



Research Article

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An Analogical Investigation of the Pythagorean Triangle: From a Mathematical Figure to an Ethical Praxis

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Abstract

This paper exposes an alternative and juxtaposed interdisciplinary view of the Pythagorean Triangle, from a mathematical point of view to ethical applicability. Pythagorean Theorem is understood as a mathematical principle ($a^2+b^2=c^2$), where the sum of the square of the shorter legs, a and b , is equal to the square of the most extended leg, the hypotenuse, c , resulted in the equation of the right triangle (Pythagorean Triangle). As an antediluvian mathematical figure, the Pythagorean Triangle's beauty and intricacy still amazed and provoked present-day thoughts. It is indubitable that the Theorem intrigues humanity's curiosity to provide proofs of the hypothesis, as well as its application. From such a viewpoint, this paper looks into the interdisciplinary applicability of the mathematical figure of the right triangle (Pythagorean Triangle). Utilizing analogical investigation, as the prime method of this research, the paper argues that the concept of the 'right triangle' could be an analogy for 'right living.' The logic behind the Pythagorean Triangle posits inherent beauty and a transcendent possibility beyond mathematics to other disciplines like ethics. The paper offers an analogical investigation of an alternative insight for the ethical problem of 'right living.' By reflecting on the presence of the existing similar feature, the right triangle (Pythagorean Triangle) could be analogically applied to 'right living.' From extant literature, the paper concludes that uprightness and balance are two essential concepts that could propose alternative ethical views by analogy to understanding what it means for 'right living.'

Keywords: Pythagorean Triangle, Analogical Investigation, Uprightness, Balance

1. Introduction

"For a right triangle with sides of lengths a , b , c , where c is the length of the hypotenuse." – Pythagoras

Pythagorean Theorem is an antediluvian principle known from an ancient civilization in which beauty still amazed and provoked present-day thoughts. The immensity of the Theorem partly maneuvers

even the development of empirical investigations. As opined by Jacob Bronowski as cited by Maor (2019), the Theorem remains the most critical single proposition in the whole of mathematics. Geometry, for one, as the mathematics of 'land-measure,' could have been brought by the discovery of the Pythagorean Theorem or vice versa (Kaplan & Kaplan, 2011; Nowlan, 2017; Hwang et al., 2018). In either claim, it is indubitable that the Theorem intrigues humanity's curiosity to provide proofs of the hypothesis, as well as its application.

As generally known, the genesis of the term is attributed to the Greek mathematician and philosopher Pythagoras. Pythagoras was credited with many mathematical ideas, although some may have been the work of his student (Chevallard, 2015; Coakley & Karamanides, 2015; Beck et al., 2020). Legend has it that Pythagoras' discovery of the theorem went hand-in-hand with religiosity by offering an ox as a ritual sacrifice. The development and controversy of the theorem didn't end with Pythagoras himself. Validation of the hypothetical applicability continued. As a matter of fact, four hundred proofs of it are known, and the number is still growing (Samuelson, 2016; Zazkis & Zazkis, 2016; Nelsen, 2020). The history of mathematics provided that highly civilized ancient societies like Babylon, Greece, Egypt, and even some Orientals like China and India found proof to validate its certitude. Proof-finding continued from Leonardo da Vinci, a future American president James A. Garfield, and young Albert Einstein (Ratner, 2000).

Today Pythagorean Theorem is simply an algebraic relation, $a^2+b^2=c^2$, in which the sum of the square of the lengths of the legs (a and b) is equal to the square of the length of the hypotenuse (c). Albeit its simplicity, according to Lewis Carroll as cited by Klein (2016) and Su et al. (2016), "the dazzling beauty of the algebraic expression cannot be hidden." In fact, as reported by the Journal of Physics World in 2004, as cited by Maor (2019), the Pythagorean Theorem won as the fourth most beautiful equation in science.

The exquisiteness of the Pythagorean Theorem can be traced from Pythagoras' interest in mathematics, particularly the study of numbers (Janaki & Saranya, 2016; Zhmud, 2016; Sporn, 2017). Although Pythagoras' penchant for numbers began as a spiritual quest, it reached his proclivity to the search of perfection, where a number is perfect if it is the sum of its proper divisors. The pursuit of Pythagoras eventually led him to figurative numbers, where numbers are represented as dots arranged in a regular pattern. Thus, arriving at the geometrical figure of a right triangle or what is known as the Pythagorean Triangle.

The qualification of being beautiful goes further beyond the validity of the theorem, rather fitting to the criterion of symmetry as depicted in the three altitudes of the right triangle. Dimmel & Herbst (2015) stated that the right triangle always meets at one point, as do the medians and the angle bisectors. Dimmel & Herbst (2015) purported that a certain elegance to the right triangle, with its complete symmetry (no sides or vertex takes precedence over any other). There is a perfect democracy among the sides.

