An analytic logic of aggregation

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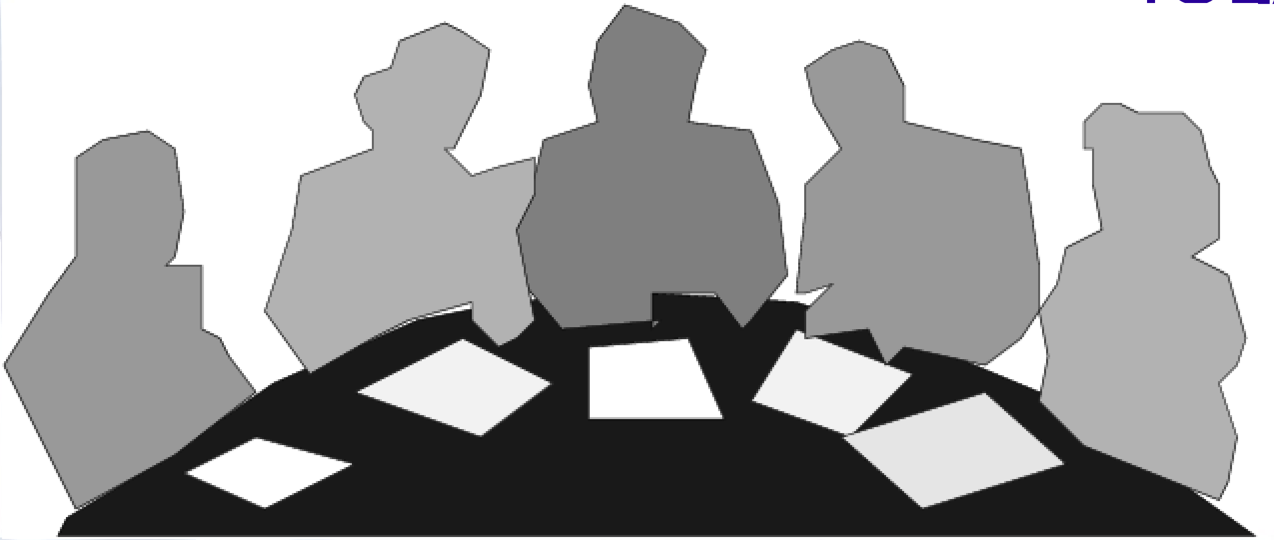
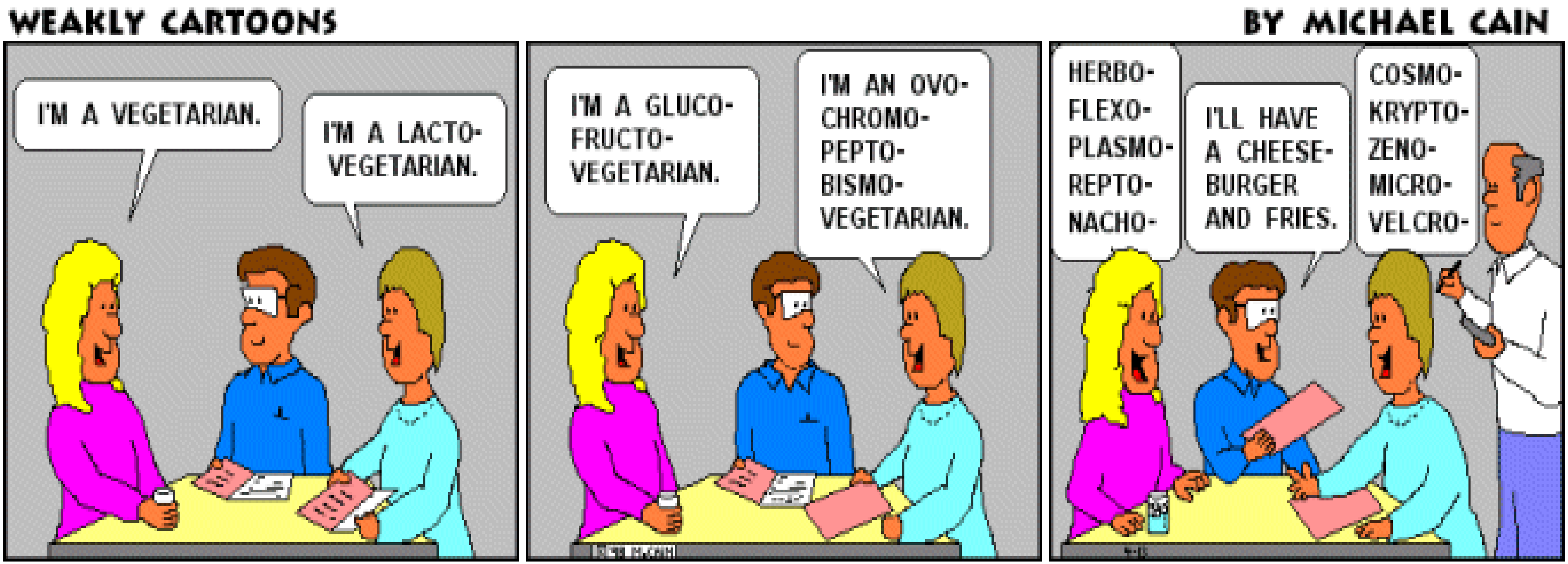
ICLA 2009, Chennai
9 January 2009




