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"An Alternate Approach to Find an Optimal Solution of a Transportation Problem."

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Abstract: *The Transportation Problem is the special class of Linear Programming Problem. It arises when the situation in which a commodity is shipped from sources to destinations. The main object is to determine the amounts shipped from each sources to each destinations which minimize the total shipping cost while satisfying both supply criteria and demand requirements. In this paper, we are giving the idea about to finding the Initial Basic Feasible solution as well as the optimal solution or near to the optimal solution of a Transportation problem using the method known as "An Alternate Approach to find an optimal Solution of a Transportation Problem". An Algorithm provided here, concentrate at unoccupied cells and proceeds further. Also, the numerical examples are provided to explain the proposed algorithm. However, the above method gives a step by step development of the solution procedure for finding an optimal solution.*
Keywords: *Transportation problem, Initial basic feasible solution, degenerate solution, Optimality.*

I. Introduction

Transportation problem is the most useful special class of Linear programming problem, which is to be used for different sources of supply to different destination of demand in such a way that the total transportation cost should be minimized. Usually the Initial Basic Feasible solution for any transportation problem is obtained by, North-west corner method (NWCM), Least-cost Method (LCM), Vogel's Approximation Method (VAM). Then after, the optimality of the given transportation problem can be checked by MODI. Since Last few years, many manufacturers had used the optimization technique most frequently in Linear Programming Problem, to solve the real world problems. For that, it is crucial to introduce new approaches that allow the model to fit in to the real world as much as possible.

Transportation problem was first discovered by F.L.Hitchcock [1] in his paper "The Distribution of a product from several sources to numerous Localities" and then after its presenting by T.C.Koopmans[2] in his historic paper "Optimum Utilization of the Transportation system." These two contributions helped in the development of transportation methods which involve a number of shipping sources and a number of destinations.Lai and Huang [1992] considered the situations where all parameters are in fuzzy number. Lai and Huang, 1992 assume that the parameters have a triangular possibility distribution. Bazarra, Jarvis and Sherali [1990] define linear programming problems with fuzzy numbers and simplex method is used for finding an optimal solution of the fuzzy transportation problem. Swarup, Gupta and Mohan [2006] explain the method to obtain sensitivity analysis or post optimality analysis of the different parameters in the linear programming problems.This paper presents a new simple approach to find the optimal solution of a Transportation problem. Proposed algorithm gives the idea about the flow of step by step procedure. Also, the numerical examples are provided here for the better explanation of Algorithm.

II. Mathematical Beyground

Let us consider the standard balanced transportation problem with m sources A_i (with supplies a_i), i ∈ I={1,2,3........,m} and n destinations B_j (with demands b_j), j∈ J={1,2,3,......,n} .
If X_{ij} = the number of load units moving from A_i to B_j , the feasible solution (x) and set of feasible solutions (X) is:
$X=\{x/\sum_{j\in J} X_{ij} = a_i, \forall i \in I; \equiv \sum_{i\in I} X_{ij} = b_j, \forall j \in J; X_{ij} \geq 0\ \forall (i,j);\ \sum a_i = \sum b_j\}$.
Mathematically the problem can be stated as minimize $z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, x_{ij}$ subject to
$\sum_{j=1}^{n} x_{ij} = a_i; for\ i = 1,2, \ldots\ldots, m\ (supply\ constraints)$ And
$\sum_{i=1}^{n} x_{ij} = b_j$ For j=1, 2... n (demand constraints) $X_{ij} \geq 0$ for all i & j.

A transportation problem is said to be balanced if the total supply from all sources equals to the total demands in all destinations i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, otherwise it is called the unbalanced transportation problem.

