An Aeroelastic Analysis of a Flexible Flapping Wing Using Modified Strip Theory

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ABSTRACT

The present study proposed a coupling method for the fluid-structural interaction analysis of a flexible flapping wing. An efficient numerical aerodynamic model was suggested, which was based on the modified strip theory and further improved to take into account a high relative angle of attack and dynamic stall effects induced by pitching and plunging motions. The aerodynamic model was verified with experimental data of rigid wings. A reduced structural model of a rectangular flapping wing was also established by using flexible multibody dynamics, so as to consider large flapping motions and local elastic deformations. Then, the aeroelastic analysis method was developed by coupling these aerodynamic and structural modules. To measure the aerodynamic forces of the rectangular flapping wing, static and dynamic tests were performed in a low speed wind-tunnel for various flapping pitch angles, flapping frequencies and the airspeeds. Finally, the aerodynamic forces predicted by the aeroelastic analysis method showed good agreement with the experimental data of the rectangular flapping wing.

Keywords: Flapping wing, unsteady aerodynamics, modified strip theory, dynamic stall, flexible-multibody dynamics, fluid-structure interaction, aeroelasticity

1. INTRODUCTION

During the past few decades, the flapping flight of birds, bats and insects has fascinated many researchers in various fields such as biology, aeromechanics and engineering because of their highly efficient maneuverability and aerodynamic benefits especially in a low Reynolds number flight regime. Numerous efforts have been made to make flapping-wing vehicles such as ornithopters or flapping micro air vehicles. Many analytical and experimental studies on flapping wings have also been performed to understand the aerodynamic characteristics and flight mechanisms of the flapping wings. However, because of the complexity of the fluid-structure interaction of a flexible flapping wing, mostly rigid wings have been considered so far, and only few studies on the aeroelastic analysis have been performed.

Actually, the biological flapping flyers have flexible wings with anisotropic flexibility in both spanwise and chordwise directions. Using the flexible wings, they can utilize complicated wing motions consisting of flapping, twisting, folding, rotating motions or area expansion and contraction [1,2]. The passive or active deformation of the wing contributes to generate appropriate aerodynamic performances according to the various flight modes. The artificial flapping flyers inspired from the biological flappers also have thin and flexible passive wings structurally similar to those of insects. Although they use only the root-flapping motion and the passive twisting motion generated by the wing flexibility, the deformation of these flapping wings is strongly coupled with aerodynamic forces, resulting in similar wing motions like the nature's flyers. In order to consider this complicated fluid-structure interaction, aerodynamic and structural analysis technologies such as CFD and FEM must be simultaneously applied. However, it is still extremely difficult and time-consuming to accurately analyze the fluid-structure interaction of the biological or artificial flexible flapping wings [3]. Therefore, for the optimal design and the real-time control of flapping-wing flight, an efficient aerodynamic model applicable to general flapping wings is indispensable, and an efficient aeroelastic analysis method should be also developed.

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Most aerodynamic models used for the previous researches can be broadly classified into the quasi-steady models and the unsteady models. The quasi-steady models assume low flapping frequencies so that unsteady wake effects can be ignored [4,5]. In the unsteady models, unsteady aerodynamic characteristics are accounted for by modeling the unsteady wake effects induced by unsteady wing motions and vortices shed from leading edges [6-8].

In the present study, an unsteady aerodynamic model is suggested, and a fluid-structure interaction analysis method for a flexible flapping wing is proposed. First, a numerical aerodynamic model based on the modified strip theory is improved to consider high relative angle of attack and dynamic stall effects due to pitching and plunging motions. This unsteady aerodynamic model is verified with experimental data of a rigid wing with heaving and pitching motions. Second, the structural dynamic model of a flexible rectangular flapping wing is established using flexible multi-body dynamics to analyze the root-flapping wing motion with large displacement and local deformation. Finally, the aeroelastic analysis method is suggested by coupling both the aerodynamic and structural models. The aeroelastic model is applied to the rectangular flapping wing, and the analysis results show the good agreement with the experimental data obtained from a low-speed wind tunnel tests under various flow conditions.

