Hana Kamasová

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AN ALGORITHM FOR THE INVERSION OF PARTITIONED MATRICES

HANA KAMASOVÁ

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1. INTRODUCTION

In paper [1] the following results were proved:

Let $\boldsymbol{A}$ be a square matrix of order $n \geqq r$ over a field of characteristic zero, partitioned into blocks $\alpha_{i,k}$ of the type $(n_i \times n_k)$,

$$\sum_{i=1}^{r} n_i = \sum_{k=1}^{r} n_k = n .$$

Let further

$$(1,1)\qquad \begin{aligned} \boldsymbol{A}_1 &= \alpha_{1,1} \\ \boldsymbol{A}_2 &= \begin{bmatrix} \alpha_{1,1}, & \alpha_{1,2} \\ \alpha_{2,1}, & \alpha_{2,2} \end{bmatrix} \\ &\vdots \\ \boldsymbol{A}_{r-1} &= \begin{bmatrix} \alpha_{1,1} & , \dots, \alpha_{1,r-1} \\ \vdots & \\ \alpha_{r-1,1}, & \dots, \alpha_{r-1,r-1} \end{bmatrix} \\ \boldsymbol{A}_r &= \boldsymbol{A} . \end{aligned}$$

Let us define the matrices $\boldsymbol{Z}_{i,k}^{(p)}$ for $i, k, p = 1, 2, \dots, r$ in the following way:

1. $\boldsymbol{Z}_{i,k}^{(1)} = \alpha_{i,k}$
2. $\boldsymbol{Z}_{i,k}^{(p)} = \boldsymbol{Z}_{i,k}^{(p-1)} - \boldsymbol{Z}_{i,p-1}^{(p-1)} \boldsymbol{Z}_{p-1}^{-1} \boldsymbol{Z}_{p-1,k}^{(p-1)}$
   for $p = 2, \dots, r$ where $\boldsymbol{Z}_{p,p}^{(p)} = \boldsymbol{Z}_p$.

For matrices $\boldsymbol{Z}_p$ we have

Theorem 1,1. *The matrices $\boldsymbol{A}_p$ are regular iff $\boldsymbol{Z}_p$ are regular for $p = 1, 2, \dots, r$.*

For the proof see [1].

For $i \leqq p$, $k \leqq p$ let us introduce $V_{i,k}^{(p)}$ as the set of subsequences of the sequence $(i, i + 1, \ldots, p - 1, p, p - 1, \ldots, k + 1, k)$ which have the following properties:

1° The first element is i, the last is k.

2° Each two neighbouring elements of the subsequences are different.

Theorem 1,2. *Let the matrices (1,1) be regular, let* $\mathbf{A}^{-1} = [\beta_{i,k}]$ *be a partitioned matrix conformal to* $\mathbf{A}$. *Then we have*

$$(1,2) \qquad \beta_{i,k} = \sum (-1)^{1+m(j_1,\ldots,j_s)} \mathbf{Z}_{j_1}^{-1} \mathbf{Z}_{j_1,j_2}^{(q_1)} \mathbf{Z}_{j_2}^{-1} \ldots \mathbf{Z}_{j_s-1,j_s}^{(q_s-1)} \mathbf{Z}_{js}^{-1},$$

$$(i = j_1, \ldots, j_s = k) \in V_{i,k}^{(r)}$$

$$g_t = \min(j_t, j_{t+1}), \quad t = 1, \ldots, s - 1$$

where $m(j_1, \ldots, j_s)$ *is the number of the elements of the sequence* $(i = j_1, \ldots, j_s = k)$.

For the proof see [1].

In this paper an algorithm for calculating the blocks of the inverse matrix of $\mathbf{A}$ is presented. It is simultaneously an algorithm for the matrices $\mathbf{Z}_{i,k}^{(p)}$ for $i \geqq p$; $k \geqq p$; $i, k, p = 2, \ldots, r$.

First, let us introduce the following sets:

For $i \leqq p \leqq k$, let us denote by $U_{i,k}^{(p)}$ the set of the subsequences of the sequence $(i, i + 1, \ldots, p - 1, p, k)$ which have the properties 1°, 2°. Let $(U_{i,k}^{(p)})_p$ denote the set of those elements from $U_{i,k}^{(p)}$ which contain the term p.

Further, for $i \geqq p \geqq k$ let $W_{i,k}^{(p)}$ denote the set of the subsequences of the sequence $(i, p, p - 1, \ldots, k + 1, k)$ which have the properties 1°, 2°.

