An Algebra for Fine-Grained Integration of XACML Policies

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Abstract

Collaborative and distributed applications, such as dynamic coalitions and virtualized grid computing, often require integrating access control policies of collaborating parties. Such an integration must be able to support complex authorization specifications and the fine-grained integration requirements that the various parties may have. In this paper, we introduce an algebra for fine-grained integration of sophisticated policies. The algebra is able to support the specification of a large variety of integration constraints. To assess the expressive power of our algebra, we prove its completeness and minimality. We then propose a framework that uses the algebra for the fine-grained integration of policies expressed in XACML. We also present a methodology for generating the actual integrated XACML policy, based on the notion of Multi-Terminal Binary Decision Diagrams.

1 Introduction

Many distributed applications such as dynamic coalitions and virtual organizations need to integrate and share resources, and these integration and sharing will require the integration of access control policies. In order to define a common policy for resources jointly owned by multiple parties applications may be required to combine policies from different sources into a single policy. Even in a single organization, there could be multiple policy authoring units. If two different branches of an organization have different or even conflicting access control policies, what policy should the organization as a whole adopt? If one policy allows the access to certain resources, but another policy denies such access, how can they be composed into a coherent whole? Approaches to policy integration are also crucial when dealing with large information systems. In such cases, the development of integrated policies may be the product of a bottom-up process under which policy requirements are elicited from different sectors of the organization, formalized in some access control language, and then integrated into a global access control policy.

When dealing with policy integration, it is well known that no single integration strategy works for every possible situation, and the exact strategy to adopt depends on the requirements by the applications and the involved parties. An effective policy integration mechanism should thus be able to support a flexible fine-grained policy integration strategy capable of handling complex integration specifications. Some relevant characteristics of such an integration strategy are as follows. First, it should be able to support 3-valued policies. A three-valued policy may allow a request, deny a request, or not make a decision about the request. In this case we say the policy is not applicable to the request. 3-valued policies are necessary for combining partially specified policies, which are very likely to occur in scenarios that need policy integration. When two organizations are merging and need policy integration, it is very likely that the organizations are unaware or might not have jurisdiction over each other resources, and thus a policy in one organization may be "NotApplicable" to requests about resources in the other organization. Second, it should allow one to specify

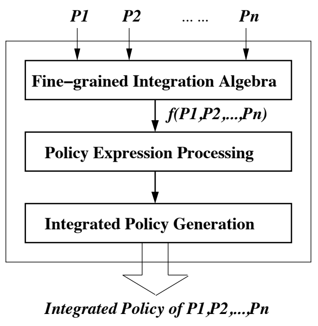

P1
P2
... ...
Pn
Fine–grained Integration Algebra
f(P1,P2,...,Pn)
Policy Expression Processing
Integrated Policy Generation
Integrated Policy of P1,P2,...,Pn

Figure 1: Policy integration

the behavior of the integrated policy at the granularity of requests and effects. In other words, one should be able to explicitly characterize a set of requests that need to be permitted or denied by the integrated policy. For example, users may require the integrated policy to satisfy the condition that for accesses to an object O_i policy P_1 has the precedence, whereas for accesses to an object O_j, policy P_2 has precedence. Third, it should be able to handle *domain constraints* requiring the integrated policy to be applied to a restricted domain instead of the original domain. And fourth, it should be able to support policies expressed in rich policy languages, such as XACML with features like policy combining algorithms.

The problem of policy integration has been investigated in previous works. The concept of policy composition under constraints was first introduced by Bonatti et al. [4]. They proposed an algebra for composing access control policies and use logic programming and partial evaluation techniques for evaluating algebra expressions. Another relevant approach is by Wijesekera et al. [15] who proposed a propositional framework for composing access control policies. Those approaches have however a number of shortcomings. They support only limited forms of compositions. For example, they are unable to support compositions that take into account policy effects or policy jumps (i.e., if P_1 permits, let P_2 makes decision, otherwise P_3 makes decision). They only model policies with two decision values, either “Permit” or “Deny”. It is not clear the scope or expressive power of their languages since they do not have any notion of completeness. They do not provide an actual methodology or an implementation for generating the integrated policies. Neither they related their languages to any language used in practice.

In this paper we propose a framework for the integration of access control policies that addresses the above shortcomings. The overall organization of our integration framework is outlined in Figure 1. The core of our framework is the *Fine-grained Integration Algebra* (FIA). Given a set of input policies $P_1, P_2, \cdots,$ P_n, one is able to specify the integration requirements for these input policies through a FIA expression, denoted as $f(P_1, P_2, \cdots, P_n)$ in Figure 1. The FIA expression is then processed by the other components of the framework in order to generate the integrated policy. We demonstrate the effectiveness of our framework through an implementation that supports the integration of XACML policies. We choose XACML because of its widespread adoption and its many features, such as attribute-based access control and 3-valued policy evaluation. We use Multi-Terminal Binary Decision Diagrams (MTBDD) [9] for representing policies and generating the integrated policies in XACML syntax. The novel contributions of this paper can be summarized as follows:

- We propose a fine-grained integration algebra for language independent 3-valued policies. This is the first time that such an algebra has been proposed.
- We introduce a notion of completeness and prove that our algebra is minimal and complete with respect to this notion.
- We propose a framework that uses the algebra for the fine-grained integration of policies expressed in XACML. The method automatically generates XACML policies as the policy integration result.

The rest of the paper is organized as follows. Section 2 introduces background information and preliminary definitions concerning XACML. Section 3 presents our fine-grained integration algebra. Section 4 discusses the expressivity of the algebra. Section 5 presents the detailed algorithms for generating well-formed integrated XACML policies. Section 6 reviews related works on policy integration. Finally, Section 7 concludes the paper.

2 Preliminaries

2.1 An Overview of XACML

XACML [1] is the OASIS standard language for the specification of access control policies. XACML policies include three main components: a *Target*, a *Rule* set and a *Rule combining algorithm*. The *Target* identifies the set of requests that the policy applies to. Each *Rule* consists of *Target*, *Condition* and *Effect* elements. The rule *Target* has the same structure as the policy *Target*. It specifies the set of requests that the rule applies to. The *Condition* element may further refine the applicability established by the target. The *Effect* element specifies whether the requested actions should be allowed ("Permit") or denied ("Deny"). The restrictions specified by the target and condition elements support the notion of attribute-based access control under which access control policies are expressed as conditions against the properties of subjects and protected objects. If a request satisfies both the rule target and rule condition predicates, the rule is applicable to the request and will yield a decision as specified by the *Effect* element. Otherwise, the rule is "NotApplicable" and the effect will be ignored. The *Rule combining algorithm* is used to resolve conflicts among applicable rules with different effects.

We now introduce an example of XACML policies that will be used throughout the paper.

Example 1 Consider a company with two departments D_1 and D_2. Each department has its own access control policies for the data under its control. Assume that P_1 and P_2 are the access control policies of D_1 and D_2, respectively, regulating access to the company's customer information. P_1 contains two rules, $P_1.Rul_{11}$ and $P_1.Rul_{12}$. $P_1.Rul_{11}$ states that the manager is allowed to read and update the customer information in the time interval [8am, 6pm]. $P_1.Rul_{12}$ states that any other staff is not allowed to read the customer information. P_2 also contains two rules, $P_2.Rul_{21}$ and $P_2.Rul_{22}$. $P_2.Rul_{21}$ states that the manager and staff can read the customer information in the time interval [8am, 8pm], and $P_2.Rul_{22}$ states that the staff cannot update the customer information. The two policies in XACML syntax can be found in Appendix. For simplicity, we adopt the following succinct representation in most discussion, where "role", "act" and "time" are attributes representing information on role, action and time, respectively.

$P_1.Rul_{11}$: role=manager, act=read or update, time= [8am, 6pm], effect= Permit.

$P_1.Rul_{12}$: role=staff, act=read, effect = Deny.

$P_2.Rul_{21}$: role=manager or staff, act=read, time = [8am, 8pm], effect = Permit.

$P_2.Rul_{22}$: role=staff, act=update, effect = Deny.

2.2 Policy Semantics

Before we introduce our algebra we need to find a suitable definition of policy semantics. Even though different approaches to the definition of such semantics are possible [4, 15], we propose a simple yet powerful notion of semantics according to which the semantics of a policy is defined by the set of requests that are permitted by the policy and the set of requests that are denied by the policy. This simple notion will provide us with a precise characterization of the meaning of policy integration in terms of the sets of permitted and denied requests.

In our work, we assume the existence of a finite set A of names. Each attribute, characterizing a subject or an object or the environment, has a name a in A, and a domain, denoted by $dom(a)$, of possible values. The following two definitions introduce the notion of access request (request, for short) and policy semantics.

Definition 1 *Let a_1, a_2, ..., a_k be attribute names, and let $v_i \in dom(a_i)$ $(1 \leq i \leq k)$. $r \equiv \{(a_1, v_1)$, $(a_2$, $v_2)$, $\cdots$, $(a_k$, $v_k)\}$ is a request.*

Example 2 Consider policy P_1 from Example 1. An example of request to which this policy applies is that of a manager wishing to read customer information at 10am. According to Definition 1, such request can be expressed as $r \equiv \{$*(role, manager), (act, read), (time, 10am)*$\}$.

