IEEE Photonics Journal
An IEEE Photonics Society Publication

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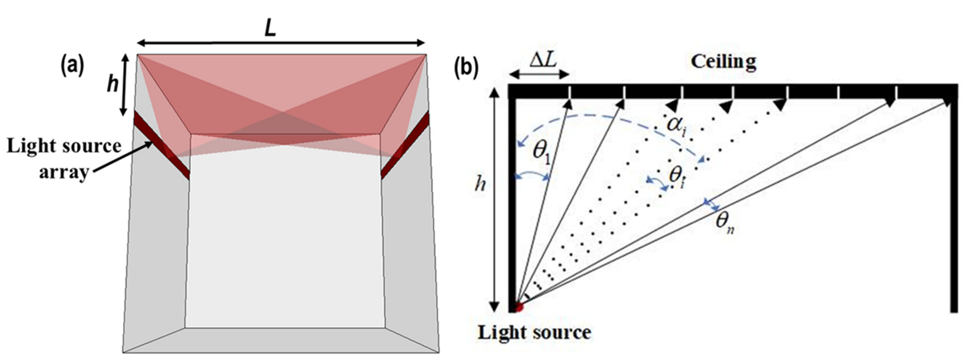
Volume 12, Number 2, April 2020

Duong Thi Giang
Thanh-Son Pham
Quang Minh Ngo, *Member, IEEE*
Van Tang Nguyen
Tran Quoc Tien
Pham Hong Duong



DOI: 10.1109/JPHOT.2020.2979609

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An Alternative Approach for High Uniformity Distribution of Indoor Lighting LED

Duong Thi Giang,[1,2] Thanh-Son Pham ,[1]
Quang Minh Ngo ,[3] *Member, IEEE*, Van Tang Nguyen,[3]
Tran Quoc Tien,[1,2] and Pham Hong Duong[1,2]

[1]Institute of Materials Science, Vietnam Academy of Science and Technology, 18 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam
[2]Graduate University of Science and Technology, Vietnam Academy of Science and Technology, 18 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam
[3]University of Science and Technology of Hanoi, Vietnam Academy of Science and Technology, 18 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam

DOI:10.1109/JPHOT.2020.2979609

Manuscript received January 17, 2020; revised February 27, 2020; accepted March 5, 2020. Date of publication March 9, 2020; date of current version March 25, 2020. This work was supported by a public grant of Ministry of Science and Technology of Vietnam under Project ĐTĐLCN.30/18.
Corresponding authors: Quang Minh Ngo; Thanh-Son Pham (e-mail: ngo-quang.minh@usth.edu.vn; sonpt@ims.vast.ac.vn).

Abstract: A high accurately theoretical analysis for high uniformity illumination distribution on the ceiling of indoor lighting LED system based on the super-positions is proposed and performed. Starting from the non-uniform illumination distribution of two LED sources, the dependences of the uniformity ratio on the height of the LED source and the length of the ceiling are investigated. The low intensity in the region near the side-wall can be avoided by rotating the LED source with a small angle, but the low uniformity illumination distribution at the center of the ceiling still exists. The aim of this work proposes the alternative design and approach for several configurations and demonstrates that can be obtained high uniform illumination distribution. The redistributed factor of the illumination through the lens is also considered and calculated. The investigation of this work will be a general guideline for designing the high uniform LED indoor lighting systems as well as being valuable materials for the LED lighting manufacturing industry.

Index Terms: LED lighting, freeform optics, indoor lighting system.

1. Introduction

As the works presented on the progress of advanced technologies and demand for environmental protection, the light-emitting diodes (LEDs) have been widely used for almost lighting systems, because of environmental benefits such as low energy consumption, compact volume, long lifespan [1]–[3]. The LED lighting system has rapidly replaced the cold cathode fluorescent and incandescent lamps which had been used for a long time. Recently, the LEDs have preferred for designing smart indoor and outdoor lighting systems due to the compact and easy to manipulate characteristics [4]–[6]. In particular, indoor lighting influences to the mood and cognitive performance of the human which reported in previous works [7]–[12]. So that controlling the intensity and color of lights are necessary for creating a positive mood and enhancing the performance of long-term memory

