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An Analysis Of The Entropy Generation In A Square Enclosure

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Abstract: The entropy generation during transient laminar natural convection in a square enclosure is numerically investigated. Two different cases are considered. The enclosure is heated either completely or partially from the left side wall and cooled from the opposite side wall. The bottom and the top of the enclosure are assumed as insulated. The Boussinesq approximation is used in the natural convection modelling. The solutions are obtained from quiescent conditions proceeded through the transient up to the steady-state. The calculations are made for the Prandtl numbers 0.01 and 1.0 and Rayleigh numbers between 10^2 – 10^8. The entropy generation and the active places triggering the entropy generation are obtained for each case after the flow and thermal characteristics are determined. It is found that the active sites in the completely heated case are at the left bottom corner of the heated wall and the right top corner of the cooled wall at the same magnitudes. In the case of partial heating, however, the active site is observed at the top corner of the heated section especially at lower Pr and Ra values.

Keywords: Enclosure, laminar natural convection, entropy generation

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Nomenclature

g	gravitational acceleration, m/s^2	X, Y	dimensionless coordinates
H	enclosure height, m	x, y	coordinates, m
k	thermal conductivity, W/mK		
L	enclosure width, m	Greek Letters	
l_h	heated part, m	α	thermal diffusivity, m^2/s
N_s	dimensionless entropy generation number	β	coefficient of thermal expansion, 1/K
P	dimensionless pressure	ρ	density, kg/m^3
p	pressure, N/m^2	θ	dimensionless temperature
Pr	Prandtl number	ν	kinematic viscosity, m^2/s
S'''_{gen}	entropy generation, W/m^3-K	ζ	dimensionless vorticity
Q	overall heat transfer rate, W	ψ	stream function, m^2/s
q	heat flux, W/m^2	Ψ	dimensionless stream function
Ra	Rayleigh number	τ	dimensionless time
T	temperature, K	Φ	viscous dissipation function, s^{-2}
t	time, s	ω	vorticity, s^{-1}
T_{avg}	average temperature ($=(T_h+T_c)/2$), K	μ	dynamic viscosity, $N\text{-}s/m^2$
ΔT	temperature difference, K	ϕ	irreversibility distribution function
U	dimensionless horizontal velocity component	Subscripts	
u	horizontal velocity component, m/s	c	cold surface
V	dimensionless vertical velocity component	h	hot surface
v	vertical velocity component, m/s		

Introduction

The phenomenon of natural convection in enclosures has attracted increasing attention in recent years. Applications extending from the double paned windows in buildings to the cooling of electronic systems are examples of natural convection systems. The comprehensive reviews of articles on natural convection were made by Catton [1], Yang [2], Ostrach [3], Kakaç and Yener [4], and Bejan [5]. In addition to the studies [6-10], Lage and Bejan [11] was investigated numerically the natural convection in a square enclosure heated and cooled in the horizontal direction in the Prandtl number range 0.01–10 and the Rayleigh number range 10^2 –10^{11}.

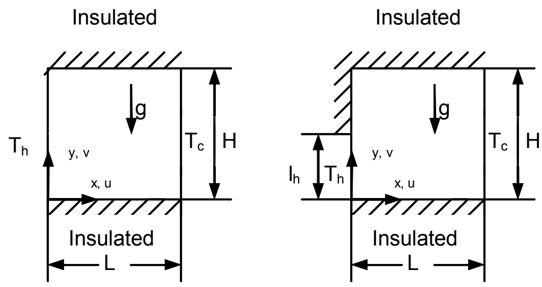
The flow and heat transfer in a square cavity with variable size heater and cooler on the vertical walls were analyzed numerically by Yücel and Türkoğlu [12]. The convective flow induced by buoyancy forces in a square enclosure due to partially heated and cooled side walls having constant size varying locations of the heater and cooler was performed by Türkoğlu and Yücel [13].

Bejan [14-16] has introduced the method of the minimization entropy generation to classical engineering topics such as heat transfer augmentation, heat exchanger design, and thermal insulation systems to evaluate the performance of the systems thermodynamics. In the literature, various studies on the entropy generation rates and the irreversibilities for the basic convective heat transfer arrangements can be found [17-20]. However, the entropy generation during the natural convection in an enclosed cavities has not received much attention since the natural convection systems do not include very high rates of heat and work transfer in which the irreversibilities can be neglected easily in comparison with the forced convection systems. Since the natural convection systems have gained increasing importance recently, there is a requirement for the second law analysis under different thermal conditions.

