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# An Analog Computer Simulation of a Closed Brayton Cycle System

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A summary of the Mathematical Model, Analog Computer Simulation, and experimental information comparison of a representative Closed Brayton Cycle System is presented. The foundation of the simulation are design, experimentation, and analysis efforts performed on full-sized Closed Brayton Cycle System components. The analog computer program shows total system interaction and component behavior, both steady-state and dynamic modes, over the entire system operating range. In order to accurately describe a complete Closed Brayton Cycle System, practical details such as auxiliary components, system parasitic losses, variation of thermodynamic properties, thermal inertia, variability of pressure losses, and transient disturbances constitute an important part of the formulation. The development of the analog computer simulation provides an economical and effective method for system performance prediction, identification of critical parameters, systems integration, and component optimization.

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# An Analog Computer Simulation of a Closed Brayton Cycle System

F. D. JORDAN M. R. HUM A. N. CARRAS



INTRODUCTION

An analog computer simulation of a representative Closed Brayton Cycle System was developed to complement the experimental investigation and analysis of compact power conversion systems. The design, performance, and operating characteristics of closed-cycle gas turbines and auxiliary components were integrated into the mathematical model and analog computer simulation. Operating experience and experimentation performed on full-size components at the Advanced Power Conversion Experimental Facility, Fort Belvoir, Virginia, and the Advanced Power Conversion Skid Experiment, San Ramon, California, provided information to validate the analog computer simulation. The influence of component interaction was determined from the compact configuration of the APCSE, where recently, a 1200-hr system demonstration test was completed.

The basic design approach was to determine or develop a general mathematical representation of the system components, then adapt the mathematical model to the specific power conversion configuration utilizing empirical scaling factors or physical constants, and finally compare the analog computer results with the experimental definition. The mathematical model and analog program were formulated in a manner to allow comparison, modification, and expansion. Both steady-state and dynamic behavior were simulated over the entire operating range. The dynamic simulation includes an investigation of two speed control modes: 1) a constant mass system using turbine bypass alone, and 2) a variable mass system utilizing working

NOMENCLATURE

$C$ = variable coefficient (subscripts 1 through 8 refer to the compressor analysis, subscripts 9 through 20 to the equations defining losses)

$F(1)$ = nonlinearity used in compressor definition

$F(2)$ = nonlinearity used in compressor definition

$H_A'$ = alternator power, Btu/sec

$H_B'$ = bearing power, Btu/sec

$H_C'$ = compressor power, Btu/sec

$H_G'$ = reduction gear, Btu/sec

$H_{LA}'$ = alternator losses, Btu/sec

$H_T'$ = turbine power, Btu/sec

$H_{WC}'$ = compressor windage losses, Btu/sec

$H_{WS}'$ = turbine windage losses, Btu/sec

$H_{wt}'$ = turbine windage losses, Btu/sec

$H_\Sigma'$ = turbine-compressor power summation, Btu/sec

$I$ = combined amount of inertia referred to turbine-compressor shaft speed, lb-sq ft

$K_1$ = proportional gain, percent/rpm

$K_1{}^1$ = integral gain, percent/rpm-sec

$K_{WL}$ = external alternator load, kw

$L$ = bypass valve position referred to full travel, percent

$M_{11}$ = compressor weight flow, lb/sec

$M_{71}$ = turbine weight flow, lb/sec

$M_{BY}$ = bypass weight flow, lb/sec

$N$ = shaft speed referred to turbine-compressor axis, rpm

$P_{11}$ = compressor inlet pressure, psia

$P_{19}$ = compressor exit pressure, psia

$P_{79}$ = turbine exit pressure, psia

$R_c$ = compressor pressure ratio

$R_T$ = turbine pressure ratio

$T_{11}$ = compressor inlet temperature, deg R

$T_{19}$ = compressor exit temperature, deg R

$T_{71}$ = turbine inlet temperature, deg R

$T_{79}$ = turbine exit temperature, deg R

$s$ = differential operator, sec$^{-1}$

$t$ = time, sec

$\Delta H_C$ = compressor enthalpy difference, Btu/lb

$\Delta T_C$ = compressor temperature difference, deg R

$\Delta\eta_C$ = compressor efficiency variation due to Reynolds number

$\eta_C$ = compressor efficiency

$\eta_c{}^*$ = compressor efficiency due to off design corrected speed and mass flow

$\tau_1$ = sensor time constant, sec

$\tau_2$ = control time constant, sec

$\tau_3$ = actuator time constant, sec

fluid admission, withdrawal, and turbine bypass. In order to accurately describe a complete Closed Brayton Cycle System, practical details such as variation of thermodynamic properties, auxiliary components, system parasitic losses, thermal inertia, variability of pressure loss, and transient disturbances constitute an important part of the formulation.

The development of the analog computer simulation provides an economical and effective method for system performance prediction, identification of critical parameters, systems integration, and component optimization. In addition, the simulation can be used to predict behavior in regimes when physical testing has not been performed.

Comparison of the analog computer projections with actual experimental information demonstrates that the simulation is accurate, repeatable, and representative (1).[1]

## BACKGROUND

The U. S. Army Engineer Reactors Group has been engaged in a continuing technology development program since 1955 to investigate Closed Brayton Cycle Systems for potential military electrical power applications in the 100 to 1000 kwe class (2). During the initial phases of the program, conceptual information was derived from commercial closed-cycle gas turbine experience in Europe, primarily, from the work of Escher Wyss Ltd. (Switzerland), and its licensees.

The Advanced Power Conversion Experimental Facility (APCEF) was the first operational Closed Brayton Cycle System in the United States and is utilized for individual component evaluation and experimentation. Since a primary objective is to demonstrate a compact, lightweight power plant configuration, system integration tests are performed at the Advanced Power Conversion Skid Experiment (APCSE).

Recently, the Closed Brayton Cycle concept has gained importance and emphasis for commercial power plant, spacecraft auxiliary power, and undersea missions (3-5). Nuclear heat source Closed Brayton Cycle Systems are inherently suitable for space-oriented electrical power, undersea, and terrestrial electrical power applications (6). The formulation of this mathematical model and analog computer simulation makes it readily adaptable for these related applications.

## DESCRIPTION OF SYSTEM

The operation of the basic recuperated

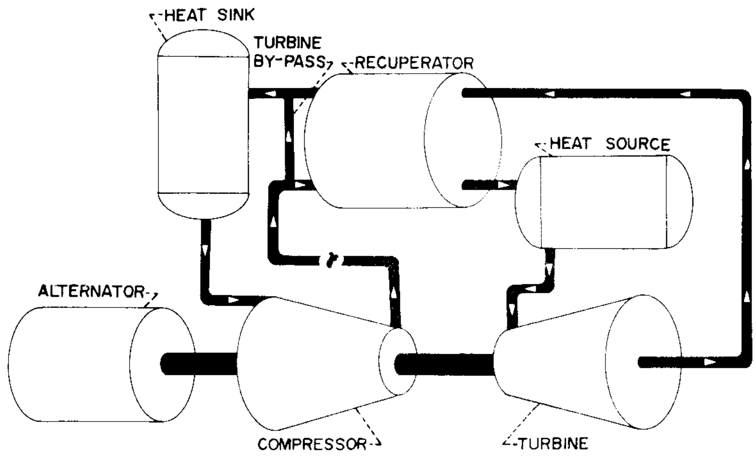


Fig.1 Recuperated Closed Brayton Cycle System

Closed Brayton Cycle System is presented schematically in Fig.1. Selecting the compressor as a starting location, the working fluid leaves the compressor discharge, flows through the high pressure side of the recuperator where heat is transferred from the turbine exhaust gas, and then proceeds to the heat source. Heat is imparted to the working fluid, expanded through the turbine to produce net work, flows through the low pressure side of the recuperator, and continues to the sink heat exchanger. Waste heat is rejected in the heat sink to a separate medium, and the working fluid then returns to the compressor inlet to complete the cycle.

