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An Algorithm for the Evaluation of Finite Trigonometric Series

Author(s): Gerald Goertzel

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3) If  $G$  has no elements whose orders divide  $n^2 - n$  or if  $G$  has no elements whose orders divide  $n - 1$  when  $\bar{n}$  is an automorphism, then  $G$  is Abelian.

It should be noted that if  $G$  is the direct product of two groups  $A$  and  $B$ , where  $A(n-1) = (1)$  and  $B(n) = (1)$ , then  $\bar{n}$  leaves  $A$  elementwise fixed and maps  $B$  into  $(1)$ . Hence any group of this type admits  $\bar{n}$  as an endomorphism, and some such restriction as in 3) is necessary if  $G$  is to be Abelian.

The proof of the proposition is as follows. Since  $\bar{n}$  is an endomorphism  $a^n b^n = (ab)^n$ . If  $a$  is cancelled on the left and  $\overline{b}$  on the right, then  $a^{n-1} b^{n-1} = (ba)^{n-1}$ . It follows that  $b^{1-n} a^{1-n} = (ba)^{1-n}$  and  $\overline{1-n}$  is an endomorphism (cf. [1]).

Then  $(a a^{n-1} b^n)(a^{1-n} b^{-n}) = (ab)^n (ab)^{1-n} = ab$ , and  $1 = a^{n-1} b^n a^{1-n} b^{-n}$ , or  $a^{n-1} b^n = b^n a^{n-1}$ . This means that  $n$ th powers commute with  $(n-1)$ st powers, whence  $G(n^2 - n)$  is Abelian (cf. [2] p. 29 Ex. 4).

Now the product of the two endomorphisms  $\bar{n}$  by  $\overline{1-n}$  is an endomorphism of  $G$  onto the Abelian group  $G(n^2 - n)$  with kernel  $G\{n^2 - n\}$ . This proves the first statement of the proposition.

Statement 2) follows from the fact that when  $\bar{n}$  is an automorphism, every element is an  $n$ th power, and therefore the equation  $a^{n-1} b^n = b^n a^{n-1}$  implies that  $G(n-1)$  is in the center of the group. It follows that  $\overline{1-n}$  is an endomorphism, mapping  $G$  onto the Abelian group  $G(n-1)$  with kernel  $G\{n-1\}$ .

Statement 3) follows immediately from 1) and 2).

We are indebted to the referee for the references to the literature.

#### References

1. J. W. Young, On the holomorphisms of a group, Trans. Amer. Math. Soc., vol. 3, 1902, pp. 186-191.
2. Hans Zassenhaus, Group Theory (English Edition), New York, 1949.

#### AN ALGORITHM FOR THE EVALUATION OF FINITE TRIGONOMETRIC SERIES

GERALD GOERTZEL, Nuclear Development Corporation of America, White Plains, N. Y.

The algorithm described below enables one to obtain the simultaneous numerical evaluation of  $C = \sum_0^N a_k \cos kx$  and  $S = \sum_1^N a_k \sin kx$  for given  $a_k$ ,  $\cos x$ , and  $\sin x$ . Tables for  $\sin kx$  and  $\cos kx$  are not needed and only about  $N$  multiplications and about  $2N$  additions or subtractions are required, so the method may be of interest to programmers of digital computers.

The algorithm is defined by

$$\begin{aligned} U_{N+2} &= U_{N+1} = 0; \\ U_k &= a_k + 2 \cos x U_{k+1} - U_{k+2}, \quad k = N, N-1, \dots, 1. \\ C &= a_0 + U_1 \cos x - U_2, \quad S = U_1 \sin x. \end{aligned}$$

To establish this result, consider

$$V_k = \sum_{j=k}^N a_j \sin(j - k + 1)x; \quad k = 1, \dots, N,$$

$$V_{N+1} = V_{N+2} = 0.$$

Then

$$a_k \sin x + 2 \cos x V_{k+1} - V_{k+2}$$

$$= a_k \sin x + \sum_{j=k+1}^N a_j [2 \cos x \sin(j - k)x - \sin(j - k - 1)x]$$

$$= a_k \sin x + \sum_{j=k+1}^N a_j \sin(j - k + 1)x = V_k,$$

whence  $V_k = U_k \sin x$  and, in particular,  $S = V_1 = U_1 \sin x$ . Furthermore

$$a_0 \sin x + V_1 \cos x - V_2 = a_0 \sin x + \sum_{j=1}^N a_j [\cos x \sin jx - \sin(j - 1)x]$$

$$= a_0 \sin x + \sum_{j=1}^N a_j \cos jx \sin x = C \sin x,$$

whence  $C = a_0 + U_1 \cos x - U_2$ .

## CLASSROOM NOTES

EDITED BY C. O. OAKLEY, Haverford College

*All material for this department should be sent to C. O. Oakley, Department of Mathematics, Haverford College, Haverford, Pa.*

### A DIRECT PROOF FOR THE LEAST SQUARES SOLUTION OF A SET OF CONDITION EQUATIONS

ERWIN SCHMID, Coast and Geodetic Survey, Washington, D. C.

The problem of finding the solution of a set of  $m$  independent "condition" equations, linear in the  $n$  variables  $v_1, \dots, v_n$ ,  $n > m$

$$(1) \quad \sum_{j=1}^n a_{ij} v_j - a_{i0} = 0, \quad i = 1, \dots, m$$

such that  $\sum_1^n v_j^2$  be a minimum is generally solved, following Lagrange, by minimizing instead, an equivalent function involving the so-called Lagrangian multipliers.

The following approach seems more direct, and generalizes a basic theorem in analytic geometry to  $n$  dimensions.

Multiplying in turn each of equations (1) by one of the  $m$  parameters