Definition 2 *Let P be a 3-valued access control policy. We define the semantics of P as a 2-tuple $\langle R_Y^P, R_N^P\rangle$, where R_Y^P and R_N^P is the set of requests that are permitted and denied by P respectively, and $R_Y^P \bigcap R_N^P = \emptyset$.*

Note that a policy P is not applicable to requests not in $R_Y^P \cup R_N^P$. P can be viewed as a function mapping each request to a value in $\{Y, N, NA\}$. Also, our approach to formulating the policy semantics is independent of the language in which access control policies are expressed. Therefore, our approach can be applied to languages other than XACML.

3 A Fine-grained Integration Algebra

The Fine-grained Integration Algebra (FIA) is given by $\langle \Sigma, \mathsf{P_Y}, \mathsf{P_N}, +, \&, \neg, \Pi_{dc}\rangle$, where Σ is a vocabulary of attribute names and their domains, $\mathsf{P_Y}$ and $\mathsf{P_N}$ are two policy constants, $+$ and $\&$ are two binary operators, and $\neg$ and Π_{dc} are two unary operators.

3.1 Policy Constants and Operators in FIA

We now describe the policy constants and operators in FIA. In what follows, $P_1 \equiv \langle R_Y^{P_1}, R_N^{P_1}\rangle$ and $P_2 \equiv \langle R_Y^{P_2}, R_N^{P_2}\rangle$ denote two policies to be combined, and $P_I \equiv \langle R_Y^{P_I}, R_N^{P_I}\rangle$ denotes the policy obtained from the combination. Operators on policies are described as set operations.

Permit policy $(\mathsf{P_Y})$. $\mathsf{P_Y}$ is a policy constant that permits everything.

Deny policy $(\mathsf{P_N})$. $\mathsf{P_N}$ is a policy constant that denies everything.

Addition $(+)$. Addition of policies P_1 and P_2 results in a combined policy P_I in which requests that are permitted by either P_1 or P_2 are permitted, requests that are denied by one policy and is not permitted by the other are denied. More precisely:

$$P_I = P_1 + P_2 \iff R_Y^{P_I} = R_Y^{P_1} \cup R_Y^{P_2} \quad \wedge \quad R_N^{P_I} = (R_N^{P_1} \backslash R_Y^{P_2}) \cup (R_N^{P_2} \backslash R_Y^{P_1})$$

A binary operator can be viewed as a function that maps a pair of values $\{Y, N, NA\}$ to one value. We give this view of addition, intersection, and two other derived binary operators to be introduced later in Table 1. A binary operator is represented using a matrix that illustrates the effect of integration for a given request r. The first column of each matrix denotes the effect of P_1 with respect to r and the first row denotes the effect of P_2 with respect to r.

$P_1 + P_2$

P_1 \ P_2	Y	N	NA
Y	Y	Y	Y
N	Y	N	N
NA	Y	N	NA

P_1 & P_2

P_1 \ P_2	Y	N	NA
Y	Y	NA	NA
N	NA	N	NA
NA	NA	NA	NA

$P_1 - P_2$

P_1 \ P_2	Y	N	NA
Y	NA	NA	Y
N	NA	NA	N
NA	NA	NA	NA

$P_1 \rhd P_2$

P_1 \ P_2	Y	N	NA
Y	Y	Y	Y
N	N	N	N
NA	Y	N	NA

Table 1: Policy combination matrix of operator $+$, $\&$, $-$, $\rhd$

One partial order on the set $\{Y, N, NA\}$ is the *information order*: $Y > NA, N > NA$, as both Y and N provide more information about a request than NA. The $+$ operator can be viewed as taking maximum on the strict order $Y > N > NA$, which can be obtaining by using the information order and prefering Y to N.

Intersection ($\&$) . Given two policies P_1 and P_2, the intersection operator returns a policy P_I which is applicable to all requests having the same decisions from P_1 and P_2. More precisely,

$$P_I = P_1 \,\&\, P_2 \iff R_Y^{P_I} = R_Y^{P_1} \cap R_Y^{P_2} \quad \wedge \quad R_N^{P_I} = R_N^{P_1} \cap R_N^{P_2}$$

The intersection operator can be viewed as taking minimum on the information order. The integrated policy makes a decision only when the two policy agrees.

Negation ($\neg$) . Given a policy P, $\neg P$ returns a policy P_I, which permits (denies) all requests denied (permitted) by P. The negation operator does not affect those requests that are not applicable to the policy. More precisely:

$$P_I = \neg P \iff R_Y^{P_I} = R_N^{P} \quad \wedge \quad R_N^{P_I} = R_Y^{P}$$

Domain projection (Π_{dc}) The domain projection operator takes a parameter, the domain constraint dc, and restricts the policy only to the set of requests identified by dc.

Definition 3 *A domain constraint dc takes the form $\{(a_1, range_1),\ (a_2,\ range_2),\ \cdots,\ (a_k,\ range_k)\}$*[1]*, where a_1, a_2, ..., a_k are attribute names, and $range_i (1 \leq i \leq k)$ are sets of values from the vocabulary Σ. Given a request $r = \{(a_{r_1}, v_{r_1}), \cdots, (a_{r_m}, v_{r_m})\}$. We say r satisfies dc if the following condition holds: for each $(a_{r_j}, v_{r_j}) \in r$ $(1 \leq j \leq m)$, if there exists $(a_{r_j}, range_i) \in dc$, then $v_{r_j} \in range_i$.*

The semantics of $\Pi_{dc}(P)$ is given by

$$P_I = \Pi_{dc}(P) \iff R_Y^{P_I} = \{r | r \in R_Y^{P} \ and \ r \ satisfies \ dc\}, R_N^{P_I} = \{r | r \in R_N^{P} \ and \ r \ satisfies \ dc\}$$

3.2 FIA expressions

The integration of policies may involve multiple operators, and hence we introduce the concept of FIA expressions.

Definition 4 *A FIA expression is recursively defined as follows:*

- *If P is policy, then P is a FIA expression.*
- *If f_1 and f_2 are FIA expressions so are $(f_1) + (f_2)$, $(f_1) \,\&\, (f_2)$, and $\neg(f_1)$.*
- *If f is a FIA expression and dc is a domain constraint then $\Pi_{dc}(f)$ is a FIA expression.*

In FIA expressions, the binary operators are viewed as left associative and unary operators are right associative. The precedence are $\neg$ and Π_{dc} together have the highest precedence, following by $\&$, and then by $+$. For example, $P_1 + \Pi_{dc}P_2 + \neg P_3 \,\&\, P_4$ is interpreted as $((P_1 + (\Pi_{dc}P_2)) + ((\neg P_3) \,\&\, P_4)$.

Theorem 1 *FIA has the following algebraic properties.*

- ***Commutativity:*** $P_1 + P_2 = P_2 + P_1$; $\quad P_1 \,\&\, P_2 = P_2 \,\&\, P_1$;
- ***Associativity:*** $(P_1 + P_2) + P_3 = P_1 + (P_2 + P_3)$; $\quad (P_1 \,\&\, P_2) \,\&\, P_3 = P_1 \,\&\, (P_2 \,\&\, P_3)$;
- ***Adsorption:*** $P_1 + (P_1 \,\&\, P_2) = P_1$; $\quad P_1 \,\&\, (P_1 + P_2) = P_1$;
- ***Distributivity:*** $P_1 + (P_2 \,\&\, P_3) = (P_1 + P_2) \,\&\, (P_1 + P_3)$; $\quad P_1 \,\&\, (P_2 + P_3) = (P_1 \,\&\, P_2) + (P_1 \,\&\, P_3)$;
  $\Pi_{dc}(P_1 + P_2) = (\Pi_{dc}P_1) + (\Pi_{dc}P_2)$; $\quad \Pi_{dc}(P_1 \,\&\, P_2) = (\Pi_{dc}P_1) \,\&\, (\Pi_{dc}P_2)$

[1]In case of an ordered domain, these sets can be represented by ranges.

- ***Complements:*** $\mathsf{P_Y} = \neg \mathsf{P_N}$; $\quad \mathsf{P_N} = \neg \mathsf{P_Y}$;
- ***Idempotence:*** $P_1 + P_1 = P_1$; $\quad P_1 \,\&\, P_1 = P_1$;
- ***Boundedness:*** $P_1 + \mathsf{P_Y} = \mathsf{P_Y}$;
- ***Involution:*** $\neg(\neg P_1) = P_1$.

3.3 Derived Operators

In this section, we introduce some commonly used operators. They are defined using the core operators.

Not-applicable policy ($\mathsf{P_{NA}}$) . $\mathsf{P_{NA}}$ is a policy constant that is not applicable for every request. It is defined as $\mathsf{P_{NA}} = \mathsf{P_Y} \,\&\, \mathsf{P_N}$.