A summary of the design parameters of major components available for the analog computer simulation is shown in Table 1.

Several factors regarding the interpretation of Table 1 should be noted. The design parameters reflect numerous modifications based on operating experience. Considering the time frame of the inception of the U. S. Army Closed Brayton Cycle Program, many of the system constraints appear conservative and well within the capabilities of present technology. The components with APCEF designations are physically separated to facilitate individual component investigation and modification. Several of the APCEF auxiliary components, in particular the heat exchangers, are overcapacity for the application in order to permit a broad experimental latitude and, therefore, should be considered only as experimental tools.

One of the basic objectives of the analog computer simulation was to define and optimize the system speed control technique. A distinguishing feature between the APCEF and the APCSE is that the compact APCSE utilizes only a constant mass control system.

The APCSE automatic speed control concept is

[1] Underlined numbers in parentheses designate References at the end of the paper.

Table 1 Design Values of Major Components Built and Tested For Army Closed-Cycle Program

TURBO-COMPRESSOR SETS (** NITROGEN GAS)

| Manufacturer | Fairchild-Hiller | Fairchild-Hiller | Fairchild-Hiller | Dresser Industries | General Electric |
|---|---|---|---|---|---|
| Designation | TCS 560-B | TCS 670-2 | TCS 670-B | CSN-2 | CSN-1A(M) |
| Shaft Output, H.P. | 330 | 497 | 575 | 592 | 480 |
| Working Fluid | ** | air or ** | ** | Air or ** | Air or ** |
| Turbine Efficiency | 79 | 81 | 85 | 74 | 86 |
| Compressor Efficiency | 78 | 79 | 79 | 77 | 81 |
| No. Comp. Stages | 2-Cent. | 2-Cent. | 2-Cent. | 11-Axial | 11-Axial |
| No. Turb. Stages | 2-Axial | 2-Axial | 2-Axial | 2-Axial | 2-Axial |
| Comp. In. Temp., °F | 100 | 130.6 | 130.6 | 130.6 | 119.6 |
| Comp. In. Press., psia | 68.6 | 119.0 | 119.0 | 119.0 | 119.0 |
| Comp. Press Ratio | 2.91 | 2.85 | 2.85 | 2.72 | 2.83 |
| Comp. Mass Flow Rate, #/sec. | 15.4 | 26.3 | 26.3 | 26.3 | 27.22 |
| Turb. In. Temp., °F | 1200 | 1200 | 1200 | 1200 | 1200 |
| Turb. In Press., psia | 184 | 311.5 | 311.5 | 294 | 302 |
| Turb. Exp. Ratio | 2.53 | 2.52 | 2.52 | 2.38 | 2.35 |
| Turb. Mass Flow Rate,#/Sec. | 15.0 | 25.65 | 25.65 | 26.0 | 26.68 |
| Shaft Speed, RPM | 18,000 | 18,338 | 18,338 | 22,000 | 22,000 |

HEAT EXCHANGERS (PRECOOLERS)

| | | Baldwin-Lima Hamilton Corp. | Griscom-Russell Co. | Stewart-Warner Corp. |
|---|---|---|---|---|
| Manufacturer | | Baldwin-Lima Hamilton Corp. | Griscom-Russell Co. | Stewart-Warner Corp. |
| Designation | | APCEF Precooler | ML-1 Precooler | APCSE Precooler |
| Description | | Finned Tube | Finned Tube | Plate-Fin |
| Flow Geometry | | Cross Flow | Cross Flow | Cross Flow |
| Heat Transfer Surface | | O.D./I.D.Finned Tube | O.D./I.D.Finned Tube | Perforated Fin |
| Heat Trans. Capacity $10^6$ BTU/hr. | | 10.73 | 8.22 | 9.30 |
| Effectiveness, % | | 97.5 | 92.0 | 94.5 |
| Core Material | | 3003 Aluminum | Aluminum | 3003 Aluminum |
| Core Dimensions, LxWxH | | 157" x 120" x 19" | 87" x 120" x 10" | 112" x 125" x 14" |
| Weight (Total), lbs. | | 13,750 | 5,290 | 4,600 |
| High Pressure Side | Working Fluid | Nitrogen Gas | Nitrogen Gas | Nitrogen Gas |
| | Flow Rate, lb./sec | 30.0 | 26.3 | 27.0 |
| | Inlet Temp., °F | 500 | 477 | 500 |
| | Inlet Press., psia | 135 | 123 | 120 |
| Low Pressure Side | Working Fluid | Ambient Air | Ambient Air | Ambient Air |
| | Flow Rate, lb./sec. | 97.2 | 68.7 | 68.7 |
| | Inlet Temp., °F | Atmospheric | Atmospheric | Atmospheric |
| | Inlet Press., psia | 14.7 | 14.7 | 14.7 |

HEAT EXCHANGERS (RECUPERATORS)

| | | | | |
|---|---|---|---|---|
| Manufacturer | | AiResearch Mfg. Div. Garrett Corp. | Ferrotherm | Griscom-Russell Co. |
| Designation | | APCEF Recuperator | GTTF Recuperator | APCSE Recuperator |
| Description | | Plate-Fin | Pin-Fin | Shell and Tube |
| Flow Geometry | | Counterflow | Counterflow | Cross-counterflow |
| Heat Transfer Surface | | Offset Rectangular Fin | Strip Pin-Fin | O.D. Finned Tube |
| Heat Transfer Capacity, $10^6$ BTU/hr. | | 17.5 | 7.75 | 10.43 |
| Effectiveness, % | | 84.6 | 83.2 | 79.0 |
| Core Material | | 347 SS | Armco Iron | 304 SS |
| Core Dimensions, LxWxH | | 36" x 36" x 36" | 47" x 30" x 27" | 50" x 47.5" dia. |
| Weight (Total), lbs. | | 13,550 | 2,500 | 5,220 |
| High Pressure Side | Working Fluid | Nitrogen Gas | Nitrogen Gas | Nitrogen Gas |
| | Flow Rate, lb/sec | 30.0 | 19.6 | 26.3 |
| | Inlet Temp., °F | 340 | 329 | 369 |
| | Inlet Press., psia | 400 | 220 | 330 |
| Low Pressure Side | Working Fluid | Nitrogen Gas | Nitrogen Gas | Nitrogen Gas |
| | Flow Rate, lb/sec | 30.0 | 19.6 | 26.3 |
| | Inlet Temp., °F | 1080 | 847 | 909 |
| | Inlet Press., psia | 140 | 83 | 123 |

based on maintaining a constant inventory of working fluid at a constant turbine inlet temperature, and controlling power output and turbine-compressor speed by automatically diverting gas from the compressor discharge to the precooler inlet through a bypass control valve, thus bypassing the heat source and the turbine (8). This concept was chosen instead of a combination variable mass system for rapid response characteristics, low auxiliary power requirements, and the elimination of the transfer and storage system (8). Testing of the constant mass speed control technique demonstrated that a 300-kw Closed Brayton Cycle System, suitable for use with a nuclear reactor, can be satisfactorily controlled automatically by means of turbine/heat source bypass-control in a manner to provide frequency regulation sufficiently precise for operating modern electronic equipment (9). The analog computer simulation predicated and visually described this experimental conclusion.

For essentially economic reasons, emphasis in the analog computer simulation was placed on two discrete configurations: (a) TCS-560B turbine-compressor set, APCEF Precooler and APCEF Recuperator, and (b) TCS-670-2, APCEF Precooler and APCEF Recuperator.

## MATHEMATICAL MODEL

The physical behavior of a system or system components can be described by governing analytical and empirical equations. The solution of these equations under variable operating conditions defines the system performance and behavior in response to prescribed input parameters.

