Geophys. J. R. astr. Soc. (1967) **12**, 259–299.

An Analysis of Annual Sea Level Variations in European Waters

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(Received 1966 July 28)

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Summary

Regression analysis has been applied to all long series of annual mean sea level heights for European stations in order to estimate the components due to secular variation, air pressure variations and the nodal tide. The multiple correlation coefficients are in general found to lie between 0·8 and 1·0.

Secular variations (rise or fall in sea level relative to the land) are well defined; they range from almost 90 cm/century fall at the head of the Gulf of Bothnia to a rise of 20 cm/century in the southern North Sea, English Channel and Mediterranean. The variations are consistent enough to enable isopleths to be constructed. The results are used to predict the probable value of mean sea level at certain stations in 1970.

Expressions are obtained for annual mean sea level anomalies, arising from variations in the air pressure distribution, which enable the computation of 'normal' mean sea level (under normal atmospheric conditions) and 'isobaric' mean sea level (on the assumption of an isobaric atmosphere). The isobaric estimates represent an approximate to the geoid in European waters, though clear indications exist of a slope due to the spatial distribution of sea water density; the principal isobaric surface slopes are found to be more than 20 cm upwards from the North Sea into the Baltic, 8 cm downward to the north along the Norwegian coast, and 10 cm downward from the English Channel into the Mediterranean.

The permanent deformation of the sea surface due to prevailing winds and air pressure gradients is well marked, rising more than 20 cm from the Mediterranean to the Gulf of Bothnia.

Estimates of the nodal tide (period 18·6 years) suggest its existence in the equilibrium form, but the degree of scatter in the results is too great to permit a more definite conclusion.

1. Introduction

In 1959 the Permanent Service for Mean Sea Level was requested by the R.E.U.N.* Commission of the International Association of Geodesy to investigate the possibility of oceanographic levelling around the European coastline. The Commission's task of carrying precise levels across national boundaries had indicated the need to know more about the slope of the mean water surface along the coastline of Europe, and in particular, whether sea level observations could be treated so as to produce geopotential surfaces in the water in the same sense that geodetic observations provide geopotential surfaces on land.

At a symposium held in Liverpool in 1959, Cahierre reported on the distribution of crude mean sea level heights in European waters, and Doodson (1960) indicated how these could be corrected for the influence of meteorological forces.

The first attempt to apply Doodson's method to this problem was reported by the present author in 1962. To ensure that the possible existence of the nodal tide (period 18·6 years) was allowed for, 19 years of data were analysed for each station. Emphasis was placed on the need for simultaneous observations, and the investigation covered sea level data for the years 1940–1958.

A valid criticism of the use of only 19 years of sea level data is that it does not provide a very accurate estimate of the secular variation of sea level. The present paper is an attempt to avoid this defect by the analysis of all available sea level data at each station, without regard to a common time span or time origin. The new approach, however, introduces its own difficulties: whereas a 19-year span of annual mean sea levels can be adequately represented by a linear form of secular variation, longer spans of data at some stations exhibit a non-linear trend; estimates of the meteorological contributions to annual sea level anomalies require a knowledge of the corresponding mean annual air pressure distribution and, particularly for the Mediterranean, the meteorological time series do not go as far back in time as the sea level data.

The results of the present paper are, in general, more reliable than those published in 1962, although appreciable differences, as for example in the isobaric sea levels, are infrequent; the geographical coverage of results in the 1962 paper, however, is slightly better than those now presented.

2. The nature of the problem

Recorded mean annual values of sea level at a given station form a time series which, if suitably analysed, are capable of providing valuable information of general

* Réseau Européen Unifié de Nivellement.

geophysical interest. The significance of such information is considerably enhanced if similar information is obtainable for other stations; assuming a uniform standard of accuracy in the data, the longer the time series the more reliable and rewarding are the results.

It therefore becomes desirable to analyse all available sea level data for all available stations in a uniform manner, so as to obtain the maximum possible information. Ideally, one would wish for at least a century of data for stations strategically situated throughout the world oceans. Lacking these, existing data must be put to full use, at the same time ensuring that the analytical processes employed give some indication of the reliability of the data. The present paper deals with data from European waters only, though the principles should be valid for all waters.

The processing methods should be designed so that the results for different stations may be compared, bearing in mind that the years for which data exist for any two stations do not, in general, coincide. These considerations determine the methods required for handling the data in their original form. The sheer volume of data to be handled, and the amount of computation required, clearly indicate the need for digital methods.

3. The factors affecting annual mean sea level

The annual variation in mean sea level at a given station is dependent upon many processes of which the major ones are:

(i) Changes in the total water balance of the seas and oceans (the eustatic change).

(ii) Movements in the land reference level relative to which sea level is measured.

(iii) Time variations in the atmospheric forces (wind stress and air pressure) affecting sea level.

(iv) Time variations in the oceanographic forces (temperature, salinity, currents) affecting sea level.

(v) Long period astronomical tides.

The time scale of these factors is of some importance, and may itself be resolved into:

(a) slowly varying changes, *e.g.* over decades, and

(b) more rapid changes which may be classed as annual anomalies, that is to say, year by year variations from the long term mean.

Consider each of the factors (i)–(v) in rather more detail.

(i) Little is yet known of the water balance of the oceans, but it is probable that the factors involved in the ice/water and rainfall/runoff/evaporation/condensation balances, on the global scale, are greater on time scale (a) than time scale (b).

(ii) Vertical movements of the Earth's crust may occur in both time scales. In scale (a) we may place the gradual sinking of land under an iceload and the resultant gradual rise when the load is removed; in the same scale we have the sinking of a solid structure in marshy surroundings. Sudden movements of the crust, generally localized but sometimes, as the result of seismic activity, over large areas, can be classified as time scale (b).

(iii) The response of the sea surface to air pressure and wind stress extends over a time scale which is only bounded at its lower end by the inertia of the body of water considered. The daily mean sea level anomaly due to meteorological forces is in general larger than the monthly anomaly, and since this progression continues up the time scale we may assume that time scale (b) is more important than time scale (a).

(iv) An argument similar to (iii) may be applied to the response of the sea surface to oceanographic forces; since the ocean is relatively more stable than the atmosphere, however, time variations due to oceanographic factors are smaller than those due to meteorological factors.

(v) The only astronomical tides with periods longer than one year, and with amplitudes of more than a few mm, are the pole tide and the nodal tide. The pole tide has a period of 1·19 years and possible amplitude of 5 mm and arises from the Chandlerian nutation; the nodal tide has a period of 18·6 years and arises from the precession of the Moon's ascending node. Due to its period, the contribution from the pole tide to an annual mean level will necessarily be extremely small, and attention need be focussed only on the nodal tide.

4. Annual mean sea level as a function of independent variables

The problem is to express annual heights of mean sea level at any given station as the dependent variables in an equation in which the independent variables represent the relevant physical factors. The independent variables should be such that numerical values can be ascribed to them for each year for which sea level data are known. The coefficients of the independent variables may then be determined statistically, using a least squares solution, thereby giving an estimate of the magnitude and time variation of the relevant physical factors.

Secular variations—3 (i), 3 (ii) (a)

Restricting considerations to one sea level station, it is impossible, from the observed data alone, to differentiate between slowly varying absolute movements of the sea and those of the adjacent land. Factors (i) and (ii) of Section 3 are therefore considered as one phenomenon, in the first instance, to which the name 'secular variation' is given. Secular variations are themselves found to vary considerably from one station to another. Assuming that, by definition, the eustatic variation should be the same at all stations, it should be possible to separate the individual absolute movements (i) of the sea surface, and (ii) of the land if a sufficiently even coverage of data are available from the world oceans.

The year of observation is the obvious choice of variable for representing the secular variation and a linear expression of the form

$$Z_Y = a_0 + a_1 Y,$$

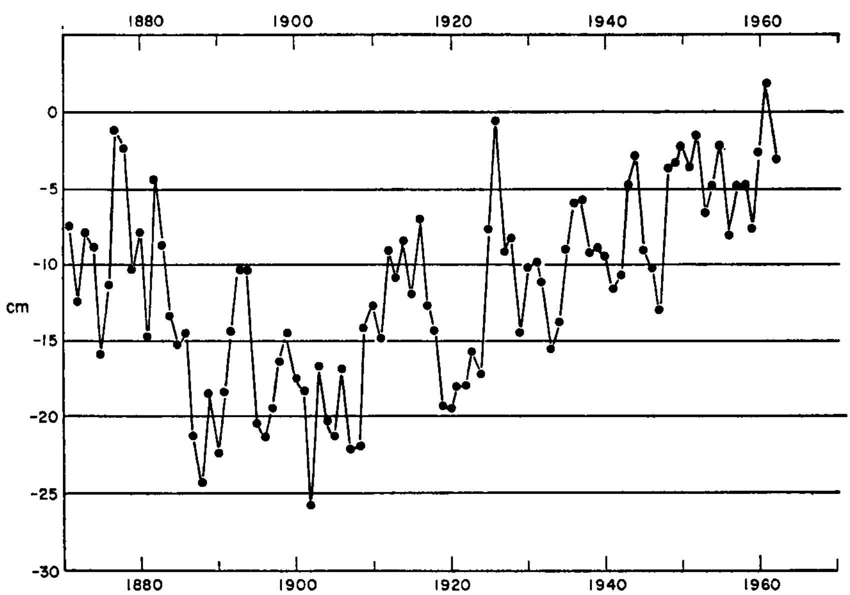

Fig. 1. Annual mean levels at Ijmuiden (150/41) illustrating non-linear trends.

where Z_Y is the annual mean value of sea level for the year 1900 + Y, is the most obvious method of representation. However, examination of the long period trend of sea level at some stations as, for example, at Ijmuiden (Fig. 1), clearly indicate that a polynomial expression may sometimes be required. The secular variation at each station has therefore been represented by

$$Z_Y = \sum_{p=0} a_p Y^p. \tag{1}$$

Clearly no geophysical phenomenon can be represented for all Y as a polynomial, since all polynomials ultimately tend to infinity, but we are entitled to use such an expression if it is restricted to the span of data used in the determination of the coefficients a_p.

Sudden, localized movements of the Earth's crust—3 (ii) (b)

No obvious variable exists to represent the phenomenon unless use is made of geodetic levelling results, which are usually insufficient for the purpose.

Meteorological contributions—3 (iii)

The response of a body of water to atmospheric forces takes the form of water movements and surface gradients, themselves functions of time, which may be deduced theoretically from a knowledge of the spatial and time distributions of the meteorological forcing functions. Such deductions, however, are extremely complex, and it is preferable to seek a more direct, albeit empirical relationship between time variations of sea level at a given station and the forcing functions.

The most direct representation would be of the form

$$Z_Y = f(\tau, B), \tag{2}$$

where τ is mean annual wind stress and B is mean annual barometric pressure. B is a most acceptable variable, since it has been extensively observed for a great many years and to a high precision. Representation of τ is more difficult; wind stress is generally accepted as being a function of wind velocity, which itself may be defined in terms of air pressure gradients. In the almost complete absence of observed annual values of wind velocity and direction over seas and oceans, the only recourse is to use pressure gradients. The method adopted here is to use pressure gradients in such a way that they represent, implicitly, both the statical pressure and wind stress effects. This is most easily done if the assumption is made that $\tau \propto W$, where W is wind velocity. To accept any non-linear law of wind stress would complicate the problem to a considerable extent, probably more than could be justified at this stage.

Using a linear law of wind stress, three variables are required: air pressure, geostrophic wind velocity and direction. In terms of air pressure, we therefore need a minimum of three stations, preferably forming an equilateral triangle covering the sea area over which the generating forces may be expected to act. Equation (2) then becomes

$$Z_Y = \sum_{r=1} b_r B_r, \tag{3}$$

where B_r is the annual mean value of atmospheric pressure at one of the r meteorological stations, and the coefficients b_r are to be determined by correlations between the two sides of equation (3). Given the geographical position of the meteorological stations, it would then be possible to interpret the coefficients in terms of the individual response of the sea to wind stress and air pressure.

Oceanographic contributions—3 (iv)

Annual mean values of temperature, salinity or density, even for the surface layers of coastal waters, are exceedingly rare, and no attempt has been made to allow for the contributions to annual mean sea level from these factors.

Table 1

Code for European sea-level stations

Norway	040	081	Narvik	68°	26′ N	17°	25′ E
		151	Heimsjo	63	26	09	07
		221	Bergen	60	24	05	18
		261	Stavanger	58	58	05	44
		301	Tregde	58	00	07	34
		311	Nevlunghavn	58	58	09	53
		321	Oslo	59	54	10	45
Sweden	050	001	Stromstad	58°	57′ N	11°	11′ E
		011	Smogen	58	22	11	13
		021	Backevik	58	22	11	15
		031	Goteborg	57	43	11	57
		041	Varberg	57	06	12	13
		051	Klagshamn	55	31	12	54
		071	Ystad	55	25	13	49
		081	Kungholmsfort	56	06	15	35
		091	Olands n. Udde	57	22	17	06
		101	Mem	58	29	16	25
		121	Landsort	58	45	17	52
		131	Nedre Sodertalje	59	12	17	37
		141	Stockholm	59	19	18	05
		161	Bjorn	60	38	17	58
		171	Nedre Gavle	60	40	17	10
		181	Draghallan	62	20	17	28
		191	Ratan	64	00	20	55
		201	Furuogrund	64	55	21	14
Finland	060	001	Kemi	65°	44′ N	24°	33′ E
		011	Oulu	65	02	25	26
		021	Raahe	64	42	24	30
		031	Ykspihlaja	63	50	23	02
		041	Pietarsaari	63	42	22	42
		051	Vaasa	63	06	21	34
		061	Ronnskar	63	04	20	48
		071	Kaskinen	62	23	21	13
		091	Reposaari	61	37	21	27
		101	Mantyluoto	61	36	21	29
		121	Rauma	61	08	21	29
		221	Lyokki	60	51	21	11
		231	Lypyrtti	60	36	21	14
		241	Turku	60	25	22	06
		271	Lemstrom	60	06	20	01
		281	Degerby	60	02	20	23
		291	Uto	59	47	21	22
		311	Jungsfrusund	59	57	22	22
		313	Stromma	60	11	22	53
		316	Hanko & Russaro	59	48	22	58
		344	Skuru	60	06	23	33
		351	Helsinki	60	09	24	58
		354	Soderskar	60	07	25	25
		361	Hamina	60	34	27	11
		364	Viipuri	60	42	28	44
U.S.S.R. (Baltic Coast)	080	061	Riga V.D.T.	56°	57′ N	24°	07′ E
		081	Daugavgriva	57	03	24	02
		111	Kolkasrags	57	48	22	38
		121	Ventspils	57	24	21	33
		151	Liepaja	56	32	20	59
		191	Pillau	54	38	19	54
Poland	110	031	Nowy Port	58°	25′ N	18°	40′ E
		071	Swinemunde	53	56	14	17

Table 1—*continued*

Germany (Baltic Coast)	120	001	Arkona	54° 41′ N	13° 26′ E
		011	Warnemunde	54 11	12 05
		021	Wismar	53 54	11 28
		031	Travemunde	53 58	10 53
		041	Marienleuchte	54 30	11 15
Denmark	130	001	Gedser	54° 34′ N	11° 58′ E
		021	Kobenhavn	55 41	12 36
		031	Hornbaek	56 06	12 28
		041	Korsor	55 20	11 08
		051	Slipshavn	55 17	10 50
		071	Fredericia	55 34	09 46
		081	Aarhus	56 09	10 13
		091	Frederikshavn	57 26	10 34
		101	Hirtshals	57 36	09 57
		121	Esbjerg	55 28	08 27
Germany (North Sea)	140	021	Bremerhaven	53° 33′ N	08° 34′ E
Netherlands	150	001	Delfzil	53° 20′ N	06° 56′ E
		011	Terschelling	53 22	05 13
		021	Harlingen	53 10	05 25
		031	Den Helder	52 58	04 45
		041	Ijmuiden	52 28	04 35
		051	Hoek van Holland	51 59	04 07
		061	Maasluis	51 55	04 15
		071	Hellevoetsluis	51 49	04 08
		081	Brouwershavn	51 44	03 54
		091	Zierikzee	51 38	03 55
		101	Vlissingen	51 27	03 26
British Isles	170	011	Aberdeen	57° 09′ N	02° 05′ W
		041	Dunbar	56 00	02 31
		053	North Shields	55 01	01 24
		071	Felixstowe	51° 56′ N	01° 19′ E
		081	Southend	51 31	00 45
		101	Sheerness	51 27	00 45
		161	Newlyn	50° 06′ N	05° 33′ W
France (West Coast)	190	091	Brest	48° 23′ N	04° 30′ W
Portugal	210	001	Cascais	38° 41′ N	09° 25′ W
		031	Lagos	37 06	08 40
France (South Coast)	230	051	Marseilles	43° 18′ N	05° 21′ E
Italy	270	061	Trieste	45° 39′ N	13° 45′ E

The nodal tide—3 (v)

The orthodox expression for a harmonic tidal constituent is $H\cos(\mu t-g)$ where H is the amplitude, μ the angular speed per unit of time t, and g is a phase lag. For the nodal tide $\mu t=N$, the mean longitude of the Moon's ascending node, and the contribution to annual mean sea level may be written

$$c_1\cos N+c_2\sin N, \tag{4}$$

where $c_1=H\cos g$, $c_2=H\sin g$.