Effect projection (Π_Y **and** Π_N) . $\Pi_Y(P)$ restricts the policy P to the requests allowed by it. It is defined as: $\Pi_Y(P) = P \,\&\, \mathsf{P_Y}$. Similarly, $\Pi_N(P)$ restricts the policy P to the requests denied by it; it is defined as $\Pi_N(P) = P \,\&\, \mathsf{P_N}$. We are overloading Π to denote both effect projection and domain projection; the meaning should be clear from the subscript.

Subtraction ($-$) . Given two policies P_1 and P_2, the subtraction operator returns a policy P_I which is obtained by starting from P_1 and limiting the requests that the integrated policy applies only to those that P_2 does not apply to. The subtraction operator is defined as:

$$P_1 - P_2 = (\mathsf{P_Y} \,\&\, (\neg(\neg P_1 + P_2 + \neg P_2))) + (\mathsf{P_N} \,\&\, (P_1 + P_2 + \neg P_2))\,.$$

To see why this is correct, observe that $\neg P_1 + P_2 + \neg P_2$ will deny a request if and only if P_1 allows it and P_2 gives NA for it. Thus $\mathsf{P_Y} \,\&\, (\neg(\neg P_1 + P_2 + \neg P_2))$ allows a request if and only if P_1 allows it and P_2 gives NA it, and is not applicable for all other requests. Similarly, $\mathsf{P_N} \,\&\, (P_1 + P_2 + \neg P_2)$ denies a request if and only if P_1 denies it and P_2 gives NA for it.

Precedence ($\triangleright$) . Given two policies P_1 and P_2, the precedence operator returns a policy P_I which yields the same decision as P_1 for any request applicable to P_1, and yields the same decisions as P_2 for the remaining requests. The precedence operator can be expressed as $P_1 + (P_2 - P_1)$. By limiting P_2 to requests that P_1 does not decide, this operator can be used as a building block for resolving possible conflicts between two policies.

4 Expressiveness of FIA

In this section, we first show that our operators can express the standard policy-combining algorithms defined for XACML policies as well as other more complex policy integration scenario. We then show that the operators in FIA are complete in that any possible policy integration requirements can be expressed using a FIA expression.

4.1 Expressing XACML Policy-Combining Algorithms in FIA

In XACML there are four standard policy-combining algorithms as follows:

Permit-overrides : The combined result is “Permit” if any policy evaluates to “Permit”, regardless of the evaluation result of the other policies. If no policy evaluates to “Permit” and at least one policy evaluates to “Deny”, the combined result is “Deny”. The combination of policies P_1, P_2,..., P_n under this policy-combining algorithm can be expressed as $P_1 + P_2 + \cdots + P_n$.

Deny-overrides : The combined result is “Deny” if any policy is encountered that evaluates to “Deny”. The combined result is “Permit” if no policy evaluates to “Deny” and at least one policy evaluates to “Permit”.

Deny-overrides is the opposite of permit-overrides. By using the combination of the negation and addition operator, we can express deny-overrides as $\neg((\neg P_1) + (\neg P_2) + \cdots + (\neg P_n))$.

First-one-applicable : The combined result is the same as the result of the first applicable policy. This combining algorithm can be expressed by using the precedence operator. Given policies P_1, P_2, ..., P_n, the expression is $P_1 \rhd P_2 \rhd \cdots \rhd P_n$.

Only-one-applicable : The combined result corresponds to the result of the unique policy in the policy set which applies to the request. Specifically, if no policy or more than one policies are applicable to the request, the result of policy combination should be "NotApplicable"; if only one policy is considered applicable, the result should be the result of evaluating the policy.

When combining policies $P_1, \cdots, P_n$ under this policy-combining algorithm, we need to remove from each policy the requests applicable to all the other policies and then combine the results using the addition operator. The final expression is : $(P_1 - P_2 - P_3 - \cdots - P_n) + (P_2 - P_1 - P_3 - \cdots - P_n) + \cdots + (P_n - P_1 - P_2 - \cdots - P_{n-1})$.

4.2 Expressing Complex Policy Integration Requirements in FIA

Our algebra supports not only the aforementioned policy-combining algorithms, but also other types of policy combining requirements, like rule constraints. A rule constraint specifies decisions for a set of requests. It may require that the integrated policy has to permit a critical request. Such an integration requirement can be represented as a new policy. Let P be a policy, and c be the policy specifying an integration constraint. We can combine c and P by using the first-one-applicable combining-algorithm. The corresponding expression is $c \rhd P$. Another frequently used operator is to find the portion of a policy P_1 that differs compared to a policy P_2, which can be expressed as: $P_1 \,\&\, (\neg P_2)$.

By using the two policy constants, we can easily modify a policy P as an *open policy* or a *closed policy*. An open policy of P allows everything that is not explicitly denied, which can be represented as $P \rhd \mathsf{P_Y}$. A closed policy of P denies everything that is not explicitly permitted, which can be represented as $P \rhd \mathsf{P_N}$.

Our algebra can also express the policy jump, a feature in the iptables firewall languages. The specific requirement is that if a request is permitted by policy P_1, then the final decision on this request is given by policy P_2; otherwise, the final decision is given by policy P_3. This can be expressed using

$$\Pi_Y(P_1 \,\&\, P_2) + \Pi_N(\neg P_1 \,\&\, P_2)) + \Pi_Y(\neg P_1 \,\&\, P_3) + \Pi_N(\neg P_1 \,\&\, \neg P_3))$$

Among the four sub-expressions, the first one gives Y when both P_1 and P_2 do so, and gives NA in all other cases. Similarly, the second sub-expression gives N when P_1 gives Y and P_2 gives N, and gives NA otherwise. The third and fourth subexpressions deal with the case that P_1 answers N.

Next, we elaborate the example mentioned in the introduction where the combination requirements are given for parts of a policy.

Example 3 Consider the policies introduced in Example 1. Assume that the policies must be integrated according to the following combination requirement: for users whose role is manager, the access has to be granted according to policy P_1; for users whose role is a staff, the access has to be granted according to policy P_2.

The resultant policy will consist of two parts. One part is obtained from P_1 by restricting the policy to only deal with managers. Such extraction can be expressed in our algebra as $\Pi_{dc_1}(P_1)$ where $dc_1 = \{(role, manager), (act, \{read,update\}), (time, [8am, 8pm])\}$. The other part is obtained from P_2 by restricting the policy to only deal with staff. Correspondingly, we can use the expression: $\Pi_{dc_2}(P_2)$ with $dc_2 = \{(role, staff), (act, \{read,update\}), (time, [8am, 8pm])\}$. Finally, we have the following expression representing the integrated policy : $\Pi_{dc_1}(P_1) + \Pi_{dc_2}(P_2)$. The integrated policy P_I is thus: $P_I.Rul_{I1}$: role=manager, act=read or update, time=[8am, 6pm], effect=Permit.

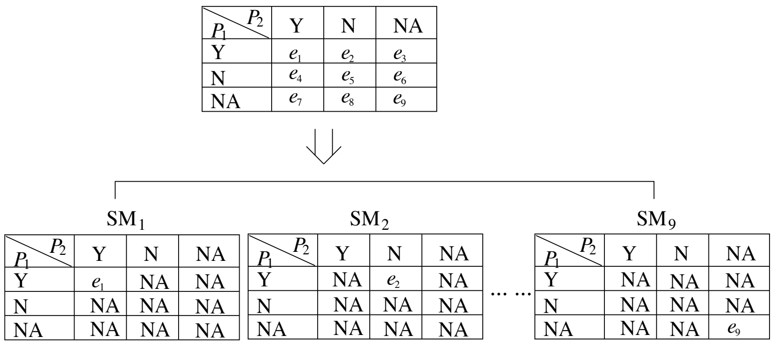
$P_I.Rul_{I2}$: role=staff, act=read, time=[8am, 8pm], effect=Permit.
$P_I.Rul_{I3}$: role=staff, act=update, effect=Deny.

4.3 Completeness

While we have shown that many policy integration scenarios can be handled by the operators in the algebra, our list of examples is certainly not exhaustive. A question of both theoretical and practical importance is whether FIA can be combined to express all possible ways of integrating policies, that is, whether FIA is *complete*. Addressing this question requires choosing a suitable notion of completeness. There are different degrees of completeness, and we show that FIA is complete in the strongest sense. First, while Table 1 gave the *policy combination matrices* for the four binary operators, many other matrices are possible, and each such matrix can be viewed as a binary operator for combining two policies. As there are three possibilities for each cell in a matrix, namely, Y, N, and NA, and there are nine cells, the total number of matrices is $3^9 = 19683$. We show that each such matrix can be expressed using $\langle \mathsf{P_N}, \mathsf{P_Y}, +, \&, \neg \rangle$. Second, when n ($n > 2$) policies are combined, policy combination can be expressed using a n-dimensional matrix. We also show that each such n-dimensional matrix can be expressed using $\langle \mathsf{P_N}, \mathsf{P_Y}, +, \&, \neg \rangle$. Finally, a fine-grained integration may use different policy combination matrices for different requests. We show that this can be handled by using the operator Π_{dc} in addition to $\langle \mathsf{P_N}, \mathsf{P_Y}, +, \&, \neg \rangle$.