The complete mathematical analysis and analog simulation of the representative Closed Brayton Cycle System includes the influence of the following variables:

1. Thermodynamic properties of the working fluid.
2. Maximum cycle temperature.
3. System pressure level.
4. System pressure losses and distribution.
5. Individual component efficiencies (turbine, compressor, and so forth).
6. Turbine-compressor set rotating speed.
7. Parasitic losses.
8. Recuperator effectiveness.
9. Heat sink characteristics.

The mathematical model is used in the analog simulation to evaluate the steady-state operating characteristics and to analyze variations of steady-state performance in the closed-loop system through selected changes of individual component characteristics and system conditions. The analog simulation is also used to explore techniques of complete system and subsystems control methods.

## ANALOG SIMULATION

### General

One approach toward a general simulation is the use of a framework that allows for the interconnection and insertion of experimental data of any number of components defined in a conventional manner. Examples of these are the actual maps of efficiency and pressure ratio versus corrected speed and mass flow for the compressor and turbine. Frequently, experimentally derived performance characteristics are not available until a time when the information is of little practical value for simulation purposes.

The basic analytical approach used herein was to formulate a mathematical model, flexible enough in scope, to define a representative Closed Brayton Cycle System and having the additional capability for correlation of component performance and system interaction influences, when available.

The basic analog computer simulation was developed with limited input information such as design parameters and system constraints. A detailed evaluation of the total mathematical model was made accounting for all equations, input variables, computational sequence, and interactions. This permits the simulation to be developed prior to, or in parallel with, experimentation and used for total system and individual component evaluation and performance predication, indication of critical parameters, analysis of control concepts, and system optimization. To obtain a more realistic representation, operating experience and experimental information is integrated and correlated to refine the simulation.

The simulation takes cognizance of effects such as duct thermal inertia, compressibility effects, and Reynolds Number variations in the turbine-compressor set. Entire components may be substituted within the simulation or discrete phenomena varied with the selected outputs under continuous monitoring over large operational band widths.

An analog computer was chosen for the simulation because of its ability to function in parallel under continuous interruption by the operator for parameter optimization, its flexible output media, low cost per solution, and availability of particular sized machines that permitted optimum price utilization of equipment for subsystem checkout.

The mathematical model may be divided into three principal, inter-related sections:

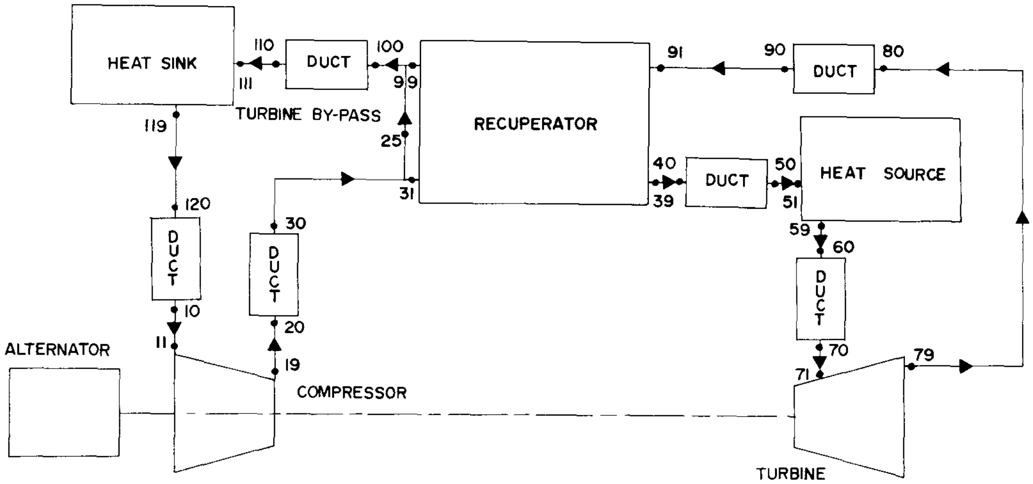


Fig.2 Closed Brayton Cycle System flow diagram

1 The Closed Brayton Cycle System components which include the turbine compressor set, auxiliary components, recuperator, heat source, heat sink, and associated ducting. A system flow schematic with appropriate station designation is shown as Fig.2.

2 The definition of parasitic and mechanical losses.

3 The control systems analysis which consists of a constant mass mode utilizing only turbine bypass flow, and a variable mass mode which employs the admission or withdrawal of system working fluid in combination with turbine bypass as the manipulated variables.

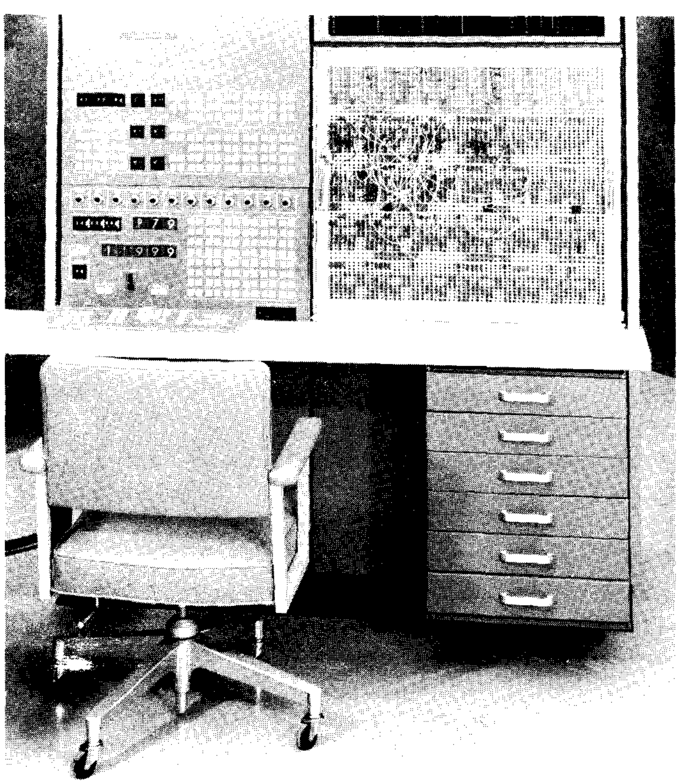

Fig.3 EAI-680 analog computing system

The analog computer simulation required that all components of the system be described in sufficient mathematical detail in order that the analog model would simulate systems operation over the full potential operating conditions.

A complete description of the actual mathematical model and the analog computer simulation, because of size and complexity, is impractical for this discussion. Therefore, a discrete portion of each of the three sections of the simulation, namely the compressor, mechanical losses and shaft dynamics, and turbine bypass control system, have been selected for definition and discussion. A detailed presentation of the entire mathematical model and analog simulation is found in reference (1).

The overall methodology is based on:

1 The separation of the three sections into groups of discrete parameters.

2 The derivation of preselected coefficients for the component defining equations having the characteristic that the variance of these will have qualitatively predictable effects on performance.

The discrete representation of each component, by definition, precludes mathematical normalization through variable manipulation and combination; however, this procedure has several advantages which include:

1 The investigation, and by proper computer simulation, the simplification of individual definitions. On the average, equipment usage for simulation purposes was reduced by about 50 percent through the use of transfer functions and the elimination of redundant definitions with no resulting loss in model accuracy for the operating range investigated.

2 The substitution of updated components into the model with a minimum of reprogramming. Representative of this was the replacement of the TCS-560B turbine-compressor set with the TCS-670-2 during the study.

3 The use of sub-programs which could be checked out individually, and resulted in a considerable time savings during the debugging phases.

4 The utilization of only the pertinent portion of the model for each particular study phase. The complete system was needed for the evaluation of steady-state characteristics; however, in the optimization of the speed control, the relatively slow response of the recuperator and ducting thermal inertia dynamics could be eliminated or linearized.

EAI TR-20, TR-48, and 231R analog computing equipment, either individually or in combinations, were used in the evaluation of various sub-programs.

