AN ALGEBRAIC APPROACH TO THE ASSOCIATION SCHEMES OF CODING THEORY *)

BY

P. DELSARTE

*) Thesis, Université Catholique de Louvain, June 1973.
Promotor: Professor Dr J. M. Goethals.

Philips Res. Repts Suppl. 1973, No. 10.

CONTENTS

List of symbols

1. INTRODUCTION 1

2. ASSOCIATION SCHEMES 8
   2.1. Definitions 8
   2.2. The Bose–Mesner algebra 9
   2.3. The eigenmatrices P and Q 12
   2.4. Examples 16
   2.5. Extensions of an association scheme 17
   2.6. Duality in association schemes 19
      2.6.1. Partitions of orthogonal matrices 19
      2.6.2. Dual of a regular scheme 21
      2.6.3. Duality in strongly regular graphs 23

3. SUBSETS IN ASSOCIATION SCHEMES 25
   3.1. Inner and outer distribution 25
   3.2. Linear programming 27
   3.3. Cliques in association schemes 29
      3.3.1. The Elias theorem 29
      3.3.2. The linear-programming bound 31
   3.4. Designs in association schemes 32
   3.5. Characteristic matrices 35

4. AN INTRODUCTION TO ALGEBRAIC CODING THEORY . 37
   4.1. The Hamming schemes 37
      4.1.1. Eigenmatrices and Krawtchouk polynomials 37
      4.1.2. Codes in Hamming schemes 40
      4.1.3. Orthogonal arrays 43
   4.2. The Johnson schemes 44
      4.2.1. Eigenmatrices and Eberlein polynomials 45
      4.2.2. Binary codes with constant weight 49
      4.2.3. t-Designs 50
   4.3. Classical inequalities for codes 52
      4.3.1. The Plotkin bound 53
      4.3.2. The Singleton bound 54
      4.3.3. The Hamming bound 55

5. POLYNOMIAL SCHEMES 56
5.1. Definitions and preliminaries 56
5.1.1. Orthogonal polynomials 56
5.1.2. The Mac Williams inequality 60
5.2. P-polynomial (= metric) schemes and codes 61
5.2.1. Preliminary results 61
5.2.2. The Hamming bound and the perfect codes 62
5.2.3. Distribution matrix of a code 65
5.3. Q-polynomial schemes and designs 70
5.3.1. Preliminary results 71
5.3.2. The Rao–Wilson bound and the tight designs 74
5.3.3. Regular designs and subschemes 79

6. ADDITIVE CODES IN HAMMING SCHEMES 84
6.1. Inner product and duality in Abelian groups 84
6.2. The Mac Williams identities on dual codes 85
6.3. Weight distribution of cosets and subschemes 88

REFERENCES 95