Summary

At each station we shall use the equation

$$Z_Y = \sum_{p=0} a_p Y^p + \sum_{r=1} b_r B_r + c_1 \cos N + c_2 \sin N + \phi_Y \tag{5}$$

where Z_Y = annual mean value of sea level, referred to some arbitrary datum (working datum), for the year Y, Y=year number relative to 1900, B_r=annual mean value of air pressure, corrected to 'mean sea level', at station r, for the year Y. N (a function of Y) and the coefficients a, b and c retain their earlier definition.

Thus the terms in equation (5) represent, successively, the secular variation, the meteorological contribution to sea level, the nodal tide and finally, ϕ_Y, the contribution to sea level from all other causes.

5. The data

These consist of (a) mean sea level data, (b) air pressure data, and (c) astronomical data.

(a) *Mean sea level data.* Monthly and annual mean values of sea level are now published triennially by the Permanent Service for Mean Sea Level. The values used in this investigation have been extracted from the following publications of the International Association of Physical Oceanography: *Publications Scientifiques*, Nos. 5 (1940), 10 (1950), 12 (1935), 19 (1958), 20 (1959), 24 (1963), 26 (1966).

The last two volumes are also issued in the Monograph series of the International Union of Geodesy and Geophysics (Nos. 21 and 30).

The data are heterogenous both in quantity and quality. Not many permanent sea level stations were operating during the last century, and very few stations in western Europe were installed prior to 1850. Data are more abundant for recent decades, and new stations have been installed so as to give a more even global coverage. Nevertheless, many stations have been ignored where the number of annual means is considered to be less than the minimum required to provide a reasonable estimate of the secular variation and nodal tide. From examination of the 1% significance values of the multiple correlation coefficient associated with equation (5), the minimum number of years is of the order of 30.

In the publications referred to above, sea level is published in a variety of units. We have standardized on the *millimetre* throughout this work.

The datum, or reference level from which mean sea level is measured, varies, in general, from station to station. Moreover, the same datum has not always been used at any one station. In obtaining a constant datum for each station for this investigation, the datum information given in Table A of the *Publications Scientifiques* has been used; it has frequently been found necessary, however, to query individual values with the national authority and in consequence, many errors in the published data have been brought to light.

Further errors in published values have also been found as a result of a verification check performed by the Permanent Service when placing the data on to punched cards. The check is an elementary though effective one, consisting of comparing the mean of the 12 published monthly values with the published annual value.

To summarize, for each station the corrected annual mean values of sea level, converted to millimetres as necessary and referred to a suitable working datum, have been placed on to punched cards; these are the quantities Z_Y of equation (5).

All sea level stations are coded for data identification; the code consists of a 3-digit country number followed by a 3-digit station number. Details of the code for European stations are given in Table 1.

(*b*) *Air pressure data.* Monthly and annual mean values of air pressure have been published in the following volumes:

World Weather Records—Smithsonian Miscell. Collections Vol. 79 (1927);

World Weather Records, 1921–1930—Smithsonian Miscell. Collections, Vol. 90 (1934);

World Weather Records, 1931–1940—Smithsonian Miscell. Collections, Vol. 105 (1947);

World Weather Records, 1941–1950—U.S. Weather Bureau (1959).

These have been augmented by data for the years 1951–1962 through the ready co-operation of national meteorological agencies.

In the earlier volumes pressures are generally given in millimetres or inches of mercury, corrected to 0°C and for gravity at 45° latitude, and at station level. In some of the later volumes pressures are given both to station level and 'sea level'; station level is not necessarily constant throughout a series.

Tables for correcting pressures from station height to mean sea level are given in the Meteorological Office *Handbook on Meterological Instruments*, Part 1, as functions of geometric height and air temperature. Tables in the *Smithsonian Meteorological Tables*, 6th revised edition, depend upon a knowledge of the geometric height, air temperature, lapse rate and humidity, and use geopotential heights for entry.

Our requirements are for the data for any station to be homogeneous (as far as possible), and preferably in modern units (the millibar). The absolute value of air pressure is less significant than the annual anomalies from a long-term average; nevertheless, all pressure data have been corrected to sea level where necessary. The Smithsonian procedure requires a knowledge of the dew point temperature, often an unknown quantity, and is tedious to apply; where necessary the Meterological Office system of correction has therefore been used, as follows:

(1) Convert station level pressure from inches or millimetres of mercury to millibars

$$1 \text{ mm} = 1{\cdot}333224 \text{ mb}$$

$$1 \text{ in} = 33{\cdot}8632 \text{ mb.}$$

Table 2

Code for air pressure stations

Baltic	$X=1$	036	Haparanda	(Sweden)
		088	Copenhagen	(Denmark)
		090	Leningrad	(U.S.S.R.)
North Sea	$X=2$	061	Bergen	(Norway)
		084	Edinburgh	(Scotland)
		127	De Bilt	(Netherlands)
Norwegian Sea	$X=3$	030	Stykkisholm	(Iceland)
		034	Bodo	(Norway)
		127	De Bilt	(Netherlands)
Biscay/Atlantic	$X=4$	124	Valentia	(Eire)
		268	Ponta Delgada	(Azores)
		269	Lisbon	(Portugal)
North Atlantic	$X=5$	030	Stykkisholm	(Iceland)
		167	Charlottetown	(Canada)
		269	Lisbon	(Portugal)

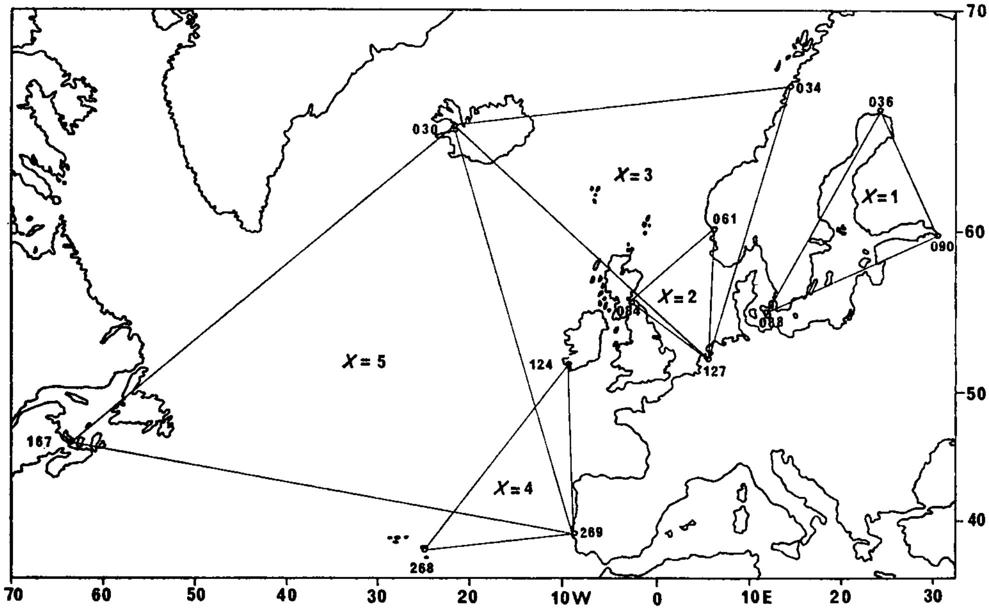

FIG. 2. Location of air pressure stations; code numbers are defined in Table 2.

(2) Convert station level pressure in millibars to pressure at mean sea level using M.O. Table LVI. These are the quantities B_r of equation (5), though in practice they are punched on cards in units of 0·1 mb and referred to a datum of 1000 mb.

The choice of triads of air pressure stations for various sea areas is not an easy one. The requirements (i) that annual air pressure values should exist for each annual value of sea level, and (ii) that the stations should cover a given sea area, as nearly as possible, by an equilateral triangle, have been rather difficult to reconcile, especially for oceanic areas. The pressure stations ultimately chosen for European waters are shown in Fig. 2 and details tabulated in Table 2.

For certain sea level stations, two triads of air pressure stations have been used, *i.e.* r in equation (5) has assumed up to six values; this is in order to permit contributions to Z_Y from distant areas. Sea level stations in the North Sea, for example, have been linked with three air pressure stations around the North Sea so as to represent the 'local' effect, as well as with three stations covering the North Atlantic so as to represent more distant effects.

Two codes have been found necessary for the pressure data. Each meteorological station has a code number ascribed to it in the publications mentioned above; these have been retained. Each triad of stations has also been coded by a two-digit number X; these details are also given in Table 2. Thus the North Sea triad ($X = 2$) consists of Bergen (61), Edinburgh (84) and De Bilt (127).

(*c*) *Astronomical data.* The only values required are those for N, the mean longitude of the Moon's ascending node; tables of N are to be found in Special Publication No. 98 of the U.S. Department of Commerce *Manual of Harmonic Analysis and Prediction of Tides* (revised 1940).

6. The analytical processes

The determination of the coefficients a, b and c in the fundamental equation (5), for each mean sea level station which has more than thirty annual means to its credit, is a considerable computational task. In European waters alone some 6000 station years of data are suitable for processing, and digital computing facilities are obviously essential.

The Permanent Service has access to an IBM 1620 Model 40K core-store capacity with disk-backing, in many respects an ideal computer for the type of work involved. Input is by punched card, and output by card or line printer.

Initially, the programme used for the regression analysis was the *Sixteen-twenty Card Regression Analysis Program* (SCRAP) from the IBM Program Library. The facilities offered by this programme are, if anything, in excess of what is required for the present investigation. It suffers from one defect, however, in that input data has to be prepared in a specialized and rather inconvenient seven-digit notation. Since the same triad(s) of meteorological stations frequently had to be associated with a different sea level station for each regression analysis it was found necessary to write an auxiliary programme (PREPSCRAP) to sort and prepare the basic data in the form and notation acceptable to SCRAP.

Subsequently a modified form of SCRAP, known as STUFF (*Sixteen Twenty Universal Function Fitter*) was used. This programme retains all the virtues of SCRAP without its restrictive input requirements, and has proved a valuable computing facility.

Input to SCRAP or STUFF comprised n cards, one card per year, containing :

Y, cos N, sin N, B_r, Z_Y, country and station code numbers.

Some of the more important output quantities of SCRAP and STUFF, both of which perform a linear regression analysis in the sense that the least-squares principle is the criterion for deciding the best fit, are :

regression coefficients and their variances, a, σ_a^2, etc.;

sum of squares of deviation removed by regression, $n(\sigma_0^2 - \sigma_\phi^2)$;

residual sums of squares of deviation, $n\sigma_\phi^2$;

square of coefficient of multiple correlation, R^2;

where n = number of data, σ = standard deviation and suffixes 0 and ϕ denote *original* and *final* respectively.

All other auxiliary computations were performed on desk machines.

7. Statistical criteria for deciding best fit

The basic equation (5) of Section 4 permits a certain amount of flexibility both in the choice of p, the degree of the polynomial expression for the secular variation, and of r, the number of air pressure stations.

In practice, as explained in Section 5, the paucity of pressure data places a very real limitation on r, and further restrictions can be applied on obvious physical grounds. Nevertheless, borderline cases arise, as may be exemplified by sea level stations in the Kattegat and on the Danish islands. Sea level at Esbjerg, in the North Sea may be expected to be affected by air pressures over the North Sea, but not by those over the Baltic. Sea level at Polish stations, on the other hand, are certainly susceptible to pressures over the Baltic and, to a lesser extent, to those over the North Sea. The proper choice of air pressure stations for sea level stations in the transitional area must therefore be made on statistical grounds.

The order of the polynomial expression for representing the secular variation of sea level must also be decided statistically. For obvious reasons, a linear expression is preferable wherever possible. But, as mentioned in Section 4, the data for certain stations show a marked curvature in the long period trend. A polynomial of order 1 and one of order 3 was therefore computed for each station. Examination of the results then indicated whether the linear or the cubic law was more appropriate, or whether a quadratic law provided the best fit.

Table 3

Coefficients and standard errors for Felixstowe (170/71)

Data: 1918–1947, 1949–1950, $n=32$

(i) Linear secular variation
(ii) Quadratic
(iii) Cubic

	a_0	a_1	a_2	a_3	b_1	b_2	b_3	b_4	b_5	b_6	c_1	c_2	R	1% *level*	σ_ϕ
(i)	88·44	1-60			−0·92	−0·14	0·91	−0·06	−0·14	−0·34	3·18	3·28	0·865	0·76	10·2
		0·27			0·32	0·34	0·41	0·17	0·49	0·27	3·37	4·32			
(ii)	277·66	−2·22	0·057		−1·25	−0·15	1·18	−0·13	−0·66	−0·51	3·15	1·59	0·878	0·78	9·8
		2·65	0·039		0·37	0·33	0·45	0·17	0·60	0·28	3·28	4·37			
(iii)	−210·18	31·63	−0·962	0·0098	−0·96	−0·35	1·24	−0·07	−0·09	−0·43	−1·63	−2·44	0·895	0·80	9·1
		19·62	0·588	0·0056	0·40	0·34	0·43	0·17	0·66	0·28	4·17	4·78			

The criteria used are as follows: (1) the total correlation coefficient (R) should be a maximum, and greater than the 1% significance level; (2) the standard deviation of the final residuals (σ_ϕ) should be a minimum; (3) the standard errors of the coefficients a, b and c should be a minimum; (4) the standard error of a_2 should be less than $|a_2|$ for a quadratic law to be accepted, and that of a_3 should be less than $|a_3|$ for a cubic law to be accepted.

In many cases, criteria (1) and (2) were found to be rather weak when deciding between a linear and a quadratic, or a quadratic and a cubic, and the greatest emphasis was therefore placed on criterion (3).

An example of the sort of decision to be made is provided by Table 3, which gives the statistical data for Felixstowe for all three polynomials. In terms of R and σ_ϕ, the cubic expression gives the best fit of the three, but the standard errors of coefficients b and c are larger in the cubic than in the linear. In this somewhat difficult case, the quadratic expression was adopted, since it came nearest to satisfying all four criteria.