In the present study, the entropy generation within a square enclosure is investigated numerically. The enclosure filled with a motionless fluid is partially or completely heated and cooled from the vertical lateral walls while the other walls are insulated. The effects of the heating completely or partially on the entropy generation are investigated by considering the Prandtl numbers of 1.0 and 0.01 and the Rayleigh numbers from 10^2 to 10^8. The governing equations for two-dimensional cartesian coordinates coupled with the Boussinesq approximation are solved numerically. Active sites for the entropy generation through the enclosure are determined after the flow characteristics are obtained.

Mathematical Formulation

The physical system considered in the present study is shown schematically in Fig.1. The physical system is composed of a closed square cavity with a completely (Fig.1a) or partially (Fig.1b) heated left side wall, completely cooled an opposite wall, insulated top and bottom walls. The enclosure is filled with a motionless fluid at a uniform temperature at the beginning. All walls are impermeable no-slip boundaries.



(a) (b)

Figure 1. Schematic of the enclosure considered in the present study a) completely and b) partially heated cases.

Assuming that the Boussinesq approximation, the dimensionless governing equations in stream-function-vorticity transport form for the two-dimensional transient incompressible buoyancy driven flow of a Newtonian fluid are written as follows

$$\frac{\partial \zeta}{\partial \tau}+\frac{\partial (U\zeta)}{\partial X}+\frac{\partial (V\zeta)}{\partial Y}=\left(\frac{\Pr}{Ra}\right)^{1/2}\left(\frac{\partial^2 \zeta}{\partial X^2}+\frac{\partial^2 \zeta}{\partial Y^2}\right)-\frac{\partial \theta}{\partial X} \tag{1}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{(Ra\Pr)^{1/2}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{2}$$

$$\zeta=\frac{\partial^2 \Psi}{\partial X^2}+\frac{\partial^2 \Psi}{\partial Y^2} \tag{3}$$

where the following dimensionless variables are used

$$(X,Y)=\frac{(x,y)}{H} \quad , \quad (U,V)=\frac{(u,v)}{(\alpha/H)(Ra\Pr)^{1/2}}$$

$$\tau=\frac{\alpha(Ra\Pr)^{1/2}}{(H^2)}t \quad , \quad \theta=\frac{T-(T_h+T_c)/2}{T_h-T_c}$$

$$P=\left(\frac{H^2}{\rho\alpha^2}\right)\left(\frac{p+\rho g y}{Ra\Pr}\right) \tag{4}$$

where Ra and Pr numbers are defined as

$$Ra=\frac{g\beta H^3(T_h-T_c)}{\alpha\upsilon} \quad , \quad \Pr=\frac{\upsilon}{\alpha} \tag{5}$$

and the dimensionless stream-function and vorticity definitions are given

$$\Psi=\frac{\psi}{\alpha(Ra\Pr)^{1/2}} \quad \text{and} \quad \zeta=\frac{\omega}{(\alpha/H^2)(Ra\Pr)^{1/2}} \tag{6}$$

Then

$$U=\frac{\partial \Psi}{\partial Y} \quad , \quad V=-\frac{\partial \Psi}{\partial X}$$

$$\zeta=\frac{\partial U}{\partial Y}-\frac{\partial V}{\partial X} \tag{7}$$

In case of the completely heated side wall, the dimensionless initial condition and boundary conditions are

$$\begin{aligned}
&U=V=\Psi=\theta=0 && \text{at } \tau=0\\
&U=V=\Psi=0 && \text{at the walls}\\
&\theta=0.5 && \text{at X=0 and 0<Y<1}\\
&\theta=-0.5 && \text{at X=L/H and 0<Y<1}\\
&\frac{\partial \theta}{\partial Y}=0 && \text{at Y=0 and Y=1}
\end{aligned} \tag{8}$$

In the second case, the dimensionless initial condition and boundary conditions are

$$\begin{aligned}
&U=V=\Psi=\theta=0 && \text{at } \tau=0\\
&U=V=\Psi=0 && \text{at the walls}\\
&\theta=0.5 && \text{at X=0 and } 0<Y\le 0.5\\
&\frac{\partial \theta}{\partial Y}=0 && \text{at X=0 and 0.5<Y<1}
\end{aligned}$$

$$\theta = -0.5 \qquad \text{at } X=L \text{ and } 0<Y<1$$
$$\frac{\partial \theta}{\partial Y} = 0 \qquad \text{at } Y=0 \text{ and } Y=1 \tag{9}$$