2. UNSTEADY AERODYNAMIC MODEL

The aerodynamic model suggested in this study is based on the modified strip theory, in which the total lift and thrust of the root flapping wing are obtained by integrating the sectional aerodynamic forces calculated in each section as shown in Fig.1. Although there are some limitations (an invariable span, a continuous sinusoidal motion, high aspect ratio and low relative angle of attack), this aerodynamic model is very efficient and applicable to design and performance prediction of flapping wing vehicles, especially to complicated numerical problems such as a fluid-structure interaction analysis. In this study, the modified strip model was further improved to consider high relative angle of attack and dynamic stall effects due to not only the pitching but also plunging motion, and the improved aerodynamic model was verified with experimental data.

2.1 Modified Strip Theory (MST)

The modified strip theory will be briefly described in this paper, and more details on this model can be found in DeLaurier[8]. The aerodynamic forces of each section of a root flapping wing can be represented as shown in Fig. 1, and Fig. 2 shows the aerodynamic forces and motion variables in a particular section of the wing.

If the leading edge of the wing section is treated as a reference point, then the section's motion consists of flight speed, U , a plunging motion, $\dot{h}$, and a pitching motion, $\dot{\theta}$. The flapping wing motion is assumed to be perpendicular to the flapping axis, but local deformation can be allowed.

Assuming that the relative angle of attack between the free stream velocity U and the resultant flow velocity V calculated at 1/4 chord location is small, the magnitude of the aerodynamic force due to the circulation ($d\Gamma$) can be almost the same as that of the normal circulatory force. Therefore, the section's normal force generated by the circulation around the wing can be expressed as

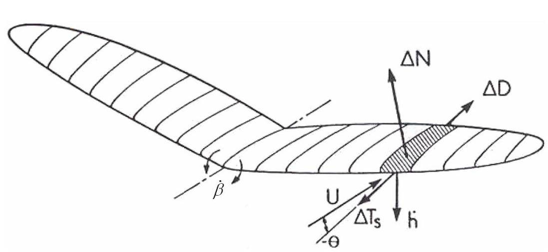


Fig. 1. Root flapping wing and aerodynamic forces [8].

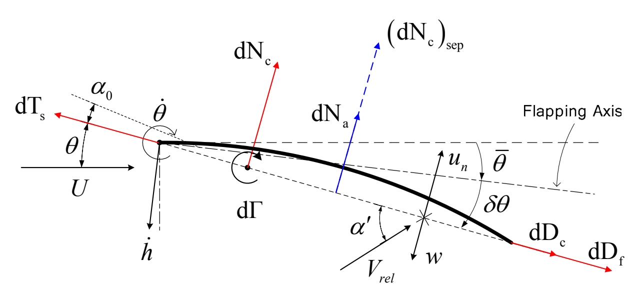


Fig. 2. Wing section aerodynamic forces and motion variables.

$$\mathrm{dN_c} = 2\pi\left(\alpha' + \alpha_0 + \bar{\theta}\right)\frac{\rho UV}{2}\mathrm{cdy} \quad (1)$$

where ρ , U , α_0 , α' are the atmospheric density, the flight speed, the angle of section's zero-lift line and the resultant flow velocity at 1/4 chord location, respectively. The other aerodynamic forces of the wing section shown in Fig. 2 can be expressed as follows:

$$\mathrm{dN_a} = \frac{\rho\pi c^2}{4}\dot{v}_2\mathrm{dy} \quad (2)$$

$$\mathrm{dD_c} = -2\pi\alpha_0\left(\alpha' + \bar{\theta}\right)\frac{\rho UV}{2}c\mathrm{dy} \quad (3)$$

$$\mathrm{dT_s} = \eta_s 2\pi\left(\alpha' + \bar{\theta} - \frac{1}{4}\frac{c\dot{\theta}}{U}\right)\frac{\rho UV}{2}\mathrm{cdy} \quad (4)$$

$$\mathrm{dD_f} = \left(C_d\right)_f \frac{\rho V_a^2}{2}\mathrm{cdy} \quad (5)$$

where dN_a , dD_c , dT_s and dD_f are the additional force due to apparent mass, the chordwise force due to the camber, the leading edge suction force and the chordwise friction drag due to viscosity, respectively.