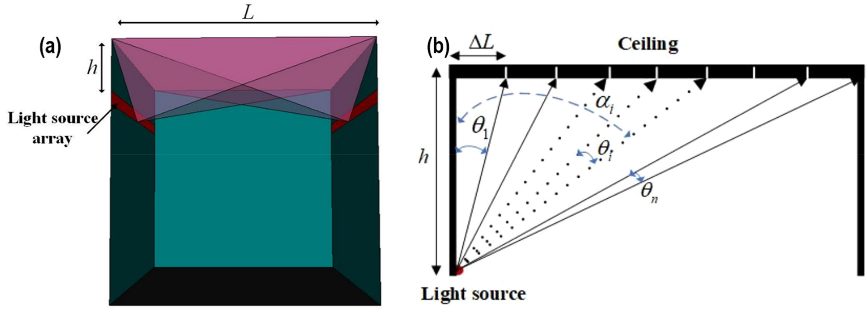

Fig. 1. Schematic of the lighting system (a) Lighting system with two light source arrays mounted on the side-walls and (b) the cross-section from the perpendicular view of the left source.

and problem-solving tasks. Although several studies focused on the design and implementation of smart lighting systems using LEDs [13]–[15], the non-uniform radiation pattern (of the illumination) is still a drawback of the proposed LED lighting systems.

LEDs tend to produce a circular pattern with high intensity at the center and low intensity at the edge of the illuminated area. To avoid the non-uniform radiation distribution of the LED lighting systems, there have been several improvement methods to redistribute the LED patterns [16]–[18]. Both outdoor and indoor lightings, the high uniform illuminances are required for friendly with human eyes which leads to improving cognitive performance. By adding a secondary optical element, the redistribution of the light pattern with high uniformity could be achieved [19]–[23]. In general, the original LEDs are considered as the light sources which have Lambertian characteristics. According to the design reported by Hu *et al.* [24], the illuminance spatial distribution of an LED unit, which was defined by the combination of an LED light source and an optical lens. Tsung *et al.* proposed the design for direct-lit LED backlighting illumination using the Lagrange interpolation to obtain the high uniformity of light distribution [15], Zhu *et al.* designed a freeform optical system for achieving the uniform distribution for road illumination application [25]. Whang *et al.* proposed a uniform illumination system by surface-tailored lens and configurations of LED arrays [26]. A freeform surface lens to obtain the uniform illumination was also proposed by Zhenrong *et al.* [27]. A 3-D free-form lens for uniform rectangular illumination was proposed by Liou *et al.* [28]. Recently, our research group has also presented a simply designed lens for human-centric lighting using LEDs [29]. However, these previous works have not provided a general approach to design a secondary optical lens for indoor illumination systems. Therefore, the general guidelines for designing the uniform indirect illumination have studied continuously.

In this work, we propose a method to accurately calculate the illuminating distribution on the ceiling based on the super-positions of the two side-wall LED system. We start from the highly non-uniform of the illumination distribution on the ceiling from two LED sources. The dependences of the uniformity ratio on the height of the LED position and the length of the ceiling are investigated. Although the LED source is rotated with a small angle to overcome the low intensity in the region near the wall, the low uniformity illumination distribution at the central region of the ceiling still exists. The main of our work proposes the alternative model and approach for several illumination configurations of two LED sources and demonstrates that can be obtained high uniform illumination distribution. The investigation of this work will be a general guideline for designing the LED indoor lighting systems which have high uniformity. This would be useful in practical efficient lighting, friendly with human eyes as well as being valuable materials for the LED lighting manufacturing industry.

2. Model and Theoretical Analysis

The schematic of an indirect lighting system shows in Fig. 1. Two arrays of the light sources are mounted in two side-walls as shown in Fig. 1(a). The lights from two linear sources illuminate to

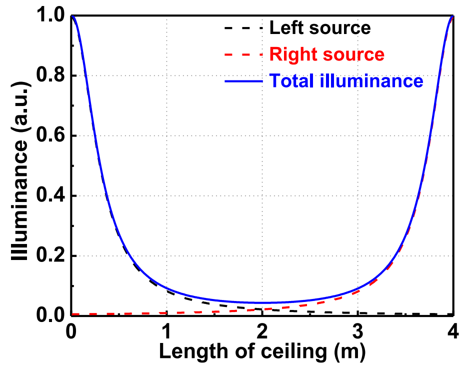

Fig. 2. Light distribution on the ceiling in case of two light sources attached in the two side-walls for $h = 0.3$ m and $L = 4$ m.