An EAI 680 Analog Computing System, shown in Fig.3, was used for the simulation. The equipment complement for the final version of the model consisted of 200 operational amplifiers, 40 quarter square multipliers, and 18 diode function generators. Potentiometers, fixed function generators, and other auxiliary equipment usage were normal for a simulation of this size.

The analog computer representation of the mathematical model is scaled in machine units such that any commercial equivalent analog computer could be substituted with the minimum changes in wiring diagrams and machine equations.

ANALOG COMPUTER SIMULATION

Compressor

The compressor, selected as a component example, may be defined by the following eight equations and three nonlinear functions. The subscripted $C_s$ represent the coefficients that may be varied to tailor the component results for correlation purposes.

$$\frac{\Delta H_c}{T_{11}} = C_1 F(1) \left(\frac{N}{\sqrt{T_{11}}}\right)^{C_2} \tag{1}$$

where

$$F(1) = f\left[ \frac{\dfrac{C_3 M_{11} \sqrt{T_{11}}}{P_{11}}}{\left(\dfrac{N}{\sqrt{T_{11}}}\right)^{C_4}} \right]$$

$$\eta_c = \eta_c^* + \Delta\eta_c \tag{2}$$

$$\eta_c^* = F(2) - C_5\left(\frac{N}{\sqrt{T_{11}}} - C_6\right) \tag{3}$$

where

$$F(2) = f\left[ \frac{\dfrac{C_3 M_{11} \sqrt{T_{11}}}{P_{11}}}{\left(\dfrac{N}{\sqrt{T_{11}}}\right)^{C_4}} \right]$$

$$\Delta\eta_c = C_6\left[1 - C_7\left(\frac{N}{\sqrt{T_{11}}} \times \frac{P_{11}}{T_{11}}\right) - \frac{1}{5}\right] \tag{4}$$

$$\frac{\Delta T_c}{T_{11}} = \frac{\Delta H_c}{T_{11}} \times C_8 \tag{5}$$

$$R_c = F(3) = \frac{P_{19}}{P_{11}} \tag{6}$$

where

$$F(3) = f\left(\eta_c \frac{\Delta T_c}{T_{11}}\right)$$

$$T_{19} = \frac{\Delta T_c}{T_{11}} T_{11} + T_{11} \tag{7}$$

$$\frac{H^1_c}{T_{11}} = \frac{\Delta H_c}{T_{11}} M_{11} \tag{8}$$

Background material, derivations, and a complete listing of the equations used in the mathematical model may be found in references (10,11). Fig.4, a compressor computational flow diagram based on the relationships described in the references, is included without a detailed explanation in order to indicate the completeness of the original mathematical development. The equations are sophisticated enough to allow selection between axial or centrifugal type compressors through manipulation of proper variable coefficients which appear as potentiometer settings on the analog computer model.