The criterion for attached flow over the section is assumed as follows:

$$\left(\alpha_{stall}\right)_{\min} \le \left[\alpha' + \bar{\theta} - \frac{3}{4}\left(\frac{c\dot{\theta}}{U}\right)\right] \le \left(\alpha_{stall}\right)_{\max} \quad (6)$$

where the last term in the bracket in Eq. (6) represents the dynamic stall effect induced by the pitching motion. When the attached flow range is exceeded, it is assumed that the flow is totally separated and all chordwise forces are negligible (localized post stall behavior). The normal forces in the stall condition are given by

$$\left(\mathrm{dN_c}\right)_{sep} = \left(C_d\right)_c \frac{\rho\hat{V}V_n}{2}\mathrm{cdy} \quad (7)$$

$$\left(\mathrm{dN_a}\right)_{sep} = \frac{\rho\pi c^2}{8}\dot{v}_2\mathrm{dy} \quad (8)$$

where $\hat{V}$ and V_n are the resultant flow velocity and the normal flow velocity at midchord location, respectively. Finally, the normal and horizontal forces of each section, $dN = dN_c + dN_a$, and $dF_a = dT_s - dD_c - dD_f$, can be obtained by using Eqs. (1-5). The section's instantaneous lift and thrust and moment are given as follows:

$$\mathrm{dL}(t) = \mathrm{dN}\cos\theta + \mathrm{dF_a}\sin\theta \quad (9)$$

$$\mathrm{dT}(t) = \mathrm{dF_a}\cos\theta - \mathrm{dN}\sin\theta \quad (10)$$

These aerodynamic forces and moment of the sections can be integrated along the span to obtain the whole wing's instantaneous lift, thrust and moment as follows:

$$\mathrm{L}(\mathrm{t}) = 2\int_0^{\frac{b}{2}}\cos\beta\mathrm{dL} \quad (11)$$

$$\mathrm{T}(\mathrm{t}) = 2\int_0^{\frac{b}{2}}\mathrm{dT} \quad (12)$$

where β is the section's instantaneous dihedral angle.

2.2 High relative angle of attack

According to the flight condition, the aerodynamic flow state around the flapping wing section can be severe, and thus more practical flight conditions should be considered for further application of the aerodynamic model. If the plunging velocity is increased or the free stream velocity relative to $\dot{h}$ is decreased, the wing section can be exposed to a high relative angle of attack, and in this condition, the assumption of small relative angle of attack is not valid. To consider high relative angle of attack, the aerodynamic model is improved. Using the horizontal and vertical components of the velocity at 1/4 chord location, the relative angle of attack can be defined as follows:

$$\gamma = \tan^{-1}\left(\frac{\dot{h}\cos\left(\theta-\overline{\theta}_a\right)-w+0.25c\dot{\theta}+U\sin\theta}{U\cos\theta-\dot{h}\sin\left(\theta-\overline{\theta}_a\right)}\right) \tag{13}$$

where w is the downwash velocity at 1/4 chord location induced by unsteady finite flapping wing motions. Using Eq. (1), Eq. (4) and the relative angle of attack, the section's aerodynamic forces can be expressed as

$$dN_c = 2\pi\left(\alpha'+\alpha_0+\overline{\theta}\right)\cos\gamma\frac{\rho UV}{2}cdy \tag{14}$$

$$dT_s = \eta_s 2\pi\left(\alpha'+\overline{\theta}-\frac{1}{4}\frac{c\dot{\theta}}{U}\right)\sin\gamma\frac{\rho UV}{2}cdy \tag{15}$$

$$dD_c = -2\pi\alpha_0\left(\alpha'+\overline{\theta}_a+\overline{\theta}_w\right)\cos\gamma\frac{\rho UV}{2}cdy \tag{16}$$

Other aerodynamic forces, dN_a and dD_f, are the same as in Eqs. (2) and (5), respectively.

2.3 Dynamic stall model

The unsteady effects due to the dynamic stall can be generated by the plunging motion as well as the pitching motion [9]. Thus, the unsteady effect due to the plunging motion should be considered in the dynamic stall model. However, it is not easy to define the stall condition using a mathematical function. It can probably be determined from test results of an unsteady flapping wing. In this study, we use the same assumption as that in Scherer[10], where the instantaneous maximum values of the lift coefficient could be as large as twice the maximum steady value. Using the dynamic stall effect and the relative angle of attack, in this study, three kinds of flow conditions are defined as follows:

1) Attached flow condition:

$$\left(\alpha_{stall}\right)_{\min} \le \gamma \le \left(\alpha_{stall}\right)_{\max} \tag{17}$$

2) Dynamic stall condition:

$$\left(\alpha_{dynamic}\right)_{\min} \le \left[\gamma-\frac{3}{4}\left(\frac{c\dot{\theta}}{U}\right)\right] \le \left(\alpha_{dynamic}\right)_{\max} \tag{18}$$

3) Post-stall condition: over the dynamic stall range

According to the flow conditions, the aerodynamic forces applied on the strip are different. In the attached flow range and the post-stall flow range, the aerodynamic forces are same as those defined in the modified strip theory. In the dynamic stall range, the aerodynamic forces are applied as those in the attached flow condition, but only the direction of the leading edge suction force is assumed to be perpendicular to the wing in order to take into account the unsteady effect due to the leading edge vortex [11].