Transportation Problem

Origins (i)	Destinations (j)				Supply (a_i)
	1	2	....	n	
1	X_{11} C_{11}	X_{12} C_{12}	....	X_{1n} C_{1n}	a_1
2	X_{21} C_{21}	X_{22} C_{22}	....	X_{2n} C_{2n}	a_2
3	X_{31} C_{31}	X_{32} C_{32}		X_{3n} C_{3n}	a_3
....	....	....	....	....	....
M	X_{m1} C_{m1}	X_{m2} C_{m2}	....	X_{mn} C_{mn}	a_m
Demand (b_j)	b_1	b_2	....	b_n	$\sum a_i = \sum b_j$

Proposed Algorithm:

Step: 1 Construct the transportation matrix from the given transportation problem.

Step: 2 Find an IBFS using any one of the method as NWCM, LCM, VAM.

Step:3 Then find the minimum cost only from unoccupied cells (non basic variables) from the matrix.

Step:4 Assign $+\theta$ to the minimum unoccupied cell and start to make a loop with occupied cells. Then find $\theta = min(-\theta)$ and add that min (-θ) value at $+\theta$ and subtract that min (-θ) value from $-\theta$. Then find the cost of the matrix.

Step: 5continue this process unless and until the optimality has been checked by the any one of the following criteria.

(1) Exactly min least cost as per the condition m+n-3in consecutive manner.
(2) If most min cost is skipped, and then also from the next consecutive least min cost should follow the condition m+n-3 in consecutive manner also.
(3) If two most min cost are skipped, and then also from the next consecutive least minimum cost should follow the condition m+n-3.
(4) If any one of the two most minimum costs are skipped then also number of allocations should follow the condition m+n-3.
(5) If any cost is zero in transportation matrix then also as per the criteria no-3 should satisfied.

Step: 6: Now the total minimum cost is calculated as sum of the product of cost and corresponding allocated value of Supply/demand. I.e. Total cost = $\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\ x_{ij}$

Numerical Examples(Proposed Method):

1) Consider the following Cost minimizing Transportation problem**:**

	D_1	D_2	D_3	D_4	D_5	**Supply**
S_1	3	4	6	8	9	**20**
S_2	2	10	1	5	8	**30**
S_3	7	11	20	40	3	**15**
S_4	2	1	9	14	16	**13**
Demand	**40**	**6**	**8**	**18**	**6**	**Total=78**

After Applying the Least Cost Method, for Initial Basic Feasible Solution, the Allocations are as follows.

	D_1	D_2	D_3	D_4	D_5	**Supply**
S_1	**11** 3	4	6	**9** 8	9	**20**
S_2	**22** 2	10	**8** 1	5	8	**30**
S_3	7	11	20	**9** 40	**6** 3	**15**
S_4	**7** 2	**6** 1	9	14	16	**13**
Demand	**40**	**6**	**8**	**18**	**6**	**Total=78**

The minimum cost using LCM is obtained as follows,

Min Cost:**(11*3) + (9*8) + (22*2) + (8*1) + (9*40) + (6*3) + (7*2) + (6*1)**

= 555

Now by applying the proposed Method, allocations are obtained as follows,

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	**20** 3	4	6	8	9	**20**
S_2	**4** 2	10	**8** 1	**18** 5	8	**30**
S_3	**9** 7	11	20	40	**6** 3	**15**
S_4	**7** 2	**6** 1	9	14	16	**13**
Demand	**40**	**6**	**8**	**18**	**6**	**Total=78**

Total Cost obtained by proposed method is as follows,

Total Minimum Cost = (20*3)+(4*2)+(8*1)+(18*5)+(9*7)+(6*3)+(7*2)+(6*1)

= 267

2) Consider the following Cost minimizing Transportation problem**:**

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	**7**
S_2	70	30	40	60	**9**
S_3	40	8	70	20	**18**
Demand	**5**	**8**	**7**	**14**	**Total=34**

After Applying the North West Corner Rule, for Initial Basic Feasible Solution, the Allocations are as follows.