8. Effect of air pressure terms on secular variation polynomial

One consequence of the analytical method used in this investigation is that any secular variation (or nodal oscillation) existing in the air pressure data may affect the coefficients a of the secular variation (or the coefficients c of the nodal tide).

To determine the magnitude of this possible interaction, all the pressure series were analysed for both a secular variation and a nodal oscillation. Using the regression equation

$$B_Y = \sum_{p=0} a_p' Y^p + c_1' \cos N + c_2' \sin N$$

the values of R obtained never exceeded 0·3. No significant trend or 19-yearly oscillation was found in any of the pressure series used; the polynomials $\sum_{p=0} a_p Y^p$ do not therefore need correction on this account.

9. Tabulated results

The coefficients a, b and c of equation (5) are given in Table 4 for each of the sea level stations considered. The stations are grouped under country headings, and the column blocks, reading from left to right, contain: (i) Station code number, years for which data were used, and n, the total number of data used; (ii) $a_0, \ldots, a_p$ the coefficients of Y in the secular variation polynomial; (iii) $b_1, \ldots, b_r$ the coefficients of annual mean air pressure, where the latter is in units of 0·1 mb and to a datum of 1000 mb; the air pressure station code used to identify the station is given in Table 2; (iv) c_1, c_2 the coefficients in the expression for the nodal tide; (v) R, the multiple correlation coefficient, followed by the 1% significance value appropriate to n and r; (vi) σ_0 and σ_ϕ, the standard deviations of Z_Y and ϕ_Y respectively, in mm; (vii) a_0, a_1 the coefficients in the linear version of the secular variation; these are only given when block (ii) indicates a quadratic or cubic choice.

Immediately below the coefficients a, b and c are given the corresponding standard errors.

As mentioned in Section 7, no correlation has been accepted if the value of R does not exceed the 1% significance level, and this has been ensured by omitting sea level stations for which less than 30 years of data exist. In general, the multiple correlation coefficient is high, frequently in excess of 0·9 and in the case of Vaasa (060/051) as high as 0·99. For some stations, however, R is less than 0·8.

Values of R, however, are more informative when taken in conjunction with the corresponding values of σ_0 and σ_ϕ (it should be remembered that $R^2 = 1 - \sigma_\phi^2/\sigma_0^2$).

Table 4

Coefficients for regression. (*For explanation see Section* 9)

Norway (040)

(i)			(ii)				(iii)						(iv)		(v)		(vi)		(vii)	
			0	1	2	3	034	127	030				1	2					0	1
081	1906–1930	40	653·96	−3·04			−0·97	1·12	−1·06				12·00	−19·83	0·828	0·62	77·9	43·6		
	1948–1962			0·44			0·60	0·49	0·41				10·88	11·55						
151	1928–1942	34	183·36	8·44	−0·102		−1·22	0·09	−0·69				11·56	12·82	0·863	0·69	41·8	21·1	361·51	−0·65
	1944–1962			4·20	0·047		0·29	0·28	0·24				6·87	6·20						0·50
							036	090	088	061	127	084								
301	1928–1940	33	162·97	0·45						−1·64	0·38	0·45	−0·29	9·35	0·778	0·67	23·8	14·9		
	1942–1943			0·33						0·33	0·35	0·42	4·69	4·36						
	1945–1962																			
311	1927–1941	35	331·49	−2·76			0·07	−0·50	−0·21	−1·22	2·29	−0·95	−2·90	10·09	0·906	0·73	42·5	18·0		
	1943–1962			0·46			0·44	0·48	0·52	0·74	0·53	0·65	6·12	5·39						
321	1886–1890	40	277·57	−4·11			−0·75	0·49	0·42	−2·37	1·77	0·57	7·06	2·47	0·957	0·69	96·5	28·0		
	1928–1962			0·33			0·65	0·73	0·72	1·07	0·78	0·94	8·24	8·23						

Table 4—*continued*

Sweden (050)

(i)			(ii)				(iii)						(iv)		(v)		(vi)		(vii)	
			0	1	2	3	036	090	088	061	127	084	1	2					0	1
001	1900–1923 1925–1940 1946–1961	56	240·25	−1·82 0·24			−0·13 0·47	−0·07 0·49	0·65 0·53	−2·55 0·77	1·62 0·53	0·07 0·72	2·54 5·23	2·20 5·38	0·891	0·59	52·3	23·6		
011	1911–1962	52	213·11	−2·37 0·26			−0·27 0·39	0·27 0·41	−0·01 0·47	−2·35 0·68	1·68 0·47	0·24 0·57	2·29 4·79	10·90 4·88	0·916	0·62	52·4	21·0		
021	1895–1928	34	244·51	−2·68 0·50			−0·44 0·56	−0·61 0·61	2·02 1·12	−2·69 1·14	0·42 0·86	−0·27 0·90	18·27 6·06	10·81 6·11	0·849	0·74	34·5	18·2		
031	1887–1900 1902–1962	75	235·19	−1·46 0·22			0·22 0·53	−0·24 0·53	−0·10 0·64	−3·01 0·92	2·24 0·63	0·10 0·77	11·83 6·03	−0·47 5·89	0·829	0·52	58·1	32·5		
041	1887–1962	76	171·86	−0·58 0·14			−0·55 0·33	−0·20 0·33	0·06 0·40	−1·79 0·58	1·86 0·40	−0·27 0·49	5·01 3·75	4·14 3·65	0·858	0·52	39·8	20·4		
051	1930–1962	33	203·52	−0·46 0·66			0·58 0·61	−1·63 0·65	−0·39 0·67	−1·44 0·96	2·10 0·74	0·05 0·82	−1·75 7·38	14·37 7·39	0·847	0·75	40·8	21·7		
071	1887–1962	76	298·92	0·86 0·17			0·07 0·42	−1·49 0·41	−0·33 0·51	−1·04 0·72	1·51 0·50	−0·11 0·60	2·95 4·74	−2·67 4·62	0·819	0·52	45·0	25·8		
081	1887–1962	76	321·37	−0·32 0·20			−0·02 0·47	−1·41 0·47	−0·62 0·57	−1·55 0·82	1·91 0·56	0·34 0·69	2·42 5·31	−3·29 5·17	0·798	0·52	47·8	28·8		
091	1923–1962	40	26·64	10·07 3·91	−0·124 0·045		0·71 0·58	−1·70 0·57	−1·02 0·60	−2·82 0·90	3·26 0·66	0·42 0·72	−2·40 6·79	17·05 6·59	0·918	0·71	55·5	21·9	227·13	−0·57 0·50
101	1887–1924	38	244·25	−2·97 0·65			−1·03 0·84	0·08 0·91	0·51 1·56	−2·00 1·60	2·03 1·13	−1·11 1·26	15·08 7·88	−22·08 8·72	0·782	0·70	42·8	26·7		
121	1887–1962	76	445·43	−3·02 0·18			−0·49 0·44	−0·94 0·43	−0·28 0·53	−2·39 0·75	2·26 0·52	0·35 0·63	0·55 4·92	−0·13 4·79	0·948	0·52	84·5	26·8		
131	1871–1962	92	506·67	−3·42 0·15			−0·41 0·44	−1·04 0·45	−0·31 0·56	−2·27 0·78	2·35 0·54	−0·05 0·67	4·07 4·90	−8·71 4·84	0·957	0·47	102·7	30·0		
141	1889–1962	74	516·21	−3·97 0·19			−0·40 0·45	−0·97 0·44	−0·48 0·53	−2·50 0·77	2·60 0·53	0·27 0·64	2·12 5·04	0·29 4·89	0·963	0·52	100·5	27·0		
161	1892–1962	71	577·49	−5·90 0·21			−0·55 0·47	−0·79 0·47	0·00 0·56	−3·10 0·81	2·36 0·56	0·53 0·68	−0·60 5·27	−0·79 5·30	0·979	0·53	137·6	28·0		
171	1896–1962	67	541·53	−5·84 0·26			−0·29 0·54	−0·94 0·53	0·40 0·66	−3·44 0·92	2·32 0·64	0·68 0·76	−1·99 6·26	−1·43 6·10	0·971	0·55	131·4	31·7		
181	1898–1962	65	701·29	−7·74 0·26			−0·73 0·54	−0·54 0·54	0·13 0·65	−3·34 0·92	2·54 0·63	0·44 0·77	1·23 6·30	1·39 6·03	0·982	0·56	163·2	31·0		
191	1892–1962	71	624·90	−7·54 0·23			−1·20 0·52	0·16 0·51	0·63 0·62	−3·89 0·87	2·47 0·61	0·87 0·73	0·76 5·76	−0·96 5·79	0·984	0·53	171·5	30·7		
201	1916–1962	47	646·41	−8·93 0·41			−1·23 0·55	0·88 0·55	0·60 0·64	−4·70 1·00	2·66 0·63	1·27 0·81	1·71 6·96	6·81 6·55	0·982	0·64	143·6	26·9		

Table 4—*continued*

Finland (060)

(i)			(ii)				(iii)						(iv)		(v)		(vi)		(vii)	
			0	1	2	3	036	090	088	061	127	084	1	2					0	1
001	1920–1962	43	445·58	−6·74			−1·90	1·49	0·81	−4·30	2·77	1·00	−0·71	12·83	0·971	0·67	117·8	28·2		
				0·50			0·60	0·59	0·71	1·05	0·69	0·86	7·49	7·56						
011	1899–1962	74	558·57	−6·09			−1·23	−0·43	1·87	−4·29	2·58	0·43	−7·59	−12·42	0·964	0·52	150·8	40·3		
				0·28			0·68	0·65	0·80	1·14	0·79	0·95	7·51	7·28						
021	1923–1962	40	477·76	−7·12			−0·93	0·16	0·16	−4·14	3·57	0·81	0·58	11·36	0·982	0·69	111·3	21·1		
				0·42			0·49	0·51	0·56	0·82	0·58	0·68	6·00	6·02						
031	1889–1924	36	392·61	−8·49			−3·02	0·42	1·87	−2·57	1·54	−0·49	6·87	−13·22	0·956	0·72	84·4	24·7		
				0·64			0·82	0·87	1·51	1·50	1·14	1·18	7·86	8·44						
041	1914–1962	49	588·09	−7·83			−1·16	0·47	0·96	−4·22	2·54	1·07	−4·98	7·65	0·979	0·63	130·3	26·7		
				0·37			0·52	0·54	0·61	0·89	0·62	0·75	6·41	6·40						
051	1884–1962	79	609·39	−7·54			−1·57	0·25	0·10	−2·56	2·82	−0·12	9·13	0·18	0·988	0·51	179·2	27·5		
				0·17			0·44	0·44	0·54	0·76	0·52	0·64	4·83	4·90						
061	1871–1910	57	539·90	−7·36			−2·24	0·19	−0·04	−1·08	2·46	−1·60	12·82	−21·41	0·983	0·59	144·2	26·2		
	1920–1936			0·21			0·59	0·62	1·08	1·31	0·84	1·03	6·57	5·67						
071	1927–1962	36	580·78	−7·54			−0·88	−0·46	0·49	−3·13	3·14	−0·27	−16·52	15·54	0·951	0·72	99·6	30·6		
				0·78			0·73	0·82	0·87	1·25	0·91	1·09	10·03	9·14						
091	1889–1926	38	314·01	−7·32			−1·46	−0·07	0·57	−2·38	2·36	−0·50	5·04	−12·96	0·917	0·70	83·3	33·2		
				0·74			1·06	1·10	1·96	1·98	1·44	1·50	10·27	9·94						
101	1911–1962	52	486·92	−6·30			−0·97	−0·11	0·24	−3·06	2·78	0·44	1·43	6·82	0·975	0·62	113·5	25·1		
				0·32			0·47	0·49	0·56	0·81	0·57	0·69	5·73	5·85						
121	1933–1962	30	489·45	−6·03			0·72	−1·47	−0·29	−4·49	4·10	0·66	−1·06	19·78	0·972	0·78	77·3	18·3		
				0·60			0·54	0·57	0·59	0·84	0·64	0·74	6·72	6·71						
221	1871–1936	66	446·79	−6·26	−0·039	0·0013	−1·43	−0·72	0·72	−1·65	2·61	−1·38	11·41	−8·18	0·973	0·59	106·8	24·7	381·75	−5·57
				0·48	0·008	0·0002	0·49	0·56	0·97	1·06	0·75	0·85	5·58	5·05						
231	1871–1874	65	389·73	−5·33			−0·92	−0·69	0·25	−2·16	2·49	−0·46	7·00	−13·37	0·969	0·56	103·0	25·6		
	1876–1936			0·20			0·50	0·57	0·97	1·07	0·76	0·84	5·50	5·05						

Table 4—*continued*

Finland—*continued*

(i)			(ii) 0	(ii) 1	(ii) 2	(ii) 3	(iii) 036	(iii) 090	(iii) 088	(iii) 061	(iii) 127	(iii) 084	(iv) 1	(iv) 2	(v)	(v)	(vi)	(vi)	(vii) 0	(vii) 1
241	1922–1962	41	353·01	−3·69			−0·36	−0·79	0·28	−3·98	2·75	1·24	−7·74	8·05	0·951	0·68	82·3	25·3		
				0·48			0·58	0·58	0·67	0·95	0·69	0·77	7·10	7·03						
271	1889–1936	48	426·45	−5·50			−0·82	−1·07	0·69	−2·37	2·08	−0·19	4·83	−7·16	0·944	0·64	82·2	27·1		
				0·35			0·66	0·71	1·18	1·29	0·98	1·05	7·25	6·42						
281	1924–1962	39	372·92	−4·09			0·14	−1·16	−0·46	−3·57	3·09	1·10	−0·55	10·56	0·954	0·70	78·2	23·2		
				0·50			0·55	0·56	0·62	0·92	0·66	0·77	6·87	6·70						
291	1871–1936	66	429·25	−3·92	−0·050	0·0015	−0·92	−1·22	0·24	−1·77	2·51	−0·83	3·35	−10·93	0·920	0·59	72·5	28·5	349·45	−3·15
				0·56	0·016	0·0008	0·57	0·65	1·12	1·23	0·87	0·98	6·42	5·82						0·23
311	1871–1934	64	317·88	−3·24			−1·23	−0·94	0·27	−1·56	2·52	−1·02	6·39	−14·76	0·932	0·56	69·8	25·4		
				0·20			0·50	0·57	0·98	1·06	0·75	0·82	5·39	5·15						
313	1899–1927 1929–1936	37	512·93	−4·65			−1·37	−1·98	2·55	−2·39	1·14	−0·57	9·02	5·31	0·940	0·71	70·0	23·9		
				0·47			0·68	0·75	1·25	1·44	0·97	1·07	7·84	7·00						
316	1871–1895 1897–1913 1915–1917 1919–1924 1926–1939 1943–1962	85	564·33	−3·05			−0·67	−1·22	−0·28	−2·23	2·83	−0·01	1·73	−2·53	0·955	0·49	95·8	28·4		
				0·14			0·44	0·45	0·59	0·80	0·55	0·67	4·99	4·81						
344	1900–1936	37	392·99	2·00	−0·099		−1·20	−2·34	3·23	−1·84	0·17	−0·86	12·72	−5·79	0·862	0·74	49·7	25·1	465·13	−1·61
				2·08	0·055		0·77	0·80	1·31	1·65	1·03	1·23	8·53	8·09						0·54
351	1879–1962	84	435·37	−4·08	0·025		−0·79	−1·23	0·54	−2·82	2·64	0·26	−3·30	−4·84	0·949	0·51	93·0	29·2	403·39	−3·15
				0·32	0·007		0·46	0·45	0·60	0·79	0·56	0·66	5·10	4·95						0·18
354	1871–1936	66	391·61	−2·11			−1·52	−1·19	−0·04	−1·07	3·17	−1·52	5·40	−16·50	0·885	0·55	64·1	29·9		
				0·22			0·58	0·66	1·13	1·24	0·89	0·97	6·33	5·88						
361	1929–1962	34	323·05	−1·82			−0·11	−1·83	0·16	−3·83	3·66	0·98	−5·34	11·68	0·951	0·74	78·2	23·8		
				0·70			0·61	0·70	0·70	0·99	0·76	0·90	8·03	7·65						
364	1889–1938	50	446·68	−3·54	0·063		−2·03	−1·55	1·69	−1·39	2·02	−1·20	0·50	−7·19	0·895	0·64	63·8	28·4	392·61	−1·80
				0·86	0·028		0·66	0·68	1·00	1·34	0·88	1·10	7·82	6·58						0·36