Let $(W_{i,k}^{(p)})_p$ denote the set of those elements from $W_{i,k}^{(p)}$ which contain the term p.

For $i \leqq p$; $k \leqq p$ let $(V_{i,k}^{(p)})_p$ denote the set of those elements from $V_{i,k}^{(p)}$ which contain the term p.

It is obvious that:

$$(1,3) \quad \text{For } i \leqq k, \qquad V_{i,k}^{(k)} = U_{i,k}^{(k)}.$$

$$(1,4) \quad \text{For } i \geqq k, \qquad V_{i,k}^{(i)} = W_{i,k}^{(i)}.$$

$$(1,5) \quad \text{For } i \leqq p, \qquad U_{i,p+1}^{(p)} = V_{i,p+1}^{(p+1)}.$$

$$(1,6) \quad \text{For } k \leqq p, \qquad W_{p+1,k}^{(p)} = V_{p+1,k}^{(p+1)}.$$

$$(1,7) \quad \text{For } i \leqq p;\ k \leqq p, \quad V_{i,k}^{(p)} \cup (V_{i,k}^{(p+1)})_{p+1} = V_{i,k}^{(p+1)}, \quad V_{i,k}^{(p)} \cap (V_{i,k}^{(p+1)})_{p+1} = \Phi.$$

$$(1,8) \quad \text{For } i \leqq p < k, \quad U_{i,k}^{(p)} \cup (U_{i,k}^{(p+1)})_{p+1} = U_{i,k}^{(p+1)}, \quad U_{i,k}^{(p)} \cap (U_{i,k}^{(p+1)})_{p+1} = \Phi.$$

$$(1,9) \quad \text{For } i > p \geqq k, \quad W_{i,k}^{(p)} \cup (W_{i,k}^{(p+1)})_{p+1} = W_{i,k}^{(p+1)}, \quad W_{i,k}^{(p)} \cap (W_{i,k}^{(p+1)})_{p+1} = \Phi.$$

Theorem 2,1. *Let the matrices* (1,1) *be regular. For* $i, k, p = 1, 2, \ldots, r$ *let us define the following transformation:*

(2,1) $$\beta_{i,k}^{(0)} = \alpha_{i,k}$$

(2,2) $$\beta_{p,p}^{(p)} = (\beta_{p,p}^{(p-1)})^{-1}$$

(2,3) $$\beta_{p,k}^{(p)} = -(\beta_{p,p}^{(p-1)})^{-1}\, \beta_{p,k}^{(p-1)} \qquad \text{for} \quad k \neq p\,.$$

(2,4) $$\beta_{i,p}^{(p)} = \beta_{i,p}^{(p-1)}(\beta_{p,p}^{(p-1)})^{-1} \qquad \text{for} \quad i \neq p\,.$$

(2,5) $$\beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} - \beta_{i,p}^{(p-1)}(\beta_{p,p}^{(p-1)})^{-1}\, \beta_{p,k}^{(p-1)} \quad \text{for} \quad i \neq p,\, k \neq p\,.$$

Then for $i \leqq p$; $k \leqq p$;

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1,\ldots,j_s=k)\in V^{(p)}{}_{i,k}} (-1)^{m+1}\, \mathbf{Z}_{j_1}^{-1}\mathbf{Z}_{j_1,j_2}^{(q_1)}\mathbf{Z}_{j_2}^{-1} \ldots \mathbf{Z}_{j_{s-1},j_s}^{(q_{s-1})}\mathbf{Z}_{j_s}^{-1}$$

(2,6) *where* $q_t = \min(j_t, j_{t+1})$ *for* $t = 1, \ldots, s-1$,
m is the number of terms of the sequence $(i = j_1, \ldots, j_s = k) \in V_{i,k}^{(p)}$,

(2,7) *for* $i > p$; $k > p$,

$$\beta_{i,k}^{(p)} = \mathbf{Z}_{i,k}^{(p+1)}\,,$$

(2,8) *for* $i \leqq p$; $k > p$,

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1,\ldots,j_s=k)\in U^{(p)}{}_{i,k}} (-1)^{u+1}\, \mathbf{Z}_{j_1}^{-1}\mathbf{Z}_{j_1,j_2}^{(j_1)}\mathbf{Z}_{j_2}^{-1}\mathbf{Z}_{j_2,j_3}^{(j_2)} \ldots \mathbf{Z}_{j_{s-1}}^{-1}\mathbf{Z}_{j_{s-1},j_s}^{(j_{s-1})}$$

where u is the number of terms of the sequnce
$(i = j_1, \ldots, j_s = k) \in U_{i,k}^{(p)}$.