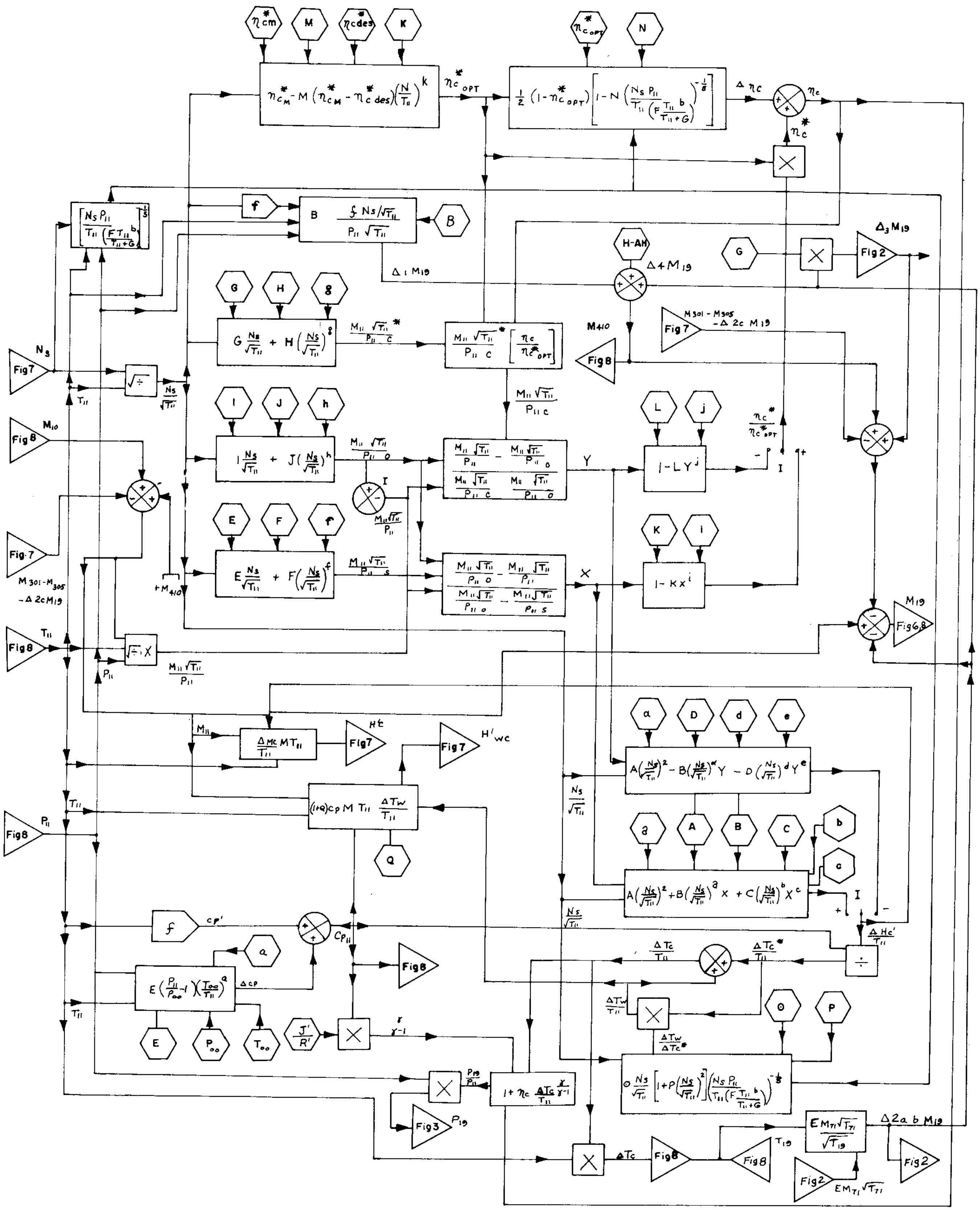

Fig.4 Compressor computational flow diagram

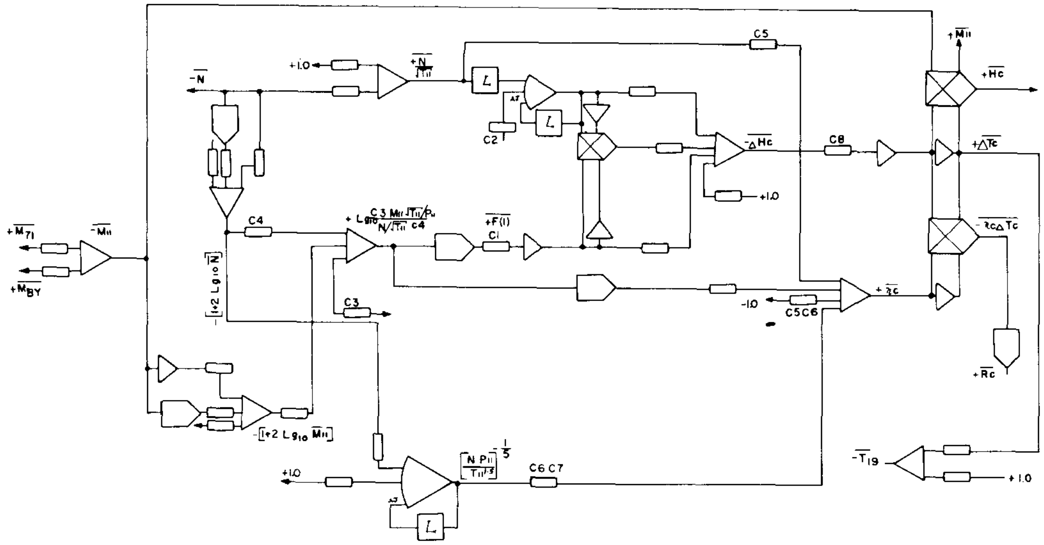


Fig.5 Compressor wiring diagram

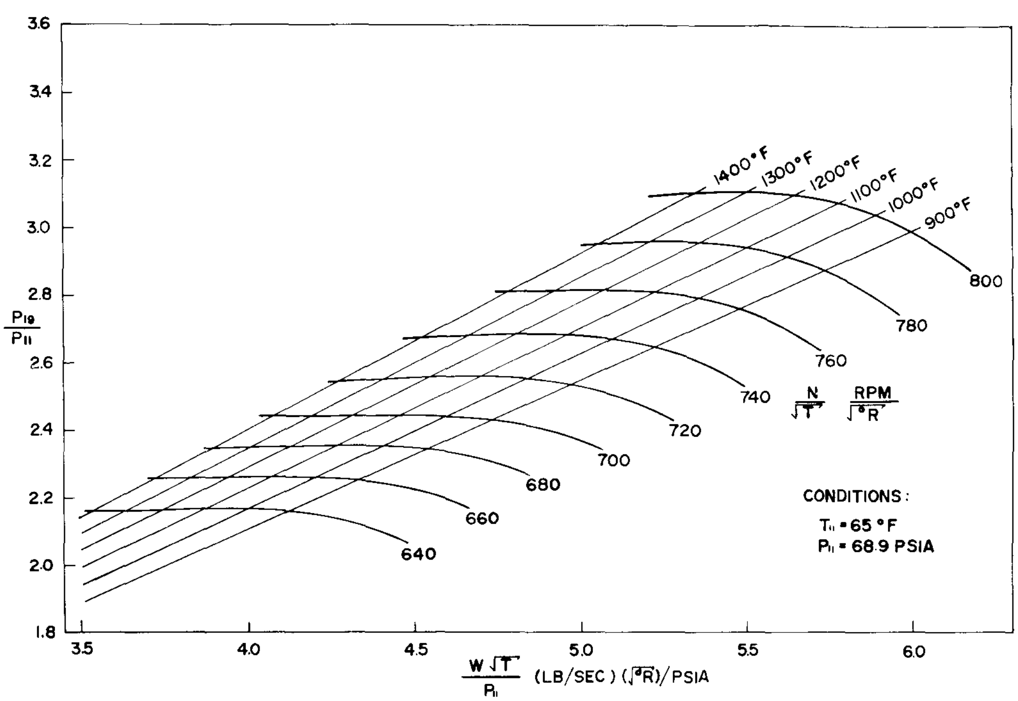


Fig.6 Computer compressor performance map (TCS-560B)

The unconventional technique of first defining enthalpy difference, equation (1), and working from there to the derivation of compressor pressure ratio (6) allows the internal computation of variations due to Reynolds number (4).

A computer wiring diagram is shown in Fig.5. The symbology may be found in reference (12). A bar quantity represents a variable in either machine units or percent full scale.

By holding selected parameters constant and varying others, it was possible to use an x-y plotter to directly define a compressor map in terms of pressure ratio, corrected mass flow and speed, with superimposed turbine inlet temperature isotherms. An example of the TCS-560B compressor performance map is shown in Fig.6. Pertinent results of the steady-state simulation of the TCS-670-2 compressor performance map are presented as Fig.7. Fig.7 also permits a comparison between computer prediction and experimentally derived

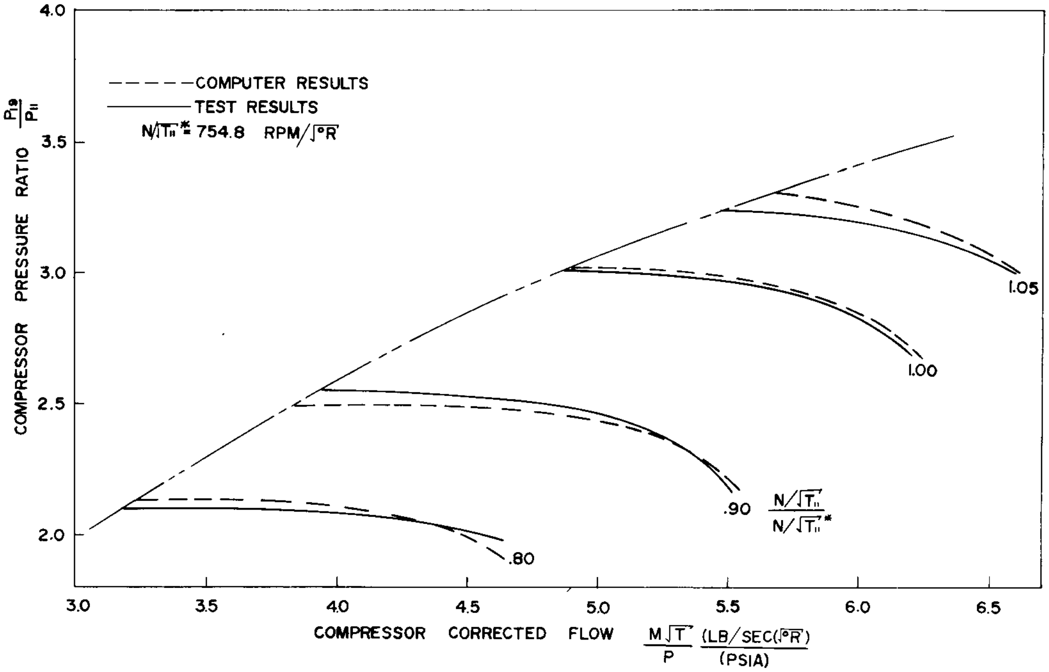


Fig.7 Computer compressor performance map (TCS-670-2), computer prediction and experimental data compared

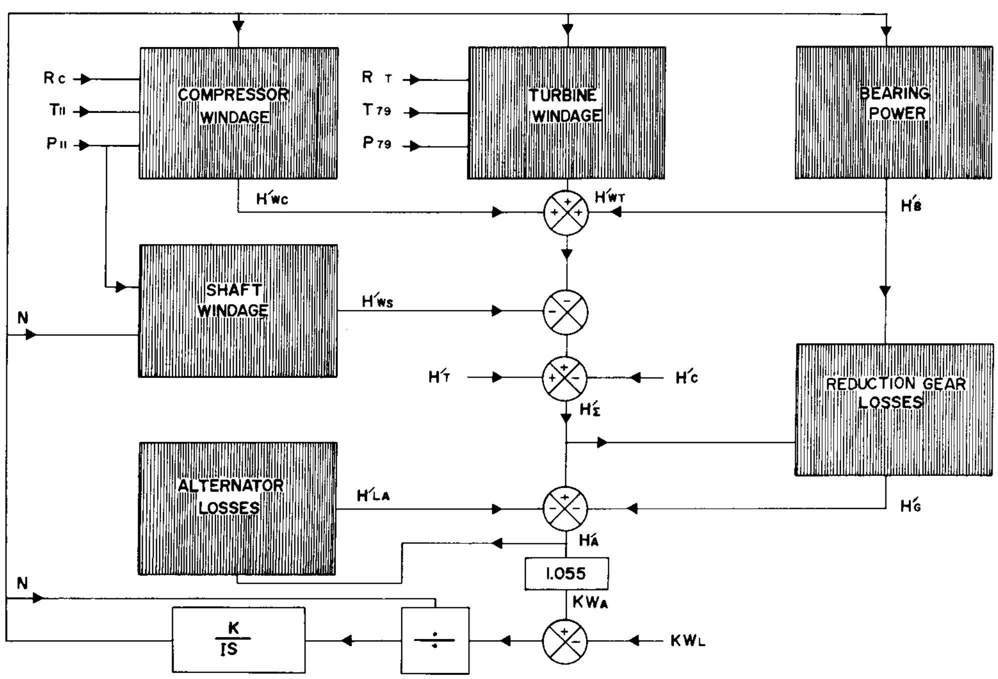


Fig.8 Block diagram of mechanical losses

data. The relative errors between computer projections and experimental data are within 2 percent.