A right triangle, as a polygon, has one angle equal to 90 degrees. It can be an isosceles triangle if the two sides that include the right angle are equal in length, while it can never be an equilateral triangle since one side is always longer than either of the two sides. The extant literature maintained that the right triangle is one of the most critical geometrical figures which, for thousands of years, had been used in many applications (Ma et al., 2015; Abánades et al., 2016)

Scriba & Schreiber (2015) narrated that land surveying and designing pyramids utilized the right triangle in ancient Egypt. While in 13th century China, the interest in right-angles triangles prompted Liu Hui to prove the correctness of algorithms in geometry and arithmetic, and algebra (Scriba & Schreiber, 2015). Going back to Greece, Plato investigates right triangles as the fairest one among polyhedral (Duhem, 2015; Iwata, 2015; Loftis & Mills, 2016; Negrepointis, 2019). The isosceles right-angles triangle is elementary triangles that construct the equilateral triangle (Burgin, 2018). Plato contends that the beauty of the right triangle is the fact of being "summetria". The term means 'the same measure' or 'of the same measure.'

Howbeit, the term 'symmetry' was unknown until Adrien-Marie Legendre, as cited by Piastra & Virga (2015) and Friedman (2018), developed the concept in the 18th century. While Plato had already

posited the immense applicability of the right triangle, the notion of the 'world of forms' closely resembles the geometrical figure (polygon). It generates an exciting idea that connects the form of a right triangle to the concept of 'form,' which for Plato is the only conceivable truth and goodness free, which is from imperfection (Paparazzo, 2015). Thus, more than a mathematical figure that fascinates mathematicians in studying trigonometry, among others, the Pythagorean triangle could propose enormous applications outside the periphery of the mathematical discipline that intersects to the realm of ethical praxis.

2. Objectives of the Study

Pythagoras' quest furthered him to discover other critical classic concepts in different fields like astronomy, philosophy, and music. However, this paper looks into the implication of the Pythagorean triangle as a mathematical figure. The concept of the right triangle could generate an analogical implication that alternatively answers the ethical problem of 'right living.' Hence, the paper surveys how the Pythagorean Triangle, as a mathematical form, could provide an alternative answer to the ethical question of what it needs to attain right-ness of living.

3. Materials and Methods

This paper utilized analogical investigation as a scientific research method (Lloyd, 1996; 2012; 2015; 2017). The paper offers an exposition of an alternative insight for the ethical problem of right living by reflecting on the presence of an existing similar feature in the Pythagorean triangle, which could be analogically applied to the question of 'right living.'

It is also important to note the limitation of the method. Although conclusions by analogy tend not to be reliable, the connections between two phenomena could, to some extent, be probable when an extensive review of similar features is facilitated (Raphals, 2017). Notable works that utilized a similar methodology include the papers of Charles Darwin, Karl Marx, J. Maxwell, N. Bohr, among others. To do this analogical investigation, extant literature is analyzed to support that the principle behind the Pythagorean Triangle could afford an analogical response of alternative insights to answer the ethical problem of the rightness of living.

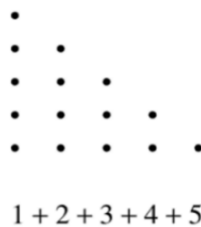


Figure 1. Pythagoras Figurative Numbers

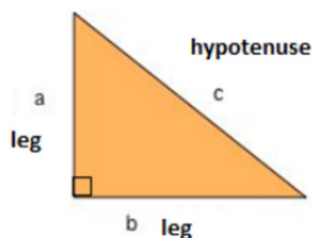


Figure 2. Pythagorean Triangle

4. Results and Discussion

4.1 Retracing the Ethical Problem of 'Right Living'

Ethics ensures a generally agreed standard of work-related behavior (Tanner et al., 2015; Valentine & Godkin, 2016). It empowers professionals to foster moral values through their work. Ethics gives a sense of justification in one's judgment (Barkan et al., 2015). It helps ensure that decisions at work are not made based on purely subjective factors (Sommers-Flanagan & Sommers-Flanagan, 2015). Without considering ethics, the practice of one's profession will fall prey to vastly conflicting individual interpretations. Such is the concern on the rightness of living. Since time immemorial, the problem of deciding for daily life is crucial to the quality of life human beings is to create (Michalos, 2017).

Ethics is an evaluation of human behaviors relating to human socialization (Cherkowski et al., 2015), organizational management (Gülcan, 2015), and proper interaction and cohabitation with other ecological beings (Al-Weshah et al., 2016). Such studies the choices people make regarding right and wrong. In other words, ethics understands the moral judgment of human actions. Here, ethics and morality are related in terms of the pertinent norm of human action influenced by a community (Bartels et al., 2015; Killen & Smetana, 2015; Pojman & Fieser, 2017).