2.4 Verification of aerodynamic models

In order to verify the aerodynamic model proposed in the present study, the aerodynamic forces calculated by the present model were compared with the experimental data of a plate wing [12]. The wing is a flat plate with $\mathrm{AR} = 6$, $c = 30mm$ and $t = 1.5mm$. This experiment was performed in a low speed wind-tunnel and the airspeed was $3.7m/s$ during the tests.

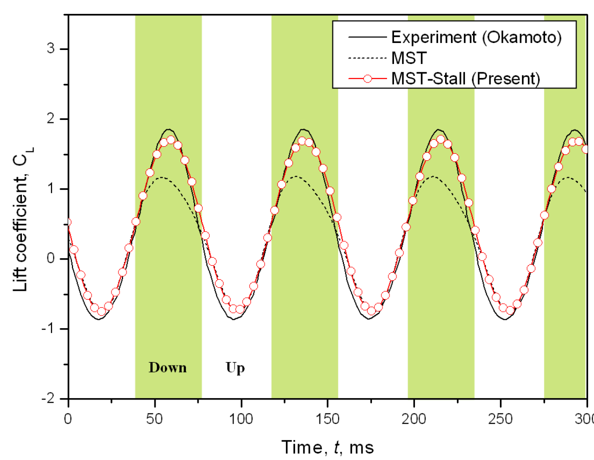
For the application of the aerodynamic model, the aerodynamic coefficients, the zero-lift angle (α_0), and the maximum and minimum stall angles should be defined before the calculation procedure of dynamic analyses. These necessary parameters can be obtained by using both the aerodynamic model and the experimental data measured from steady aerodynamic tests. In this study, the parameters are estimated by using the steady lift and drag coefficients measured in the static test, and the estimated values were as follows:

$$\eta_s = 0.18\,,\ (C_d)_c = 2.65\,,\ (C_d)_f = 0.068\,,$$

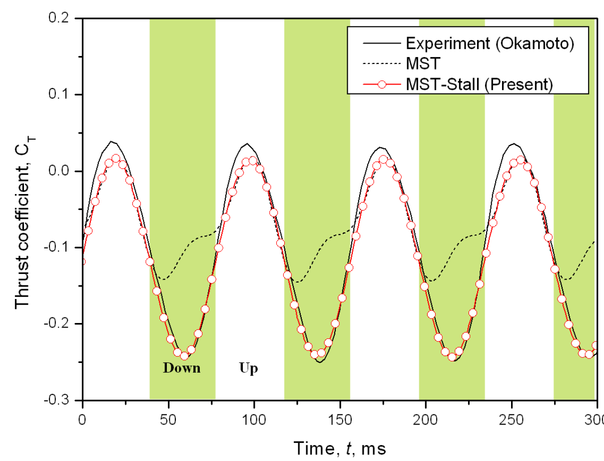
$$\alpha_0 = 0.0\deg\,,\ (\alpha_{stall})_{\max} = 8.67\deg \ \text{ and } \ (\alpha_{stall})_{\min} = -8.67\deg \tag{19}$$

Using the estimated aerodynamic parameters, the unsteady aerodynamic analysis of the same plate wing was performed. In the dynamic condition, the pitch angle of the wing was fixed at $\theta = 6\deg$, and the wing was sinusoidally oscillated in vertical direction with only the plunging motion.