Mechanical Losses and Shaft Dynamics

The representation of the mechanical losses in a functional block diagram is shown in Fig.8. The losses are defined in a manner suitable for correlating to test data. The defining equations are:

$$H^{1}_{WC} = C_9 \frac{P_{11}}{T_{19}} (1 + R_c) N^{C_{10}} \tag{9}$$

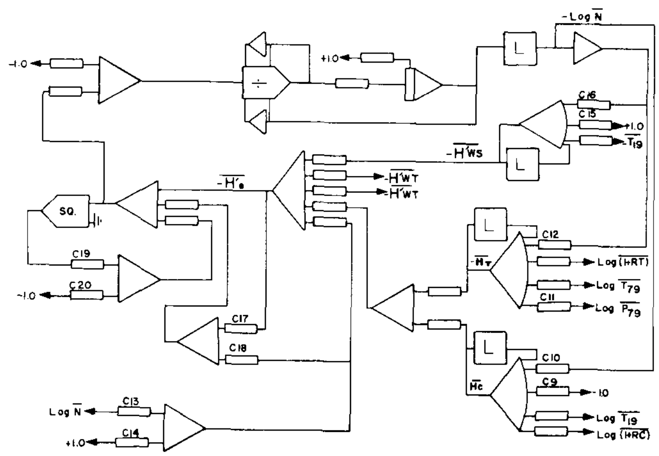


Fig.9 System losses and shaft dynamic wiring diagram

$$H'_{wt} = C_{11}\frac{P_{79}}{T_{79}}(1 + R_T)\, N^{C_{12}} \tag{10}$$

$$H'_{B} = C_{13} N^{C_{14}} \tag{11}$$

$$H'_{ws} = C_{15} N^{C_{16}} \frac{P_{11}}{T_{19}} \tag{12}$$

$$H'_{\Sigma} = H'_{T} - H'_{c} - H'_{wc} - H'_{ws} - H'_{B} - H'_{wt} \tag{13}$$

$$H'_{G} = C_{17} H'_{\Sigma} + C_{18} H'_{B} \tag{14}$$

$$H'_{LA} = C_{19} + C_{20} {H'_{A}}^{2} \tag{15}$$

$$H'_{A} = H'_{\Sigma} - H'_{G} - H'_{LA} \tag{16}$$

The summation of outputs, losses, and an assumed load allows the differential equation, defining the shaft dynamics, to be written as:

$$\frac{dN}{dt} = 2.1647 \times 10^{6} \left[ \frac{1.055\, H'_{A} - K_{WL}}{I\, N} \right] \tag{17}$$

where

$2.1647 \times 10^{6}$ = a dimensional constant, rpm$^2$-lb-sq ft/sec-kw

$1.055$ = a dimensional constant, kw/Btu/sec

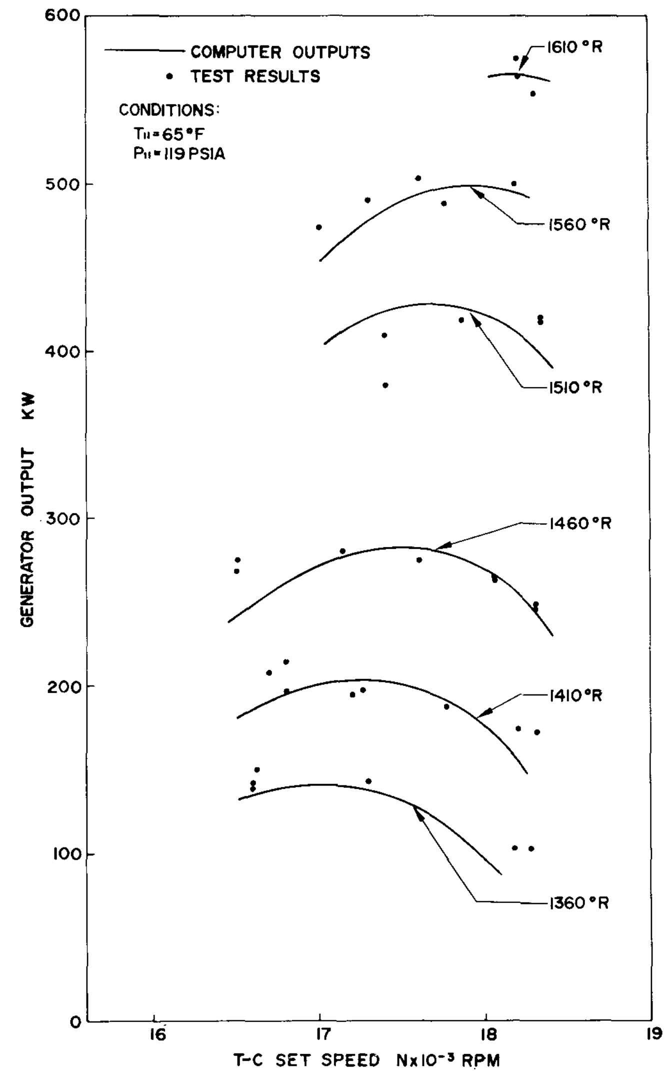


Fig.10 Overall systems performance (computer prediction)

A wiring diagram of the computer simulation of equation (17) is shown in Fig.9.

A comparison between computer predictions and experimentally derived data for the Overall System Performance is shown in Fig.10. This result of the simulation is shown as alternator power versus turbine-compressor set speed for a family of turbine inlet temperature isotherms.

Constant Mass Speed Control System

The objectives of the control study were:

1 The optimization of the proportional and integral control gains $K_1$ and $K_1^1$.

2 The determination of the effects of control system lags, represented by $\tau_1$, $\tau_2$, and $\tau_3$ upon stability.