Theorem 2 *(Binary completeness). Given any policy combination matrix M, let $M(P_1, P_2)$ denote the result of combining two policies P_1 and P_2 using M. There exists a FIA expression $f_I(P_1, P_2)$ that is equivalent to $M(P_1, P_2)$. That is, $f_I(P_1, P_2) = M(P_1, P_2)$ for any two policies P_1 and P_2.*

Proof. When all entries in M are NA, $f_I(P_1, P_2) = \mathsf{P_Y} \,\&\, \mathsf{P_N}$. For the case that M has at least one entry that is not NA, we use the divide-and-conquer methodology. We transform the problem of finding an expression for M into finding sub-expressions for each entry in M. We number the cells in M from 1 to 9, by starting from the top-left cell and going right first. We then consider nine *simple matrices* SM_1, SM_2, ..., SM_9 (see Figure 2). In each simple matrix, at most one entry is not NA.



P_1 \ P_2	Y	N	NA
Y	e_1	e_2	e_3
N	e_4	e_5	e_6
NA	e_7	e_8	e_9

SM$_1$

P_1 \ P_2	Y	N	NA
Y	e_1	NA	NA
N	NA	NA	NA
NA	NA	NA	NA

SM$_2$

P_1 \ P_2	Y	N	NA
Y	NA	e_2	NA
N	NA	NA	NA
NA	NA	NA	NA

... ...

SM$_9$

P_1 \ P_2	Y	N	NA
Y	NA	NA	NA
N	NA	NA	NA
NA	NA	NA	e_9

Figure 2: Policy combination matrix transformation

If no cell in M is NA, then let $f_i(P_1, P_2)$ denote the FIA expression corresponding to the nine simple matrices. $f_I(P_1, P_2)$ is thus the addition of the expression corresponding to each simple matrix, that is: $f_I(P_1, P_2) = f_1(P_1, P_2) + f_2(P_1, P_2) + ... + f_9(P_1, P_2)$. If M contains NA somewhere in the nine cells, then we only need to add up the expressions corresponding to the cells that are not NA. For example, if cells 1 and 4 are not NA, then $f_I(P_1, P_2) = f_1(P_1, P_2) + f_4(P_1, P_2)$.

Now our task is to find proper expressions for each $f_i(P_1, P_2)$. Let $f'_i(P_1, P_2)$ denote the expression that is equivalent to a policy matrix where the i'th cell is Y, and all other cells are either N or NA. Then when $e_i = Y$, we have $f_i(P_1, P_2) = f'_i(P_1, P_2) \,\&\, \mathsf{P_Y}$ and when $e_i = N$, we have $f_i(P_1, P_2) = \neg f'_i(P_1, P_2) \,\&\, \mathsf{P_N}$. Expressions $f'_i(P_1, P_2), 1 \le i \le 9$ are shown in Table 2.

To summarize, FIA can express any policy combination matrix. ∎

Theorem 2 has proved the completeness when considering two policies. We now extend our completeness result to any number of policies. The proof of Theorem 3 is shown in appendix.

Theorem 3 *(General completeness) Given n ($n > 2$) policies P_1, P_2, ..., P_n, let $M^*(P_1, P_2, \dots, P_n)$ be a n-dimensional policy combination matrix which denotes the combination result of the n policies. There exists a FIA expression $f_I(P_1, P_2, ..., P_n)$ that is equivalent to $M^*(P_1, P_2, ..., P_n)$.*

So far, we have proved the completeness in the scenario when there is one n-dimensional combination matrix for all requests. In the following theorem, we further consider the fine-grained integration when there are multiple combination matrices each of which is corresponding to a subset of the requests.

Definition 5 *Let $\mathcal{R}_\Sigma$ denote the set of all requests in the vocabulary Σ. A* fine-grained integration specification *is given by $[(R_1, M_1^*), (R_2, M_2^*), \cdots, (R_k, M_k^*)]$, where $R_1, R_2, \cdots, R_k$ form a partition of $\mathcal{R}_\Sigma$, i.e., $\mathcal{R}_\Sigma = R_1 \cup R_2 \cup ... \cup R_k$ ($k \geq 1$) and $R_i \cap R_j = \emptyset$ when $i \neq j$, and each $M_i^*(P_1, .., P_n)$ ($1 \leq i \leq k$) is a n-dimensional policy combination matrix. This specification asks requests in each set R_i to be integrated according to the matrix M_i^*.*

Theorem 4 *Given a fine-grained integration specification $[(R_1, M_1^*), (R_2, M_2^*), \cdots, (R_k, M_k^*)]$, if for each R_i, there exists $dc_{i,1}, \cdots, dc_{i,m_i}$ such that $R_i = R(dc_{i,1}) \cup \cdots \cup R(dc_{i,m_i})$(where $R(dc_{i,j})$ denotes the set of requests satisfying $dc_{i,j}$), then there exists a FIA expression $f_I(P_1, P_2, ..., P_n)$ that achieves the integration requirement.*

Proof. We first use the domain projection operator Π_{dc} to project each policy according to $dc_{1,1}, \cdots, dc_{k,m_k}$. For requests in each $R(dc_{i,j})$, there is one fixed M_i^*. By Theorem 3, there is a FIA expression (denoted as $f_{i,j}$) for integrating policies $\Pi_{dc_{i,j}}(P_1)$, ..., $\Pi_{dc_{i,j}}(P_n)$ according to M_i^*. Finally, f_I is the addition of all $f_{i,j}$'s. ∎

We note that the above theorem requires that each R_i in the partition to be expressible in finite number of domain constraints. This condition is always satisfied when the set of all requests is finite. However, when the set if infinite, it may not hold. For example, if one requires all requests that satisfy a predicate uses one combination matrix and all other requests use another matrix, but the predicate is uncomputable, then we cannot generate a FIA expression to achieve this. However, we believe that requirements that are likely to occur in practice will satisfy the condition that each R_i is expressible in finite number of domain constraints.

4.4 Minimal Set of Operators

Recall that FIA has $\{\mathsf{P_Y}, \mathsf{P_N}, +, \&, \neg, \Pi_{dc}\}$. The operator Π_{dc} is needed to deal with fine-grained integration. Operators $\{\mathsf{P_Y}, \mathsf{P_N}, +, \&, \neg\}$ are complete in the sense that any policy combination matrix can be expressed using them. A natural question is among the set $\Theta = \{\mathsf{P_N}, \mathsf{P_Y}, \mathsf{P_{NA}}, +, \&, \neg, \Pi_Y, \Pi_N, -, \triangleright\}$,

P_1 \ P_2	Y	N	NA
Y	$f'_1 = P_1 \,\&\, P_2$	$f'_2 = P_1 \,\&\, (\neg P_2)$	$f'_3 = \neg(\neg P_1 + P_2 + (\neg P_2))$
N	$f'_4 = (\neg P_1) \,\&\, P_2$	$f'_5 = (\neg P_1) \,\&\, (\neg P_2)$	$f'_6 = \neg(P_1 + P_2 + (\neg P_2))$
NA	$f'_7 = \neg(\neg P_2 + P_1 + (\neg P_1))$	$f'_8 = \neg(P_2 + P_1 + (\neg P_1))$	$f'_9 = \mathsf{P_Y} - (P_1 + P_2)$

Table 2: Expressions for $f'_i(P_1, P_2)$

what subsets are *minimally complete*. We say a subset of Θ is minimally complete, if operators in the subset are sufficient for defining all other operators in Θ, and any smaller subset cannot define all operators in Θ. The following theorem answers this question. The only redundancy in $\{\mathsf{P_Y}, \mathsf{P_N}, +, \&, \neg\}$ is that only one of $\mathsf{P_Y}$ and $\mathsf{P_N}$ is needed.

Theorem 5 *Among the 10 operators in Θ, there are 12 minimally complete subsets. They are the 12 elements in the cartesian product* $\{\neg\} \times \{\mathsf{P_Y}, \mathsf{P_N}\} \times \{\Pi_Y, \Pi_N, \&\} \times \{+, \triangleright\}$.

Details of the proof are in the appendix. Here, we summarize the key lemmas that lead to the result:

- The policy constant $\mathsf{P_N}$ cannot be expressed using $\Theta \setminus \{\mathsf{P_Y}, \mathsf{P_N}\}$. When given $\neg$, $\mathsf{P_Y}$ and $\mathsf{P_N}$ can be derived from each other.
- The unary operator $\neg$ cannot be expressed using $\Theta \setminus \{\neg\}$.
- The binary operator $\&$ cannot be expressed using $\Theta \setminus \{\&, \Pi_Y, \Pi_N\}$. Given $\neg$, Π_Y and Π_N can be expressed from each other. Π_Y can be expressed using $\{\&, \mathsf{P_Y}\}$. And $\&$ can be expressed using $\{\mathsf{P_N}, +, \neg, \Pi_N\}$.
- The binary operator $+$ cannot be expressed using $\Theta \setminus \{+, \triangleright\}$. However, $+$ can be expressed using $\{\mathsf{P_N}, \neg, \&, \triangleright\}$ or $\{\mathsf{P_N}, \neg, \Pi_N, \triangleright\}$, and $\triangleright$ can be expressed using $\{\mathsf{P_N}, +, \neg, \&\}$.