The entropy generation per unit volume at an arbitrary point in the medium is given by [14-16]

$$S'''_{gen} = -\frac{1}{T^2}\mathbf{q}\cdot\nabla T + \frac{\mu}{T}\Phi \tag{10}$$

where T is the local absolute temperature, **q** is the heat flux vector, and Φ is the viscous dissipation function. The volumetric entropy generation rate for a two-dimensional flow in cartesian coordinates becomes

$$S'''_{gen} = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T}\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\} \tag{11}$$

By using the same dimensionless parameters given in Eq.(4), Eq.(11) takes the following dimensionless form:

$$N_s = \left[\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2\right] + \phi\left\{2\left[\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right] + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right\} \tag{12}$$

where N_s the entropy generation number, is the dimensionless volumetric entropy generation rate and written explicitly as

$$N_s = S'''_{gen}\frac{T_{avg}^2 H^2}{k\,\Delta T^2} \tag{13}$$

where T_{avg} is the average temperature ($(T_h+T_c)/2$), ΔT is the temperature difference (T_h-T_c).

The coefficient of the second term on the right-hand side of the Eq.(12), ϕ, is defined as irreversibility distribution function and represents the relative importance of fluid friction and heat transfer on the entropy generation. In the present study, ϕ is obtained as

$$\phi = \frac{\mu\alpha^2 T_{avg} Ra \Pr}{kH^2\,\Delta \mathrm{T}^2} \tag{14}$$

Eq.(12) is used to derive irreversibility profiles (or contours) for the present physical problem after the velocity and temperature fields are determined. The dimensionless governing equations given by equations (1)-(3) and equations (12)-(14) show that θ, ζ and Ψ are dependent variables; X, Y and τ are independent variables, and the equations depend on the Pr and Ra numbers. In this study, the effect of completely or partially heated wall on the rate of entropy generation are investigated by considering the boundary conditions given in the Eqns.(8) and (9), respectively.

Numerical Solution

The present problem is solved numerically using finite volume method (FVM) coupled with power-law scheme for the convective terms. The resulting system of linear equations are solved by Gauss-Seidel iteration method. The solutions are started from quiescent conditions proceeded through the transient up to the steady-state case. A computer program was written and compared with the isotherms and stream function solutions obtained by those of reported by Lage and Bejan [11] under the case of an enclosure with differentially heated side walls. The present and the benchmark results are summarized in Table 1 for the steady–state values of the Nusselt number at more relevant grids for each case.

Table 1. The Summary of the present and benchmark results for steady-state Nusselt values

Pr	Ra	Benchmark values by Lage and Bejan [11]	Grid	Present values at steady-state
0.01	10^2	1.00	40 x 40	1.004
	10^3	1.05	40 x 40	1.080
	10^4	1.50	40 x 40	1.593
	10^5	2.77	70 x 70	2.778
1.0	10^5	4.9	90 x 90	4.674
	10^6	9.2	100 x 100	9.194
	10^7	17.9	100 x 100	17.897
	10^8	31.8	200 x 200	31.784

Results And Discussion

The entropy generation represented by N_s has been calculated for certain combinations of Prandtl and Rayleigh numbers as Pr = 1.0 for Ra = 10^5, 10^6, 10^7 and 10^8 and Pr = 0.01 for Ra = 10^2, 10^3, 10^4 and 10^5 .

Fig.2 demonstrates the irreversibilities due to heat transfer at the different time steps for the combination of Prandtl number 1.0 and Rayleigh number 10^5 for completely and partially heated walls in Fig.2a,b respectively. In the completely heated case (Fig.2a) the entropy generation is concentrated along the heated and cooled walls where the temperature gradient is maximum. The active sites originate at the left bottom corner and at the right top corner. The active sites are at the top corner of the heated section and cooled wall in the case of partial heating (Fig.2b).

Fig.3 shows the transient entropy generation due to fluid friction at the same Pr and Ra numbers. For both cases, the contours of the entropy generation due to fluid friction are observed at the center of the vertical walls.