WEAKLY CARTOONS
BY MICHAEL CAIN
I'M A VEGETARIAN.
I'M A LACTO-VEGETARIAN.
I'M A GLUCO-FRUCTO-VEGETARIAN.
I'M AN OVO-CHROMO-PEPTO-BISMO-VEGETARIAN.
HERBO-FLEXO-PLASMO-REPTO-NACHO-
I'LL HAVE A CHEESE-BURGER AND FRIES.
COSMO-KRYPTO-ZENO-MICRO-VELCRO-


KEEGAN


Hickory smoked with
LOVIN' CARE
Little Pigs
genuine pit
Bar-B-Q
Sandwiches
SPECIAL TONIGHT
VEGETARIANS
EAT FOR FREE
HAHAHA!!!

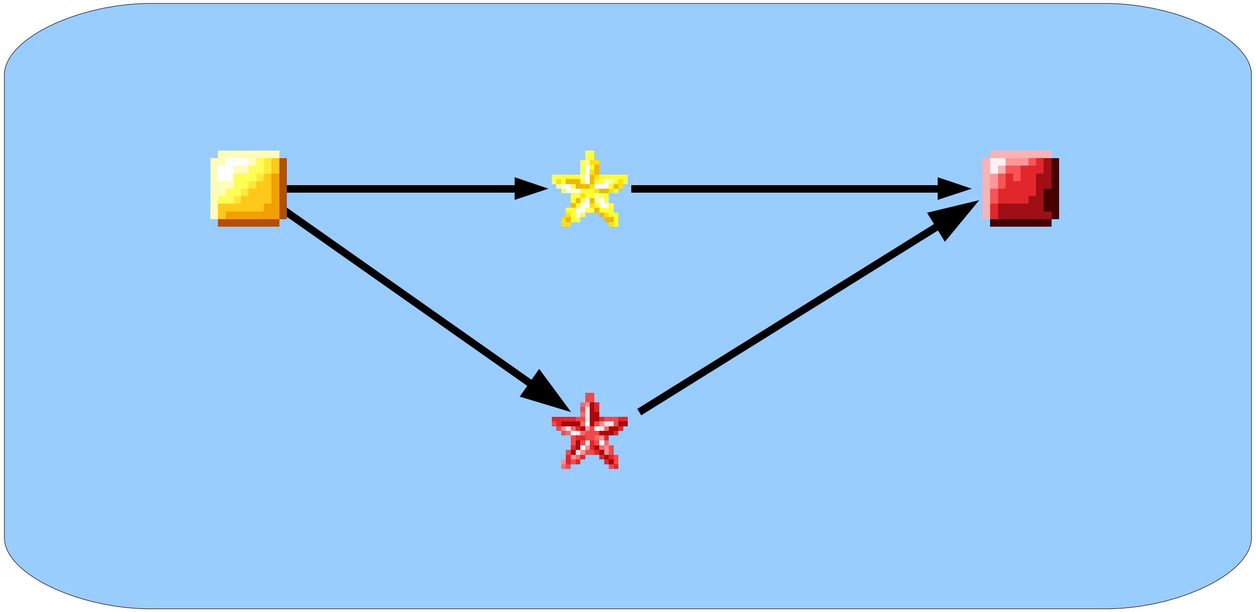
Hierarchy

Agreement

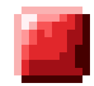
Subordination

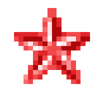
Basic Preference Logic

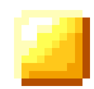
- Start with a basic preference order ≤ (reflexive and transitive) over a set of objects (or states):

