega_0) \dot{x}_3 \\
 y &= x_1 + x_2 + x_3
 \end{aligned} \tag{31}$$

By combining the similar terms in each line of (31), the following can be obtained:

$$\begin{aligned}
 \ddot{y} &= -\omega_0^2 y - 2\xi_0 \omega_0 \dot{y} + bu + \omega(x, \dot{x}) \\
 \omega(x, \dot{x}) &= -\sum_{i=1}^3 [(\omega_i^2 - \omega_0^2) x_i - 2(\xi_i \omega_i - \xi_0 \omega_0) \dot{x}_i] \\
 b &= \sum_{i=1}^3 b_i = 0.012972
 \end{aligned} \tag{32}$$

After this transformation, the proposed algebraic substitution method and its structure can convert (32) into to a cascaded integral system according to ((8)-(17)), which will not be repeated here. Then, the ADRC controller can be directly and easily used to control the converted cascaded integral system. As long as the coefficients  $\omega_0^2, 2\xi_0 \omega_0$  in (32) change in a certain range, the ADRC controller can always suppress the oscillation of the flexible arm well.

The adjusting parameters of the ADRC controller are taken as  $h_0 = 0.05$ ;  $\beta_{01} = 100$ ,  $\beta_{02} = 200$ ,  $\beta_{03} = 800$ , and  $\delta = 0.05$ ;  $\alpha_1 = 0.75$ ,  $\alpha_2 = 1.25$ ,  $\beta_1 = 100$ ,  $\beta_2 = 10$ , and  $\delta_1 = 0$ ;  $b_0 = 0.003$  with a trial and error method. As for the  $r$ , it can take a proper big value  $r = 2.5$  for a good control effect. Although the flexible arm model has  $b = 0.012972$ , the  $b$  is not necessarily known in a real system. Thus, it is better to regard the  $b$  as an adjustable parameter. The dividing frequency of the flexible arm model is taken as  $\omega_1^2 = 0$ ,  $\omega_2^2 = 4.2849$ , and  $\omega_3^2 = 561.69$  according to (30), respectively. The simulation results are shown in Figure 9. The oscillation can be suppressed quickly without overshoot, and the control signal is small and reasonable.

If the frequencies of the flexible arm model are changed into  $\omega_1^2 = 0$ ,  $\omega_2^2 = 1$ ,  $\omega_3^2 = 16$  or  $\omega_1^2 = 0$ ,  $\omega_2^2 = 9$ , and  $\omega_3^2 = 1600$ , the simulation results are very similar to that of Figure 9. The controller parameters for the three cases are fixed, but the oscillation mode change is so large, which shows that the ADRC controller plays a very good antioscillation and adaptive effect with the proposed noncascaded integral ADRC control system.

## 7. Conclusions

This paper begins with an introduction to the ADRC method and its typical form for the cascaded integral system. Since the real systems do not necessarily have the cascaded integral form, this paper proposes an algebraic substitution method and its structure, which can convert a noncascaded integral system into the cascaded integral form. By this way, the converted system can be controlled directly and easily using



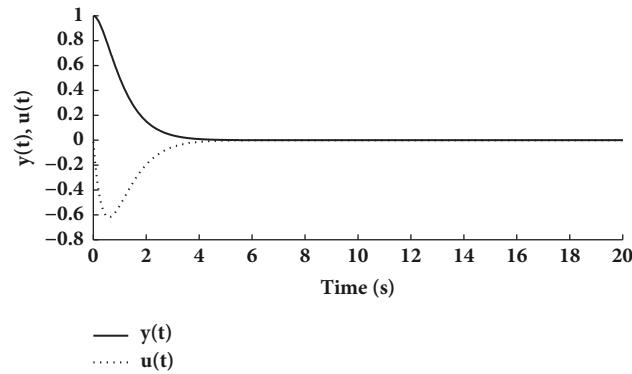


FIGURE 9: The oscillation control of a flexible arm under  $\omega_1^2 = 0$ ,  $\omega_2^2 = 4.2849$ , and  $\omega_3^2 = 561.69$  with the noncascaded integral ADRC control system.

the ADRC method. Meanwhile, a numerical example and the oscillation control of a flexible arm are simulated to demonstrate the conversion and its control effect. In addition, the control effect of the numerical example is compared between the converted ADRC control system and the PID control system under a variety of disturbances while keeping the two controllers and their parameters invariable.