(2,9) *For* $i > p$; $k \leqq p$,

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1,\ldots,j_s=k)\in W^{(p)}{}_{i,k}} (-1)^{w}\, \mathbf{Z}_{j_1,j_2}^{(j_2)}\mathbf{Z}_{j_2}^{-1}\mathbf{Z}_{j_2,j_3}^{(j_3)}\mathbf{Z}_{j_3}^{-1} \ldots \mathbf{Z}_{j_{s-1},j_s}^{(j_s)}\mathbf{Z}_{j_s}^{-1}$$

where w is the number of terms of the sequence
$(i = j_1, \ldots, j_s = k) \in W_{i,k}^{(p)}$.

Particularly, for $p = r$ *the partitioned matrix* $[\beta_{i,k}^{(r)}] = \mathbf{A}^{-1}$.

Proof by induction.

1. For $p = 1$, $\beta_{1,1}^{(0)} = \mathbf{Z}_1$ is regular and we have by (2,1)–(2,5)

$$\beta_{i,k}^{(1)} = \begin{bmatrix} \mathbf{Z}_1^{-1}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{13}^{(1)}, & \dots & -\mathbf{Z}_1^{-1}\mathbf{Z}_{1r}^{(1)} \\ \mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_2, & \mathbf{Z}_{23}^{(2)} & & \mathbf{Z}_{2r}^{(2)} \\ \vdots & \vdots & & & \vdots \\ \mathbf{Z}_{r1}^{(1)}\mathbf{Z}_1^{-1} & \mathbf{Z}_{r2}^{(2)}, & \dots\dots & & \mathbf{Z}_{rr}^{(2)} \end{bmatrix}.$$

For $p = 2$, $\beta_{2,2}^{(1)} = \mathbf{Z}_2$ is regular by Theorem 1,1 and we obtain

$$\beta_{i,k}^{(2)} = \begin{bmatrix} \mathbf{Z}_1^{-1} + \mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}, \\ -\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_2^{-1}, \\ \mathbf{Z}_{31}^{(1)}\mathbf{Z}_1^{-1} - \mathbf{Z}_{32}^{(2)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_{32}^{(2)}\mathbf{Z}_2^{-1}, \\ \vdots & \vdots \\ \mathbf{Z}_{r1}^{(1)}\mathbf{Z}_1^{-1} - \mathbf{Z}_{r2}^{(2)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_{r2}^{(2)}\mathbf{Z}_2^{-1}, \end{bmatrix.}$$

$$\begin{bmatrix} -\mathbf{Z}_1^{-1}\mathbf{Z}_{13}^{(1)} + \mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{23}^{(2)}, & \dots & -\mathbf{Z}_1^{-1}\mathbf{Z}_{1r}^{(1)} + \mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{2r}^{(2)} \\ -\mathbf{Z}_2^{-1}\mathbf{Z}_{23}^{(2)}, & \dots\dots & -\mathbf{Z}_2^{-1}\mathbf{Z}_{2r}^{(2)} \\ \mathbf{Z}_3, & \dots\dots & \mathbf{Z}_{3r}^{(3)} \\ & & \\ \mathbf{Z}_{r3}^{(3)}, & \dots\dots & \mathbf{Z}_{rr}^{(3)} \end{bmatrix}.$$

It is obvious that Theorem 2,1 is true for $p = 1, 2$.

2. Let us suppose that our theorem is true for p; then $\beta_{p+1,p+1}^{(p)} = \mathbf{Z}_{p+1}$, therefore by Theorem 1,1 $\beta_{p+1,p+1}^{(p)}$ is regular.