In summary, among the 10 operators in Θ, for completeness, we must have $\neg$, one in $\{\mathsf{P_Y}, \mathsf{P_Y}\}$, one in $\{\Pi_Y, \Pi_N, \&\}$, and one in $\{+, \triangleright\}$. There are 12 combinations. It is not difficult to verify that every such combination is in fact complete. For example, once we have $\{\neg, \mathsf{P_N}, \Pi_N\}$, adding $+$ allows us to derive $\&$, and then derive $\triangleright$, adding $\triangleright$ allows us to derive $+$ and then $\&$. There are thus 12 minimally complete subsets in Θ.

5 Integrated Policy Generation

In this section, we present an approach to automatically generate the integrated policy given the FIA policy expression. Internally, we represent each policy as a Multi-Terminal Binary Decision Diagram (MTBDD) [9], and then perform operations on the underlying MTBDD structures to generate the integrated policy. We have chosen a MTBDD based implementation of the proposed algebra because (i) MTBDDs have proven to be a simple and efficient representation for XACML policies [8] and (ii) operators in FIA can be mapped to efficient operations on the underlying policy MTBDDs. Our approach consists of three main phases:

1. **Policy representation**: For each policy P_i in the FIA expression $f(P_1, P_2, ..., P_n)$, we construct a policy MTBDD, T^{P_i}.
2. **Construction of the integrated policy MTBDD**: We combine the individual policy MTBDD structures according to the operations in the FIA expression to construct the *integrated policy MTBDD*.
3. **Policy generation**: The *integrated policy MTBDD* is then used to generate the actual integrated XACML policy.

5.1 Policy Representation

Recall from section 2 that we characterize a policy P as a 2-*tuple* $\langle R_Y^P, R_N^P \rangle$, where R_Y^P is the set of requests permitted by the policy, and R_N^P is the set of requests denied by the policy. Alternatively, we can define P as a function $P : R \to E$ from the domain of requests R onto the domain of effects E, where $E = \{Y, N, NA\}$.

An XACML policy can be transformed into a compound Boolean expression over request attributes [2]. A compound Boolean expression is composed of atomic Boolean expressions (AE) combined using the logical operations $\vee$ and $\wedge$. Atomic Boolean expressions that appear in most policies belong to one of the following two categories: (i) one-variable equality constraints, $a \rhd c$, where a is an attribute name, c is a constant, and $\rhd \in \{=, \neq\}$; (ii) one-variable inequality constraints, $c_1 \lhd a \rhd c_2$, where a is an attribute name, c_1 and c_2 are constants, and $\lhd, \rhd \in \{<, \leq, >, \geq\}$.

Example 4 Policy P_1 from Example 1 can be defined as a function :

$$P_1(r) = \begin{cases} Y, & \textit{if}\, role = manager \wedge (act = read \vee act = update) \wedge 8am \leq time \leq 6pm \\ N, & \textit{if}\, role = staff \wedge act = read \end{cases}$$

where r is a request of the form $\{(role, v_1), (act, v_2), (time, v_3)\}$.

We now encode each *unique* atomic Boolean expression AE_i in a policy into a Boolean variable x_i such that: $x_i = 0$ if AE_i is `false`; $x_i = 1$ if AE_i is `true`. To determine *unique* atomic Boolean expressions we use the following definition.

Definition 6 *Two atomic Boolean expressions* $a_i \rhd_i c_i$ *and* $a_j \rhd_j c_j$ *are* equal *iff* $a_i = a_j \wedge \rhd_i = \rhd_j \wedge c_i = c_j$. *Two atomic Boolean expressions* $c_{i1} \lhd_i a_i \rhd_i c_{i2}$ *and* $c_{j1} \lhd_j a_j \rhd_j c_{j2}$ *are* equal *iff* $a_i = a_j \wedge \lhd_i = \lhd_j \wedge \rhd_i = \rhd_j \wedge c_{i1} = c_{j1} \wedge c_{i2} = c_{j2}$.

Example 5 The Boolean encoding for policy P_1 is given in Table 3.

x_0	x_1	x_2	x_3	x_4
$role = manager$	$role = staff$	$act = read$	$act = update$	$8am \leq time \leq 6pm$

Table 3: Boolean encoding for P_1

Using the above Boolean encoding, a policy P can be transformed into a function $P : B^n \mapsto E$, over a vector of Boolean variables, $\vec{x} = x_0, x_1, \cdots, x_n$, onto the finite set of effects $E = \{Y, N, NA\}$, where n is the number of unique atomic Boolean expressions in policy P. A request r corresponds to an assignment of the Boolean vector $\vec{x}$, which is derived by evaluating the atomic Boolean expressions with attribute values specified in the request.

Example 6 After Boolean encoding, the policy P_1 is transformed into the function :

$$P_1(\vec{x}) = \begin{cases} Y, & \textit{if}\, x_0 \wedge (x_2 \vee x_3) \wedge x_4 \\ N, & \textit{if}\, x_1 \wedge x_4 \end{cases}$$

The transformed policy function can now be represented as a MTBDD. A MTBDD provides a compact representation of functions of the form $f : \mathbb{B}^n \mapsto \mathbb{R}$, which maps bit vectors over a set of variables ($\mathbb{B}^n$) to a finite set of results ($\mathbb{R}$). The structure of a MTBDD is a rooted acyclic directed graph. The internal (or non-terminal) nodes represent Boolean variables and the terminals represent values in a finite set. Each non-terminal node has two edges labeled 0 and 1 respectively. Thus when a policy is represented using a MTBDD, the non-terminal nodes correspond to the unique atomic Boolean expressions and the terminal nodes correspond to the effects. Each path in the MTBDD represents an assignment for the Boolean variables along the path, thus representing a request r. The terminal on a path represents the effect of the policy for the request represented by that path. Note that different orderings on the variables may result in different MTBDD representations and hence different sizes of the corresponding MTBDD representation. Several approaches for determining the variable ordering that results in an optimally sized MTBDD can be found in

[10]. For examples discussed in this paper, we use the variable ordering $x_0 \prec x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$. The MTBDD of the policy P_1 is shown in Figure 3, where the dashed lines are 0-edges and solid lines are 1-edges.

Compound Boolean expression representing the policies to be integrated may have atomic Boolean expressions with matching attribute names but overlapping value ranges. In such cases, we need to transform the atomic Boolean expressions with overlapping value ranges into a sequences of new atomic Boolean expressions with disjoint value ranges, before performing the Boolean encoding. A generic procedure for computing the new atomic Boolean expression is described below.

Assume that the original value ranges of an attribute a are $[d_1^-, d_1^+]$, $[d_2^-, d_2^+]$, ..., $[d_n^-, d_n^+]$ (the superscript '-' and '+' denote lower and upper bound respectively). We sort the range bounds in an ascending order, and then employ a plane sweeping technique to obtain the disjoint ranges: $[d_1'^-, d_1'^+]$, $[d_2'^-, d_2'^+]$, ..., $[d_m'^-, d_m'^+]$, which satisfy the following three conditions: (i) $d_i'^-, d_i'^+ \in D$, $D = \{d_1^-, d_1^+, ..., d_n^-, d_n^+\}$; (ii) $\cup_{i=1}^m [d_i'^-, d_i'^+] = \cup_{j=1}^n [d_j^-, d_j^+]$; and (iii) $\cap_{i=1}^m [d_i'^-, d_i'^+] = \emptyset$.

Consider policy P_2 from Example 1. We can observe that the atomic Boolean expression $8am \leq time \leq 6pm$ in P_1 refers to the same attribute as in the atomic Boolean expression $8am \leq time \leq 8pm$ in P_2 and their value ranges overlap. In order to distinguish these two atomic Boolean expressions during the later policy integration, we split the value ranges and introduce the new atomic Boolean expression $6pm \leq time \leq 8pm$. The expression $8am \leq time \leq 8pm$ in P_2 is replaced with $(8am \leq time \leq 6pm \vee 6pm \leq time \leq 8pm)$. Boolean encoding is then performed for the two policies by considering unique atomic Boolean expressions across both policies.