The research results show the following: (1) This converting method is feasible not only in theory but also in simulation. (2) For the converted cascaded integral system, the ADRC can achieve a nonovershoot tracking control while satisfying the rapidity under many forms of disturbances, and the ADRC controller also has a better control effect than that of the PID controller. (3) For the noncascaded integral system with inaccurate model and parameter, the ADRC can still achieve the conversion and its control effect as long as the system order can be determined. (4) The model and parameter error can also be seen as a disturbance and then be estimated and compensated by the ESO and compensator. This is also an advantage of the proposed method. (5) This paper presents an approach to transform the noncascaded integral system into the cascaded integral form when necessary. (6) This converting method solves the ADRC control problem of some noncascaded integral systems in theory and simulation. It also expands the application scope of the ADRC method.

The theory is a guide before practice. In future, the converting method for non-PID control system should also be researched, and the application to a more complex system can be implemented.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

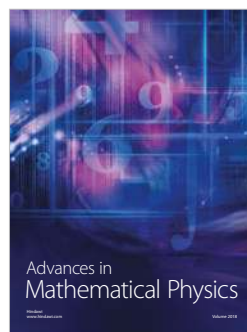
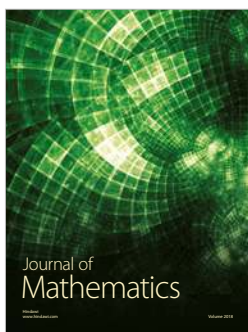
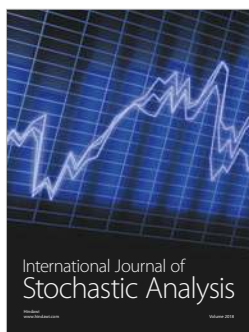
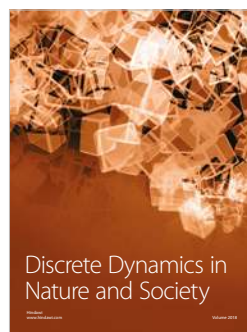
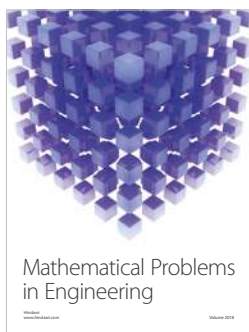
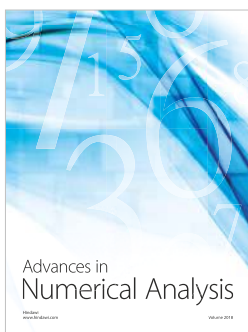
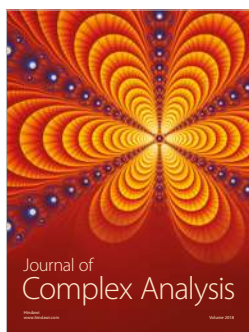
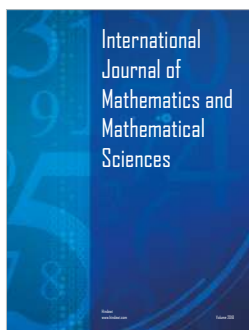
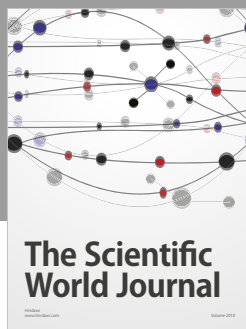
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