The word "ethics" is a translation of the Greek word (ἦθος) "ethos," which implies basically, "customs" or "rules" of human conduct. In contrast, the word "moralis" (morals), which is the origin of the term "morality," is the Latin translation of "ethos." The two terms, therefore, are etymologically synonymous. However, the modern understanding of the usage of these words may signify partly different aspects. On the one hand, Sommers-Flanagan & Sommers-Flanagan (2015) and Pearson (2017) purported that ethics is the philosophical query of understanding the essence of human conduct (e.g., what makes the action of abortion wrong?). On the other hand, Bartels et al. (2015) and Dedeke (2015) opined that morality presents varying beliefs that may govern a person in ethical decision-making (e.g., for natural law principle, abortion is ethically wrong, but for cultural relativism, abortion may be morally permissible). The study of ethics and morality would help us develop a sound moral compass to help us come up with the best decisions.

Moreover, ethics is in a dilemma in which a difficult choice has to be made between two or more alternatives, especially equally desirable ones. Thus, there is a lot to consider in figuring out the rightness of living. Judgments about right and wrong infuse in everyday life could be challenging to do. Forming ethical decisions necessitates thoughtfulness to the ethical insinuations of problems and situations (Sommers-Flanagan & Sommers-Flanagan (2015) and involves practice (Dedeke, 2015). Understanding the nuances of the right-ness of living for ethical decision-making is, therefore, essential.

For Confucius, as one of the early Oriental thinkers to tackle the problem, the concern dwelled on living the right way as provided in the golden rule (Mooney & Williams, 2016). In the Analects, living the right way is described as the development of a set of typical behaviors associated with the moral ideal of the "way" of the "gentleman" (Ruin, 2018). This line of thought echoed the brand of ethics, which the Greek philosopher Aristotle proposed, namely the Virtue Ethics.

Virtue ethics emphasized the role of character formation in living one's life rather than doing one's duty or acting to bring about functional consequences. As cited by Sanderse (2015) and Konch & Panda (2019), Aristotle illustrated the acquisition of character excellence by habituation. Character excellence and habit are two crucial terms needed to consider in understanding the rightness of living. On the one hand, the word character means the development of personality and resulted in the application of virtues. On the other hand, the word habit means that individual human acts are being carried out frequently.

Habit and virtue are closely connected, for virtues are good acts habitually put into actions. The equivalent opposite of virtue is vice, which means a bad habit. Virtues are acquired through constant practice, which, in turn, develops our character excellence. That is why virtues are being put into action habitually, for once a person ceases in carrying out or put into disuse, virtues will be lost. Moreover,

Aristotle, as cited by Besser & Slote (2015) and Hawking et al. (2017), suggested that virtues are in the middle between too much and too little.

4.2 'Right Living' in the light of the Pythagorean Triangle (Right Triangle): Uprightness and Balance

The very ethical problem of 'right living' is grounded on the nuances of meaning by which the term is understood. The quest to know how we could attain the right living continues since the former's abstractness needs to be concretized by utilizing analogical means. As purported by Lloyd (2017):

"If we learn to understand our dependence on analogies, images and the like, and to appreciate how inappropriate quests for certainty may be, we may have a better chance to understand others, and indeed ourselves, as well as to learn from them how to expand our own intellectual and imaginative horizons. By understanding analogy better, the hope is we shall be able to understand ourselves and one another more sympathetically (page 249)." (non-Italics ours)

In this context, the mathematical principle of the Pythagorean Triangle could be investigated as an analogy to the rightness of living. Here we will try to analyze the analogical relationship between the right living and Pythagorean triangle.

For Pythagoras, as cited by McKirahan (2017), the mathematical figures are intelligible with the aid of the philosophical principle. The Geometrical forms provide an intelligible guide to the understanding matter at hand. While Plato, as cited by Paparazzo (2015), added that geometry at an exceptional level would assist one to come to apprehend the (Form of) Good, this thought transcends the mathematical principle to another field, to philosophy. Pythagoras himself left a philosophy of knowledge that defined reality via mathematics. Niess (2000) purported that Pythagoras seek to decipher the mathematical code by which truth is structured. One of which indeed is the geometrical figure of a right triangle. From this standpoint, it would be essential to wonder why the right triangle is called as such.

Etymologically, the term 'right' descends from the Latin word 'rectus', which means 'upright.' The term 'right' describes the 90-degree position of the triangle's angle from the ground (see figure2). The vertical perpendicular angle to a horizontal baseline connotes the upright posture of the right triangle, making the term 'uprightness' one of the essential attributes of the right triangle.