the ceiling and then the scattered lights will illuminate the room. The ceiling is made by a material with diffuse reflection property as a Lambertian surface for obtaining the comfortable lighting. In the longitudinal direction, the light sources are arranged in such that uniform distribution is achieved. In this paper, we consider only the transverse direction. Fig. 1(b) is the schematic cross-section of the left lighting system. First, we start from array of light sources of uniform cylindrical distribution attached to the side-wall while each element has a uniform spherical distribution. However, in the perpendicular direction, the light pattern is a circular shape. The light rays from each source will illuminate over the ceiling but in this work we assume that the uniform distribution of the light source in the longitudinal direction so the light rays can simplify for more convenient of calculation. The distance between the light sources to the ceiling is h. The length of the ceiling is L and dividing into n parts of n x ΔL. The angles of θ_{i} and α_i form between ΔL_{i} and the light source and between the right-edge ΔL_i and the side-wall, respectively. Therefore, the light source pattern is divided into n small angles. When the light source has distributed uniformly and n is large enough, the light intensity in the ΔL_i seems to be proportional to θ_i.

The angle formed between the left side-wall and the right edge ray of ΔL_{i} (α_i) can be expressed as

$$\tan(\alpha_i) = i \cdot \frac{\Delta L}{h} \tag{1}$$

where h is the distance between the light source and the ceiling.

The angle of θ_i can be calculated (by this equation $\theta_i = \alpha_i - \alpha_{i-1}$)

$$\theta_i = \arctan\left(\frac{i \cdot \Delta L}{h}\right) - \arctan\left(\frac{(i-1) \cdot \Delta L}{h}\right) \tag{2}$$

As the assumption, the light source has a spherical distribution, the flux density from the source is equally in all directions and the light distribution on the ceiling is proportional to the angle of $i^{th} \Delta L$. Consequently, the light distribution can be formed as shown in Fig. 2. The normalized illuminance on the ceiling for h of 0.3 m and L of 4 m was calculated. As Fig. 2 shown, the light distributions in regions near the side-walls are higher several times compared to that in the parts of the ceiling far from the side-walls. Therefore, using a spherical pattern light source is not efficient for uniform illumination of the indirect lighting system.

In the actual system, the LEDs are used to replace the spherical pattern light source. Most of the LED light patterns have Lambert's distribution, it can be expressed as

$$I_{LED} = I_0 \sin(\alpha_i) \tag{3}$$

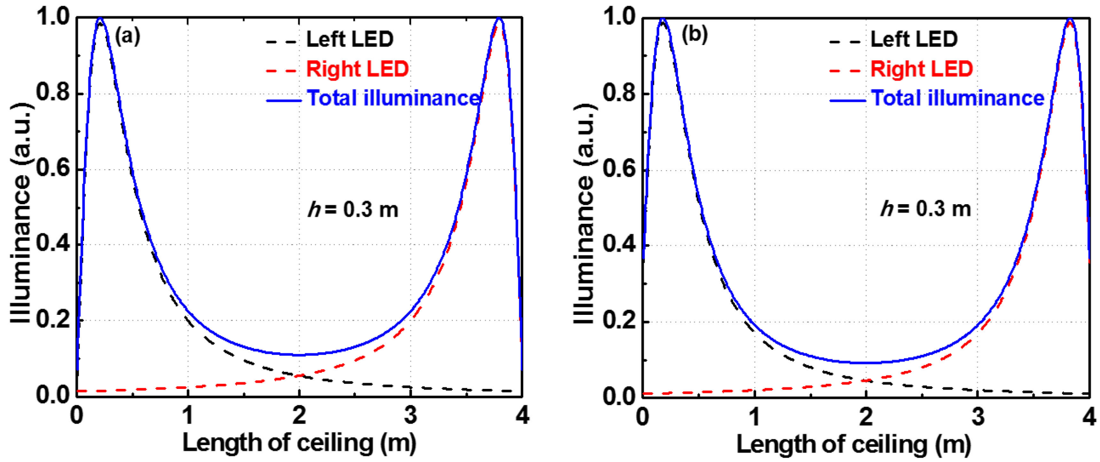

Fig. 3. Light distribution on the ceiling in case of two LED sources mounted on the two side-walls for $h = 0.3$ m and $L = 4$ m. (a) the horizontal LED's source and (b) the LED's line illuminates to the midpoint of the ceiling.

where I_0 is the maximum illumination intensity. The illumination from the LED to the ceiling is the product of Lambert's LED and the spherical pattern light source.

$$I = I_{LED} \cdot \theta_i = I_0 \sin(\alpha_i)\, \theta_i \tag{4}$$

where I is the illumination distribution of an LED on the ceiling at the position ith ΔL.