	D_1	D_2	D_3	D_4	Supply
S_1	**5** 19	**2** 30	50	10	**7**
S_2	70	**6** 30	**3** 40	60	**9**
S_3	40	8	**4** 70	**14** 20	**18**
Demand	**5**	**8**	**7**	**14**	**Total=34**

The minimum cost using NWCM is obtained as follows,

Min Cost: (5*19) + (2*30) + (6*30) + (3*40) + (4*70) + (14*20)

=**1015**

Now by applying the proposed Method, allocations are obtained as follows,

	D_1	D_2	D_3	D_4	Supply
S_1	**5** 19	30	50	**2** 10	**7**
S_2	70	**2** 30	**7** 40	60	**9**
S_3	40	**6** 8	70	**12** 20	**18**
Demand	**5**	**8**	**7**	**14**	**Total=34**

Total Cost obtained by proposed method is as follows,

Total Minimum Cost = (19*5)+(2*10)+(2*30)+(7*40)+(6*8)+(12*20)

= 743

3) Consider the following Cost minimizing Transportation problem**:**

	D_1	D_2	D_3	Supply
S_1	6	4	1	**50**
S_2	3	8	7	**40**
S_3	4	4	2	**60**
Demand	**20**	**95**	**35**	**Total=150**

After Applying the North West Corner Rule, for Initial Basic Feasible Solution, the Allocations are as follows.

	D_1	D_2	D_3	Supply
S_1	[20] 6	[30] 4	1	**50**
S_2	3	[40] 8	7	**40**
S_3	4	[25] 4	[35] 2	**60**
Demand	**20**	**95**	**35**	**Total=150**

The minimum cost using NWCM is obtained as follows,

Min Cost :(20*6) + (30*4) + (40*8) + (25*4) + (35*2)

= 730

Now by applying the proposed Method, allocations are obtained as follows,

	D_1	D_2	D_3	Supply
S_1	6	[15] 4	[35] 1	**50**
S_2	[20] 3	[20] 8	7	**40**
S_3	4	[60] 4	2	**60**
Demand	**20**	**95**	**35**	**Total=150**

Total Cost obtained by proposed method is as follows,

Total Minimum Cost = (15*4) + (35*1) + (20*3) + (20*8) + (60*4)

= 555

4) Consider the following Cost minimizing Transportation problem**:**

	D_1	D_2	D_3	D_4	Supply
S_1	4	3	0	5	**8**
S_2	9	7	3	2	**2**
S_3	1	5	4	3	**9**
S_4	6	8	24	16	**2**
Demand	**5**	**7**	**6**	**3**	**Total=21**

After Applying the North West Corner Rule, for Initial Basic Feasible Solution, the Allocations are as follows.

	D_1	D_2	D_3	D_4	Supply
S_1	[5] 4	[3] 3	0	5	**8**
S_2	9	[2] 7	3	2	**2**
S_3	1	[2] 5	[6] 4	[1] 3	**9**
S_4	6	8	24	[2] 16	**2**
Demand	**5**	**7**	**6**	**3**	**Total=21**

The minimum cost using NWCM is obtained as follows,

Min Cost: (5*4) + (3*3) + (2*7) + (2*5) + (6*4) + (1*3) + (2*16)

= 112

Now by applying the proposed Method, allocations are obtained as follows,

	D_1	D_2	D_3	D_4	Supply
S_1	[2] 4	[6] 3	0	5	**8**
S_2	9	[1] 7	3	[1] 2	**2**
S_3	[3] 1	5	[6] 4	3	**9**
S_4	6	8	24	[2] 16	**2**
Demand	**5**	**7**	**6**	**3**	**Total=21**

Total Cost obtained by proposed method is as follows,

Total Minimum Cost = (2*4) + (6*3) + (1*7) + (1*2) + (3*1) + (6*4) + (2*16)

= 94

III. Conclusion

The main aim of this paper is to deriving the optimal transportation cost by using the above proposed method with less number of steps and it's very easy to understand. Based on the optimal solution it allows us to taking a decision effectively. Thus, there are possible extensions to improve our algorithm. The decision maker goes through all the steps of algorithm which makes our approach very useful to be applied in a lot of real world problems.

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