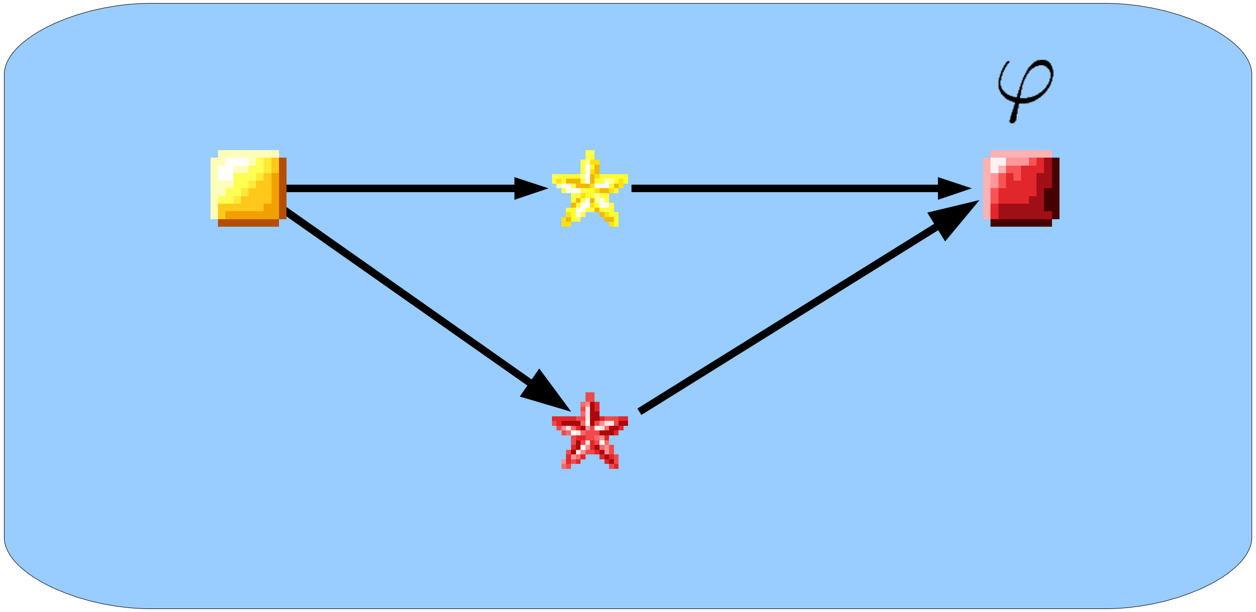


Basic Preference Logic

$\mathfrak{M},$  $\models \Diamond^{\leq}\varphi$

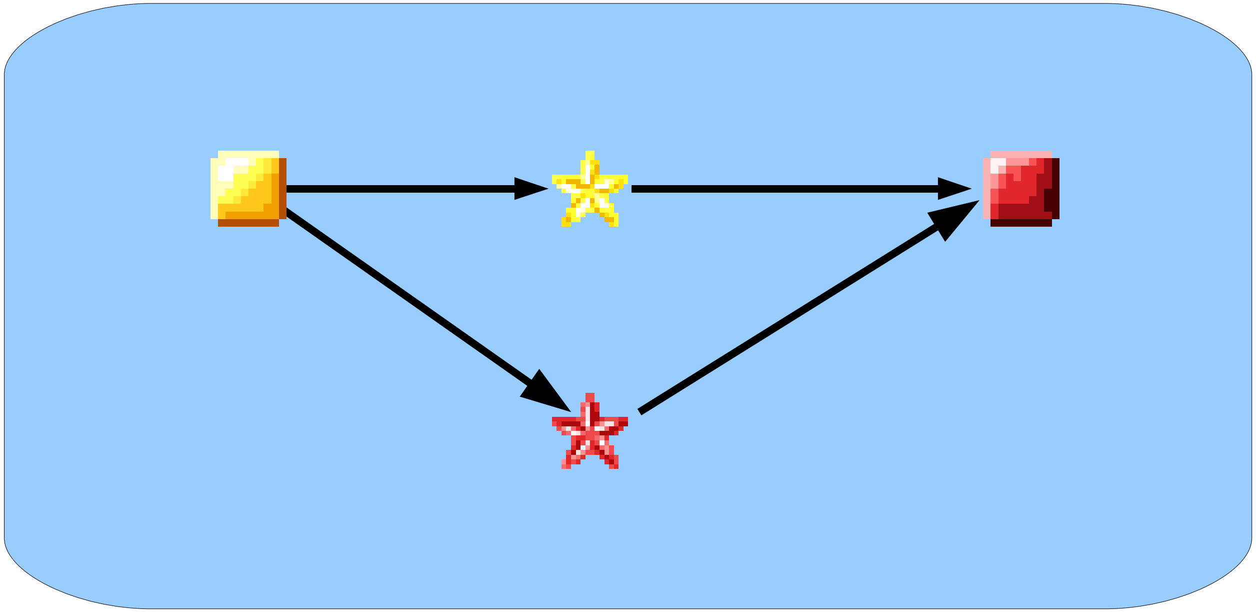
$\mathfrak{M},$  $\models \Diamond^{<}\varphi$

$\mathfrak{M},$  $\models \Diamond^{<}\varphi$



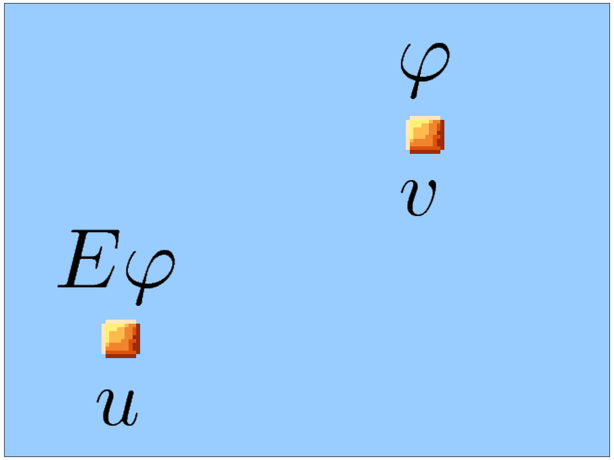
Preference Logic

- Are stars preferred to squares?
- Is red preferred to yellow?