Fig.4 summarizes the effect of Rayleigh number on the entropy generation due to heat transfer at the steady-state. The isotherms in the enclosure develope rapidly with increasing Rayleigh number. The active sites originate at the left bottom corner and at the right top corner for completely heated

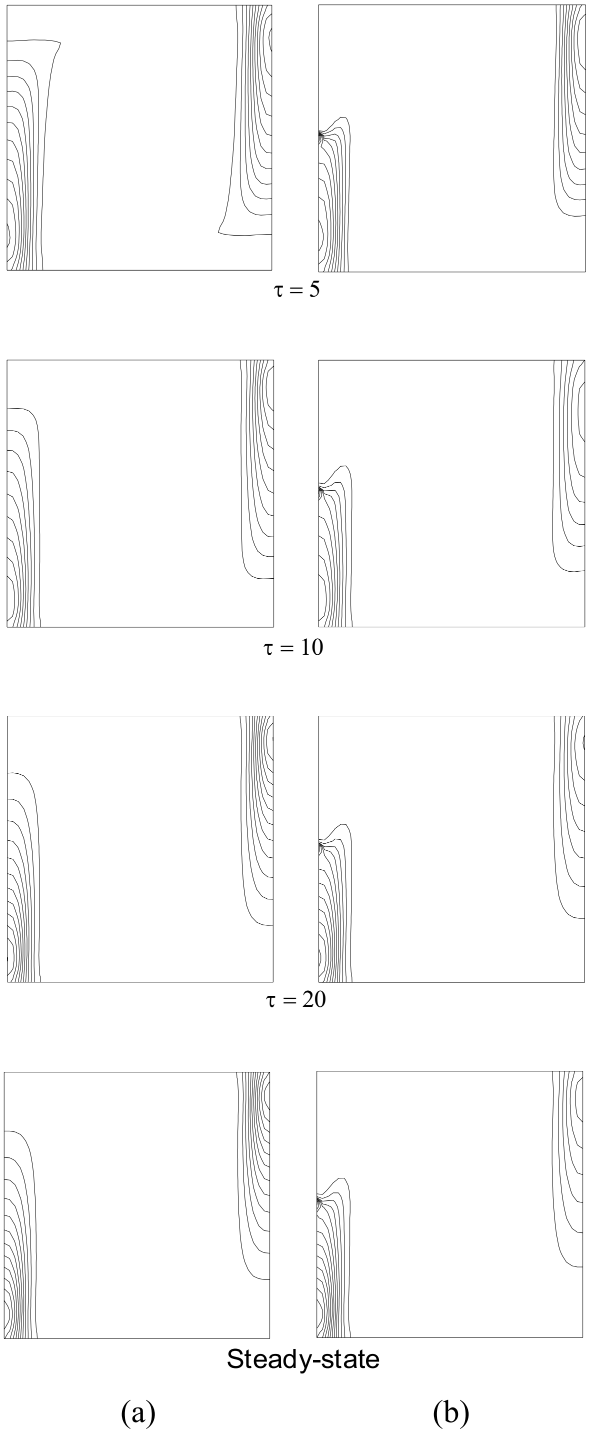


Figure 2. Transient change in entropy generation due to heat transfer from τ=5 up to steady-state for a) completely and b) partially heated cases (Pr=1.0, Ra= 10^5 , $\phi=10^{-10}$ and 90x90 grid).