Example 7 *By introducing another atomic Boolean expression* $6pm \leq time \leq 8pm$*, i.e.* x_5*, the transformed function for policy* P_2 *is :*

$$P_2(\vec{x}) = \begin{cases} Y, & \textit{if } (x_0 \vee x_1) \wedge x_2 \wedge (x_4 \vee x_5) \\ N, & \textit{if } x_1 \wedge x_3 \end{cases}$$

Using the same variable ordering $x_0 \prec x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$ *we construct the MTBDD for* P_2 *shown in Figure 3.*

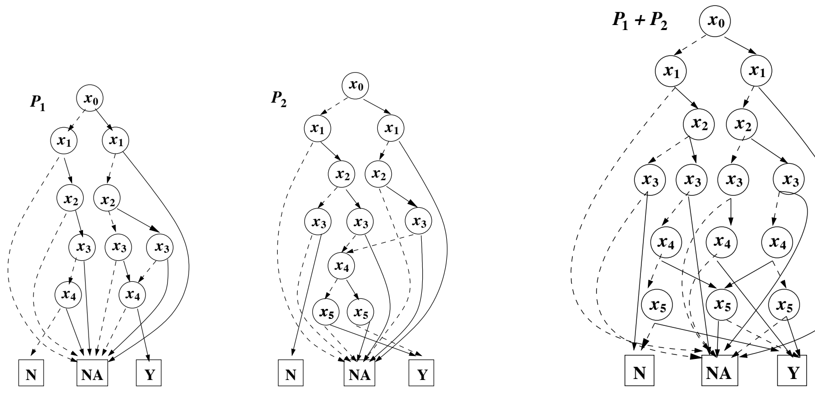


Figure 3: MTBDDs of P_1, P_2 and $P_1 + P_2$

5.2 Construction of Integrated Policy MTBDD

Given the FIA expression $f(P_1, P_2, ..., P_n)$ and the MTBDD representations $T^{P_1}, T^{P_2}, ..., T^{P_n}$ of the policies P_1,P_2,...,P_n respectively, we construct the integrated policy MTBDD T^{P_I}, by performing the operations (specified in f) on the individual policy MTBDDs.

Procedure Apply($Node_1, Node_2, OP$)
Input : $Node_1, Node_2$ are MTBDD nodes, OP is a policy operation

```
1.  initiate $Node_I$ // $Node_I$ is the combination result
2.  if $Node_1$ and $Node_2$ are terminals then
3.      $Node_I \leftarrow (Node_1\ OP\ Node_2, null, null)$
4.  else
5.      if $Node_1.var = Node_2.var$ then
6.          $Node_I.var \leftarrow Node_1.var$
7.          $Node_I.left \leftarrow$ Apply($Node_1.left, Node_2.left, OP$)
8.          $Node_I.right \leftarrow$ Apply($Node_1.right, Node_2.right, OP$)
9.      if $Node_1.var$ precedes $Node_2$.var then
10.         $Node_I.var \leftarrow Node_1.var$
11.         $Node_I.left \leftarrow$ Apply($Node_1.left, Node_2, OP$)
12.         $Node_I.right \leftarrow$ Apply($Node_1.right, Node_2, OP$)
13.     if $Node_2.var$ precedes $Node_1$.var then
14.         $Node_I.var \leftarrow Node_2.var$
15.         $Node_I.left \leftarrow$ Apply($Node_2.left, Node_1, OP$)
16.         $Node_I.right \leftarrow$ Apply($Node_2.right, Node_1, OP$)
17. return $Node_I$
```

Figure 4: Description of the `Apply` procedure

Operations on policies can be expressed as operations on the corresponding policy MTBDDs. The `negation` $\neg$ operation can be performed by interchanging the terminal Y and N. Many efficient operations have been defined and implemented for MTBDDs [9]. In particular, we use the `Apply` operation defined on MTBDDs to perform the FIA binary operations $\{+, -, \&, \triangleright\}$. We introduce a new MTBDD operation called `Projection` to perform the domain projection operation Π_{dc} defined in FIA.

The `Apply` operation combines two MTBDDs by a specified binary arithmetic operation. A high level description of the `Apply` operation is shown in Figure 4, where *var*, *left*, *right* refer to the variable, left child and right child of a MTBDD node, respectively. The `Apply` operation traverses each of the MTBDDs simultaneously starting from the root node. When the terminals of both MTBDDs are reached, the specified operation is applied on the terminals to obtain the terminal for the resulting combined MTBDD. A variable ordering needs to be specified for the `Apply` procedure.

Example 8 The integrated MTBDD T^{P_I} for the policy expression $f(P_1, P_2) = P_1 + P_2$ is obtained by using MTBDD operation `Apply(`$T^{P_1}.root$`,` $T^{P_2}.root$`, +)`, where "root" refers to the root node of the corresponding MTBDD. Figure 3 shows the integrated policy MTBDD. The same variable ordering $x_0 \prec x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$ has been used in the construction of the integrated policy MTBDD.

The procedure for performing the effect projection operation is the following. For Π_Y, those paths in T^P that lead to N are redirected to the terminal NA. Similarly, for Π_N, those paths in T^P that lead to Y are redirected to the terminal NA.

For the domain projection operation with domain constraint *dc*, we traverse the policy MTBDD from the top to the bottom and check the atomic Boolean expression associated with each node (denoted as $Node$). There are two cases. If the atomic Boolean expression of $Node$ contains an attribute specified in dc, we simply replace the attribute domain with the new domain given by dc. Otherwise, it means $Node$ represents an attribute no longer applicable to the resulting policy, and hence we should remove it. After removing $Node$, we need to adjust the pointer from its parent node by redirecting it to $Node$'s left child which leads to the path when N is not considered. After all nodes have been examined, those nodes that have no incoming edges are also removed. If the projection operation contains both types of constraints, we apply one constraint first and then apply the other by using previous two algorithms.

Thus, given any arbitrary FIA expression $f(P_1, P_2, ..., P_n)$, we can use a combination of the `Apply`, `not`, `Projection` MTBDD operations on the policy MTBDDs to generate the integrated policy MTBDD. An example is given below.

Example 9 Consider the FIA policy expression for the only-one-applicable policy combining algorithm together with the domain constraint $dc = \{(role, manager), (act, \{read, update\}, (time, [8am, 8pm])\}$. Here, $f(P_1, P_2) = \Pi_{dc}((P_1 - P_2) + (P_2 - P_1))$. The integrated MTBDD can be obtained by using the `Apply` and `Projection` operations as follows :

`Projection`(`Apply`(`Apply`($T^{P_1}.root, T^{P_2}.root, -$), `Apply`($T^{P_2}.root, T^{P_1}.root, -$), $+$), dc).

5.3 XACML Policy Generation

In the previous section, we have presented how to construct the integrated MTBDD given any policy expression f. Though such integrated MTBDD can be used to evaluate requests with respect to the integrated policy, they cannot be directly deployed in applications using the access control system based on XACML. Therefore, we develop an approach that can automatically transform MTBDDs into actual XACML policies. The policy generation consists of three steps :

1. Find the paths in the combined MTBDD that lead to the Y and N terminals, and represent each path as a Boolean expression over the Boolean variable of each node.
2. Map the above Boolean expressions to the Boolean expressions on actual policy attributes.
3. Translate the compound Boolean expression obtained in step 2 into a XACML policy.

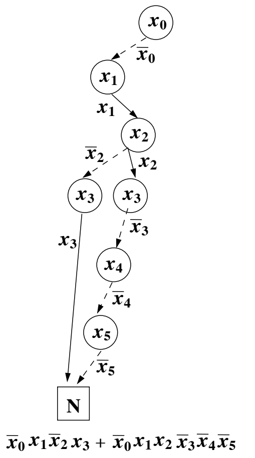
We first elaborate on step 1. In the MTBDD, each node has two edges, namely *0-edge* and *1-edge*. The 0-edge and 1-edge of a node labelled x_i correspond to *edge-expressions* $\bar{x}_i$ and x_i respectively. A path in the MTBDD corresponds to an expression which is the conjunction of *edge-expressions* of all edges along that path. We refer to this as a *path-expression*. Those paths leading to the same terminal correspond to the disjunction of *path-expressions*.

Next, we replace Boolean variables in the path-expressions with the corresponding atomic Boolean expressions by using the mapping built in the Boolean encoding phase. During the transformation in each path-expression, we need to remove some redundant information. For instance, the resulting expression may contain an attribute with both equality and inequality functions like $(role = manager) \wedge (role \neq staff)$. In that case, we only need to keep the equality function of the attribute.

The last step is to generate the actual XACML policy from the compound Boolean expression obtained in previous step. Specifically, for each path-expression whose evaluation is Y, a permit rule is generated; and for each path-expression whose evaluation is N, a deny rule is generated. Attributes that appear in conditions of the rules in original policies still appear in conditions of the newly generated rules, and attributes that appear in targets in the original policies still appear in targets in the integrated policy. Here we do not distinguish the policy target with rule target. Instead, all targets appear as rule targets.

Note that the number of rules generated for the integrated policy depends on the number of paths leading to $Y(N)$ which may be exponential in the number of nodes in the MTBDD. To address this issue, we propose to leverage existing BDD path-minimization techniques [6, 7] that minimize the number of paths leading to one-terminal in a BDD along with logic minimization techniques [5]. Our preliminary results by using Espresso [5] show a significant reduction (from 75% to 99%) in the number of rules generated.