In the unsteady aerodynamic analyses using the aerodynamic model, two kinds of aerodynamic models were used and their accuracies were compared with the experimental data. The first one was 'MST' (Modified Strip Theory) based on the assumption of low relative angle of attack, and the second one was 'MST-Stall' suggested in this study for the consideration of the high relative angle of attack and the dynamic stall model. Fig. 3 shows the comparison between the experimental data (solid line) and the analytical values (dashed line and solid line with circular symbols) of time histories of the aerodynamic lift and thrust coefficients. These results clearly demonstrate that 'MST-Stall' provides more accurate prediction. In fully attached flow range during the up-stroke phase, the aerodynamic force coefficients can be accurately predicted by using both the aerodynamic models. However, in the stall flow range during the down-stroke phase, there are great disparities between the experimental data and the analysis data obtained by using 'MST'(dashed line). This result means that the 'MST' model is not enough to predict the unsteady aerodynamic characteristics of the oscillating wing especially exposed to higher relative angle and dynamic stall phenomena. This deficiency of the aerodynamic model can be satisfied with the 'MST-Stall' model (solid line with circular symbols) which accurately predicts the aerodynamic performance during the entire strokes. Therefore, it is evident from these results that the high relative angle of attack and the dynamic stall effect due to the plunging motion should be taken into account for the prediction of unsteady aerodynamic characteristics of flapping wings.



a) Attached flow range



a) Attached flow range

Fig. 3. Time histories of lift coefficients of the flat plate.

3. AEROELASTIC ANALYSIS

The aeroelastic analysis of the flexible flapping wing can be performed by interactively solving the unsteady aerodynamic model and the structural dynamic model suggested in this study. Thus, in order to solve the fluid-structure interaction problem, a coupling method between the aerodynamic and the structural models should be achieved. In the present work, the dedicated coupling method is developed and the aeroelastic analysis results are compared with experimental data obtained from a low-speed wind tunnel test of a rectangular flapping wing.

3.1 Fluid-structure interaction

The fluid-structure interaction of the flapping wing is analyzed by coupling the aerodynamic model and the structural model which is established by using flexible multibody dynamics. At a time step t_i, the motion and deformation data of the wing calculated by the structural module are used as the input values of the aerodynamic model to obtain the aerodynamic force and moment of the wing, and then the structural dynamic responses are also calculated by using the obtained aerodynamic outputs. This iterative calculation is successively performed until the structural response satisfies a convergence criterion in which the difference of the total elastic deformation is restricted to be less than 10^{-4}. In this analysis, it is assumed that the pitching motion generated by the flapping motion is a sinusoidal passive motion as follows:

$$\theta_z(t) = \theta_0 + \theta_1 \sin(2\pi f_z \cdot t) \tag{20}$$

where, θ_0 and θ_1 are the mean value and the amplitude of the flapping angle, respectively. The flight condition parameters, such as air density ρ, viscosity μ, flight speed U, aspect ratio AR, flapping frequency ω and the aerodynamic parameters of the rectangular wing, are time-invariant values. The time integration of the coupling problem is conducted by using Newmark beta method, and the final calculation time is decided by the period of the steady mean pitch angle $\bar{\theta}$.

3.2 Aeroelastic analysis results

The aeroelastic analysis results of a rectangular flapping wing, which has a rectangular wing shape with the aspect ration of 5.58 and the half span of 54cm, were compared with experimental data obtained using a low-speed wind tunnel with a test section of $762 \times 1016 \times 1524 mm^3$. In the wind tunnel test, the flapping driving device is fixed on a test stand consisting of two load cells which are perpendicular in each direction, so that the vertical and horizontal constraint forces produced by the flapping motion can be simultaneously measured. The mean value and the amplitude of the flapping angle in Eq. (20) are $\theta_0 = 3.16°$ and $\theta_1 = 33.13°$. The flapping angle $\bar{\theta}_a$ is varied from $0°$ to $20°$, the flight speed U is $6 m/s$ and the flapping frequency f_z is $4 Hz$.