3 The investigation of the influence of turbine bypass valve offset on overall performance.

The constant mass control system simulation

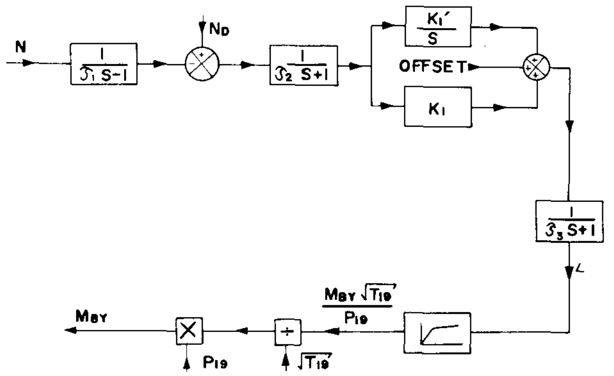


Fig.11 Block diagram, constant mass speed control system

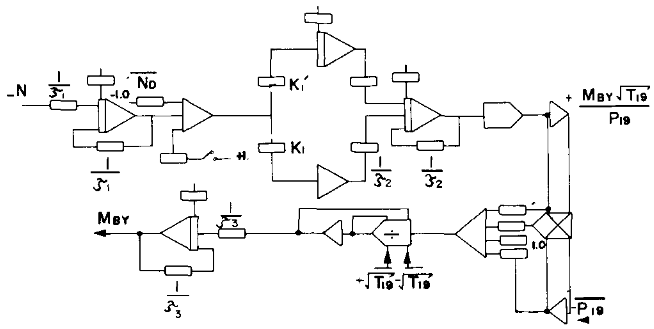


Fig.12 Wiring diagram, constant mass speed control system

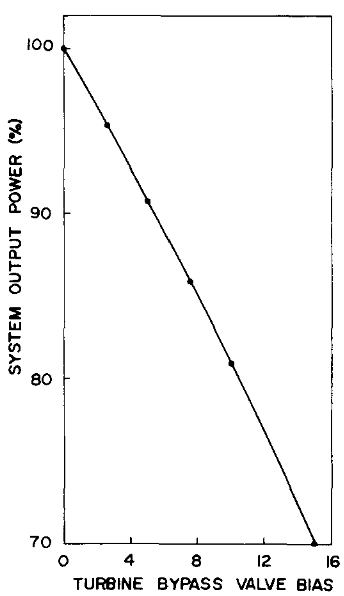


Fig.13 Percent reduction in performance versus turbine bypass valve bias

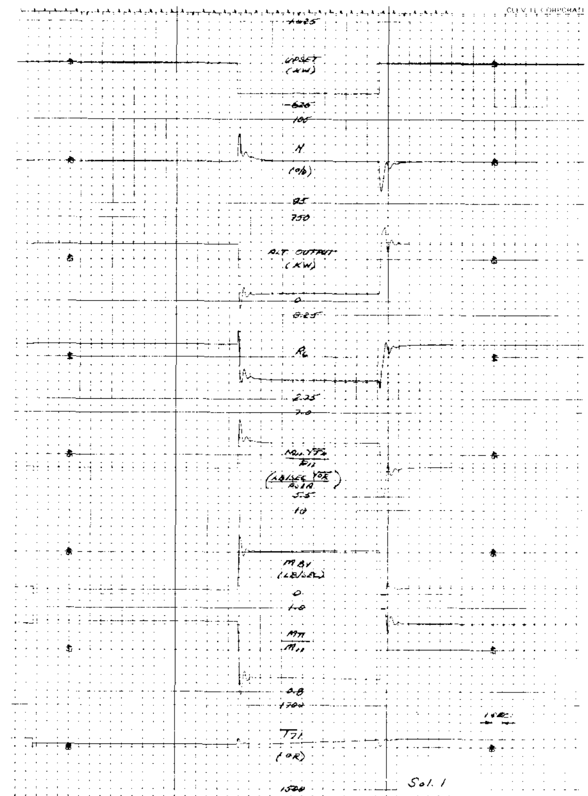

Fig.14 Speed control study results, solution no. 1

was of an existing hardware configuration at the APCEF, consisting of a combination of electrical and pneumatic elements. A block diagram illustrating the constant mass control system is shown in Fig.10. Sensed shaft speed--a d-c voltage--was compared to a desired set point speed, the difference converted to an a-c voltage which was fed into a conventional proportional-integral control. Current—the control output—was converted into a pneumatic pressure which, in turn, acted on the pilot of the turbine bypass control valve servo system.

As shown in Fig.10, the sensor, control, and actuator time constants are represented as first-order lags with respective values of $\tau_1$, $\tau_2$, and $\tau_3$ sec. Bandwidths were estimated as ranging from 12.5 to 50 msec for $\tau_1$ and $\tau_2$ and from 100 to 400 msec for $\tau_3$. The proportional and integral gains represented by $K_1$ and $K_1^{1}$ could be varied independently and were assumed to have no practical limits.

Actual valve characteristics in terms of flow corrected to inlet conditions, represented as $M_{BY} \sqrt{T_{19}}/P_{19}$ versus percentage lift (L), were functionally included in the simulation. Clearing the corrected flow term yielded the manipulated variable which, in turn, was fed back into the main body of the simulation.