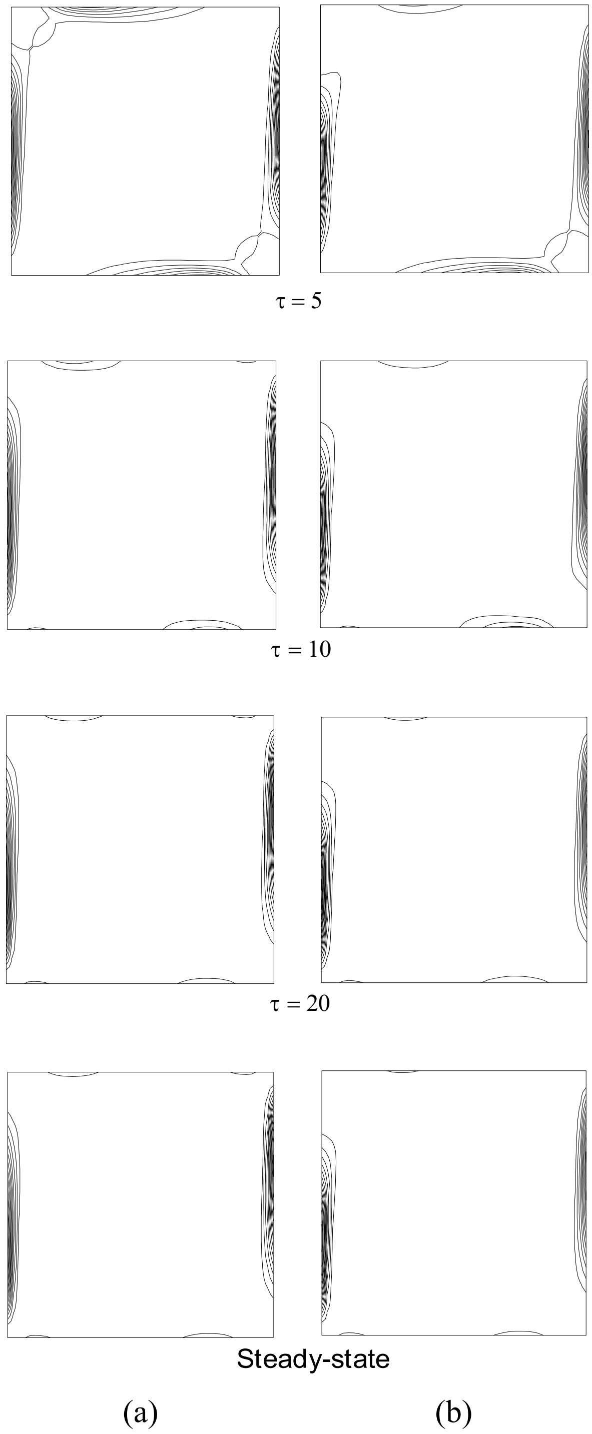


Figure 3. Transient change in entropy generation due to fluid friction from τ=5 up to steady-state for a) completely and b) partially heated cases (Pr=1.0, Ra= 10^5, $\phi=10^{-10}$ and 90x90 grid).

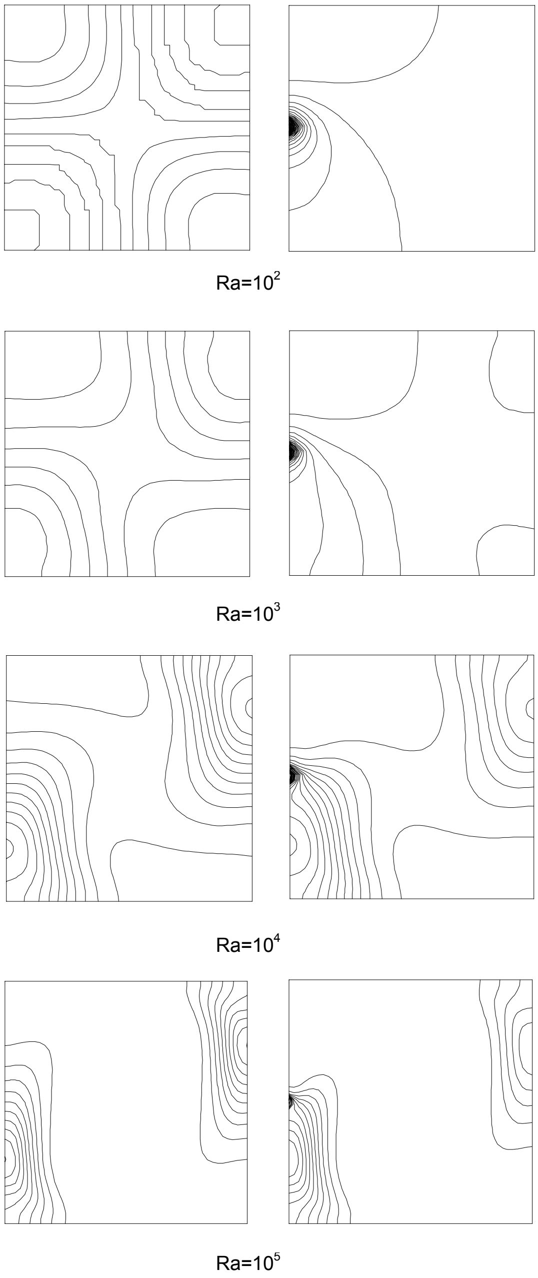


Figure 4. The effect of Rayleigh number given on the entropy generation due to heat transfer at steady-state for a) completely and b) partially heated cases (Ra= 10^2, $\phi=10^{-13}$ 40x40 grid,Ra= 10^3 ,$\phi=10^{-12}$, 40x40 grid, Ra= 10^4, $\phi=10^{-11}$, 40x40 grid, Ra= 10^5, $\phi=10^{-10}$, 70x70 grid).