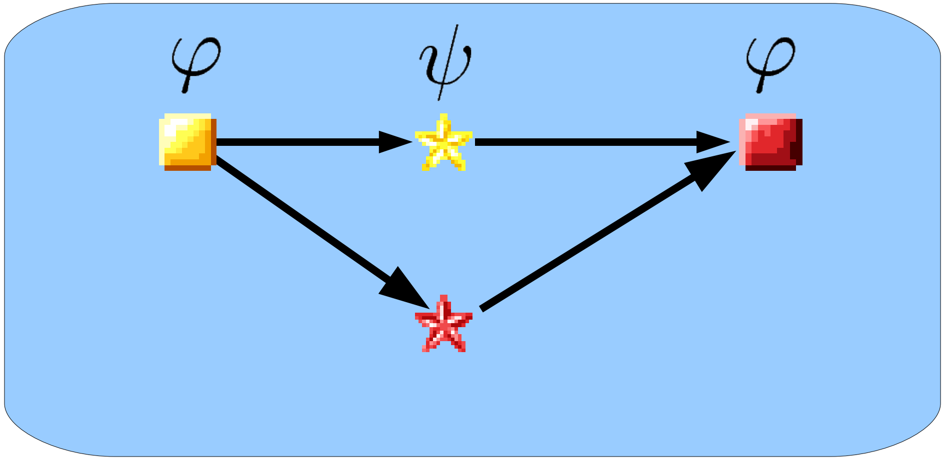
Preference Logic

- Existential modality:
  - $\mathfrak{M}, u \models E\varphi \quad \text{iff} \quad \exists v : \mathfrak{M}, v \models \varphi$
  - $U\varphi \quad \Leftrightarrow \quad \neg E \neg \varphi$



Preference Logic

$$
\begin{array}{ll}
P\varphi & U(\neg\Diamond^{<}\top \rightarrow \varphi) \\
P(\neg\varphi|\psi) & U((\psi \wedge \neg\Diamond^{<}\psi) \rightarrow \neg\varphi) \\
\varphi <_{\exists\exists} \psi & E(\varphi \wedge \Diamond^{<}\psi) \\
\varphi \not<_{\forall\forall} \psi & \neg U(\psi \rightarrow \neg\Box^{\leq}\varphi)
\end{array}
$$



Aggregation Logic

- Language: $p \in \mathrm{PROP}, i \in \mathrm{NOM}, a \in \mathrm{AGE}$

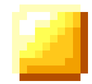
$$\varphi \quad := \quad p \mid i \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle t \rangle^{\leq}\varphi \mid \langle t \rangle^{<}\varphi \mid E\varphi$$
$$t \quad := \quad a \mid t \parallel t \mid t/t$$

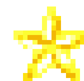
- Models: $\mathfrak{M} = \langle W, I, V \rangle$
  - $I(a) = \leq_a$
  - $\langle u, v \rangle \in I^{<}(t) \Leftrightarrow \langle u, v \rangle \in I(t) \,\&\, \langle v, u \rangle \notin I(t)$
  - $I(t_1 \parallel t_2) = I(t_1) \cap I(t_2)$
  - $I(t_1/t_2) = (I(t_1) \cap I(t_2)) \cup I^{<}(t_1)$