Table 4—*continued*

U.S.S.R. (Baltic Coast) (080)

(i)			(ii)				(iii)						(iv)		(v)		(vi)		(vii)	
			0	1	2	3	036	090	088	061	127	084	1	2					0	1
061	1873–1874 1876–1914 1918 1921–1923 1925–1926 1929–1936	55	330·16	−0·25 0·39			−0·29 1·07	−1·67 1·15	−1·05 2·17	−1·37 2·02	1·06 1·75	1·88 1·58	−9·19 10·40	−16·06 10·53	0·689	0·60	61·8	44·9		
081	1875–1916 1923–1938	58	455·97	1·73 0·37	−0·147 0·017		−1·72 0·78	−0·76 0·76	−1·66 1·22	0·61 1·43	3·33 1·12	−1·41 1·19	11·10 7·86	−30·22 7·66	0·895	0·60	74·9	33·3	447·74	−0·37 0·46
111	1884–1893 1901–1913 1926–1936	34	313·27	2·80 0·74	0·236 0·062	−0·0085 0·0022	−2·57 0·71	−0·15 0·79	−1·35 1·34	1·05 1·45	2·82 1·12	−2·25 1·14	4·38 7·88	−15·41 9·40	0·923	0·78	49·1	18·9	321·49	1·03 0·41
121	1873–1895 1897–1902 1904–1913 1922–1936	54	410·61	1·67 0·36	−0·079 0·019		−0·95 0·89	−1·22 0·93	−0·23 1·75	−0·91 1·70	3·08 1·45	−1·72 1·37	8·43 9·17	−30·23 8·82	0·839	0·62	64·1	34·9	305·35	0·92 0·36
151	1871–1893 1896–1913 1921–1938	59	367·62	1·00 0·30	−0·049 0·015		−0·03 0·80	−1·70 0·72	0·80 1·23	−3·05 1·59	1·76 1·08	0·56 1·26	−0·30 8·70	−19·48 7·41	0·852	0·60	63·2	33·1	287·44	0·53 0·26
191	1898–1939 1941–1943	45	211·59	0·80 0·37			−0·80 0·62	−1·50 0·64	−0·64 0·94	−0·11 1·26	2·54 0·79	−1·07 1·03	10·14 7·41	−9·52 6·59	0·847	0·65	49·8	26·4		

Poland (100)

			0	1	2	3	036	090	088	061	127	084	1	2				
031	1886–1938	53	300·87	0·56 0·31			−0·49 0·59	−1·88 0·59	−0·09 0·88	−0·46 1·24	2·15 0·77	−0·88 0·98	6·17 6·70	−4·94 5·98	0·849	0·61	50·7	26·8
071	1871–1942	72	323·14	0·75 0·15			−0·14 0·44	−1·60 0·46	−0·80 0·70	0·39 0·89	1·79 0·57	−1·13 0·73	8·01 4·82	−6·85 4·52	0·827	0·53	42·9	24·0

Table 4—*continued*

Germany (Baltic Coast) (120)

(i)			(ii)				(iii)						(iv)		(v)		(vi)		(vii)	
			0	1	2	3	036	090	088	061	127	084	1	2					0	1
001	1882–1910 1912–1914 1919–1934	48	265·37	0·07 0·29			−0·69 0·56	−0·92 0·64	−1·75 1·08	1·42 1·25	2·13 0·83	−1·91 0·92	12·00 6·25	−10·69 6·19	0·767	0·64	38·1	24·5		
011	1882–1939	58	218·72	1·48 0·20			−0·03 0·44	−1·07 0·46	−1·04 0·68	0·59 0·96	1·56 0·55	−0·94 0·73	10·31 4·94	0·69 4·57	0·859	0·59	42·1	21·5		
021	1882–1939	58	187·03	1·49 0·21			0·17 0·46	−1·06 0·48	−0·80 0·72	0·08 1·01	1·34 0·58	−0·50 0·77	8·94 5·19	−3·56 4·81	0·840	0·59	41·7	22·6		
031	1871–1923 1925–1935 1937–1943	71	267·88	0·34 0·49	−0·136 0·017	0·0041 0·0006	−0·03 0·57	−0·09 0·62	−1·15 0·94	−0·15 1·21	1·49 0·76	−0·35 1·01	−9·02 6·72	2·60 6·05	0·905	0·57	73·2	31·2	77·36	2·34 0·30
041	1882–1921 1923–1938 1940 1943	58	271·75	0·98 0·20			−0·54 0·44	−0·31 0·48	−2·00 0·70	1·13 1·02	1·11 0·59	−0·67 0·79	9·36 5·13	−7·47 4·68	0·797	0·59	36·7	22·2		

Denmark (130)

(i)			(ii)				(iii)						(iv)		(v)		(vi)		(vii)	
			0	1	2	3	036	090	088	061	127	084	1	2					0	1
001	1898–1962	65	153·62	1·04 0·21			0·28 0·42	−1·25 0·42	−0·49 0·51	−0·49 0·71	1·00 0·49	0·13 0·59	−0·14 4·93	1·46 4·73	0·759	0·56	37·3	24·3		
021	1889–1962	74	176·34	0·23 0·16			−0·10 0·37	−1·16 0·36	−0·47 0·44	−0·69 0·63	1·24 0·44	0·11 0·53	6·09 4·17	−0·52 4·04	0·779	0·52	35·6	22·3		
031	1898–1962	65	174·56	−0·03 0·17			−0·34 0·35	−0·80 0·35	−0·18 0·42	−1·23 0·58	1·74 0·41	−0·16 0·49	4·26 4·08	0·68 3·92	0·841	0·56	37·2	20·1		
041	1897–1962	66	154·50	−0·31 0·51	0·019 0·009		−0·16 0·32	−0·86 0·32	−0·16 0·40	−0·48 0·53	1·08 0·39	−0·22 0·45	6·54 3·73	1·78 3·59	0·833	0·57	32·6	18·2	126·30	0·78 0·16
051	1896–1962	67	135·81	0·88 0·14			0·19 0·31	−1·21 0·30	−0·24 0·37	−0·40 0·52	1·09 0·36	−0·31 0·43	6·86 3·56	2·43 3·47	0·833	0·54	32·5	18·0		
071	1889–1962	74	137·47	1·03 0·14			0·04 0·33	−0·74 0·32	−0·61 0·40	0·17 0·56	0·71 0·39	−0·36 0·47	6·34 3·72	1·83 3·60	0·802	0·52	33·4	19·9		
081	1888–1962	75	153·42	0·51 0·11			0·03 0·27	−0·71 0·26	−0·15 0·32	−0·52 0·46	0·84 0·32	−0·23 0·39	4·14 3·04	2·38 2·94	0·767	0·52	25·6	16·4		
091	1893–1957 1959–1962	69	65·89	−0·81 0·41	0·025 0·007		−1·61 0·30	0·76 0·41	0·32 0·45	−0·29 0·17	0·08 0·43	0·17 0·27	7·75 3·28	4·08 3·62	0·809	0·56	29·8	17·4	91·93	0·32 0·14
101	1892–1962	71	211·69	−0·04 0·19			−0·83 0·43	−0·37 0·42	−0·08 0·52	−0·76 0·73	1·25 0·51	−0·54 0·61	9·80 4·83	0·49 4·86	0·742	0·53	38·5	25·8		
							030	269	167	061	127	084								
121	1889–1962	74	193·35	1·48 0·15			−0·47 0·18	0·10 0·37	0·04 0·29	−3·10 0·32	1·13 0·43	0·71 0·47	7·85 3·63	5·24 3·79	0·914	0·56	48·7	19·8		

Table 4—*continued*

Germany (North Sea) (140)

(i)			(ii)				(iii)						(iv)		(v)		(vi)		(vii)	
			0	1	2	3	036	090	088	061	127	084	1	2					0	1
021	1898–1943	46	239·42	4·32 1·46	−0·083 0·034		−0·42 0·32	−0·27 0·99	−0·18 0·54	−3·28 0·70	0·53 0·79	1·64 1·02	−6·60 7·28	−18·84 7·94	0·777	0·67	40·4	25·5	124·95	0·82 0·39

Netherlands (150)

(i)			(ii)				(iii)						(iv)		(v)		(vi)		(vii)	
			0	1	2	3	030	269	167	061	127	084	1	2					0	1
001	1874–1962	89	257·66	1·44 0·24	0·042 0·015	−0·0005 0·0002	−0·45 0·22	0·41 0·45	−0·36 0·39	−3·10 0·40	−0·72 0·57	2·39 0·62	7·20 4·64	−0·37 4·73	0·878	0·51	57·0	27·3	278·01	1·71 0·15
011	1921–1946 1948–1962	41	−109·41	11·90 2·49	−0·121 0·030		−0·15 0·26	0·21 0·79	0·15 0·33	−1·53 0·47	−0·17 0·60	0·68 0·55	−6·62 5·05	6·90 5·56	0·828	0·70	30·5	17·1	49·44	1·77 0·38
021	1874–1962	89	260·45	1·35 0·11			−0·27 0·17	0·70 0·31	−0·23 0·25	−2·30 0·28	−0·05 0·40	0·81 0·43	3·86 3·19	−1·08 3·31	0·909	0·48	47·2	19·7		
031	1874–1962	89	275·33	1·45 0·12			−0·33 0·18	0·16 0·35	0·01 0·29	−1·69 0·32	−0·57 0·45	0·98 0·49	9·17 3·61	−1·35 3·75	0·867	0·48	44·9	22·3		
041	1884–1962	79	494·56	0·40 0·53	0·085 0·030	−0·0008 0·0004	−0·46 0·26	−0·41 0·57	−0·63 0·44	−2·04 0·48	−0·79 0·65	1·19 0·71	14·72 5·34	1·14 5·72	0·882	0·54	63·9	30·1	553·19	2·36 0·24
051	1874–1962	89	406·38	2·46 0·12			−0·35 0·18	0·04 0·35	−0·29 0·28	−1·73 0·32	−0·91 0·45	1·09 0·48	−2·56 3·57	0·11 3·71	0·943	0·48	66·2	22·1		
061	1874–1936	63	274·30	2·20 0·23	−0·054 0·013		0·05 0·24	1·47 0·45	−0·55 0·46	−1·96 0·51	−1·16 0·66	0·90 0·76	11·74 5·35	−12·33 5·36	0·896	0·58	54·2	24·1	203·96	1·73 0·23
071	1874–1962	89	181·43	1·74 0·14			0·12 0·22	0·93 0·41	−0·21 0·33	−1·35 0·37	−0·84 0·52	0·69 0·56	3·65 4·19	−9·00 4·35	0·877	0·48	54·0	25·9		
081	1874–1962	89	120·08	0·67 0·20	0·025 0·005		0·05 0·20	0·90 0·37	−0·18 0·32	−1·33 0·35	−0·18 0·49	0·20 0·53	−2·25 3·84	−8·46 4·04	0·894	0·49	52·8	23·6	132·53	1·48 0·14
091	1874–1962	89	82·30	1·21 0·25	0·014 0·006		0·25 0·24	1·09 0·47	−0·33 0·40	−1·04 0·42	−0·22 0·61	−0·09 0·66	7·09 4·82	−2·96 5·07	0·858	0·50	57·6	29·6	89·21	1·65 0·16
101	1890–1962	73	280·87	3·04 0·19			−0·32 0·21	−0·62 0·50	0·40 0·34	−1·25 0·37	−0·33 0·51	0·58 0·55	11·05 4·22	10·85 4·43	0·929	0·53	62·3	23·0		

Table 4—*continued*

British Isles (170)

(i)			(ii)				(iii)						(iv)		(v)		(v)		(vii)	
			0	1	2	3	030	269	167	061	127	084	1	2					0	1
011	1874–1914 1916–1929 1932–1962	86	236·50	0·51 0·52	0·035 0·010	−0·0004 0·0002	−0·42 0·16	−0·02 0·29	−0·55 0·26	−0·39 0·27	0·52 0·40	−0·80 0·43	6·34 3·16	0·57 3·18	0·858	0·52	35·4	18·2	255·05	0·78 0·11
041	1914–1950	37	168·55	0·13 0·24			−0·41 0·14	−0·56 0·47	0·27 0·25	0·06 0·29	0·43 0·33	−0·68 0·41	12·56 3·04	14·38 4·22	0·852	0·71	20·5	10·8		
053	1906–1962	57	295·58	2·78 0·30			−0·31 0·26	−1·28 0·84	0·66 0·43	−0·37 0·49	−0·23 0·68	0·41 0·72	20·71 5·70	12·99 6·17	0·837	0·59	49·0	26·8		
071	1918–1947 1949–1950	32	277·66	−2·22 2·65	0·057 0·040		−0·13 0·17	−0·66 0·60	−0·51 0·28	−1·25 0·37	−0·15 0·33	1·18 0·45	3·15 3·28	1·18 4·37	0·878	0·78	20·4	9·8	88·44	1·60 0·27
081	1929–1962	34	−175·59	10·40 3·48	−0·076 0·038		−0·04 0·24	0·19 0·70	0·00 0·35	0·01 0·40	−1·03 0·56	0·68 0·53	4·64 4·76	−9·04 5·03	0·914	0·76	34·9	14·5	−119·17	3·43 0·42
101	1874–1881 1883–1959	85	221·54	1·86 0·24	0·018 0·007		0·00 0·23	−0·12 0·47	0·55 0·39	−0·68 0·43	−0·80 0·58	1·05 0·66	3·00	2·10	0·900	0·49	63·0	27·4	224·84	2·37
							030	269	167	124	268	(269)								
161	1915–1962	48	431·05	2·15 0·14			−0·15 0·12	−0·80 0·39	0·03 0·19	−0·88 0·12	−0·74 0·24		6·16 2·66	1·42 3·05	0·953	0·64	37·5	11·3		
France (West Coast) (190)																				
			0	1	2	3	030	269	167	124	268		1	2					0	1
091	1894–1943 1953–1961	59	466·27	2·08 0·26			0·01 0·25	−0·70 0·85	0·18 0·50	−1·20 0·24	−0·05 0·50		11·49 5·90	3·14 6·89	0·843	0·58	51·5	27·6		
France (South Coast) (230)																				
			0	1	2	3				124	268	269								
051	1894–1936 1938–1951 1954–1958	62	433·10	0·35 0·51	0·026 0·009					−0·59 0·17	0·23 0·31	−1·74 0·51	−7·43 3·97	−9·59 4·15	0·869	0·53	40·2	19·9	430·79	1·69 0·15
Italy (270)																				
			0	1	2	3				124	268	269								
061	1905–1914 1921 1927–1936 1945–1962	39	563·37	1·40 0·32						−1·02 0·25	−0·25 0·94	−1·36 0·91	−5·33 6·98	−5·23 7·94	0·891	0·61	44·6	24·7		

