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An alternative tool to reservoir eutrophication index: fuzzy synthetic evaluation

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Abstract

A general methodology for fuzzy synthetic evaluation is developed and illustrated with a case study of trophic status assessment for Fei-Tsui Reservoir in Taiwan. The historical data base was collected from the management agency of Fei-Tsui Reservoir from 1987 to 1995. The results of this investigation show that the long-term change of water quality and the overturn phenomena cannot be observed with the Carlson index from 1987 to 1992 but is expressed by fuzzy synthetic evaluation. Fuzzy synthetic evaluation is better suited than the Carlson index to rating the trophic status of self-sustaining lakes.

Introduction

The trophic state index is often based on total phosphorous (TP) concentration, chlorophyll a (chl a) concentration, and Secchi disk depth (SD). Of these three factors, chl a plays the most important role, followed by TP. Since eutrophication involves complex changes in the water, the results obtained from using only one parameter may easily mislead or bias the user. For this reason, the multivariable trophic state indexing methods were developed. The most commonly used multivariable index is the Carlson index (Carlson[1]), Multivariable indexing allows for a more thorough investigation of water quality and a more continuous description of the eutrophication process, but several problems are inherent in this method. The first problem is that geographical and atmospheric factors influence eutrophication, causing the standards for each area to differ. Consequently, the three aforementioned indices are heavily regional in nature. Another problem involves unreasonable classification standards. For example, the Carlson index gives TSI =49 and TSI=50 different classifications but TSI=31 and TSI=49 the same. A further problem with multivariables is that it gives different weights to each parameter. Thus, the scientific community has been unable to agree upon a single, reliable trophic state index.

Methodology

Eutrophic, mesotrophic, and oligotrophic standards defined by OECD is unclear; there is a fuzzy zone between the eutrophic, mesotrophic, and oligotrophic states. This is a feature of the problem which can be solved by fuzzy theory. Based on the OECD report (Rast[6]), this study defines three membership functions for the 3 factors (TP, chl a, and SD) that are relative to the water quality index.

1. Determination of membership function

The status of the lake is eutrophic if TP is greater than 40 ug/L according to OECD standards. If TP is lower than 8 ug/L, the status of the reservoir is oligotrophic; and if TP is exactly equal to 20 ug/L, the lake is classified as mesotrophic. Several keypoints defining the value of the membership functions are set as $\mu_{pe}(40)=1$, $\mu_{pe}(20)=0$, $\mu_{pm}(8)=0$, $\mu_{pm}(20)=1$, $\mu_{pm}(40)=0$, $\mu_{po}(8)=1$, $\mu_{po}(20)=0$ where p represents phosphorous; e, eutrophic; m, mesotrophic; and o, oligotrophic. After selecting the keypoints, a membership function should be selected to fit these data sets. The half-T-shaped distribution function is chosen as the TP membership function, and the parameter k must be fixed to improve the similarity between the membership function and real water quality condition. When TP ranges from 8 to 12 ug/L, it is difficult to classify the water body as either oligotrophic or mesotrophic; also, when TP ranges from 28 to 40 ug/L, it cannot be characterized as mesotrophic or eutrophic. The degree of fuzziness for the two states is assigned to a maximum value of 0.5 when TP concentration is exactly equal to 10 and 34 ug/L. Thus, parameter k is determined by eqns (1) and (4), The modified TP membership functions are listed below:

$$\mu_{pe}(x)=((x-20)/2)^{k1} \qquad \text{if} \quad 20 < x < 40 \tag{1}$$
$$\mu_{pe}(x)=1 \qquad \text{if} \quad 40 < x \tag{2}$$
$$\mu_{pm}(x)=1-\mu_{p1}(x)-\mu_{p3}(x) \tag{3}$$
$$\mu_{po}(x)=((20-x)/12)^{k2} \qquad \text{if} \quad 8 < x < 20 \tag{4}$$
$$\mu_{po}(x)=1 \qquad \text{if} \quad x < 8 \tag{5}$$
$$x = 34 , \ \mu_{pm}(x)=\mu_{pe}(x)=0.5 , \ k_1 = 1.943 \tag{6}$$
$$x = 10 , \ \mu_{pm}(x)=\mu_{po}(x)=0.5 , \ k_2 = 3.802 \tag{7}$$