Fig. 3(a) shows the illumination distribution from two LED sources on the ceiling for the same configuration of Fig. 2. The LED source has a half angle of 120°. The total illumination on the ceiling has a form of batwing shape. The illumination distribution of this configuration is similar to the previous one as shown in Fig. 2, but the difference of spherical pattern light source and the LEDs with Lambert's distribution lead to form a peak of illumination intensity at the distance to the side-wall of around 0.5 m. The peak's intensity is ~10 times higher compared to that in the center of the ceiling. Due to Lambert's characteristics, the region near the wall has an ultra-low intensity and creating a dark region on the ceiling. To solve this problem, the LED is rotated with a small angle so that the LED's line illuminates to the midpoint of the ceiling. Fig. 3(b) shows the illumination on the ceiling when the LED's line illuminates to the midpoint, it corresponds to the angle of 5°. The light intensity in the region near the side-wall is improved while the uniformity of other region does not change. If we rotate the LED source with a large angle, this is not shown exhibit the dark region near the side-wall disappeared but led to reducing the uniformity ratio which is defined by the ratio of the illuminance intensities at the center and its maximum on the ceiling, respectively.

In the actual design, the length of ceiling L and the position of LED source h are not fixed. Next, we consider the dependence of the illumination distribution on h and L. Fig. 4(a) shows the light distribution on the ceiling for $h = 0.3$ m and $L = 8$ m. As we have seen, a non-uniform distribution is obtained, the illuminance is lower at the center of the ceiling compared to the case of $h = 0.3$ m and $L = 4$ m. When the length of the ceiling L is longer, the distance from LED's source to the center of the ceiling is also longer then lead to an angle θ_i (the angle formed between LED's source and ΔL_i) smaller and resulting in lower illumination intensity. The uniformity ratio is inversely proportional to the length of the ceiling. As Fig. 4(b) shown, by keeping the length of ceiling L of 8 m and increasing the height h to 0.8 m, the uniform illumination at the center of the ceiling is better than that of the height h of 0.3 m. Which means that, with the given length of the ceiling L and increases the height h, the reduced difference between θ_i at the center and it's at maximum is achieved. Fig. 4(c) shows the illuminance for the height of 0.8 m and the length of the ceiling of 4 m. The high uniformity ratio is achieved when the angle illuminates from the LED source to the ceiling is more evenly. The mapping of the uniformity ratio depends on the height and the length of