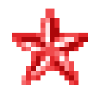
Semantics

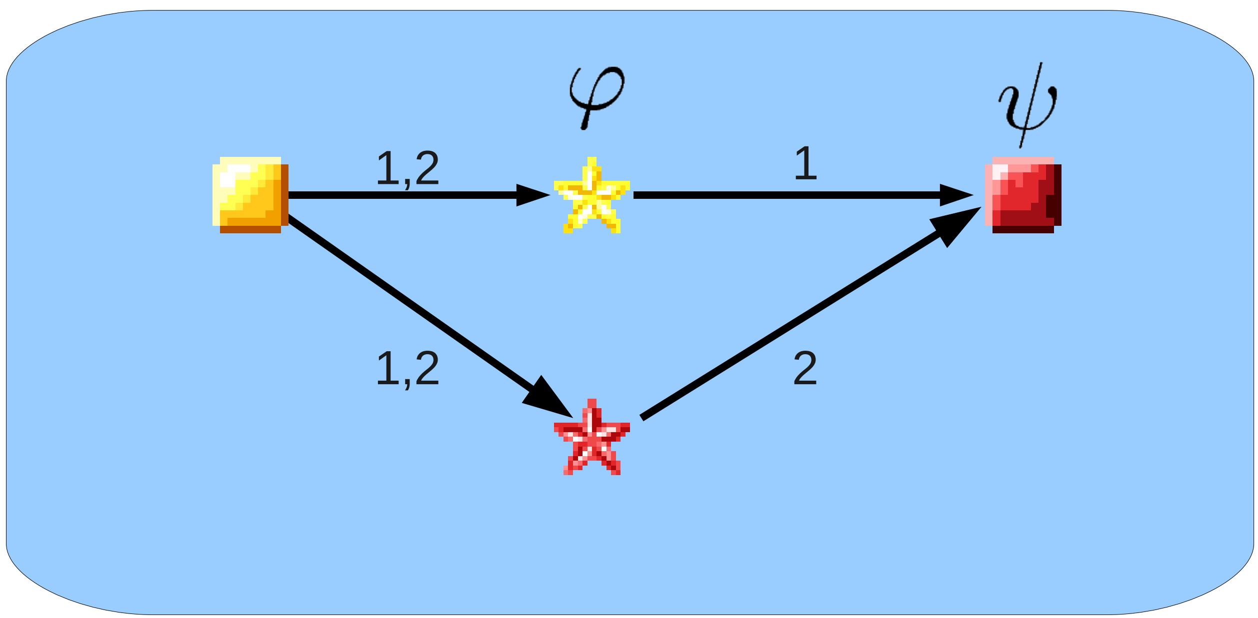
- $\mathfrak{M}, u \models \langle t \rangle^{\leq} \varphi \quad \text{iff} \quad \exists v : uI(t)v \,\&\, \mathfrak{M}, v \models \varphi$
  - There is a state at least as good as the current state where φ holds.
- $\mathfrak{M}, u \models \langle t \rangle^{<} \varphi \quad \text{iff} \quad \exists v : uI^{<}(t)v \,\&\, \mathfrak{M}, v \models \varphi$
  - There is a state strictly better than the current state where φ holds.
- $\mathfrak{M}, u \models E\varphi \quad \text{iff} \quad \exists v : \mathfrak{M}, v \models \varphi$