where ***p*** represents total phosphorous; ***e***, eutrophoic; ***m***, mesotrophic; and ***o***, oligotrophic. The membership functions for chl a and SD are similarly given as:

$$\mu_{ce}(x)=((x-5)/5)^{1.943} \qquad \text{if} \quad 5 < x < 10 , \tag{8}$$
$$\mu_{ce}(x)=1 \qquad \text{if} \quad 10 < x \tag{9}$$
$$\mu_{cm}(x)=1-\mu_{ce}(x)-\mu_{co}(x) \tag{10}$$
$$\mu_{co}(x)=((5-x)/12)^{3.802} \qquad \text{if} \quad 8 < x < 20 , \tag{11}$$
$$\mu_{co}(x)=1 \qquad \text{if} \quad x < 8 \tag{12}$$
$$\mu_{sde}(x)=((3-x)/1.3)^{2.642} \qquad \text{if} \quad 1.7 < x < 3.0 \tag{13}$$
$$\mu_{sde}(x)=1 \qquad \text{if} \quad 1.7 > x \tag{14}$$
$$\mu_{sdm}(x)=1-\mu_{sde}(x)-\mu_{sdo}(x) \tag{15}$$
$$\mu_{sdo}(x)=((x-3)/1.5)^{2.235} \qquad \text{if} \quad 3.0 < x < 4.5 \tag{16}$$
$$\mu_{sdo}(x)=1 \qquad \text{if} \quad x > 4.5 \tag{17}$$

where ***sd*** represents secchi disk depth; ***c***, chl a. A final fuzzy membership matrix used for fuzzy synthetic evaluation is derived from synthesizing the trophic membership functions of the 3 factors (TP, chl a, and SD). The matrix can be formulated as the following:

$$R = \begin{bmatrix} \mu_{pe} & \mu_{pm} & \mu_{po} \\ \mu_{ce} & \mu_{cm} & \mu_{co} \\ \mu_{sde} & \mu_{sdm} & \mu_{sdo} \end{bmatrix} \tag{18}$$

2. Determination of weights

Chlorophyll a is the most important factor in the determination of trophic status (Mineeva[4]); total phosphorus is ranked as the second (Nedoma[5]). Therefore, in this study we assume an initial judgment matrix ***J*** for these factors listed below:

$$J = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 2 & 1 & 3 \\ \frac{1}{2} & \frac{1}{3} & 1 \end{bmatrix} \tag{19}$$

A normalized matrix ***D*** is determined by normalizing the initial matrix ***J*** column-by-column. Then, the final weighted vector ***W*** can be derived from summing the elements of each row of matrix ***D*** and normalizing again to this vector.

$$D = \begin{bmatrix} 0.2857 & 0.2727 & 0.3333 \\ 0.5714 & 0.5455 & 0.5 \\ 0.1429 & 0.1818 & 0.1667 \end{bmatrix} \tag{20}$$

$$W = \begin{bmatrix} w_p & w_c & w_{sd} \end{bmatrix} = \begin{bmatrix} 0.2972 & 0.5390 & 0.1638 \end{bmatrix} \tag{21}$$

3. Fuzzy synthetic evaluation

In this study, the evaluation set ***U***={oligotrophic, mesotrophic, eutrophic} contains 3 levels, and the factor set ***V***={chl a, TP, SD} has 3 factors. The evaluation matrix of 3 factors is $\boldsymbol{R}=(u_{ij})_{n\times m}$; the weights set is $\boldsymbol{W}=\{w_p, w_c, w_e\}$. When ***W*** and ***R*** are given, fuzzy synthetic evaluation can be performed following eqn (22):

$$B = W \times R = \begin{bmatrix} W_p & W_c & W_{sd} \end{bmatrix} \times \begin{bmatrix} \mu_{pe} & \mu_{pm} & \mu_{po} \\ \mu_{ce} & \mu_{cm} & \mu_{co} \\ \mu_{sde} & \mu_{sdm} & \mu_{sdo} \end{bmatrix} = \begin{bmatrix} b_o & b_m & b_e \end{bmatrix} \tag{22}$$