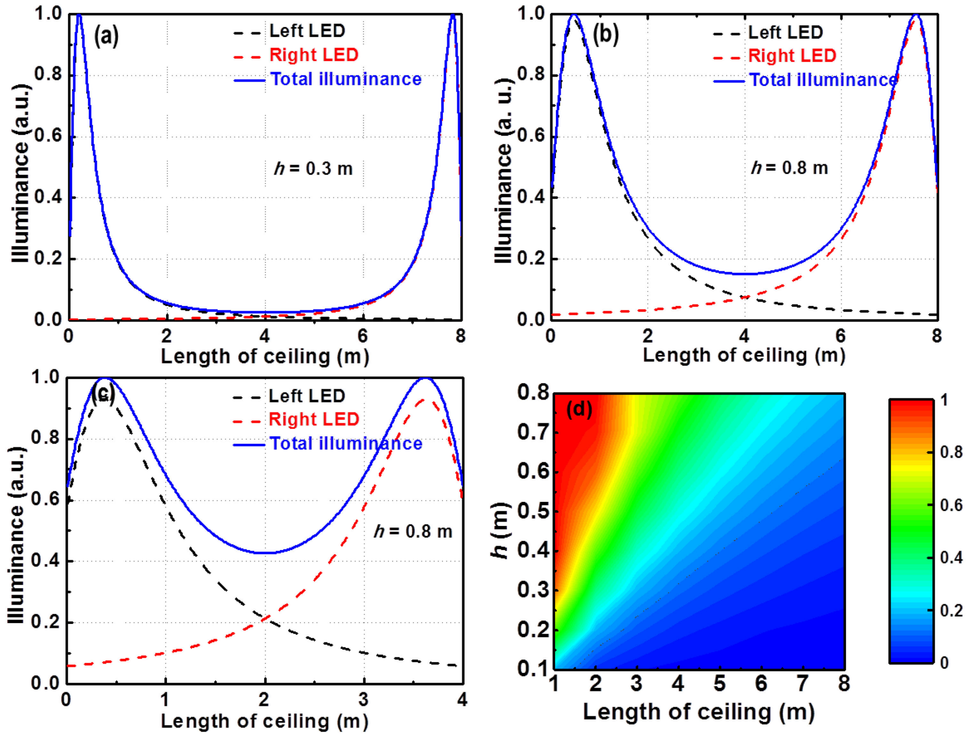

Fig. 4. Light distribution on the ceiling in case of two LED sources located on the two side-walls: (a) $h = 0.3$; $L = 4$m; (b) $h = 0.3$; $L = 8$ m; (c) $h = 0.8$, $L = 4$ m; and (d) Illumination intensity at the central point of the ceiling depending on h and L.

ceiling is performed in Fig. 4(d). To calculate this ratio we sweep h from 0.1 to 0.8 m and L from 1 to 8 m. As it is shown, the uniformity ratio is proportional to the ratio of the height over the length of ceiling. If we want a high uniformity ratio, a high ratio of h over L should be chosen. But the small value of h is expected for the indoor lighting system. So there is a trade-off between uniformity ratio and height.

The non-uniform of illumination on the ceiling is the problem of the LED wall system. For creating a friendly LED lighting system with human eyes, it is required to design the high uniformity distribution on the ceiling equally as much as possible. In this aspect, a secondary optical lens is added after the LEDs to modulate the light pattern of the LED source in achieving a high uniformity illumination.

3. An Alternative Approach for the Uniform Illuminance Lighting System

As the results shown in Section 2, we achieved the non-uniform illumination distribution. For the smart and friendly with the human applications, the uniform light distribution should be expected; therefore, the alternative design and theoretical approach that show the uniform illumination distribution are proposed here. A secondary lens is added after the LED to redistribute the light intensity of the LED source to achieve the uniform total illumination.

Fig. 5(a) shows the illumination distribution from the LED source through the lens system on the ceiling. The black dot line is the illumination distribution from the left side-wall while the red dot line presents for the right side-wall. In this configuration, the light intensity from a single LED source on the ceiling linearly decreases following the length of ceiling. At the position near the side-wall, the intensity is greater than that at the position far from the side-wall. In our designed LED system, two side-walls are used; the total illumination distribution on the ceiling is overlapping from both left and

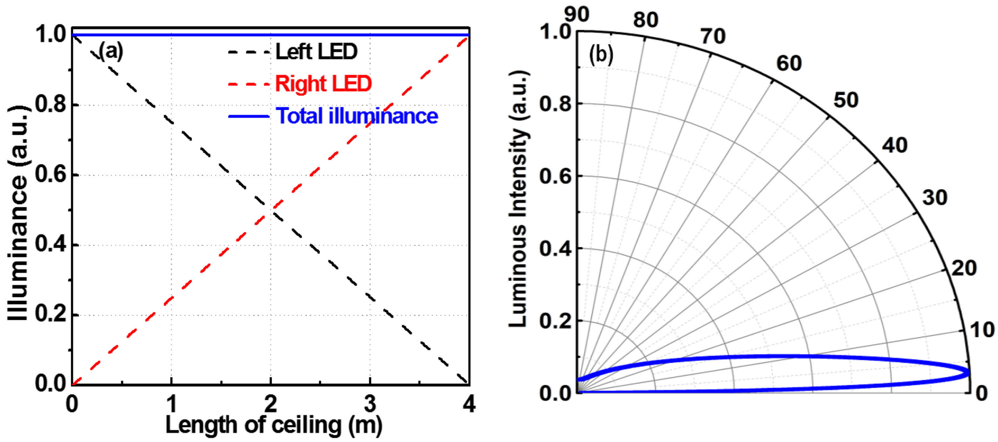

Fig. 5. (a) Illumination distribution of LED source for uniform distribution and (b) luminous intensity of LED.

right illuminations. Therefore, the distribution of illumination from each side is not uniform but they are designed for achieving uniform overlapping illumination.

The light distribution on the ceiling of the LED system with the secondary lens can be express as

$$\begin{cases} F_l = F_0 \left(1 - \frac{i}{n}\right) \\ F_r = F_0 \left(\frac{i}{n}\right) \end{cases} \tag{5}$$

where F_l and F_r are the illumination distribution on the ceiling of the left and right lighting systems, respectively. F_0 is the maximum illumination distribution. We can see that with any value of i and at any position on the ceiling the total illumination overlapping from left and right side-walls has a constant of F_0. F_0 is also the maximum illumination intensity of a LED source.