Significantly, the other related definitions of the concept of 'upright-ness' project its adherence to rectitude, righteousness, honest, or justness, and accordance with what is right (Dictionary.com, 2020). The angle of the right triangle posited 'balance' as the geometric law of the Pythagorean Theorem declares that the square of the hypotenuse of the right-angled triangle is equal to the sum of the squares of the other two sides. As purported by Plato, right-angled elementary triangles construct other solid shapes (equilateral triangle, square, or rectangle), thereby emphasizing symmetry and proportion (Paparazzo, 2015).

Antoshkin et al. (2015) and Fouze & Amit (2017) described that the mathematical equation of the Pythagorean Theorem highlights the elements of symmetry and proportion inherent to the right triangle. From which the geometric beauty of the right triangle is dominantly based. Hence, pointing out the significance of 'balance' as another essential attribute of a right triangle.

As a mathematical principle, the Pythagorean Triangle postulated the concept of 'rightness' as 'uprightness' and 'balance.' Uprightness and balance are two essential concepts that could be alternative ethical views by analogy to understand 'right living.' As opined by Yu et al. (2016), being upright is a moral posture described as being awake. It allows a person to be grounded with principles. With the same respect to the Confucian and Aristotelian notion of virtue, uprightiness is a synonymous term because it denotes conforming to the standard of right and wrong (Juurikkala, 2018). Of which, the opposite concept is bendiness, which is the tendency to distort and stoop.

For Confucius, as cited by Mooney & Williams (2016), Ruin 2018), and Radice & Goldin (2017), on the one hand, uprightiness could mean the pursuit of the heavenly way without the slightest deflection.

On the other hand, Aristotle defined uprightness as the equilibrium point between a deficiency and an excess of a trait (Konch & Panda, 2019; McKirahan, 2017).

The balance point of virtue lies in the middle that promotes individual and collective wellbeing. Aristotle termed this the golden mean. The golden mean is "the desirable middle between two extremes" (Konch & Panda, 2019; McKirahan, 2017; Sanderse, 2015). Here we see the connection and relatedness between 'uprightness' and 'balance.'

Balance, as pointed out earlier, emphasized symmetry and proportion. In the case of the Pythagorean Theorem, balance is the equivalence between the sum of the square of a and b and the sum of the square of c ($a^2+b^2=c^2$). The presence of balance generates symmetry and proportionality in the right triangle, which served as the basis of defining the latter as a perfect triangle. Hence, here we understand that 'perfection' is a foreseeable tacit of balance. Without balance, perfection is likely unattainable (Johari et al., 2018; Shaker, 2015; Suvorov & Suvorova, 2015; 2016).

Moreover, 'balance' inevitably served as the foundation of 'uprightness.' By maintaining the balance, the 'upright' posture is attained in the case of the right triangle. Such insight could be analogically implied to 'right living.' The perfect life doesn't mean the absence of struggle and protection against adversaries; instead, it is the ability to live a balance-life (Gavino, 2015; Jing, 2019; Weinstein, 2016). One example is the case of married life in which "*couple has to find equilibrium between themselves as spouses and their relationship to their children*" (Kahambing et al., 2019). Balance is a proportionate sharing of weight, enabling one to prevail upright and steady. Albeit, the irony of 'balance' is that it seemed to be the very goal to achieve. Instead, it is both the consequence of simplicity and the means to achieve uprightness. For instance, the plainness of the Pythagorean Theorem produced a subtle figure of a right triangle. At the same time, it recognizes 'balance' as the sheer quality of the Pythagorean equation that creates the right triangle. The same with life itself, the simpler and subtler life becomes, the greater the ability to maintain balance and thereby enabling uprightness.

5. Conclusion and Implication

Pythagoras, more than a mathematician, was a philosopher who was fond of the search for truth. His significant works in different fields are always geared towards perfection. No wonder why Pythagoras was claimed as a mystical and upright teacher. As mentioned above, Pythagoras mathematics is related to spirituality. In fact, Pythagoras' study of numbers is anchored on the belief that the whole cosmos is a scale. Pythagoras' study of geometry is to ground relationships among nature and humans, where conscientiousness and uprightness in the affairs of life would seem to be distinguishing characteristics of a wise man thus, transforming Pythagoras into an analogical ethicist.

The simplicity of the Pythagorean Theorem is an analogy of the simplicity and subtleness of life: the ability to live a simple lifestyle without losing its inherent beauty and sophistication. While balance and uprightness, being essential attributes of the Pythagorean triangle, also provided analogical implications to the ethical problem of 'right living.'

By balance, we mean the ability to take care and foster oneself, to know priorities, to form a resilient mindset, to expect the unexpected, and to sustain a positive mental attitude, in other words, to maintain personal equilibrium. By uprightness, we mean an affirmed position to choose what is ethically right for oneself. Thus, this paper concludes that 'uprightness' and 'balance' are two essential concepts that could propose alternative ethical views by analogy to understanding what it means for 'right living.'

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