The assessment result of the trophic status is determined by the maximum value of the 3 values (b_o, b_m, and b_e). The results of fuzzy synthetic evaluation is summed up with a eutrophication index (***EI***). The ***EI*** value is calculated by eqn (23) and can be plotted versus time to reveal the trend of water quality.

$$EI = 1 \times b_o + 2 \times b_m + 3 \times b_e \tag{23}$$

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Results and discussion

A case study of fuzzy synthetic evaluation of trophic status has been implemented at Fei-Tsui Reservoir, situated in northern of Taiwan. Fei-Tsui Reservoir (surface = 10.24 km^2; mean depth = 39.68 m; hydraulic retention time = 150.8 days) is one of the most detailed, surveyed reservoirs in Taiwan. The historical data base was collected from the management agency of Fei-Tsui Reservoir from 1987 to 1995.

Evaluation results for water quality data of Fei-Tsui Reservoir from January 1987 to June 1996 are summarized in Fig. 3. The pattern of eutrophication occuring around October reveals an overturn effect. Overturn is the circulation of water and nutrients in a lake or reservoir caused by a drop in temperature in the spring and the fall. Because colder water has a higher density, surface water sinks as warmer, deeper water rises to the top, causing nutrients from the bottom of the lake to rise with it and thereby bringing about eutrophication. In Taiwan, winter temperatures do not fall below 4 ℃, so overturn occurs only in the fall. When the pollution loading is low, the effect of overturn on eutrophication becomes significant.

Table 1. Yearly averages of membership values for trophic status

	individual membership value			
year	oligotrophic	mesotrophic	eutrophic	level
1987	0.432	0.568	0.000	mesotrophic
1988	0.001	0.881	0.118	mesotrophic
1989	0.073	0.927	0.000	mesotrophic
1990	0.121	0.879	0.000	mesotrophic
1991	0.209	0.791	0.000	mesotrophic
1992	0.226	0.683	0.091	mesotrophic
1993	0.303	0.697	0.000	mesotrophic
1994	0.280	0.720	0.000	mesotrophic
1995	0.369	0.631	0.000	mesotrophic

The nutrients in the reservoir were self-supplied, the overturn effect was very obvious in early years. Yearly averages show that the mesotrophic value of the membership function decreased while the oligotrophic value increased (see Table 1). These results reveal that good pollution control practices enable the nutrients in a reservoir to be depleted gradually and thus improve water quality. After the nutrient supply is exhausted, overturn becomes less obvious, explaining the absence of said effect for 1993 and 1994.

Eutrophication of Fei-Tsui Reservoir lasted for three months (from Oct.-Dec.) in 1992. Because the reservoir experienced unnaturally high temperatures that winter (water temperature was above 20 ℃), the algae that grew abundantly in October continued to flourish in the following two months and caused eutrophication to be prolonged. The water condition reverted to normal in January 1993 when the temperature decreased.

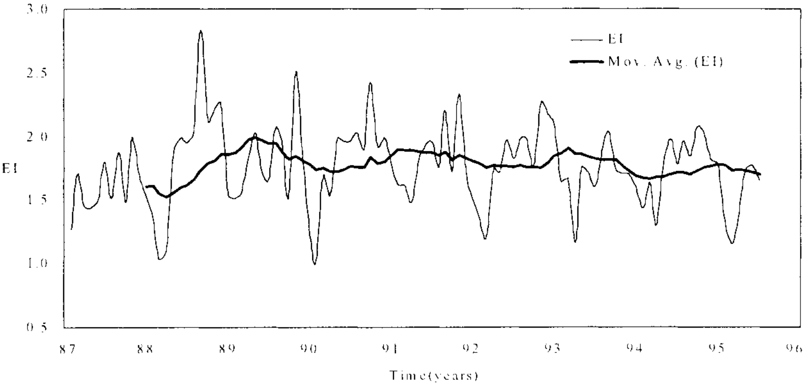

Figure 1: The trend of eutrophication assessed by fuzzy synthetic evaluation for Fei-Tsui Reservoir

The highest eutrophic conditions occurred in the first 6 years (1987-1992) according to the fuzzy synthetic evaluation, peaking in August 1988 and at the end of 1992. When the conventional TSI method is applied to the data sets, the reservoir is classified as mesotrophic for October in 1988, 1990, and 1991. However, fuzzy synthetic evaluation classifies the water body as eutrophic for the same years. Moreover, the water quality for some months was classified as mesotrophic by fuzzy synthetic evaluation, whereas the Carlson index conferred an oligotrophic status. All of the conflicting data for trophic states are listed in Table 2.