Aggregation Logic

$\mathfrak{M},$  $\models \langle 1 \parallel 2 \rangle^{\leq} \varphi$

$\mathfrak{M},$  $\models \langle 1/2 \rangle^{<} \psi$

$\mathfrak{M},$  $\not\models \langle 1/2 \rangle^{<} \psi$



Preference Logic

$$
\begin{array}{ll}
P^t \varphi & U(\neg \langle t \rangle^{<} \top \rightarrow \varphi) \\
P^t(\neg \varphi | \psi) & U((\psi \wedge \neg \langle t \rangle^{<} \psi) \rightarrow \neg \varphi) \\
\varphi <^t_{\exists\exists} \psi & E(\varphi \wedge \langle t \rangle^{<} \psi) \\
\varphi \not<^t_{\forall\forall} \psi & \neg U(\psi \rightarrow \neg [t]^{\leq} \varphi)
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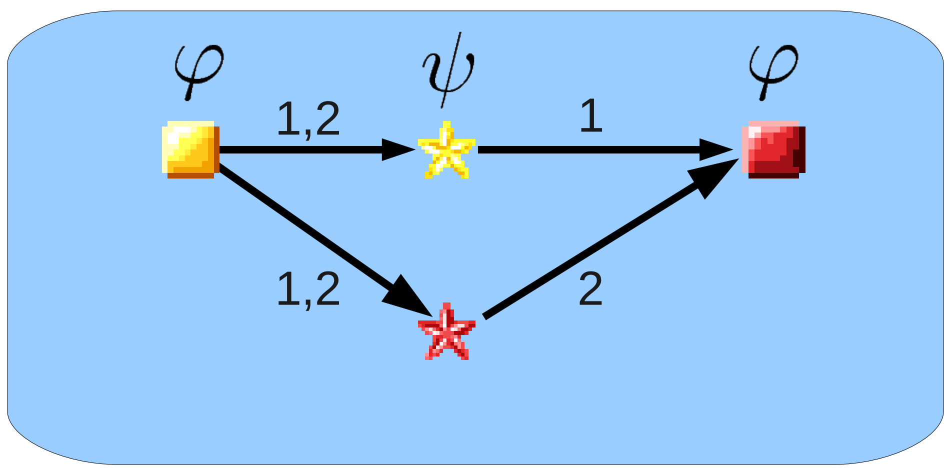
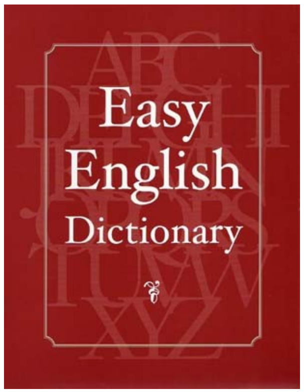
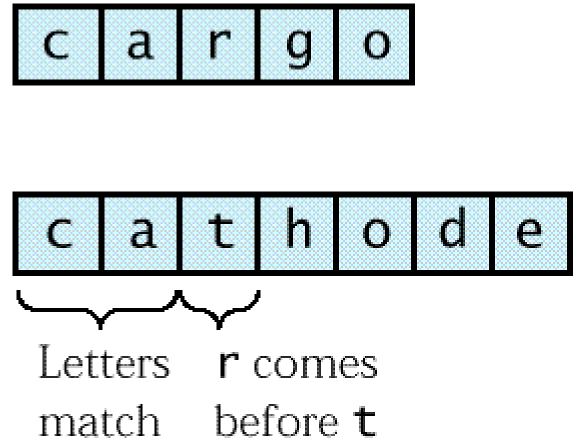
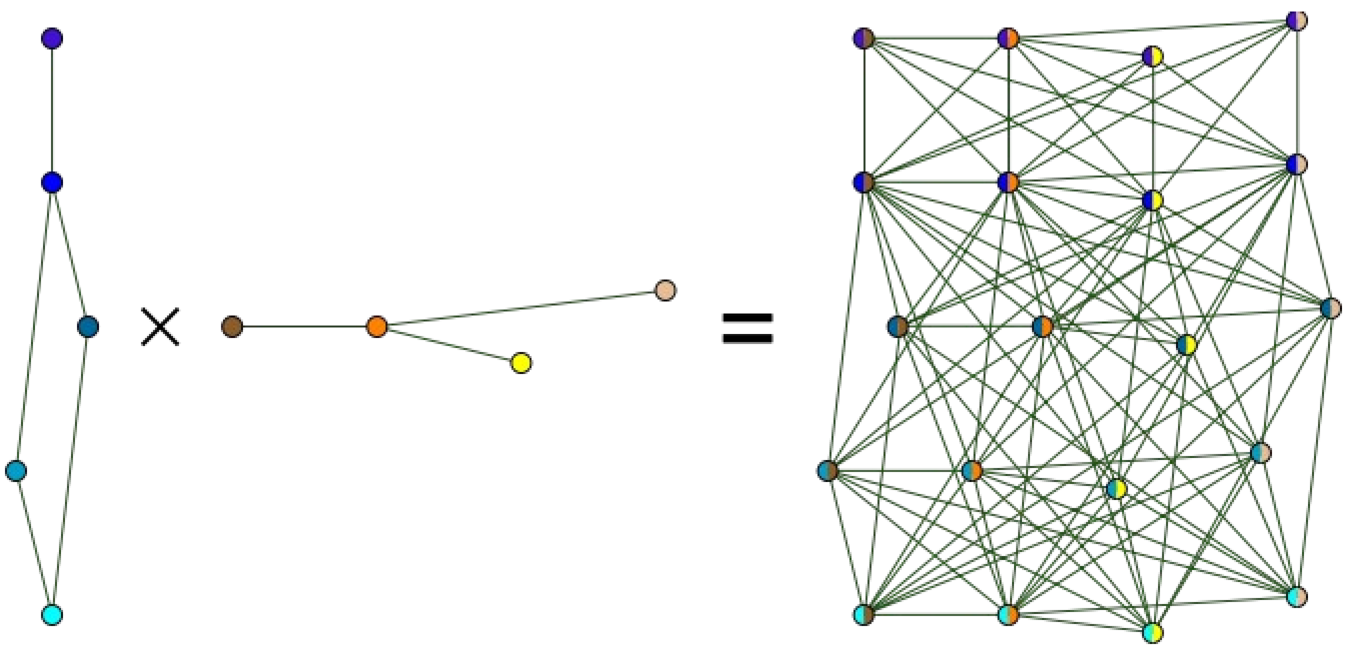