Based on the illumination distribution in Fig. 5(a) we can design an asymmetric lens along the side-wall using the method reported [29]. Fig. 5(b) shows the luminous intensity of the LED with a lens presenting in angle scale, the angle formed by the horizontal direction and the light ray from the LED to the ceiling. It shows a large factor for a small angle and reduces when the angle increases. In the region near the wall corresponding to the angle of around 90 degrees, a high redistributed factor is required.

To satisfy the linear distribution as presented in Fig. 4, the light from the LED sources have to redistribute with the redistributed factors as shown below.

$$\begin{cases} L_l = \frac{F_l}{I} = \frac{F_0\left(1-\frac{i}{n}\right)}{I_0 \sin(\alpha_i)\left(\arctan\left(\frac{i\cdot\Delta L}{h}\right)-\arctan\left(\frac{(i-1)\cdot\Delta L}{h}\right)\right)} \\ L_r = \frac{F_r}{I} = \frac{F_0\left(\frac{i}{n}\right)}{I_0 \sin(\alpha_i)\left(\arctan\left(\frac{i\cdot\Delta L}{h}\right)-\arctan\left(\frac{(i-1)\cdot\Delta L}{h}\right)\right)} \end{cases} \tag{6}$$

Fig. 6 shows the redistributed factor through the lens. The light intensity from the LED source is non-uniform with a peak near the side-wall as shown in Fig. 3(b) and decline quickly at the position far from the side-wall. So we need to reduce the light intensity at the peak and enhance it at the center of ceiling. We can use that distribution as a guideline to design the lens for the LED system aiming to achieve the uniform illumination of the backlight system.

In general, for more convenient and easy to design the lens to obtain the uniform distribution, the light through the lens can have several types of distribution. The distribution follows the Logistic function as expressed by

$$f(x) = \frac{L}{1 + e^{-k(x-x_0)}} \tag{7}$$

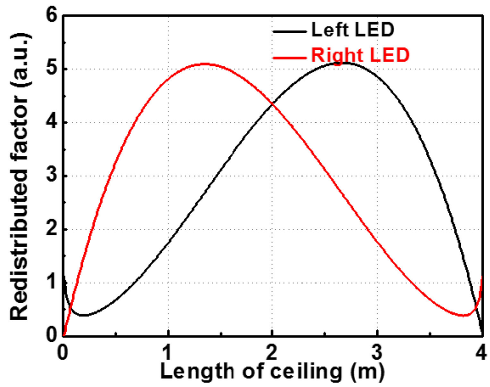

Fig. 6. The redistributed factor of the illumination through the lens.