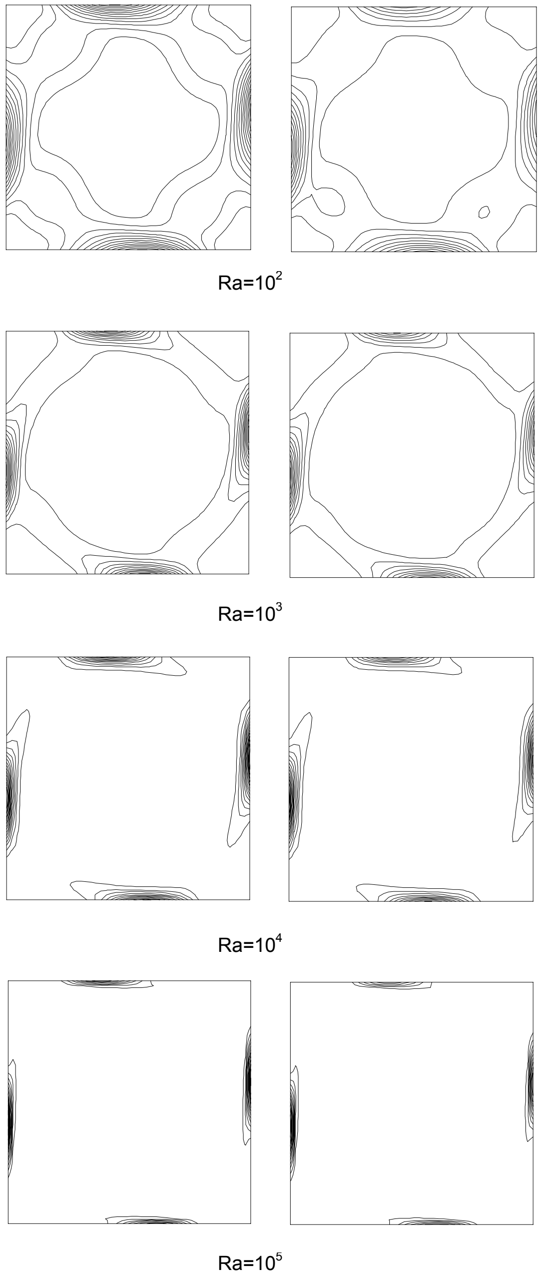


Figure 5. The effect of Rayleigh number given on the entropy generation due to fluid friction at steady-state for a)completely and b) partially heated cases (Ra= 10^2, $\phi=10^{-13}$,40x40 grid, Ra= 10^3 ,$\phi=10^{-12}$, 40x40 grid, Ra= 10^4 ,$\phi=10^{-11}$, 40x40 grid, Ra= 10^5 , $\phi=10^{-10}$,70x70 grid).

case (Fig.4a) while the active sites are on the heated part and on the cooled wall as seen in (Fig.4b). When Rayleigh number increases, the temperature gradient concentrates along the vertical wall; therefore, entropy generation due to heat transfer concentrates along the vertical walls.

The entropy generation due to fluid friction for various Rayleigh values is given in Fig.5. The structure of the contours keep the similar distribution of both cases (Fig.5a,b). Since all of the walls are no-slip boundaries, the contours of the entropy generation number are observed at the center of all walls.

The magnitude of entropy generation due to fluid friction irreversibilities is negligible with respect to the entropy generation due to the heat conduction. Therefore the total value of the entropy generation number is the same with that of the heat conduction.

Conclusion

The study investigates the entropy generation during transient laminar natural convection in a square enclosure with a completely or partially heated left side wall, completely cooled an opposite wall, insulated top and bottom walls. At the beginning, the enclosure is occupied by motionless fluid. The initial temperature is uniform and equal to the enclosure average. All surfaces are rigid no-slip boundaries. The entropy generation numbers and active sites have been determined. The active sites, i.e., the spots at which the entropy generation initiates due to irreversibilities representing the energy loss regions. In the case of completely heated wall, the active site for the entropy generation due to heat transfer is observed at the lower left corner of the heated wall and the upper right corner of the cooled wall at the same magnitude. However, in the case of partial heating, the most effective site is found at the upper corner of the heated part of the side wall.

The irreversibilities are dominant due to heat transfer whereas fluid friction irreversibilities have been found negligible as it is expected for the natural convection. Therefore, total value of the entropy generation number has the same distribution and value with the entropy generation due to heat transfer.

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