As illustration, take the cases of Vaasa and Dunbar (170/041). The relevant figures to be compared are:

	n	R	1% level	σ_0	σ_ϕ
Vaasa	79	0·99	0·51	179	28
Dunbar	37	0·85	0·71	21	11

In terms of R alone, sea level variations at the Finnish station would appear to be more closely represented by the regression equation than would those at the English station. But the high value of R for Vaasa is primarily due to the fact that much of the large variance in the observed annual means is due to a large secular variation (−7·5 mm/year) which has been successfully removed by the regression

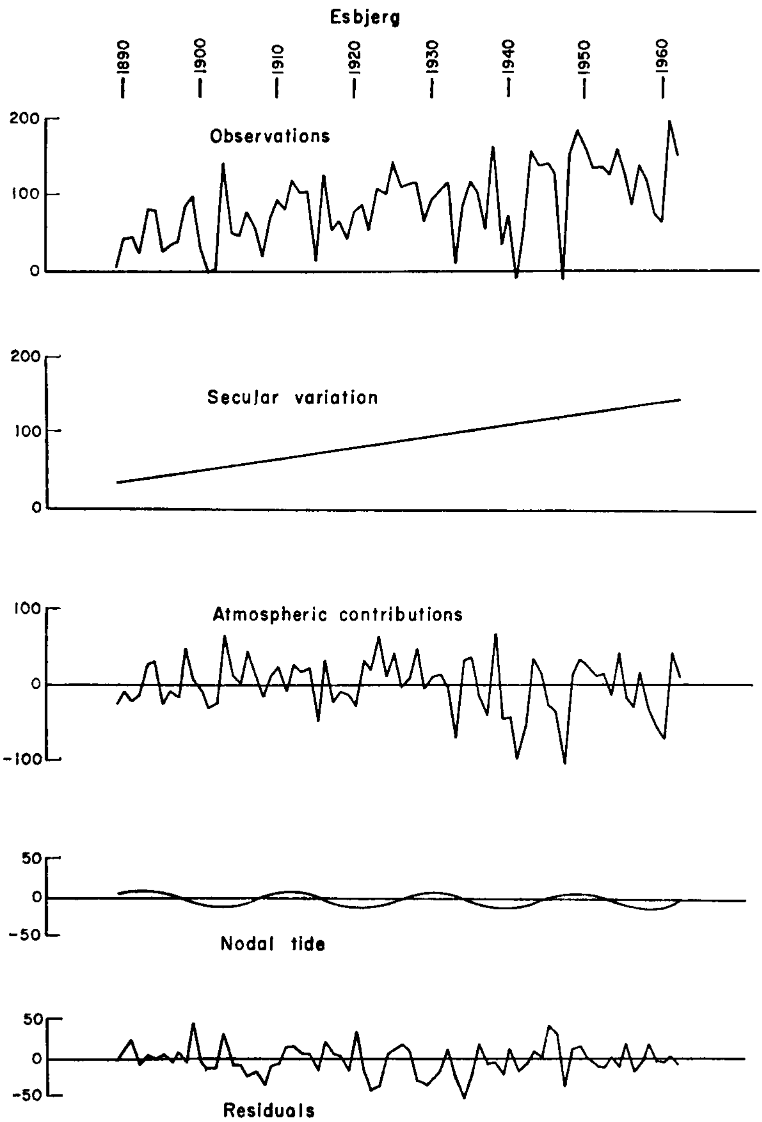

FIG. 3. Components of sea level variation at Esbjerg (130/121); units are mm.

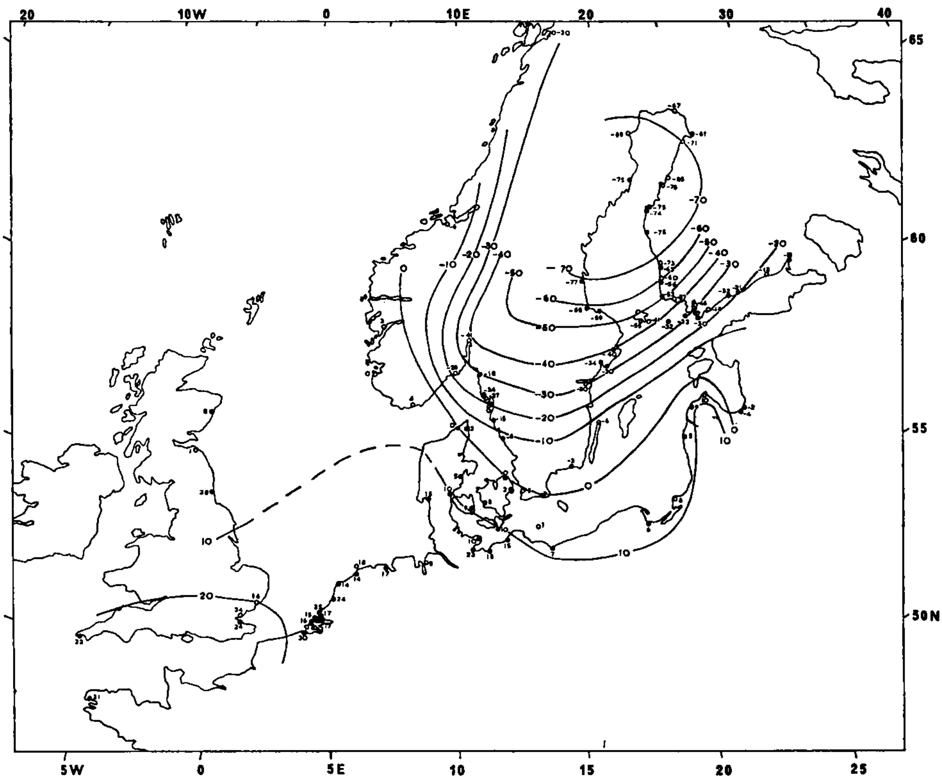

FIG. 4. Isopleths of secular variation in cm/century; positive values denote sea level rise.

(the equivalent variation at Dunbar is only 0·1 mm/year). A better criterion for the success of the regression equation is the value of σ_ϕ; on this basis, the correlation for Dunbar is superior to that for Vaasa.

For the purposes of this study, therefore, σ_0 and σ_ϕ are together more useful indicators than R alone. Very high values of σ_0 (up to 180 mm) are found at Baltic stations and, as mentioned above, are associated with large secular variations. Small values of σ_0 may accordingly be associated with small secular changes. But at very few stations are σ_ϕ as small as at Dunbar (11 mm), more often being of the order of 20 mm. It so happens that the three stations in European waters having the lowest values of σ_ϕ are Dunbar (11 mm), Felixstowe (10 mm) and Newlyn (11 mm); these three gauges are the only ones operated by the Ordnance Survey, and the inescapable conclusion is that these results are a measure of the accuracy of the observations. Within limits the converse may also be held to be true, *i.e.* high values of σ_ϕ are suggestive of poor observations.

A graphical representation of the various components of Z_Y is given in Fig. 3 for Esbjerg (130/121). To prepare such a diagram, it is first necessary to re-write equation (5) in the form

$$Z_Y = \sum_{p=0} a_p Y^p + \sum_{r=1} b_r \bar{B}_r + \sum_{r=1} b_r(B_r - \bar{B}_r) + c_1 \cos N + c_2 \sin N + \phi_Y, \qquad (6)$$

where $\bar{B}_r$ is the long term mean value of B_r. The first two terms in (6) now represent the secular variation in sea level, above the working datum of the sea level observations; the third term represents the meteorological contributions to sea level relative to the long term mean situation.

Table 5

Linear secular variation (a_1) and its standard deviation in mm/year

			a_1	σ_{a_1}	Data span
Norway	040	081	−3·04	0·44	1906–1962
		151	−0·65	0·50	1928–1962
		221	0·28	0·22	1904–1962
		261	0·00	0·31	1917–1962
		301	0·45	0·33	1928–1962
		311	−2·76	0·46	1927–1962
		321	−4·11	0·33	1886–1962
Sweden	050	001	−1·82	0·24	1900–1961
		011	−2·37	0·26	1911–1962
		021	−2·68	0·50	1895–1928
		031	−1·46	0·22	1887–1962
		041	−0·58	0·14	1887–1962
		051	−0·46	0·66	1930–1962
		071	0·86	0·17	1887–1962
		081	−0·32	0·20	1887–1962
		091	−0·57	0·50	1923–1962
		101	−2·97	0·65	1874–1924
		121	−3·02	0·18	1887–1962
		131	−3·42	0·15	1871–9162
		141	−3·97	0·19	1889–1962
		161	−5·90	0·21	1892–1962
		171	−5·84	0·26	1896–1962
		181	−7·74	0·26	1898–1962
		191	−7·54	0·23	1892–1962
		201	−8·93	0·41	1916–1962
Finland	060	001	−6·74	0·50	1920–1962
		011	−6·09	0·28	1889–1962
		021	−7·12	0·42	1923–1962
		031	−8·49	0·64	1889–1924
		041	−7·83	0·37	1914–1962
		051	−7·54	0·17	1884–1962
		061	−7·36	0·21	1871–1936
		071	−7·54	0·78	1927–1962
		091	−7·32	0·74	1889–1926
		101	−6·30	0·32	1911–1962
		121	−6·03	0·60	1933–1962
		221	−5·57	0·20	1871–1936
		231	−5·33	0·20	1871–1936
		241	−3·69	0·48	1922–1962
		271	−5·50	0·35	1889–1936
		281	−4·09	0·50	1924–1962
		291	−3·15	0·23	1871–1936
		311	−3·24	0·20	1871–1934
		313	−4·65	0·47	1899–1936
		316	−3·05	0·14	1871–1962
		344	−1·61	0·54	1900–1936
		351	−3·15	0·18	1879–1962
		354	−2·11	0·22	1871–1936
		361	−1·82	0·70	1929–1962
		364	−1·80	0·36	1889–1938
U.S.S.R. (Baltic coast)	080	061	−0·25	0·39	1873–1936
		081	−0·37	0·46	1875–1938
		111	1·03	0·41	1884–1936
		121	0·92	0·36	1873–1936
		151	0·53	0·26	1871–1938
		191	0·80	0·37	1898–1943
Poland	110	031	0·56	0·31	1886–1938
		071	0·75	0·15	1871–1942

Table 5—*continued*

			a_1	σ_{a_1}	Data span
Germany (Baltic coast)	120	001	0·07	0·29	1882–1934
		011	1·48	0·20	1882–1939
		021	1·49	0·21	1882–1939
		031	2·34	0·30	1871–1943
		041	0·98	0·20	1882–1943
Denmark	130	001	1·04	0·21	1898–1962
		021	0·23	0·16	1889–1962
		031	−0·03	0·17	1898–1962
		041	0·78	0·16	1897–1962
		051	0·88	0·14	1896–1962
		071	1·03	0·14	1889–1962
		081	0·51	0·11	1888–1962
		091	0·32	0·14	1893–1962
		101	−0·04	0·19	1892–1962
		121	1·48	0·15	1889–1962
Germany (North Sea)	140	021	0·82	0·39	1898–1943
Netherlands	150	001	1·71	0·15	1874–1962
		011	1·77	0·38	1921–1962
		021	1·35	0·11	1874–1962
		031	1·45	0·12	1874–1962
		041	2·36	0·24	1884–1962
		051	2·46	0·12	1874–1962
		061	1·73	0·23	1874–1936
		071	1·74	0·14	1874–1962
		081	1·48	0·14	1874–1962
		091	1·65	0·16	1874–1962
		101	3·04	0·19	1890–1962
British Isles	170	011	0·78	0·11	1874–1962
		041	0·13	0·24	1914–1950
		053	2·78	0·30	1906–1962
		071	1·60	0·27	1918–1950
		081	3·43	0·42	1929–1962
		101	2·37	0·17	1874–1959
		161	2·15	0·14	1915–1962
France (West coast)	190	091	2·08	0·26	1894–1961
Portugal	210	001	1·08	0·22	1894–1962
		031	1·61	0·42	1909–1961
France (South coast)	230	051	1·69	0·15	1894–1958
Italy	270	061	1·40	0·32	1905–1962

The secular variation has not been similarly corrected for the long term mean values of the nodal terms because (a) the corrections are very small, and (b) the validity of the coefficients in the nodal term is open to criticism (see Section 13).

10 (a). Linear secular variations

The average (linear) rates of rise of sea level relative to the land for European stations are summarized in Table 5 as values of a_1. These values enable the construction of Fig. 4 which shows the isopleths of rise or fall of sea level for the waters of northern Europe. This area is effectively defined by the distribution of available sea level data; it is bounded by Narvik in the north, Viipuri in the east, Brest in the south and Newlyn in the west. The only long series of data extant for the Mediterranean are those for Marseilles and Trieste.

In drawing the isopleths as continuous lines over such a large area it is assumed that the phenomenon or phenomena they represent are also continuous. The most obvious discontinuity which could exist arises from local land movement, though in this context the word 'local' could embrace more than one station. This argument may cast doubt upon the validity of the isopleths connecting the British Isles to the Continent, for example. Nevertheless, the isopleths exhibit few kinks, nor do the results for many individual stations depart markedly from the isopleths.

The outstanding features of Fig. 4 are the negative isopleths centred over Fennoscandia which indicate the well known result of continuing land uplift following deglaciation. Since this phenomenon dominates most of the available data, the contents of Table 5 cannot be expected to give much useful information on eustatic changes. The most profitable way of interpreting these data is to assume a value for the eustatic rise of sea level, to correct the values of a_1 accordingly, and to examine the results in terms of land movement. This involves the further assumption that the eustatic rise is uniform over the area under consideration. Accepting a eustatic rise of 1 mm/year, as proposed by Gutenburg (1941) and Munk & Revelle (1952), the rate of land uplift is given by $(1-a_1)$ mm/year.

Fig. 5 compares the values of $1-a_1$ in units of cm/century with those obtained by Gutenburg, which were also deduced from sea level data but from a shorter span of time and without the benefit of meteorological corrections. Satisfactory agreement is evident for the general outlines. In particular, the tendency for the line of zero movement to follow the Latvian coastal contour is repeated. However, Gutenburg's kink in the same zero line parallel to the Danish coastline is not repeated.

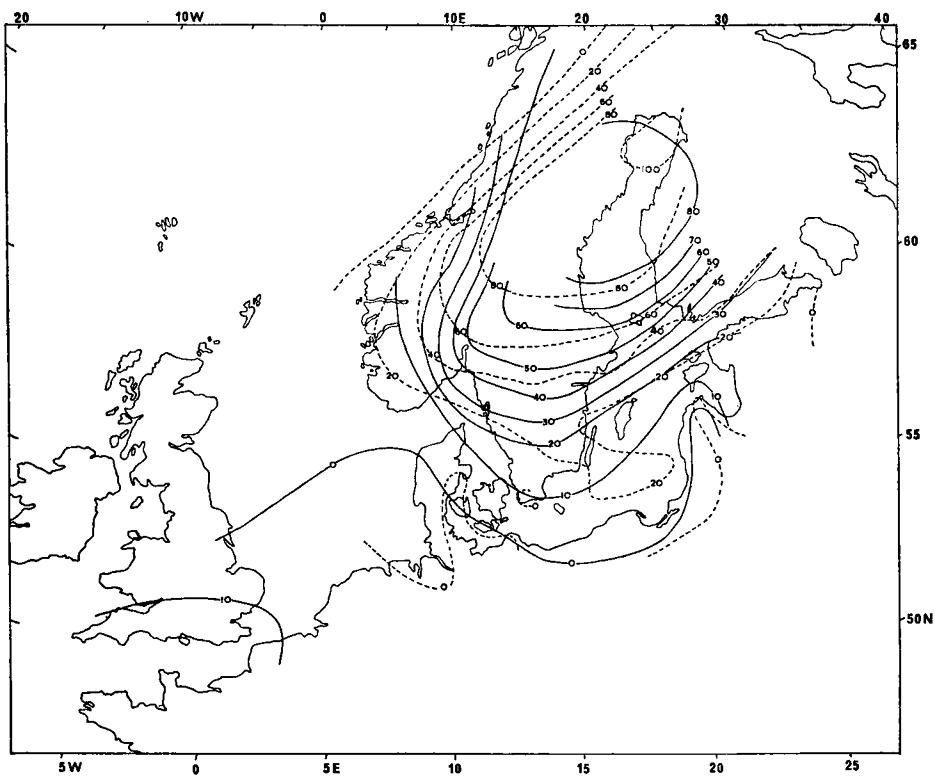

FIG. 5. Isobases in cm/century; positive values denote land uplift.
Rossiter, ———— ; Gutenberg (1941), - - - - - .