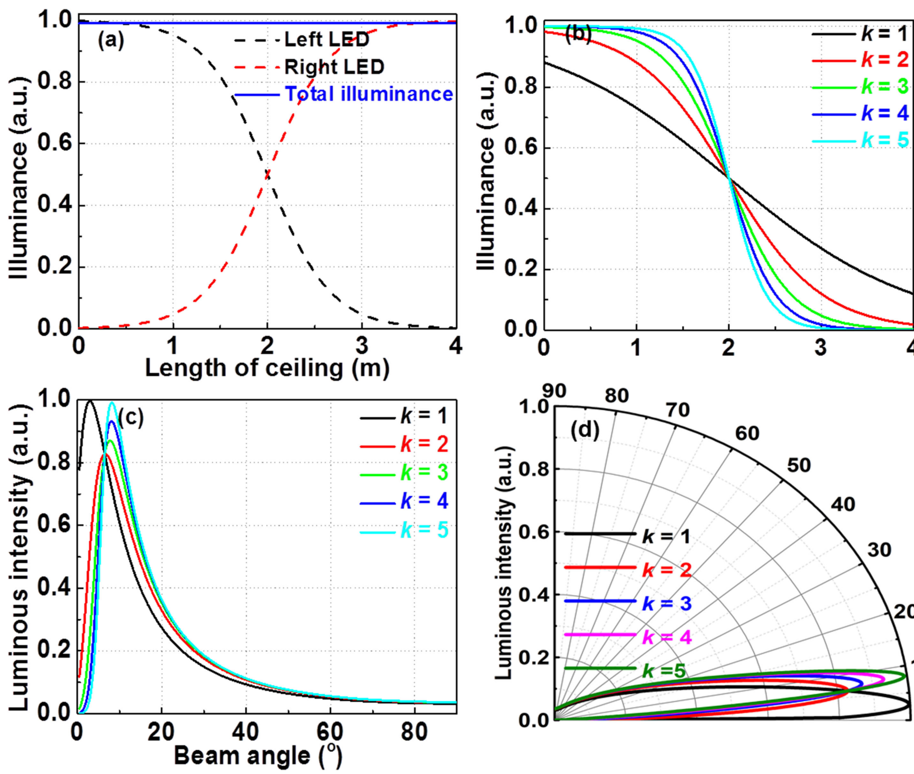

Fig. 7. (a) The light distribution for both left and right LEDs with $k = 3$; (b) the light distribution of the left LED source following Logistic function for several k; and (c) and (d) luminous intensities as the function of beam angle of the left LED source for several k.

where L is the curve's maximum value; k is the Logistic growth rate; x is the position on the ceiling (m); x_0 is the x-value at the midpoint of ceiling (m).

Fig. 7(a) illustrates the distributed illumination of the light through the secondary lens on the ceiling for the left and right sides for $k = 3$. The illuminations of the left and right sides result from Logistic's function. Due to the symmetric system, the total uniform illuminance overlapped from both sides is achieved. Fig. 7(b) shows the illumination distribution of the left side LED source for several k. We can see that the distribution illumination has the different shape. When k is small, the distribution curve becomes the similarly linear. When k increases from 1 to 5, the slope of