Which aggregation operators can we represent?

Lexicographic (re)ordering







Can we get rid of hierarchies?

Arrow's Theorem

Axiomatization

- Basic Hybrid logic
- Preference axioms, $M \in \{\langle t \rangle^{\leq}, \langle t \rangle^{<}, E\}$

$$\vdash i \rightarrow Mi$$
$$\vdash MMi \rightarrow Mi$$
$$\vdash i \rightarrow (\langle t \rangle^{<} \varphi \leftrightarrow \langle t \rangle^{\leq}(\varphi \wedge \neg \langle t \rangle^{\leq} i))$$

- Aggregation axioms

$$\vdash \langle t_1 \parallel t_2 \rangle^{\leq} i \leftrightarrow \langle t_1 \rangle^{\leq} i \wedge \langle t_2 \rangle^{\leq} i$$
$$\vdash i \rightarrow (\langle t_1 / t_2 \rangle^{\leq} j \leftrightarrow ((\langle t_1 \rangle^{\leq} j \wedge \langle t_2 \rangle^{\leq} j) \vee \langle t_1 \rangle^{\leq} (j \wedge \neg \langle t_1 \rangle^{\leq} i)))$$

Labelled Sequent Calculus

$$\frac{k : \langle t_1 \rangle^{\leq} i, k : \langle t_2 \rangle^{\leq} i, \Gamma \Rightarrow \Delta}{k : \langle t_1 \parallel t_2 \rangle^{\leq} i, \Gamma \Rightarrow \Delta} \parallel L$$

$$\frac{\Gamma \Rightarrow \Delta, k : \langle t_1 \rangle^{\leq} i \quad \Gamma \Rightarrow \Delta, k : \langle t_2 \rangle^{\leq} i}{\Gamma \Rightarrow \Delta, k : \langle t_1 \parallel t_2 \rangle^{\leq} i} \parallel R$$

$$\frac{k : \langle t_1 \rangle^{\leq} i, k : \langle t_2 \rangle^{\leq} i, \Gamma \Rightarrow \Delta \quad k : \langle t_1 \rangle^{\leq} i, \Gamma \Rightarrow \Delta, i : \langle t_1 \rangle^{\leq} k}{k : \langle t_1 / t_2 \rangle^{\leq} i, \Gamma \Rightarrow \Delta} /L$$

$$\frac{\Gamma \Rightarrow \Delta, k : \langle t_1 \rangle^{\leq} i \quad i : \langle t_1 \rangle^{\leq} k, \Gamma \Rightarrow \Delta, k : \langle t_2 \rangle^{\leq} i}{\Gamma \Rightarrow \Delta, k : \langle t_1 / t_2 \rangle^{\leq} i} /R$$