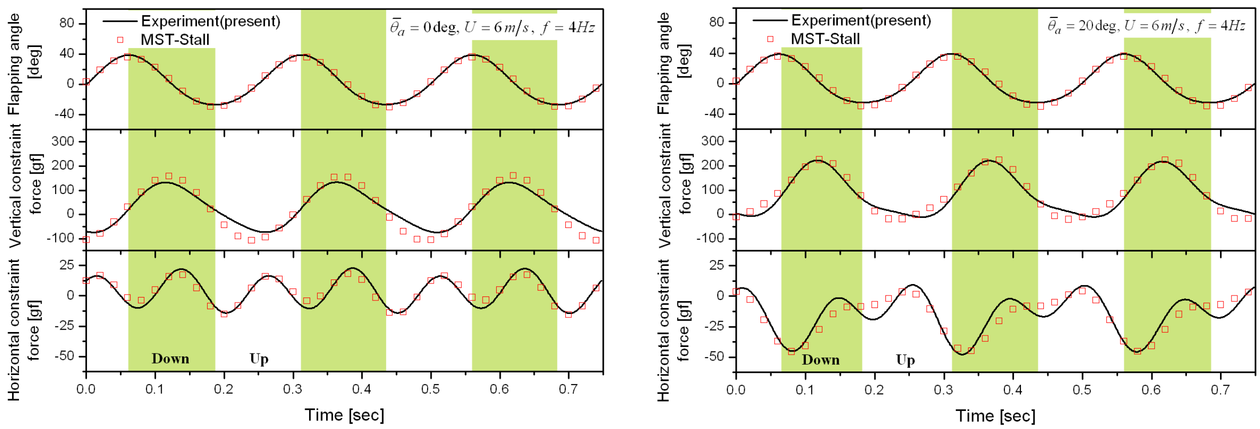
In the manner similar to the flat plate, the aerodynamic parameters of the rectangular flapping wing are determined from steady lift and drag coefficient results of the same wing by minimizing the estimation error between the aerodynamic model and experimental data, and the estimated values are as follows:

$$\eta_s = 0.04,\ (C_d)_{c1} = 2.1,\ (C_d)_{c2} = 2.9,$$

$$\alpha_0 = 0.0\,\deg,\ (C_d)_f = 0.014,\ (\alpha_{stall})_{\max} = 7.2\,\deg \text{ and } (\alpha_{stall})_{\min} = -9.6\,\deg \tag{21}$$

where $(C_d)_{c1}$ and $(C_d)_{c2}$ are the crossflow drag coefficients of upper and lower surfaces, respectively, which are different owing to the asymmetric surface conditions.

Figure 4 shows the comparisons of the vertical and horizontal constraint forces between the measured and the calculated data in $\bar{\theta}_a = 0°$ (Fig. 4-a) and $\bar{\theta}_a = 20°$ (Fig, 4-b). It is evident from the figure that the aeroelastic analysis results obtained by using 'MST-Stall' provide good agreements with the experimental data. As shown in Fig. 4-a, in the symmetric flapping condition, the symmetric vertical constraint force was produced as a sinusoidal function with the same frequency as that of the flapping angle. The maximum value was generated at the middle of the down-stroke where the resultant velocity V was maximized. The horizontal constraint force was also varied sinusoidally but the main frequency was two times larger than that of the flapping angle, and the maximum was occurred at the middle phases of the up- and down-strokes. In the asymmetric flapping condition (Fig. 4-b), the main frequency of the vertical constraint force was the same as that of the flapping angle but the mean value of the vertical constraint force was about 87gf. During the down-stork, the maximum values of the vertical and horizontal constraint forces were generated at the middle stroke like the symmetric flapping condition, but during the up-stroke, there were some distortions in both constraint forces. These results clearly demonstrate that the wing deformation such as bending and pitching motions was passively produced by the interaction between the fluid and the structure. Additionally, it is worth noting that the constraint force is the sum of the aerodynamic force and the inertial force of the wing, therefore, it is not easy to distinguish these forces from the experimental results. One of the advantages of the aeroelastic analysis method proposed in this study is that the aerodynamic force can be successfully extracted from the constraint force by decoupling the aerodynamic and the inertial results.



a) $\bar{\theta}_a = 0°$, $U = 6m/s$, $f = 4Hz$ b) $\bar{\theta}_a = 20°$, $U = 6m/s$, $f = 4Hz$

Fig. 4. Comparisons of constraint forces between experiment and analysis.

4. CONCLUSION

Aeroelastic analyses of a flexible flapping wing were performed by using a fluid-structure interaction analysis method. An improved unsteady aerodynamic model based on the modified strip theory was suggested with the consideration of the high relative angle of attack and the dynamic stall effects owing to not only the pitching but also plunging motion. The accuracy of the improved aerodynamic model was verified with steady and unsteady aerodynamic data. To take into account the elastic deformation as well as the large flapping motion, the flexible multibody dynamics was applied for a structural dynamic model of the flapping wing. Finally, the aeroelastic analyses of the rectangular flapping wing were efficiently performed for various test conditions. From the analysis results, it is evident that the aeroelastic model suggested in the present work compared well with the experimental results. The fluid-structure interaction analysis method of a flexible flapping wing can be effectively applied to other studies such as an optimal flapping wing design and a controller design of flapping-wing flight in the future works.

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