Example 10 Consider policies P_1 and P_2 in Example 1. Figure 5 shows an example of $P_1 + P_2$. The left part of the figure shows the paths leading to the N terminal and the corresponding Boolean expressions which



PolicyId=P1+P2
    <RuleId=R1 Effect=Deny>
        <Target>
            <Subject role=staff>
            <Action act=update>
        </Target> </Rule>
    <RuleId=R2 Effect=Deny>
        <Target>
            <Subject role=staff>
            <Action act=read>
        </Target>
        <Condition time $\neq [8am, 6pm]$ AND time $\neq [6pm, 8pm]$>
    </Rule>
    <RuleId=R3 Effect=Permit> $\cdots$

Figure 5: The integrated XACML policy representing $P_1 + P_2$

can be transformed as follows: $(role = staff \wedge act = update) \vee (role = staff \wedge act = read \wedge time \neq [8am, 6pm] \wedge time \neq [6pm, 8pm])$

The right part of the figure shows the corresponding deny rules for the integrated policy.

6 Related Work

In the literature, many efforts have been devoted into policy composition [3, 4, 12, 14, 15]. Few approaches have been proposed for dealing with the fine-grained integration of XACML policies. Approaches most closest to ours are by Mazzoleni et al. [11], Bonatti et al. [4], Wijesekera et al. [15] and Backes et al. [3].

Mazzoleni et al. [11] have proposed an extension to the XACML language, called *policy integration preferences*, using which a party can specify the approach that must be adopted when its policies have to be integrated with policies by other parties. They do not discuss mechanisms to perform such integrations. Also, the integration preferences discussed in this work are very limited and do not support fine-grained integration requirements. Bonatti et al. [4] have proposed an algebra for combining authorization specifications that may originate from heterogenous independent parties. They model an access control policy as a set of ground (variable-free) authorization terms, where an authorization term is a triple of the form (subject, object, action). They propose an implementation of their algebra based on logic programming and partial evaluation techniques. Unlike our work which can handle 3-valued policies, their model does not explicitly support negative authorizations. Also, our implementation is based on representations used in model checking techniques which have been proven to be very efficient. In addition, we also provide mechanisms to synthesize a concrete integrated policy resulting from the evaluation of a policy expression. Wijesekera et al. [15] have proposed a propositional algebra for access control. They model policies as nondeterministic transformers of permission set assignments to subjects and interpret operations on policies as set-theoretic operations on the transformers. Their work does not discuss an implementation for the algebra. Backes et al. [3] have proposed an algebra for combining enterprise privacy policies. They define conjunction, disjunction and scoping operations on 3-valued EPAL [13] policies. In contrast to our work, the *don't care* value is treated as a special value that can only be used by the default rulings of a policy and they do not have an implementation of their algebra.

7 Conclusion

In this work we have proposed an algebra for the fine-grained integration of language independent policies. Our operations can not only express existing policy-combining algorithms but can also express any arbitrary combination of policies at a fine granularity of requests, effects and domains, as we have proved in the completeness theorem. Based on this algebra, we propose a framework for integration of XACML policies. We also discuss the generation of an actual XACML policy representing the integrated policy corresponding to a FIA policy expression.

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Appendix

Theorem 3 *Given n ($n \geq 2$) policies P_1, P_2, ..., P_n, let $M^*(P_1, P_2, \dots, P_n)$ be a n-dimensional policy combination matrix which denotes the combination result of the n policies. There exists a FIA expression $f_I(P_1, P_2, ..., P_n)$ that is equivalent to $M^*(P_1, P_2, ..., P_n)$.*

Proof. We prove this theorem by induction. The base case is when $n = 2$, which is true according to Theorem 2.

Assuming that when $n = k - 1$ the corollary holds, we now consider the case when $n = k$. As shown in Table 4, $M^*(P_1, ..., P_k)$ has 3^k entries in total, each of which is denoted as $e_{i,j}$ $(1 \leq i \leq 3, 1 \leq j \leq 3^{k-1})$. Take entries $e_{i,1}$ to $e_{i,3^{k-1}}$ as a *(k-1)*-dimensional policy combination matrix, and we have three such *(k-1)*-dimensional policy combination matrices corresponding to the policy P_k's effect. Based on the assumption, we obtain the FIA expression for each cell for the $k - 1$ policies as shown in the column of $f^{k-1}(P_1$, ..., $P_{k-1})$.

$P_1, P_2, ..., P_{k-1}$	P_k	M^*	$f^{k-1}(P_1, P_2, ..., P_{k-1})$
$Y, Y, ..., Y$	Y	$e_{1,1}$	$f^{k-1}_{1,1}(P_1, P_2, ..., P_{k-1})$
...	...	...	...
$NA, NA, ..., NA$	Y	$e_{1,3^{k-1}}$	$f^{k-1}_{1,3^{k-1}}(P_1, P_2, ..., P_{k-1})$
$Y, Y, ..., Y$	N	$e_{2,1}$	$f^{k-1}_{2,1}(P_1, P_2, ..., P_{k-1})$
...	...	...	...
$NA, NA, ..., NA$	N	$e_{2,3^{k-1}}$	$f^{k-1}_{2,3^{k-1}}(P_1, P_2, ..., P_{k-1})$
$Y, Y, ..., Y$	NA	$e_{3,1}$	$f^{k-1}_{3,1}(P_1, P_2, ..., P_{n-1})$
...	...	...	...
$NA, NA, ..., NA$	NA	$e_{3,3^{k-1}}$	$f^{k-1}_{3,3^{k-1}}(P_1, P_2, ..., P_{k-1})$

Table 4: n Policies

Next, we extend $f^{k-1}(P_1, ..., P_{k-1})$ to $f^k(P_1, ..., P_k)$ for each cell in M^* (in what follows we use f^{k-1} and f^k for short). According to the effect of P_k and $e_{i,j}$, we summarize the expressions of f^k in Table 5. Note that we do not need to consider the cell where $e_{i,j}$ is NA.

P_k	$e_{i,j}$	$f^k_{i,j}$
Y	Y	$f^{k-1}_{i,j}\&(P_k\&\mathsf{P_Y})$
Y	N	$f^{k-1}_{i,j}\&[\neg(P_k\&\mathsf{P_Y})]$
N	Y	$f^{k-1}_{i,j}\&[\neg(P_k\&\mathsf{P_N})]$
N	N	$f^{k-1}_{i,j}\&(P_k\&\mathsf{P_N})$
NA	Y	$f^{k-1}_{i,j}$
NA	N	$f^{k-1}_{i,j}$

Table 5: n Policies

Finally, we add up f^k for all the cells and obtain the expression $f(P_1, P_2, ..., P_k)$.

We have shown that the corollary holds for $n = 2$, and we have also shown that if the corollary holds for $n = k - 1$ then it holds for $n = k$. We can therefore state that it holds for all n. ∎

Theorem 5 *Among the 10 operators in Θ, there are 12 minimally complete subsets. They are the 12 elements in the cartesian product* $\{\neg\} \times \{\mathsf{P_Y}, \mathsf{P_Y}\} \times \{\,\Pi_Y, \Pi_N,\ \&\,\} \times \{\,+, \rhd\}$.
Proof.

- The policy constant $\mathsf{P_N}$ cannot be expressed using $\Theta \setminus \{\mathsf{P_Y}, \mathsf{P_N}\}$. When given $\neg$, $\mathsf{P_Y}$ and $\mathsf{P_N}$ can be derived from each other: $\mathsf{P_Y} = \neg\mathsf{P_N}$ and $\mathsf{P_N} = \neg\mathsf{P_Y}$.

  We need to show that, there does not exist a policy expression using operators in $\Theta \setminus \{\mathsf{P_Y}, \mathsf{P_N}\}$ that is equivalent to $\mathsf{P_N}$. Consider the information ordering among the three values: $Y > NA$ and $N > NA$. The key observation is that the operators in $\Theta \setminus \{\mathsf{P_Y}, \mathsf{P_N}\}$ are all non-increasing in the information ordering.

  Suppose, for the sake of contradiction, that a policy expression $f(P_1, P_2, \cdots, P_n)$ constructed from $\Theta \setminus \{\mathsf{P_Y}, \mathsf{P_N}\}$ and policy variables $P_1, \cdots, P_n$ is equivalent to $\mathsf{P_N}$. Then this must mean that no matter what actual policies are used to instantiate $P_1, \cdots, P_n$, the result is $\mathsf{P_N}$. Let $e_0 = f(\mathsf{P_{NA}}, \mathsf{P_{NA}}, \cdots, \mathsf{P_{NA}}) = \mathsf{P_N}$. We now use a structural induction to show that e_0 must gives NA for every request; thus contradiction. For the base case, we have policy constant $\mathsf{P_{NA}}$, this is true. For the unary operators, if e gives NA for a request, then $\Pi_Y(e), \Pi_N(e)$, and $\neg(e)$ are also NA. For the binary operators $+, -, \&$, and $\rhd$, if both operands are NA for a request, then the result is also NA for the request.

- The unary operator $\neg$ cannot be expressed using $\Theta \setminus \{\neg\}$.

  The key observation is that without $\neg$, one cannot switch Y and N.