Proof of 2,6:

a) Let $i < p + 1$; $k < p + 1$; then by (2,5)

$$\beta_{i,k}^{(p+1)} = \beta_{i,k}^{(p)} - \beta_{i,p+1}^{(p)}(\beta_{p+1,p+1}^{(p)})^{-1}\,\beta_{p+1,k}^{(p)} = \sum_{V^{(p)}{}_{i,k}} (-1)^{m_1+1}\,\mathbf{Z}_i^{-1} \dots \mathbf{Z}_k^{-1} -$$

$$- \Big(\sum_{U^{(p)}{}_{i,p+1}} (-1)^{u+1}\,\mathbf{Z}_i^{-1} \dots \mathbf{Z}_{j_{s-1},p+1}^{(j_{s-1})}\Big)\,\mathbf{Z}_{p+1}^{-1}\Big(\sum_{W^{(p)}{}_{p+1,k}} (-1)^{w}\,\mathbf{Z}_{p+1,j_2}^{(j_2)} \dots \mathbf{Z}_k^{-1}\Big) = I + II\,.$$

After performing the multiplication in II we obtain obviously a sum where we summarize over all sequences of the set $(V_{i,k}^{(p+1)})_{p+1}$. The sign of every summand in II is $(-1)^{u+w}$. Since the term $p + 1$ is included both in number u and in number w, it is clear that $(-1)^{u+w} = (-1)^{v+1}$ where v is the number of terms of the sequence from $(V_{i,k}^{(p+1)})_{p+1}$.

Since $U_{i,p+1}^{(p)}$ contains only increasing sequences and $W_{p+1,k}^{(p)}$ contains only decreasing sequences, we obtain for every $Z^{(q_t)}_{j_t, j_{t+1}}$ in II clearly $q_t = \min(j_t, j_{t+1})$. By relation (1,7) we can therefore write

$$\beta^{(p+1)}_{i,k} = \sum_{\substack{(i=j_1,\ldots,j_s=k)\in V_{i,k}(p+1)\\ q_t = \min(j_t, j_{t+1})}} (-1)^{m+1} Z^{-1}_{j_1} Z^{(q_1)}_{j_1,j_2} \ldots Z^{-1}_{j_s} .$$

b) Let $i = p + 1$; $k = p + 1$; then

$$\beta^{(p+1)}_{p+1,p+1} = (\beta^{(p)}_{p+1,p+1})^{-1} = Z^{-1}_{p+1} .$$

c) Let $i = p + 1$; $k < p + 1$; then

$$\beta^{(p+1)}_{p+1,k} = -(\beta^{(p)}_{p+1,p+1})^{-1} \beta^{(p)}_{p+1,k} = -Z^{-1}_{p+1}\Big(\sum_{W_{p+1,k}(p)} (-1)^w Z^{(j_1)}_{p+1,j_1} \ldots Z^{-1}_k \Big),$$

therefore by (1,6)

$$\beta^{(p+1)}_{p+1,k} = \sum_{V_{p+1,k}(p+1)} (-1)^{m+1} Z^{-1}_{p+1} \ldots Z^{-1}_k .$$

d) Let $i < p + 1$; $k = p + 1$; then

$$\beta^{(p+1)}_{i,p+1} = \Big(\sum_{U_{i,p+1}(p)} (-1)^{u+1} Z^{-1}_i \ldots Z^{(j_{s-1})}_{j_{s-1},p+1} \Big) Z^{-1}_{p+1}$$

and by (1,5) we get

$$\beta^{(p+1)}_{i,p+1} = \sum_{V_{i,p+1}(p+1)} (-1)^{m+1} Z^{-1}_i \ldots Z^{-1}_{p+1} .$$

Proof of (2,7):

Let $i > p + 1$; $k > p + 1$; then

$$\beta^{(p+1)}_{i,k} = \beta^{(p)}_{i,k} - \beta^{(p)}_{i,p+1}(\beta^{(p)}_{p+1,p+1})^{-1} \beta^{(p)}_{p+1,k} = Z^{(p+1)}_{i,k} - Z^{(p+1)}_{i,p+1} Z^{-1}_{p+1} Z^{(p+1)}_{p+1,k} = Z^{(p+2)}_{i,k} .$$

Proof of (2,8):

a) Let $i < p + 1 < k$; then

$$\beta^{(p+1)}_{i,k} = \sum_{U_{i,k}(p)} (-1)^{u_1+1} Z^{-1}_i Z^{(i)}_{i,j_2} \ldots Z^{-1}_{j_{s-1}} Z^{(j_{s-1})}_{j_{s-1},k} -$$

$$- \Big(\sum_{U_{i,p+1}(p)} (-1)^{u_2+1} Z^{-1}_i \ldots Z^{(j_{s-1})}_{j_{s-1},p+1} \Big) Z^{-1}_{p+1} Z^{(p+1)}_{p+1,k} =$$

$$= \sum_{U_{i,k}(p)} (-1)^{u_1+1} Z^{-1}_i Z^{(i)}_{i,j_2} \ldots Z^{(j_{s-1})}_{j_{s-1},k} + \sum_{(U_{i,k}(p+1))_{p+1}} (-1)^{u_3+1} Z^{-1}_i \ldots Z^{-1}_{p+1} Z^{(p+1)}_{p+1,k}$$

where $u_3 = u_2 + 1$ is the number of terms of the sequence $(i = j_1, \ldots, j_s = k) \in$ $\in (U^{(p+1)}_{i,k})_{p+1}$.