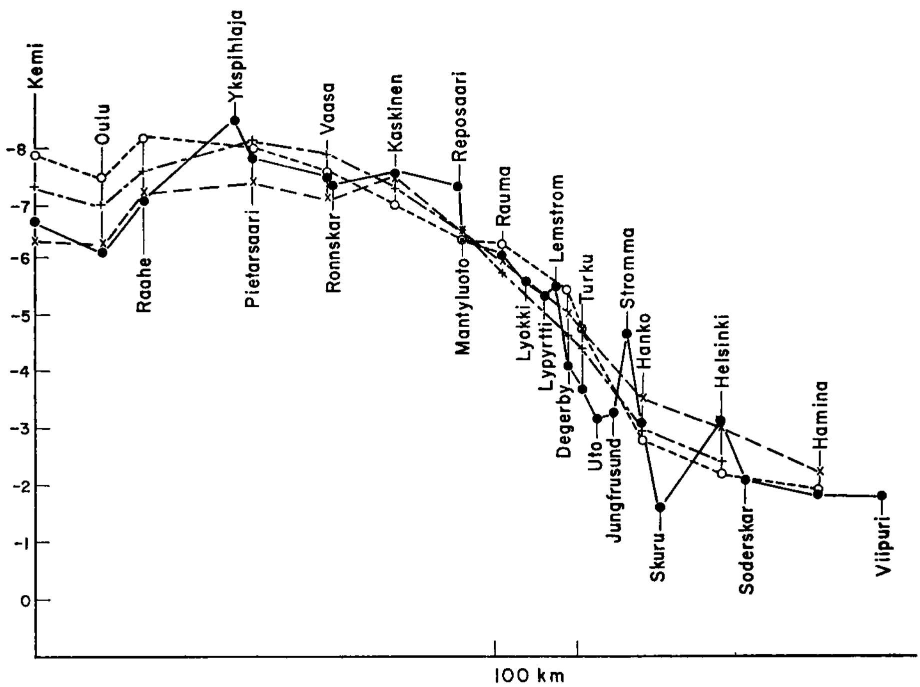

FIG. 6. The secular variation of sea level along the Finnish coast; units are mm/year. Rossiter, ——●——; Lisitzin (1964), ——+——; Hela (1953), -- × --; Kääriäinen, ---○---.

The maximum uplift, which may be considered as being effective during the first half of the twentieth century, is centred in the northern part of the Gulf of Bothnia, and amounts to more than 90 units. From this area, a line of maximum uplift runs south-west through Sweden. The line of zero movement follows the southern shores of the Baltic, and clearly separates the Scandinavian contours from those to the south. The rather sparse data from southern England and France follows the trend indicated by the Netherlands' data to suggest that in these areas a fairly uniform lowering of land of the order of 10 units exists.

Returning to Fig. 4 and considerations of mean secular variations of sea level relative to the land (a_1), and examining the data for each country in turn, some general comments can be made. The author, however, is extremely conscious of the need for a detailed knowledge of the history of the sea level recordings and datum-fixing operations, and of the local geological structure, in adequately interpreting the apparent mean secular variations. For this reason, the hope is expressed that national authorities will go further with the estimates given in this paper than the author dare do.

(a) *Norway.* Small positive values of a_1 are found in the west. Results for Bergen (221) and Stavanger (261) which did not allow for meteorological corrections support the positioning of the zero isopleth. At Narvik, in the north, and in the Oslo Fjord, sea level is falling at the rate of 30 units.

(b) *Sweden.* Values of a_1 range from zero in the south to -80 or -90 units in the Gulf of Bothnia. The results for the Bothnian stations do not progress very smoothly along the coast, and have made the positioning of the -60, -70 and -80 isopleths rather doubtful. The value of -18 units for Smogen must also be considered anomalous.

(c) *Finland*. The results show a high degree of consistency along the entire length of the Finnish coast. Lisitzin (1964) has compared the results of various sea level studies by Hela (1953), Rossiter (1960) and herself with the results of precise geodetic levelling reported by Kääriäinen. These are reproduced in Fig. 6, except that the values of Table 4 replace the present author's original estimates, which were based on only 19 years of data.

It is worth noting that the present investigation has utilized data from twelve stations additional to those used by previous authors, thereby providing a more detailed picture of the coastal distribution of variation. However, data from some of these stations provide the source of most of the anomalies; in particular, secular variations of −46 and −16 units at Stromma and Skuru respectively must be considered doubtful.

With these exceptions, the various estimates given in Fig. 6 now present a reasonably coherent picture; the author's earlier (1960) values for stations south of Kaskinen are shown to have been underestimates of the secular variation, thus confirming Lisitzin's 1964 criticism.

(d) *U.S.S.R.* From the entrance to the Gulf of Riga, to the Gulf of Danzig, a uniform rise of between 5 and 10 units is evident. This changes to a fall of a few units at the head of the Gulf of Riga, and requires the sharp bend in the zero and 10 unit isopleths of Fig. 4.

Apart from the tilting due to the Fennoscandian land uplift, this area shows the greatest tilting in all of north-west Europe.

(e) *Poland, Baltic Germany and Baltic Denmark*. These areas all lie outside the influence of land uplift, the rise of sea level being most prominent (more than 15 units) in Lübeck Bay.

(f) *North Sea Denmark and Germany, and the Netherlands*. These stations present a quite coherent picture of sea level rising at a rate of 14–24 units, though there are some anomalous values. That for Bremerhaven (140/21) of 8 units seems small, whereas 30 units for Vlissingen (150/101) is so high as to suggest local land subsidence relative to the other Netherlands stations.

The cautionary remark at the beginning of this section is particularly relevant in the case of the Netherlands. At seven of the eleven stations (including Vlissingen), a linear expression for the secular variation was found to give a poor fit compared with a quadratic or cubic, as the annual plots of sea level of Ijmuiden (150/41) in Fig. 1 show.

(g) *British Isles*. Despite the paucity of data, the results are, in the main, compatible with one another. Along the shores of the English Channel the sea level rise is a little more than 20 units, but clear evidence is provided of a north–south land tilt in England and Scotland. The value of 28 units for North Shields (170/53) must be viewed as evidence of land subsidence in that area (a mining region).

(h) *France, Portugal and Italy*. Referring to Table 5, the results for the five stations in these countries show reasonable agreement amongst themselves. The rise of 21 units at Brest (190/91) is in accord with the rise of 22 units at Newlyn (170/161) on the opposite shore of the Channel. The values of 11 and 16 units rise for Cascais (210/1) and Lagos (210/31) respectively have been computed without the benefit of meteorological corrections; nevertheless, they differ little from the rises of 17 and 14 units at Marseilles (230/51) and Trieste (170/61) respectively.

These results sugest an area of maximum secular variation centred on the Channel, of the order of 20 units. South of the Channel, the variation is quite uniform at about 15 units, from Portugal through to the Adriatic.

Table 6

Secular variation, in mm/year, at certain stations for certain epochs

Station	Code No.	Secular variation in 1880	1900	1920	1940	1960	Data span
Helsinki	060/351	−5	−4	−3	−2	−1	1879–1962
Travemunde	120/031	11	0	0	9	—	1871–1943
Delfzijl	150/001	−1	1	2	2	1	1874–1962
Ijmuiden	041	—	0	3	3	2	1884–1962
Brouwershavn	081	1	1	2	3	4	1874–1962
Zierikzee	091	1	1	2	2	3	1874–1962
Aberdeen	170/011	−1	1	1	1	0	1874–1962
Sheerness	101	1	2	3	3	4	1874–1959

10 (b). Non-linear secular variations

Of the ninety-two stations considered, twenty-six required a quadratic or cubic expression in order that the secular variation should satisfy the statistical criteria discussed in Section 7. It is of interest to examine whether any general deduction can be made from these results.

There are at least three reasons why a non-linear distribution should be required:

(i) accidental correlation with large anomalies of mean sea level;

(ii) discontinuities in the sea level data, either real (sudden land movement) or fictitious (lack of good datum control);

(iii) a real and gradual non-linear secular variation.

The first of these possibilities is most likely to occur when the span of data is short; the need for a quadratic expression for Felixstowe (34 years of data) is a case in point. To minimize this possibility, we shall consider only those stations for which more than 70 years of data have been used. These eight stations are listed in Table 6, in which are tabulated the secular variation for the years 1880, 1900, 1920, 1940 and 1960, in mm/year; these values have been derived from the expression $a_1 + 2a_2 Y + 3a_3 Y^2$.

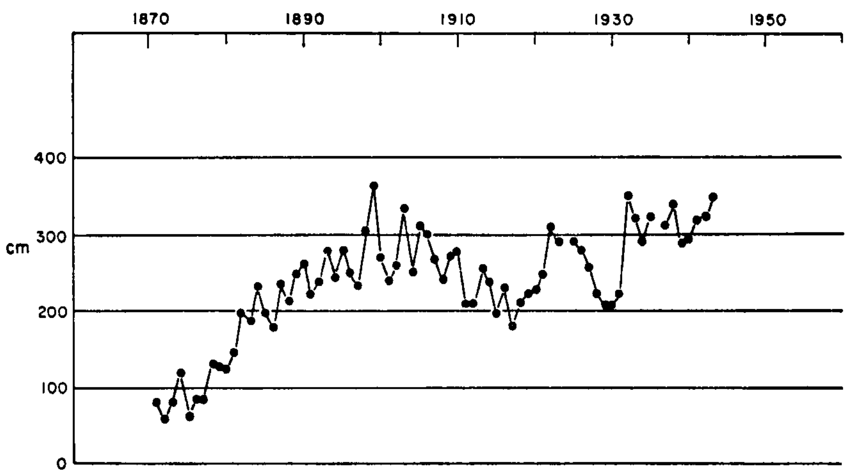

FIG. 7. Annual mean levels at Travemunde (120/31), illustrating apparent cyclical trend.

At most of these stations there is a tendency for the secular variations to increase in the positive sense, *i.e.* the rise in sea level has been increasing, or the land subsidence has been decreasing, or both.

Only two of these stations are in the Baltic, the remainder being in the Netherlands (four) and Great Britain (two).

This tendency for the secular variation to accelerate between 1880 and 1960 is apparently contradicted, however, by the fact that twenty-three stations (including five in the Netherlands) with more than 70 years' data do *not* require non-linear expressions. This latter fact would appear to exclude the suggestion that the eustatic variation is accelerating, and leaves only the possibility that the non-linearity represents local or regional land movement. Such land movement could be sudden (over the space of not more than a few years) or gradual. In the case of Travemunde, the sea level data used (Fig. 7) clearly show an apparent cyclical movement for which a cubic expression happens to provide a best fit over the span of time considered.

It would be of interest to learn whether local knowledge of land movement at the eight stations mentioned can be linked with the interpretations, given above, of the sea level data.

11 (a). The mean sea level surface under normal atmospheric conditions

Section 10 dealt with annual mean sea level at a given station as a time dependent phenomenon. It is also of value to examine the spatial distribution of the mean sea level surface at any given epoch.

From equation (5) the best statistical estimate of h^Y, the height of mean sea level above the National Reference Plane (N.R.P.) at any station for the year $1900+Y$ is given by

$$h^Y = \Sigma a_p Y^p + \Sigma b_r B_r + \delta,$$

where δ is the height of the working datum, in mm, above N.R.P.

Table 7

Estimated values of mean sea level in 1970, *in cm above National Reference Plane*

Norway	Narvik	−3·3	North Norwegian Normal Null
	Heimsjo	−1·4	Norwegian Normal Null
Denmark	Hirtshals	−5·1	Danish Normal Null
	Esbjerg	15·5	Danish Normal Null
Netherlands	Delfzijl	0·9	Normaal Amsterdam Peil (REUN Phase B)
	Den Helder	−5·4	
	Ijmuiden	−0·6	
	Hoek van Holland	3·3	
	Hellevoetsluis	6·6	
	Brouwershavn	−7·2	
	Zierikzee	−3·6	
	Vlissingen	−0·7	
British Isles	Aberdeen	51·9	Ordnance Datum Newlyn (3rd levelling)
	Dunbar	44·3	
	Felixstowe	11·8	
	Southend	20·4	
	Sheerness	16·2	
	Newlyn	11·4	
France	Brest	17·9	Zéro du Nivellement Général de la France
	Marseilles	13·4	Zéro du Nivellement Général de la France

Under *normal* atmospheric conditions $B_r = \bar{B}_r$, and the best estimate becomes

$$h_n^Y = \Sigma a_p Y^p + \Sigma b_r \bar{B}_r + \delta. \quad (7)$$

Equation (7) can be applied, in practice, for a number of purposes. Possibly the most important is for determining the best value of mean sea level at a given station when preparing tidal predictions. For this purpose, however, it would be advisable to replace the term $\Sigma a_p Y^p$ by its linear version, since it is inadvisable to extrapolate the secular variation foiwards in time on anything other than a linear law. Values of h_n^{70}, the height of mean sea level in 1970, using a linear law, are given in Table 7 for those stations which normally feature in tide tables. Interpolations of h_n^{70} between neighbouring stations should be used with caution, however, for reasons which will now be discussed.

11 (b). The mean sea level surface under isobaric conditions

For the purpose of 'oceanographic levelling', *i.e.* the use of mean sea level data to carry levels across straits and along coastlines, it is necessary to correct the mean sea level heights for influences which affect the slope of the sea surface but which have no appreciable influence on the slope of the land; these are the influences which cause the mean surface of the sea to depart from the theoretical definition of the geoid, the equipotential surface of the Earth's gravitation and rotation.

The mean sea surface slope between two stations at any given epoch may depend, amongst other factors, upon: (a) the differential response at the two stations due to air pressure and wind stress, including the effects of radiation stress (wave 'set-up' or 'set-down'); (b) the distribution of the density of sea water (steric slope); (c) in the case of stations on opposite sides of a strait, the curvature of the time-mean current through the strait; (d) in the case of stations along a coastline, the coastwise distribution of longshore mean current; (e) the distribution of shallow water tides.

Only for case (a) can an attempt be made to use the results of the regression equation (5). What is probably the most important contribution, the steric slope, would require considerable effort in data collection of temperatures and salinities for its success. Bowden, Lisitzin and others have indicated the order of magnitude of the steric slope in certain European waters, and in the hope that an accurate and comprehensive picture of the steric surface will eventually be obtained the contributions (a) have been allowed for as follows.

From equation (7), the best estimate of the height of mean sea level above N.R.P. at any station under *isobaric* conditions is given by

$$h_i^Y = \Sigma a_p Y^p + \bar{P} \Sigma b_r + \delta. \quad (8)$$

The choice of a pressure $\bar{P}$ to represent an isobaric atmosphere depends upon the proportion of the Earth's surface over which the atmosphere is to be considered isobaric. The smaller the area, the less difficult the choice becomes; for European waters $\bar{P}$ has been taken as 1015 mb.