  Suppose, for the sake of contradiction, that $e_0(P)$ is equivalent to $\neg P$. Let P be a policy that returns Y on q_1 and N on q_2. Then $e_0(P)$ must return N on q_1 and Y on q_2, i.e., it must give (N, Y) on q_1 and q_2. We use structural induction to show that the result $e_0(P)$ gives for q_1 and q_2 must be among (Y, N), (Y, Y), (N, N), (NA, NA), (Y, NA), (NA, N). That is, if the answer for q_1 is N, then the answer for q_2 must be N, and if the answer for q_2 is Y, the answer for q_1 must be Y. Hence contradiction. For the base case, this holds for P and the three constants $\mathsf{P_Y}, \mathsf{P_N}, \mathsf{P_{NA}}$. One can verify that the six pairs are closed under $\Pi_Y, \Pi_N, +, -,\ \&\ , \rhd$.

- The binary operator $\&$ cannot be expressed using $\Theta \setminus \{\,\&\,, \Pi_Y, \Pi_N\}$. Given $\neg$, Π_Y and Π_N can be expressed from each other: $\Pi_Y(P) = P \,\&\, \neg\mathsf{P_N}$, $\Pi_N(P) = \neg\Pi_Y(\neg P)$. Π_Y can be expressed using $\{\,\&\,, \mathsf{P_Y}\}$ by definition, and $\&$ can be expressed using $\{\mathsf{P_N}, +, \neg, \Pi_N\}$.

  Assume, for the sake of contradiction, that $e_0(P_1, P_2)$ is equivalent to $P_1 \& P_2$. Let P_1 be a policy that returns (Y, Y) on q_1 and q_2, and P_2 returns (Y, N) on q_1 and q_2. Then $e_0(P_1, P_2)$ must return (Y, NA) on q_1 and q_2. We show this is not possible. The key insight here is that without $\&\,, \Pi_Y, \Pi_N$, one cannot get information asymmetry Y or N for one request and NA for another from symmetric policies.

  We use a structural induction to show that the result $e(P_1, P_2)$ gives for q_1 and q_2 must be among the following set: $\{(Y, N), (Y, Y), (NA, NA), (N, Y), (N, N)\}$. This holds for all $P_1, P_2, \mathsf{P_Y}, \mathsf{P_N}, \mathsf{P_{NA}}$. One can verify that the set is closed under $\neg, +, -, \rhd$.

  Given $\{\mathsf{P_N}, +, \neg\}$, $\&$ can be expressed using either Π_Y or Π_N:

  $$P_1 \,\&\, P_2 = (P_1 \sqcap P_2) + \neg(\neg P_1 \sqcap \neg P_2),$$

  where

  $$P_1 \sqcap P_2 = \Pi_Y(P_1) - (\Pi_N(\mathsf{P_N} + P_2)),$$

  and $-$ can be defined using Π_Y, Π_N as:

  $$P_1 - P_2 = (\Pi_Y(\neg(\neg P_1 + P_2 + \neg P_2))) + (\Pi_N(P_1 + P_2 + \neg P_2))\,.$$

The effect of $P_1 \sqcap P_2$ is to authorize any request that is authorized by both P_1 and P_2, and to be NA for all other requests.

- The binary operator $+$ cannot be expressed using $\Theta \setminus \{+, \triangleright\}$. However, $+$ can be expressed using $\{\mathsf{P_N}, \neg, \&, \triangleright\}$ or $\{\mathsf{P_N}, \neg, \Pi_N, \triangleright\}$, and $\triangleright$ can be expressed using $\{\mathsf{P_N}, +, \neg, \&\}$.

  Suppose, for the sake of contradiction, that an expression $e_0(P_1, P_2)$ constructed from P_1, P_2 and $\Theta \setminus \{+, \triangleright\}$ is equivalent to $P_1 + P_2$. Let P_1 be a policy that returns (Y, NA) on q_1 and q_2, and P_2 be a policy that returns (NA, N) on q_1 and q_2. Then $e_0(P_1, P_2)$ must return (Y, N) on q_1 and q_2.

  We use a structural induction to show that the result $e(P_1, P_2)$ gives for q_1 and q_2 must be among $(Y, Y), (Y, NA), (N, N), (N, NA), (NA, Y), (NA, N), (NA, NA)$. Hence contradiction. This is satisfied by $P_1, P_2, \mathsf{P_Y}, \mathsf{P_N}, \mathsf{P_{NA}}$. One can verify that these seven values are closed under $\neg, \Pi_Y, \Pi_N, -, \&$.

  The $+$ operator can be expressed using $\{\mathsf{P_N}, \neg, \&, \triangleright\}$:

$$P_1 + P_2 = (P_1 \& \mathsf{P_Y}) \triangleright (P_2 \& \mathsf{P_Y}) \triangleright (P_1 \& \mathsf{P_N}) \triangleright (P_2 \& \mathsf{P_N}).$$

  Similarly, the $+$ operator can be expressed using $\{\mathsf{P_N}, \neg, \Pi_N, \triangleright\}$:

$$P_1 + P_2 = (\Pi_Y P_1) \triangleright (\Pi_Y P_2) \triangleright (\Pi_N P_1) \triangleright (\Pi_N P_2).$$

  Recall that the operator $\triangleright$ is defined using $+$ and $-$ as $P_1 \triangleright P_2 = P_1 + (P_2 - P_1)$, and $P_1 - P_2 = (\mathsf{P_Y} \& (\neg(\neg P_1 + P_2 + \neg P_2))) + (\mathsf{P_N} \& (P_1 + P_2 + \neg P_2))$. Thus, $\triangleright$ can be expressed using $\{\mathsf{P_N}, -, \neg, \&\}$.

In summary, among the 10 operators in Θ, for completeness, we must have $\neg$, one in $\{\mathsf{P_Y}, \mathsf{P_Y}\}$, one in $\{\Pi_Y, \Pi_N, \&\}$, and one in $\{+, \triangleright\}$. There are 12 combinations. It is not difficult to verify that every such combination is in fact complete. For example, once we have $\{\neg, \mathsf{P_N}, \Pi_N\}$, adding $+$ allows us to derive $\&$, and then derive $\triangleright$, adding $\triangleright$ allows us to derive $+$ and then $\&$. There are thus 12 minimally complete subsets in Θ. ∎

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          <SubjectMatch MatchId="Name-Match">
            <AttributeValue DataType="#string">
            manager </AttributeValue>
            <SubjectAttributeDesignator AttributeId="role"
            DataType="#string"/ >
          </SubjectMatch>
        </Subject>
      </Subjects>
      <Actions>
        <Action>
          <ActionMatch MatchId="function:string-equal">
            <AttributeValue DataType="#string">
            read </AttributeValue>
            <ActionAttributeDesignator AttributeId="act"
            DataType="#string"/ >
          </ActionMatch>
        </Action>
        <Action>
          <ActionMatch MatchId="function:string-equal">
            <AttributeValue DataType="#string">
            update </AttributeValue>
            <ActionAttributeDesignator AttributeId="act"
            DataType="#string"/ >
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        </Action>
      <Actions>
    <Condition>
    <Apply FunctionId="Time-In-Range">
      <Apply FunctionId="Time-one-and-only">
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    </Apply>
    </Condition>
    </Target>
  <Rule RuleId="Rul_12" Effect="Deny">
    <Target>
      <Subjects>
        <Subject>
          <SubjectMatch MatchId="Name-Match">
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      <Actions>
        <Action>
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            read </AttributeValue>
            <ActionAttributeDesignator AttributeId="act"
            DataType="#string"/ >
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        </Action>
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    </Target>
  </Rule>
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Figure 6: Policy P_1

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               <SubjectMatch MatchId="Name-Match">
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                  <SubjectAttributeDesignator AttributeId="role"
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               </SubjectMatch>
            </Subject>
            <Subject>
               <SubjectMatch MatchId="Name-Match">
                  <AttributeValue DataType="#string"> staff
                  </AttributeValue>
                  <SubjectAttributeDesignator AttributeId="role"
                  DataType="#string"/ >
               </SubjectMatch>
            </Subject>
         </Subjects>
         <Actions>
            <Action>
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                  <AttributeValue DataType="#string">
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                  <ActionAttributeDesignator AttributeId="act"
                  DataType="#string"/ >
               </ActionMatch>
            </Action>
         <Actions>
      <Condition>
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         <Apply FunctionId="Time-one-and-only">
            <SubjectAttributeDesignator AttributeId="time"
                  DataType="#time"/ >
         </Apply>
         <AttributeValue DataType="#time">
                  08:00 </AttributeValue>
         <AttributeValue DataType="#time">
                  20:00 </AttributeValue>
      </Apply>
      </Condition>
      </Target>
   <Rule RuleId="Rul_12" Effect="Deny">
      <Target>
         <Subjects>
            <Subject>
               <SubjectMatch MatchId="Name-Match">
                  <AttributeValue DataType="#string">
                  staff </AttributeValue>
                  <SubjectAttributeDesignator AttributeId="role"
                  DataType="#string"/ >
               </SubjectMatch>
            </Subject>
         </Subjects>
         <Actions>
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                  update </AttributeValue>
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            </Action>
         </Actions>
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```

Figure 7: Policy P_2