Using relation (1,8) we obtain

$$\beta^{(p+1)}_{i,k} = \sum_{U_{i,k}(p+1)} (-1)^{u+1} Z^{-1}_i \ldots Z^{(j_{s-1})}_{j_{s-1},k} .$$

b) Let $i = p + 1 < k$; then

$$\beta^{(p+1)}_{p+1,k} = -\mathbf{Z}^{-1}_{p+1}\mathbf{Z}^{(p+1)}_{p+1,k} = \sum_{U_{p+1,k}(p+1)} (-1)^{u+1}\, \mathbf{Z}^{-1}_{p+1} \ldots \mathbf{Z}^{(p+1)}_{p+1,k}\,,$$

for $U^{(p+1)}_{p+1,k}$ contains only one element – the sequence $(p + 1, k)$.

Proof of (2,9):

a) Let $i > p + 1 > k$; then

$$\beta^{(p+1)}_{i,k} = \sum_{W_{i,k}(p)} (-1)^{w_1}\, \mathbf{Z}^{(j_2)}_{i,j_2}\mathbf{Z}^{-1}_{j_2} \ldots \mathbf{Z}^{(k)}_{j_{s-1},k}\mathbf{Z}^{-1}_{k} -$$

$$- \mathbf{Z}^{(p+1)}_{i,p+1} \sum_{W_{p+1,k}(p)} (-1)^{w_2}\, \mathbf{Z}^{(j_2)}_{p+1,j_2} \ldots \mathbf{Z}^{(k)}_{j_{s-1},k}\mathbf{Z}^{-1}_{k} =$$

$$= \sum_{W_{i,k}(p)} (-1)^{w_1}\, \mathbf{Z}^{(j_2)}_{i,j_2} \ldots \mathbf{Z}^{-1}_{k} + \sum_{(W_{i,k}(p+1))_{p+1}} (-1)^{w_3}\, \mathbf{Z}^{(p+1)}_{i,p+1} \ldots \mathbf{Z}^{-1}_{k}$$

where $w_3 = w_2 + 1$ is the number of terms of the sequence $(i = j_1, \ldots, j_s = k) \in \in (W^{(p+1)}_{i,k})_{p+1}$.

By (1,9) we obtain

$$\beta^{(p+1)}_{i,k} = \sum_{W_{i,k}(p+1)} (-1)^{w}\, \mathbf{Z}^{(j_2)}_{i,j_2} \ldots \mathbf{Z}^{-1}_{k}\,.$$

b) Let $i > p + 1 = k$; then

$$\beta^{(p+1)}_{i,p+1} = \mathbf{Z}^{(p+1)}_{i,p+1}\mathbf{Z}^{-1}_{p+1}\,.$$

For $p = r$, formulas (2,6) for $\beta^{(r)}_{i,k}$ and (1,2) for $\beta_{i,k}$ are identical, thus the partitioned matrix $[\beta^{(r)}_{i,k}] = \mathbf{A}^{-1}$.

3. EXAMPLE

An algorithm for the calculation of the inverse matrix of a quasi-triangular matrix.

Theorem 3,1. *Let the matrices (1,1) be regular. Let $\alpha_{i,k} = 0$ for $i > k$. Let $\mathbf{A}^{-1} = [\beta_{i,k}]$ be a partitioned matrix conformal to $\mathbf{A}$. Then*

(3,1) $$\mathbf{Z}^{(p)}_{i,k} = \alpha_{i,k} \quad \textit{for} \quad i \geqq p;\ k \geqq p;\ p \geqq 2\,,$$

(3,2) $$\beta_{i,k} = \sum_{(i=j_1,\ldots,j_s=k)\in V_{i,k}(k)} (-1)^{m+1}\, \alpha^{-1}_{j_1,j_1}\alpha_{j_1,j_2}\alpha^{-1}_{j_2,j_2} \ldots \alpha^{-1}_{j_s,j_s} \quad \textit{for} \quad i \leqq k$$

where m is the number of terms of the sequence $(i = j_1, \ldots, j_s = k) \in V^{(k)}_{i,k}$,

(3,3) $$\beta_{\iota,k} = 0 \quad \textit{for} \quad i > k\,.$$

For the proof see [2].