An example of the calculation of h_i^{50} is given in Table 8 for Esbjerg.

In order to relate mean sea level values in one country with those in another, it is necessary to know the relationships between the various National Reference Planes used. Under the auspices of the International Association of Geodesy attempts are being made to establish such relationships, but the task is immensely complicated and rather slow. The international reference plane chosen is N.A.P. (Normaal Amsterdam Peil), and relationships are computed in geopotential units. A brief history of this geodetic work has been written by Simonsen (1962).

Table 8

Computation of h_n^Y, h_i^Y for Esbjerg (130/121), 1950

$Y=50$

Working datum is 50 mm below Danish Normal Null, i.e. $\delta=-50$ mm

$$\begin{array}{lll} a_0=193\cdot35 & b_1=-0\cdot47 & \bar{B}_1=59 \\ a_1=\ \ 1\cdot48 & b_2=\ \ 0\cdot10 & \bar{B}_2=187 \\ & b_3=\ \ 0\cdot04 & \bar{B}_3=139 \\ & b_4=-3\cdot10 & \bar{B}_4=113 \\ & b_5=\ \ 1\cdot13 & \bar{B}_5=154 \\ & b_6=\ \ 0\cdot71 & \bar{B}_6=125 \\ & \Sigma b_r=-1\cdot59 & \bar{P}=150 \end{array} \left.\right\} \text{in units of 0·1 mb. and referred to a datum of 1000 mb}$$

$$\begin{array}{rl|rl} \Sigma a_p Y^p = & 267\cdot3 & \Sigma a_p Y^p = & 267\cdot3 \\ \Sigma b_r \bar{B}_r = & -91\cdot0 & \bar{P}\Sigma b_r = & -238\cdot5 \\ \delta = & -50\cdot0 & \delta = & -50\cdot0 \\ h_n^{50} = & 126 \text{ mm} & h_i^{50} = & -21 \text{ mm} \end{array}$$

It had been the author's intention to reduce the sea level data to the common datum of N.A.P., and to discuss the results on that basis. Unfortunately this is not possible, at present, for many of the stations involved. Wherever possible N.A.P. has been used, but in general the values of h_i^{50} have been interpreted below on a national basis. Where N.A.P. has not been used, it has been assumed that N.R.P. was a horizontal plane at epoch 1950 (an assumption which is not necessarily true in all countries).

Table 9 lists the values of h_n^{50}, h_i^{50}, $h_n^{50}-h_i^{50}$ all to N.R.P. together with a definition of the latter. Where possible, the correction to N.A.P. is given, together with the adjusted value of h_i^{50} above N.A.P. The epoch 1950 (more properly, 1950·5) has been chosen to coincide, as nearly as possible, with the epoch about which international geodetic levelling in Europe has been centred. At many sea level stations, however, the span of data ended before 1950. The values for all such stations are asterisked to indicate that they involve extrapolation of the computed secular variations; moreover, only the linear version of the variations has been used for these stations.

The difference $h_n^{50}-h_i^{50}$ represents the permanent contribution to mean sea level due to the normal air pressure distribution, and it will be shown in Section 11(c) that they are so coherently distributed as to justify the acceptance of h_i^{50} as a better approximation to the geoid than h_n^{50}. It must be borne in mind, however, that a closer approximation will require corrections for items (b) and (e) listed earlier. Until this is done, and in particular until reliable corrections are available for the stationary steric slope, it is not reasonable to state how closely the plane of h_i^{50} approximates to the geoid; the following summaries, therefore, do not necessarily imply errors in geodetic levelling.

Norway. Apart from Oslo (321), all the Norwegian data can be referred to N.A.P. and to this datum the isobaric surface h_i^{50} is tilted downwards to the north by 8 cm between Nevlunghavn (311) and Narvik (81). By comparison, the normal surface h_n^{50} referred to Norwegian Normal Null for the four southernmost stations is virtually horizontal.

Sweden. Only five stations in Sweden have been connected by R.E.U.N. to N.A.P. The need for such a connection in interpreting the sea level data is clearly shown in the upper part of Fig. 8; here the values of h_i^{50} are plotted against distance along the coastline relative to both N.R.P. and N.A.P.

To N.A.P., the isobaric surface appears to slope upwards from Smogen (11) in the east to Furuogrund (201) at the head of the Gulf of Bothnia by 24 cm with a particularly sudden rise in the last 100 km.

In a pilot study of the contributions to sea level from the sea water density distribution, Bowden (1960) arrived at a steric slope which has been plotted on Fig. 8. More recently Lisitzin (1962) has derived an estimate of the steric slope for that part of the Baltic between Gedser, just within the Baltic proper, and Degerby, at the entrance to the Gulf of Bothnia. This is also shown in Fig. 8, and is similar to Bowden's estimate. It will be seen that the overall magnitude of both isobaric and steric slopes are of the same order of magnitude, though the local distribution is different in the two cases.

Finland. The Finnish stations are of two kinds; the major stations, for which careful bench mark levelling has been effected, and for which modern sea level data exist, and the secondary stations (some of them on islands) for which the connection to N.R.P. is approximate and for which modern sea level data do not exist. All major stations have been connected to N.A.P., and the isobaric surface is illustrated in Fig. 8.

Apart from anomalous values of h_i^{50} above N.R.P. at Ykspihlaja (31), and Stroma (313)—both of them secondary stations—the Finnish data are distributed smoothly along the coastline.

The isobaric surface relative to N.A.P. is even smoother, varying by only a few cm from a mean of N.A.P. + 8 cm.

The steric slope along this coastline has been computed by Lisitzin (1957–1958), and is also shown on Fig. 8. It is greater, both in the Gulf of Bothnia and the Gulf of Finland, than the isobaric slope.

U.S.S.R., Poland and Germany. Mean sea level data for all these stations end more than 20 years ago, and the definition of N.R.P. is somewhat doubtful. It would

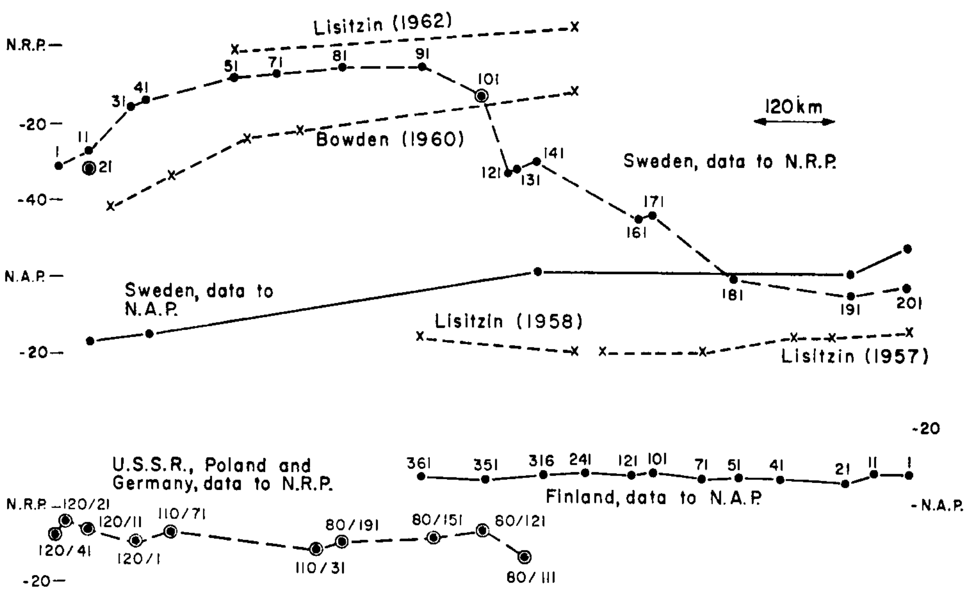

FIG. 8. The isobaric sea level slope in 1950, along various coastlines. Related to National Reference Plane, —·—●—·—; related to R.E.U.N. Plane (N.A.P.), ——●——; where data involve extrapolation forward to 1950, ⊙; steric slopes, - - - × - - -.

Table 9

Mean sea level, 1950·5, *in cm above National Reference Plane*

	h_n^{50}	h_i^{50}	$h_n^{50}-h_i^{50}$	National Reference Plane	Correction to N.A.P.	h_i^{50} above N.A.P.
Norway						
81	3	−13	16	North Norwegian Normal Null	−10·1	−24
151	1	−12	13		−9·2	−21
301	1	−4	5		−12·5	−16
311	0	−9	9	Norwegian Normal Null	−7·1	−16
321	−1	−11	9			
Sweden						
1	−21	−31	10			
11	−18	−27	9		10·1	−17
*21	−20	−32	12			
31	−4	−16	11			
41	−4	−14	10	Zero of precision levelling system	−0·7	−15
51	−1	−8	7			
71	1	−7	7			
81	4	−5	9			
91	7	−5	12			
*101	1	−13	14	Local datum		
121	−20	−33	13			
131	−19	−32	13			
141	−17	−30	13		31·2	1
161	−30	−45	15	Zero of precision levelling system		
171	−30	−44	14			
181	−45	−61	16			
191	−47	−65	17		64·9	0
201	−45	−63	18		69·6	7
Finland						
1	−22	−41	19		49·2	8
11	−20	−39	19		47·1	8
21	−24	−42	18		48·5	6
*31	−32	−54	21			
41	−28	−44	16		51·2	7
51	−24	−41	17		48·5	7
*61†	−23	−41	18			
71	−22	−39	17		45·6	7
*91†	−21	−38	17			
101	−19	−34	15		43·1	9
121	−17	−33	16		40·6	8
*221†	−15	−31	16			
*231†	−14	−29	15	Normal Null		
241	−8	−23	15		32·0	9
*271†	−13	−28	15			
281†	−11	−24	13			
*291†	−6	−20	14			
*311†	−4	−19	15			
*313†	−8	−26	18			
316	−2	−16	14		24·6	8
*344	2	−14	16			
351	−1	−16	15		22·7	7
*354†	−1	−18	17			
361	1	−15	16		23·5	8
*364	6	−11	18			
U.S.S.R. (Baltic)						
*61	147	140	7			
*81	137	122	15			
*111	1	−13	14	Undefined datum		
*121	8	−6	14			
*151	5	−8	13			
*191	2	−9	11	German Normal Null		

* Values involve extrapolation of secular variation. † Approximate connection to N.R.P.

Table 9—*continued*

	h_n^{50}	h_t^{50}	$h_n^{50}-h_t^{50}$	National Reference Plane	Correction to N.A.P.	h_t^{50} above N.A.P.
Poland						
*31	−1	−11	10	German Normal Null		
*71	0	−6	6	German Normal Null		
Germany (Baltic)						
*1	−2	−9	6	German Normal Null		
*11	−1	−5	4	German Normal Null		
*21	−2	−5	3	German Normal Null		
*31	−1	−3	2	German Normal Null		
*41	−4	−7	3	German Normal Null		
Denmark						
1	7	3	4	Danish Normal Null	−8·4	−5
21	4	−2	6	Danish Normal Null	−5·2	−7
31	2	−7	9	Danish Normal Null	−7·0	−14
41	7	2	5	Danish Normal Null	−10·1	−8
51	4	0	4	Danish Normal Null	−10·2	−10
71	4	2	2	Danish Normal Null	−13·4	−11
81	1	−3	4	Danish Normal Null	−10·9	−14
91	−4	−10	6	Danish Normal Null	−11·1	−21
101	−5	−14	9	Danish Normal Null	−9·3	−23
121	13	−2	15	Danish Normal Null	−15·8	−18
Germany (North Sea)						
*21	7	−5	12	German Normal Null		
Netherlands						
1	−2	−13	11	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
11	−4	−10	6	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
21	−2	−13	11	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
31	−8	−15	7	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
41	−5	−12	8	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
51	−1	−8	7	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
*61	6	−2	9	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
71	3	−2	5	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
81	−9	−16	7	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
91	−6	−12	6	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
101	−7	−10	3	Normaal Amsterdam Peil (R.E.U.N. Phase B)		
British Isles						
11	51	43	8	OrdnanceDatumNewlyn (3rd geodetic levelling)	−22·6	21
41	44	41	3	OrdnanceDatumNewlyn (3rd geodetic levelling)	−22·6	18
53	37	40	−3	OrdnanceDatumNewlyn (3rd geodetic levelling)	−22·6	17
71	10	9	1	OrdnanceDatumNewlyn (3rd geodetic levelling)	−22·6	−14
81	14	16	−1	OrdnanceDatumNewlyn (3rd geodetic levelling)	−22·6	−7
101	13	14	−1	OrdnanceDatumNewlyn (3rd geodetic levelling)	−22·6	−8
161	7	12	−5	OrdnanceDatumNewlyn (3rd geodetic levelling)	−22·6	−10
France (West coast)						
91	14	15	−2	Zéro du Nivellement Général de la France	−12·8	3
France (South coast)						
51	11	15	−4	Zéro du Nivellement Général de la France	−27·2	−12
Italy						
61	158	164	−6	Level Hopfner (1949)	−192·0	−28

* Values involve extrapolation of secular variation

seem reasonable, however, to assume that with the exception of the U.S.S.R. stations 61 and 81, all the values of h_i^{50} refer to German Normal Null. No connection to N.A.P. is available.

Fig. 8 shows that from Kolkasrags (080/11) in the east to Marienleuchte (120/41) in the west, the isobaric surface is fairly regular, with no clear indication of a pronounced slope. In the absence of further information regarding the nature of the N.R.P., further comment is pointless.

The solitary value of h_i^{50} for Bremerhaven (140/21) in the North Sea is similar to the average for the German Baltic stations. Compared with values for the Danish and Netherlands stations, however, it appears to be some 10 cm too high; this may possibly be accounted for in terms of the N.R.P.–N.A.P. relationship.

Denmark. Using the values of h_i^{50} referred to N.A.P., a downward slope of the isobaric surface is evident from both the eastern and western stations.

In the Kattegat and the area of the Danish islands, the mean slope is 16 cm in 340 km, or 1 cm in 21 km. Bowden's estimate of the corresponding steric slope was 1 cm in 18 km.

The North Sea slope upwards from Hirtshals (101) to Esbjerg (121) of 5 cm is not explained by Bowden's steric slope, which is 2 cm downwards. It is possible that the higher sea level at Esbjerg is a purely local phenomenon arising from the shallow water tides experienced there.

Netherlands. There is little evidence of an isobaric slope along this coastline. The general level of h_i^{50} is at N.A.P. -12 cm with notable exceptions at Maasluis (61) and Hellevoetsluis (71). At both stations the value of N.A.P. -2 cm is compatible with their location upriver, where the water density is likely to be less than at the other stations.

British Isles. Apart from the anomalous value at Felixstowe (71) the main feature of the isobaric surface is a positive slope to the north along the east coast of 28 cm between the mouth of the Thames and Aberdeen. It has been suggested (Edge 1959) that a substantial proportion of this slope may be due to a systematic error in British levelling.

The connection between Ordnance Datum (Newlyn) and N.A.P. is based on the connection between O.D.N. and the zero of the French Levelling system (N.G.F.) derived by Cartwright & Crease (1963) from a study of sea level data on opposite shores of the Strait of Dover.

France and Italy. The results for Brest (190/91) and Marseilles (230/51), referred to N.A.P., indicate a downward isobaric slope into the Mediterranean of 15 cm. This is of the same order of magnitude, and in the same sense, as the steric slope reported by Lisitzin (1965). This slope is continued by a further fall of 16 cm between Marseilles and Trieste (270/61) at the head of the Adriatic.