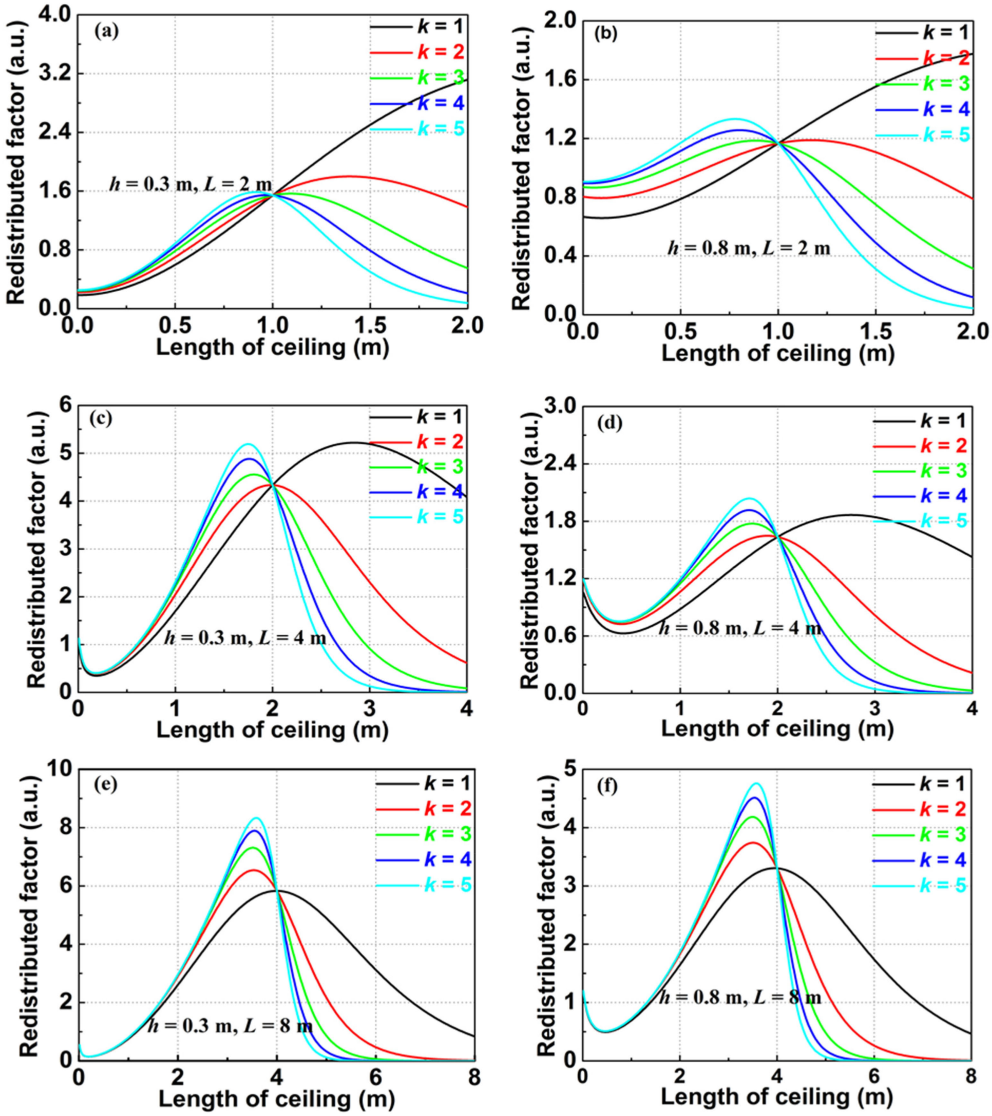

Fig. 8. The redistributed factors of the secondary optical lens for several configurations: (a) $h = 0.3$ m, $L = 2$ m; (b) $h = 0.8$ m, $L = 2$ m; (c) $h = 0.3$ m, $L = 4$ m; (d) $h = 0.8$ m, $L = 4$ m; (e) $h = 0.3$ m, $L = 8$ m; and (f) $h = 0.8$ m, $L = 8$ m.

curve increases. But with any k, the total overlapping illumination is a constant, which means that the uniform illumination on the ceiling is always achieved. Figs. 7(c) and 7(d) show the luminous intensity depending on the angle formed by the horizontal direction and the light ray from the LED to the ceiling (beam angle).

To satisfy the illuminance distribution as presented in Fig. 7, the lights from the LED sources have to redistribute with the redistributed factors as shown in below equations.

$$\begin{cases} RD_l = \frac{f_{(x)l}}{I} = \frac{\frac{1}{1+e^{-k\left(i\cdot\Delta L - x_0\right)}}}{I_0 \sin(\alpha_i)\left(\arctan\left(\frac{i\cdot\Delta L}{h}\right) - \arctan\left(\frac{(i-1)\cdot\Delta L}{h}\right)\right)} \\ RD_r = \frac{f_{(x)r}}{I} = \frac{\frac{1}{1+e^{k\left(i\cdot\Delta L - x_0\right)}}}{I_0 \sin(\alpha_i)\left(\arctan\left(\frac{i\cdot\Delta L}{h}\right) - \arctan\left(\frac{(i-1)\cdot\Delta L}{h}\right)\right)} \end{cases} \quad (8)$$

where $f_{(x)l}$ *and* $f_{(x)r}$ is the light distribution for the left and right LED.

Fig. 8(a) shows the redistributed factor of illumination on the ceiling when a secondary optical lens added in the system of $h = 0.3$ m and $L = 2$ m. For different k, the redistributed illumination from the lens is different. As k decreases, the peak shifts to the right and decreases the sharpness. Fig. 8(b) shows the redistributed factor for $h = 0.8$ m and $L = 2$ m. As it is shown the distributed factor decreases compared to the case of $h = 0.3$ m and $L = 4$ m, because of the higher uniform factor of this configuration before adding the lens. In cases of Figs. 8(c) and 8(d) show redistributed factor for $h = 0.3$ m and 0.8 m, while L increase to 4 m respectively. As mentioned above, the longer length of ceiling leads to the low uniformity ratio, so that a high redistributed factor is needed in this case. The trending peak in that two configurations are similar to Figs. 8(a) and 8(b) but with a higher value. That shows more clearly when further increases the length to 8 m as shown in Figs. 8(e) and 8(f) the redistribution factors are sharp a larger value. The other lengths of the ceiling whose redistributed factors are not shown exhibit the same tendency. The design of the lens would be easily constructed using the redistributed factor. For some value of the redistributed factors, constructing the lens is hard such as the peak of redistributed factor is too sharp. Depending on the real condition and typical application the specific k-value would be chosen for optimum the benefit of the LED system.

4. Conclusions

In this paper, a new model and approach for designing the LED wall systems with high uniformity of backlight illumination for several configurations were presented. For the given wall washing lighting system, we proposed the guiding to design the adding lens for LEDs for achieving the high uniformity illumination. Depending on the purpose of the lighting system, the illumination of the LED on the ceiling could be designed followed the Logistic function with growth rate coefficient k. The numerical analysis was performed to prove the efficiency of this method. With the various k, the intensity illuminations on the ceiling have been shown. Although the illuminance intensity from a LED line of a side-wall is various, but the total unity uniform illumination has been achieved on the ceiling. This work might provide a convenient and time-saving method to design the LED backlight system with a high uniformity of illumination.

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