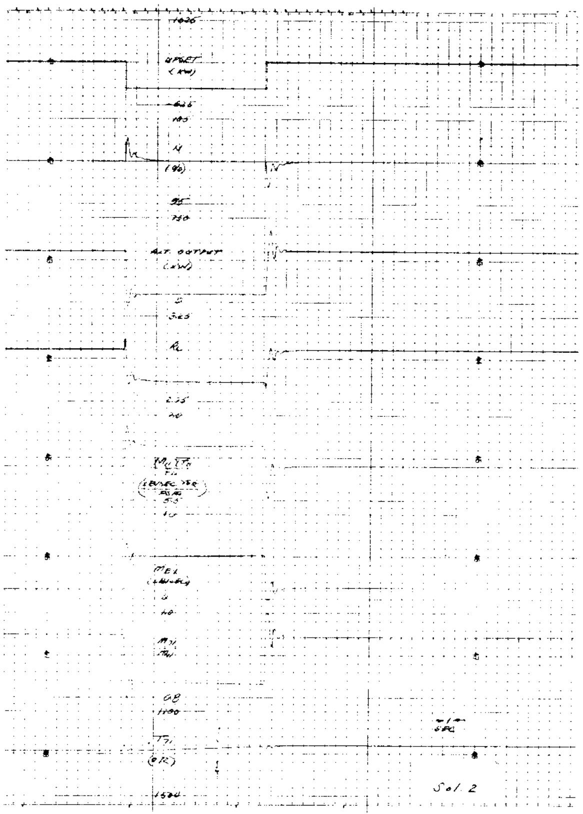


Fig.15 Speed control study results, solution no. 2

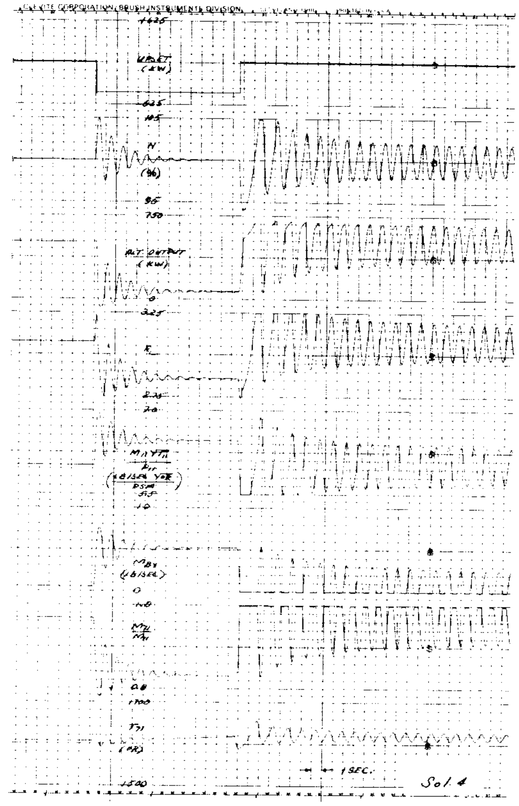


Fig.17 Speed control study results, solution no. 4

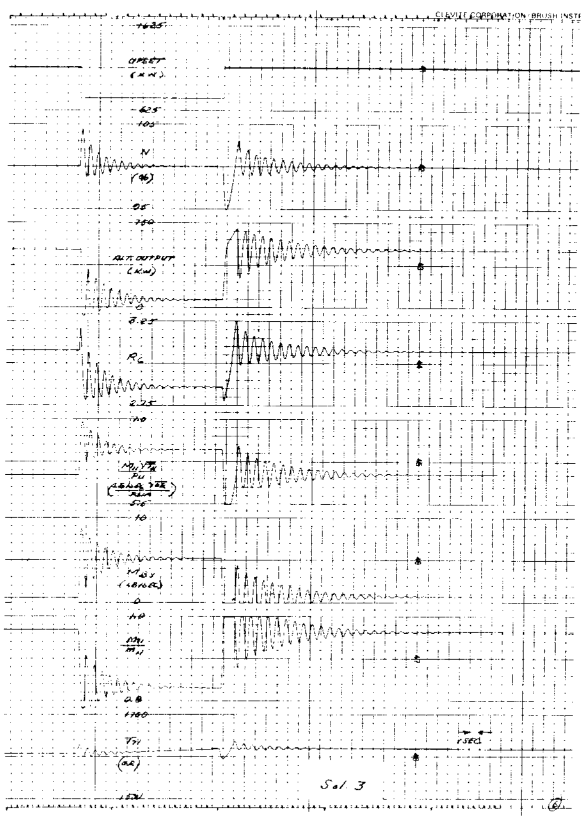


Fig.16 Speed control study results, solution no. 3

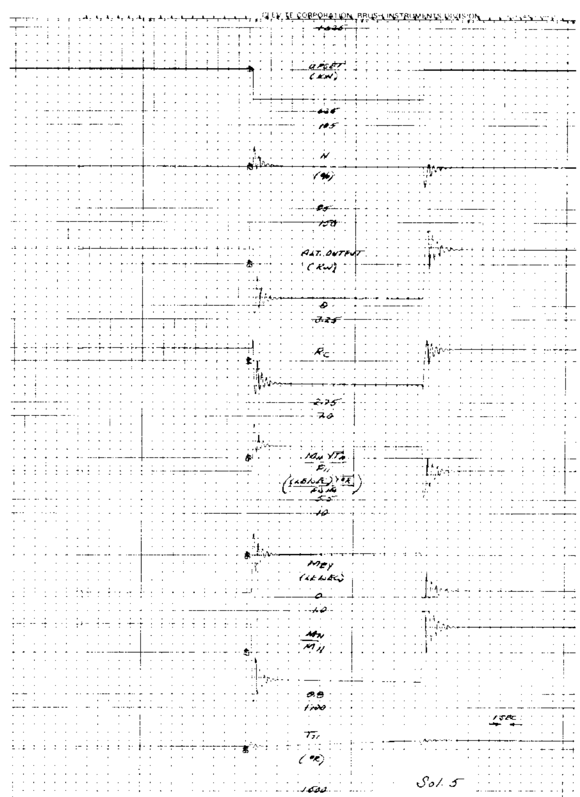


Fig.18 Speed control study results, solution no. 5

Table 2

| Figure Number | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|
| Solution Number | 1 | 2 | 3 | 4 | 5 |
| Offset (%) | 5 | 10 | 5 | 5 | 5 |
| K (%/RPM) | .1 | .1 | .1 | .1 | .2 |
| $K_1^{\ 1}$ (%/RPM SEC) | .1 | .1 | .1 | .1 | .2 |
| $\tau_1$ (SEC) | .0125 | .0125 | .05 | .0125 | .0125 |
| $\tau_2$ (SEC) | .0125 | .0125 | .05 | .0125 | .0125 |
| $\tau_3$ (SEC) | .1 | .1 | .1 | .4 | .1 |

A wiring diagram of the constant mass system is shown in Fig.12. The potentiometers, representing gains and time constants, are labeled as such, while those representing scale factors are physically represented with the settings omitted.

For the isochronous system represented, a zero error condition, where actual and desired speed are identical, can occur at various turbine bypass valve lift positions dependent on the load. Two phenomena that should be minimized are integrator windup, occurring when the valve is closed and desired speed is greater than actual speed, and on-off control occurring when the bypass valve cycles between an open and closed position. This can be accomplished by a valve offset or bias, so selected as to be finite at maximum power. A reduction in maximum power, depending on the degree of offset, results. The overall system power level in percent versus the offset position for one model configuration is shown in Fig.13.

A series of transients were performed on the simulation using an eight channel strip chart recorder as an output device. The upset means were 95 percent changes in load applied as a step. In order of appearance, the recorded variables are the upset, shaft speed (N), alternator output, compressor pressure ratio ($R_c$), compressor corrected mass flow ($M_{11}\ \sqrt{T_{11}}/P_{11}$) turbine bypass flow ($M_{BY}$), turbine to compressor mass flow ratio ($M_{71}/M_{11}$), and turbine inlet temperature ($T_{71}$).

A representative solution, as shown in Fig. 14, starts initially with the turbine-compressor set at a full power steady-state condition. Upon application of the negative load upset, shaft speed increases and is sensed by the control which opens the turbine bypass valve. Compressor pressure ratio (Channel 4) increases due to the momentary increase in speed, decreases as the control reduces speed, and stabilizes at a value commensurate to the mass flow continuity resulting from control action and a new turbine inlet temperature. Turbine inlet temperature (Channel 8) shows a rapid variation because of the changing compressor discharge temperature and a slower change caused by transients in the hot and cold side entrance temperatures of the recuperator.

The recuperator dynamics are slow and, if represented by a first-order lag, would have a time constant of approximately 14 sec. In most cases, the traces are not allowed to continue long enough for a final recuperator steady-state condition. The positive upset, with load reapplied, demonstrates the same general characteristics.

Five representative solutions have been selected for discussion and are shown in Figs.14 through 18, respectively. The degree of offset, gains, and time constants are as specified in Table 2. The proportional and integral gains $K_1$ and $K_1^{\ 1}$ are defined as lumped parameters with units of (percent/rpm) and (percent/rpm-sec), respectively. The indicated units refer to percent full travel of the turbine bypass valve and speed error as referred to the turbine-compressor rotor axis.

Solution 1, selected as both optimum and base conditions, shows a maximum speed deviation of 3.6 percent for the positive upset. The damping characteristics are ideal. This solution was made with a 5 percent offset, proportional and integral gains of 0.1, (percent/rpm) and (percent/rpm-sec), respectively, 12.5 msec for the sensor and control time constants, and 100 msec for the bypass valve actuator time constant.

Solution 2 shows the result of doubling the offset to 10 percent of turbine bypass valve travel. Speed deviations for the positive upset are reduced from 3.6 to 3.0 percent. The tradeoff is a reduction of 9.8 percent in maximum power for the 0.6 percent less speed deviation.

Solution 3, performed with time constants of 50 msec for the control and sensor and with all other coefficients identical to those of Solution 1, is highly oscillatory and represents the upper limits of these valves.

Solution 4 shows the result of increasing the actuator time constant to 400 msec. Marginal stability is encountered in the positive transient with the system going into a limit cycle.

Solution 5 is a repeat of Solution 1 with the exception of doubled valves for the integral and proportional gain. This increased gain reduces the speed deviations to 2.4 percent. Some oscillations, at approximately 2 Hz, appear but are attenuated in approximately 3 sec.

## SUMMARY AND DISCUSSION

The general mathematical model (1,10) was adapted to represent a particular Closed Brayton Cycle System. The analog computer program duplicates in all significant respects the operation of an actual power conversion system. The analog simulation can be used to accomplish the following:

1 The optimization of the total system design.

2 The simulation of steady-state and transient behavior in regimes where physical testing has not been performed.

3 The rapid evaluation of the influences of changing system components.

4 The identification of critical parameters.

5 The investigation of important problems such as system control requirements and techniques.

Practical design considerations dictate a timely formulation of the mathematical model even though the initial definition is not complete. This permits a checkout and evaluation of the concept of the mathematical model and analog computer program at an early stage. If the definition is flexible enough, experimental information and operating experience can be correlated to refine the simulation. The simulation can thus be used in parallel to experimental testing of the actual power conversion configuration and can be used for system performance projection.

The general mathematical model is sufficiently definitive to evaluate a number of system component configurations. Representative of such configurations are single or multiple-shaft turbine-compressors having either radial or axial compressors and multi-stage turbines employing gas-lubricated bearings and driving either a directly coupled high speed, high frequency alternator, or a low speed alternator through a reduction gear. In addition, other practical details as seal leakage, internal leakage in the recuperator, cooling flows for the alternator gas bearing feeds, windage losses for each rotating component, and so forth, can be considered in the simulation.

The formulation of the general mathematical model and analog computer simulation makes it readily adaptable to related compact, Closed Brayton Cycle power conversion systems.

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