A research report (Karydis[2]) reveals that a reservoir water body does not stabilize until the nutrients that existed before the reservoir began operation are exhausted. At least 5 years are needed before the water quality becomes more stable. This phenomenon is also proven by

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this investigation using fuzzy synthetic evaluation. Eutrophication of Fei-Tsui Reservoir caused by overturn stopped occurring after 1991, which is exactly 5 years from the first year of operation (1987). However, the Carlson index is unable to reveal the same information.

Table 2. Comparison of the Carlson index with fuzzy synthetic evaluation

Date	Parameter			Carlson Index		Fuzzy Synthetic Evaluation	
	SD*	TP*	chl a*	TSI	trophic state	EI	trophic state
Oct,89	3.00	13.50	14.47	48.12	mesotrophic	2.51	eutrophic
Oct,91	2.80	9.10	14.39	46.57	mesotrophic	2.33	eutrophic
Oct,92	3.10	8.30	19.83	46.70	mesotrophic	2.27	eutrophic
Nov,92	4.00	6.50	13.14	42.97	mesotrophic	2.18	eutrophic
Dec,92	4.40	5.00	11.69	40.90	mesotrophic	2.10	eutrophic
Mar,87	2.90	14.70	1.29	40.78	mesotrophic	1.45	oligotrophic
Apr,87	3.00	14.20	1.88	41.68	mesotrophic	1.44	oligotrophic
Jan,89	5.20	12.60	2.47	39.37	oligotrophic	1.51	mesotrophic
Jun,93	5.96	11.30	2.90	38.72	oligotrophic	1.61	mesotrophic
Dec,87	2.00	17.10	1.71	44.20	mesotrophic	1.54	oligotrophic
Dec,88	2.00	17.10	1.71	44.20	mesotrophic	1.54	oligotrophic

* SD represents Secchi disk depth (m); TP, total phosphorous (ug/L); chl a, chlorophyll a (ug/L)

A comparison of fuzzy synthetic evaluation with Carlson indexing reveals that the latter is more sensitive to changes in SD and TP. The Carlson index provides an average index for SD, TP, and chl a but does not weigh each factor by its relative influence on eutrophication. As a consequence, the effects of SD and TP become amplified. The eutrophic status conferred on the reservoir for November 1988 using this index is due to the extremely low SD value. Considering individual factors alone, however, the water is mesotrophic according to TP and oligotrophic based on chl a. Therefore, the unlimited amount of influence that an individual factor may exert on the index and the failure to weigh each factor, which may create inaccurate results, are the disadvantages of this method. The low SD and high TP values obtained in November 1988 may not necessarily have been indications of eutrophication but rather were caused by a high concentration of inorganic suspended solids. In fuzzy synthetic evaluation, a comparison of membership values shows that the level of eutrophication in the reservoir decreased annually. Neither this trend nor overturn is observable with the Carlson index.

Thus, fuzzy synthetic evaluation is better suited than the Carlson index to rating the trophic status of self-sustaining lakes.

Conclusions

In this application, an alternative tool to reservoir eutrophication index is developed and the utility of the tool is compared with the Carlson index. The conclusions are summarized as the following:

1. The result of fuzzy synthetic evaluation shows that the water quality of the Fei-Tsui Reservoir was unstable, which was caused by a short-term overturn effect from 1987 to 1992. However, the Carlson index is unable to reveal the same information.
2. Fuzzy synthetic evaluation can be used as an alternative tool in trophic state assessment, especially for self-sustaining reservoirs.
3. Fuzzy synthetic evaluation is more sensitive to a variation in water quality than the Carlson index.

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