Taken together, the results for Newlyn (N.A.P. -10 cm) and Brest (N.A.P. $+3$ cm) indicate a 13 cm slope of the isobaric surface across the entrance to the English Channel. Between Ramsgate and Dunkerque, Cartwright and Crease deduced a comparable slope of 7 cm in the same sense, composed of a geostrophic component of $5\frac{1}{2}$ cm due to the residual current, and $1\frac{1}{2}$ cm due to the curvature of the tidal streams; the steric slope was negligible.

On the basis of continuity of volume between the two sections, the geostrophic slope on the Newlyn–Brest line is unlikely to be more than $1\frac{1}{2}$ cm. The steric slope is also unlikely to be more than a few cm, leaving some 10 cm to be explained. There may be an appreciable contribution, however, from the strongly curved tidal streams around the two peninsulars concerned.

To summarize, the topography of the isobaric sea surface in European waters exhibits well defined slopes in the Baltic proper, the channel between the Baltic and the North Sea, along the east coast of the British Isles, and between the Atlantic and the central Mediterranean. Some of these slopes may be explained in terms of the stationary distribution of water density.

In the Gulfs of Bothnia and Finland the levels are approximately N.A.P. +8 cm, decreasing to N.A.P. in the main body of the Baltic, and even further to N.A.P. − 15 cm in the North Sea. A further decrease to N.A.P. − 24 cm occurs northward along the Norwegian coast. In the southern North Sea the isobaric levels are approximately N.A.P. − 10 cm. An average value for the northern Mediterranean is N.A.P. − 20 cm.

11 (c). The permanent sea level slope due to normal atmospheric conditions

The distribution of $h_n^{50} - h_i^{50}$ tabulated in Table 9 and illustrated in Fig. 9 represents the long term mean deformation of the sea surface corresponding to the long term mean air pressure distribution. The approximate courses of the 1010, 1012 and 1014 mb isobars are also shown. The deformation is quite regular, and depicts a slope, positive to the northeast. The lowest levels are in the Mediterranean (−6 cm), the western English Channel (−4 cm) and along the east coast of the British Isles (0 cm); the highest levels are along the northern coast of Norway (16 cm) and in the Gulfs of Bothnia (18 cm) and Finland (16–18 cm). Local anomalies, which may be due to poor mean sea level data or to local topographical peculiarities, are found at North Shields, in the Netherlands Delta area, at Tregde in southern Norway, and in the Gulf of Riga. The distribution is complex in the region of the Danish islands, and the lines in the North Sea cannot be located with any great exactitude.

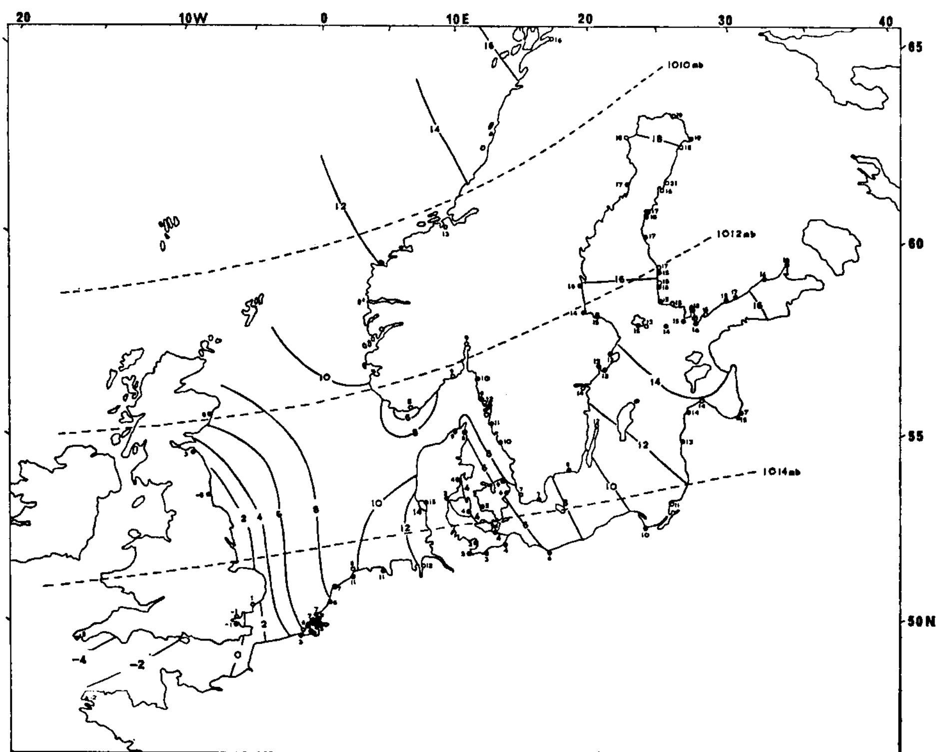

Fig. 9. Mean deformation of the sea surface due to the long term mean air pressure distribution; units are cm.

The mean air pressure distribution, in which the pressures decrease to the north, would be expected to result in sea level being higher to the north. The associated east-going wind field would also be expected to set up a stationary gradient of the sea surface, positive to the east. The combination of these two effects is indeed reflected very clearly in Fig. 9. The maximum sea level gradients, which are those in the North Sea, in the Kattegat and the main body of the Baltic, are all approximately normal to the resultant of the statical pressure gradient and the wind stress.

12. The nodal tide

In the classical theory of tides, the equilibrium tide is that which would exist in a deep global ocean if the water did not possess inertia. Under certain circumstances, which will be discussed later, the equilibrium tide and the real, observed tide exhibit common features.

The harmonic development of the tide generating potential, as given by Doodson (1921), indicates the existence of a tidal oscillation with a period equal to that of the precession of the Moon's nodes. Its equilibrium elevation is given by

$$\zeta = \tfrac{9}{8}\frac{M}{E}\,e\left(\frac{e}{\rho}\right)^3(\sin^2\lambda-\tfrac{1}{3})\cos N'\times 0{\cdot}06552, \tag{9}$$

where M, E = masses of the Moon and Earth respectively,

e = mean radius of the Earth,

ρ = mean distance between centres of Moon and Earth (e/ρ is the lunar parallax),

λ = latitude,

$N' = -N$, where N is the longitude of the Moon's ascending node.

Taking $M/E \sim 1/81{\cdot}5$, $e \sim 6{\cdot}37\times 10^6$ m, $e/\rho \sim 1/60{\cdot}26$, equation (9) becomes $\zeta = 0{\cdot}0263\ (\sin^2\lambda-\tfrac{1}{3})\cos N'$ metres.

For $|\lambda| > 35^\circ{\cdot}3$, $\sin^2\lambda > \tfrac{1}{3}$ and $\zeta = 26{\cdot}3(\sin^2\lambda-\tfrac{1}{3})\cos N$ mm,

$|\lambda| < 35^\circ{\cdot}3$, $\sin^2\lambda < \tfrac{1}{3}$ and $\zeta = 26{\cdot}3(\tfrac{1}{3}-\sin^2\lambda)\cos(N+180)$ mm.

Thus if the equilibrium nodal tide exists in nature, the analytical results should give values of g, the phase lag, near to zero for latitudes greater than $35^\circ{\cdot}3$, and near to 180° for stations between these parallels.

It should be noted that the amplitude of

$$26{\cdot}3\,|\sin^2\lambda-\tfrac{1}{3}|\ \text{mm}$$

needs to be reduced by the factor 0·7 to allow for the effect of a yielding Earth. On this assumption,

$$\zeta = \bar{H}\cos(N-\bar{g})$$

$$\left.\begin{aligned} \text{where}\quad \bar{H} &= 18{\cdot}4\ (\sin^2\lambda-\tfrac{1}{3})\ \text{mm},\ \bar{g}=0^\circ \text{ when } |\lambda|>35^\circ{\cdot}3,\\ \bar{H} &= 18{\cdot}4\ (\tfrac{1}{3}-\sin^2\lambda)\ \text{mm},\ \bar{g}=180^\circ \text{ when } |\lambda|<35^\circ{\cdot}3. \end{aligned}\right\} \tag{10}$$

The terms representing the nodal tide in the regression equation (5) are

$$c_1\cos N + c_2\sin N,$$

where

$$c_1 = H\cos g,\ c_2 = H\sin g$$

and H and g are the statistical estimates of the observed amplitude and phase lag corresponding to the equilibrium values $\bar{H}$ and $\bar{g}$ of equations (10). It is well known that, in general, the harmonic constants of the observed tide have only a loose relationship with those of the equilibrium tide. Only in the case of the long period tides is the equilibrium form likely to be observed in nature, and Proudman (1960) has shown that this should theoretically be true for the nodal tide.

Unfortunately, theory also indicates that interactions amongst the fundamental harmonic tidal constituents can produce perturbations of the nodal tide (Rossiter 1962). Such interactions are most prominent in shallow water areas with large tidal ranges; one such major tri-linear interaction is that between the principal lunar semi-diurnal tide (M_2), the lunisolar declinational diurnal tide (K_1) and the lunar declinational diurnal tide (0_1). The bi-linear interaction of M_2 with itself can also produce a perturbation which effectively reduces the amplitude of the equilibrium nodal tide by 0·74 of the amplitude of M_4, the lunar quarter-diurnal tide.

It is obvious from Fig. 10, which shows the distribution of H and g for the stations considered, that there is a considerable degree of scatter. This was equally obvious from an examination of the standard deviations of the coefficients c_1 and c_2 in Table 4.

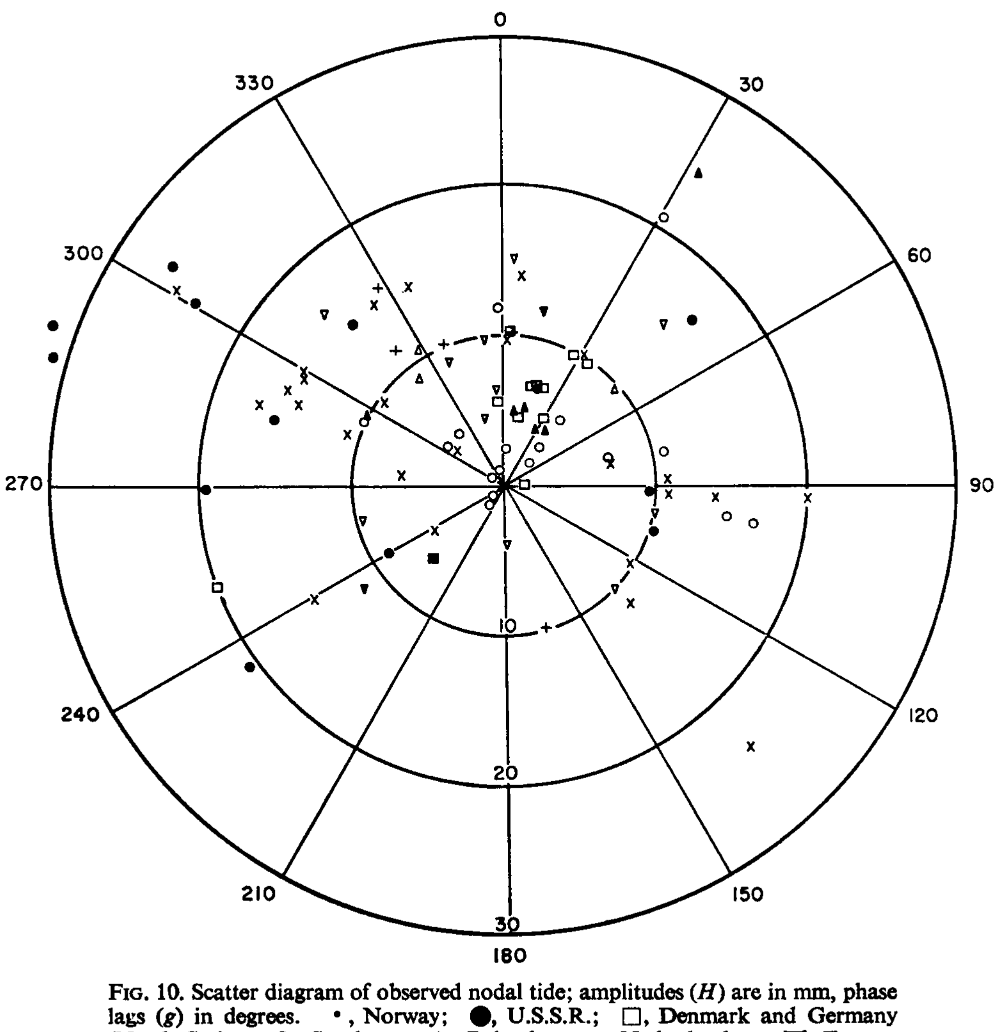

FIG. 10. Scatter diagram of observed nodal tide; amplitudes (H) are in mm, phase lags (g) in degrees. •, Norway; ●, U.S.S.R.; □, Denmark and Germany (North Sea); ○, Sweden; △, Poland; ▽, Netherlands; ▼, France; ×, Finland; +, Germany (Baltic); ▲, British Isles; ■, Italy.

Only by considering the average estimates for groups of stations is any coherent picture likely to emerge. A weighted vectorial average of H and g has therefore been computed for each country; the weight applied for each station was taken as the inverse of the standard deviation of the unexplained residual, *i.e.* $1/\sigma_\phi$. Table 10 lists the mean values of H and g for each country. Although the scatter has now been (inevitably) reduced, the results are hardly convincing even when allowance is made for the considerable variations in the number of station years between one country and another.

Table 10

Mean nodal tide, by countries

	Number of years	H (mm)	g (degrees)
Norway	182	7·4	58
Sweden	1119	4·5	32
Finland	1316	2·4	339
U.S.S.R.	305	19·9	284
Poland	125	9·3	320
Germany (Baltic)	293	8·2	331
Denmark	700	6·3	19
Germany (North Sea)	46	19·9	251
Netherlands	879	4·6	346
British Isles	379	8·2	24
France (Atlantic)	59	11·9	15
France (Mediterranean)	62	12·1	232
Italy	39	7·5	224

Considering the phase lags, the countries for which g differs by appreciably more than 30° from zero are Norway, U.S.S.R., Germany (North Sea), France and Italy, all of which represent rather insubstantial data. The most substantial data produce values of 32° (Sweden), 21° (Finland), 19° (Denmark) and 346° (Netherlands).

Considering the amplitudes, even for those countries which contribute substantial data, there is little evidence of the dependence upon latitude which the equilibrium distribution indicates. According to equation (10), this variation should range from 9 mm in latitude 65° to 3 mm in latitude 45°.

One is therefore reduced to averaging the whole of the data to produce a single estimate of the nodal tide for north-European waters. This is not surprising, since the exercise is to determine an oscillation with an amplitude well below the noise level, at any one station, of the crude data. Again, weighting station results with $1/\sigma_\phi$, the results are $H = 4{\cdot}4$ mm, $g = 354°$.

That the phase lags are so close to zero is support for the existence of an equilibrium nodal tide. The fact that the amplitudes are only about two-thirds of the corresponding mean value of $\bar{H}$ could possibly be confirmation of the bi-linear interaction perturbation mentioned above.

One may conclude that the data provide some support for the existence of an equilibrium nodal tide, but that for convincing proof, further evidence is required from other sea areas.

Acknowledgments

The author wishes to acknowledge his indebtedness to the U.S. Weather Bureau and the many national and local authorities for freely supplying data; to Dr O. Simonsen of the Danish Geodetic Institute for guidance on R.E.U.N. matters; to Mr G. W. Lennon, of the Tidal Institute, for valuable comments and criticisms on the data handling problems and the interpretation of results; and to Miss J. P. Davies and Mrs E. D. Corkan who attended to all the laborious details of the work.

The research was materially assisted by funds from UNESCO, the International Association of Physical Oceanography, and the Federation of Astronomical and Geophysical Services.

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