Lemma. *Let the matrices* (1,1) *be regular, let* $\alpha_{i,k} = 0$ *for* $i > k$. *Then* $\beta_{i,k}^{(p)} = 0$ *for* $i > k$; $p = 1, 2, \ldots, r$.

Proof. For $\beta_{i,k}^{(p)}$ formulas (2,6)–(2,9) are true for $p = 1, 2, \ldots, r$. In the case of a quasi-triangular matrix, these formulas have by (3,1)–(3,3) the following form:

(2,6)′ For $i \leqq k \leqq p$,

$$\beta_{i,k}^{(p)} = \sum_{V_{i,k}(k)} (-1)^{m+1} \alpha_{i,i}^{-1} \ldots \alpha_{k,k}^{-1} ;$$

for $k < i \leqq p$, $\beta_{i,k}^{(p)} = 0$.

(2,7)′ For $i > p$; $k > p$,

$$\beta_{i,k}^{(p)} = \mathbf{Z}_{i,k}^{(p+1)} = \alpha_{i,k} .$$

(2,8)′ For $i \leqq p$; $k > p$,

$$\beta_{i,k}^{(p)} = \sum_{U_{i,k}(p)} (-1)^{u+1} \alpha_{i,i}^{-1} \ldots \alpha_{j_{s-1},k} .$$

(2,9)′ For $i > p$; $k \leqq p$,

$$\beta_{i,k}^{(p)} = \sum_{W_{i,k}(p)} (-1)^{w} \alpha_{i,j_2} \alpha_{j_2,j_2}^{-1} \ldots \alpha_{k,k}^{-1} .$$

Since $W_{i,k}^{(p)}$ contains only decreasing sequences, it is $j_t > j_{t+1}$ for all $t = 1, \ldots, s - 1$; thus $\alpha_{j_t,j_{t+1}} = 0$. Therefore $\beta_{i,k}^{(p)} = 0$ for $i > k$.

The transformation formulas:

$$\beta_{i,k}^{(0)} = \alpha_{i,k} ,$$

$$\beta_{p,p}^{(p)} = (\beta_{p,p}^{(p-1)})^{-1} ,$$

$$\beta_{p,k}^{(p)} = -(\beta_{p,p}^{(p-1)})^{-1} \beta_{p,k}^{(p-1)} \quad \text{for} \quad k > p ,$$

$$\beta_{p,k}^{(p)} = 0 \quad \text{for} \quad k < p ,$$

$$\beta_{i,p}^{(p)} = \beta_{i,p}^{(p-1)} (\beta_{p,p}^{(p-1)})^{-1} \quad \text{for} \quad i < p ,$$

$$\beta_{i,p}^{(p)} = 0 \quad \text{for} \quad i > p ,$$

$$\beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} - \beta_{i,p}^{(p-1)} (\beta_{p,p}^{(p-1)})^{-1} \beta_{p,k}^{(p-1)} \quad \text{for} \quad i < p < k ,$$

$$\beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} \quad \text{for} \quad i > p;\ k > p \quad \text{and for} \quad i < p;\ k < p .$$

References

[1] *H. Kamasová, A. Šimek:* Metoda inverze matice rozdělené na bloky, Aplikace matematiky, 2, 14 (1969), 105–114.
[2] *H. Kamasová, A. Šimek:* Inversion of quasi-triangular matrices, Aplikace matematiky, 2, 15 (1970), 146–148.

Souhrn

ALGORITMUS PRO INVERZI BLOKOVÝCH MATIC

HANA KAMASOVÁ

V článku je uveden algoritmus pro výpočet inverzní matice k blokové matici rozdělené na $r \times r$ bloků.

Věta. *Nechť matice* (1,1) *jsou regulární. Pro* $i, k, p = 1, 2, \ldots, r$ *definujme transformace* (2,1)–(2,5). *Potom platí* (2,6)–(2,9). *Specielně pro* $p = r$ *bloková matice* $[\beta_{i,k}^{(r)}] = \mathbf{A}^{-1}$.

Author's address: Hana Kamasová, Vysoká škola chemicko-technologická, Technická 1905, Praha 6.