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# AN ANALYTIC TECHNIQUE FOR ROUTER COMPARISON

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AN ANALYTIC TECHNIQUE FOR ROUTER COMPARISON

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INTRODUCTION

Design automation of electronic systems is generally separated into a number of distinct areas of effort. Breuer [1] has divided design automation into the areas of logic synthesis, gate simulation, partitioning, placement, routing, and fault detection and diagnosis. While this separation may not be complete or entirely accurate, these functions generally must be performed.

At some point in the design process the components have been chosen, the logical interconnections specified, and the components placed on a printed circuit board; the next step is the physical interconnection of electrically common elements. Numerous techniques have been proposed to solve the resulting interconnection or routing problem, with almost endless minor variations on these techniques possible [3].

Lee's algorithm [2] is one of the few true algorithms applicable to the routing problem; it guarantees that an interconnection between two points will be found if a satisfactory path exists. It is an exhaustive algorithm, as many true algorithms are, exploring all possible paths. Typically it is quite slow in execution because all possible paths are explored in parallel. A number of modifications to this basic technique have been proposed; most change the algorithm to a heuristic (it can no longer guarantee a solution will be found) in order to realize a gain in execution speed.

There are also many heuristics of very different kinds of solving the routing problem; nearly any possible technique for tracing lines is feasible, but results cannot be guaranteed. However, many heuristics run very rapidly in comparison to exhaustive algorithms and appear to do a "reasonable" interconnection job.

One general classification scheme applicable to most routers is whether they are depth- or breadth-first routers. That is, do they explore a path to a great depth choosing to explore another when failure is encountered, or do they explore multiple paths in parallel, stopping when the endpoint is reached by any path. Lee's algorithm is a breadth-first algorithm. Most heuristics use depth-first approaches which attempt to reduce execution time.

There has been some experimentation with polyrouters, i.e., two or more routers operating in tandum. Often a fast heuristic depth-first router is used to make as many connections possible, followed by a slower, more expensive and exhaustive router attempting only those connections that could not be made by the first state of the router.

Most of the routers discussed in current literature are experimentally evaluated to give some indication of their success, with performance generally expressed as a percentage completion figure for the number of wires routed on a board, compared to the number of wires attempted. There are few analytical tools, however, to indicate or predict router performance.

This paper introduces a model for printed circuit boards which can be used to predict the probability that a router will successfully make a connection. The model reflects certain characteristics of the circuit board that is being routed and the model incrementally changes as the board is routed. Routing procedures typically have certain parameters which influence selection of the set of paths that are explored, and determine the order of exploration. These parameters obviously influence the probability that a particular connection can be made. These parameters may also be used to formulate a model of the behavior of a particular routing procedure.

The purpose of the board and router models proposed here is to allow (comparative) predictions of router performance. More specifically, it is assumed that the features of a router which select the set of paths explored should impact the probability that a router will be successful, as well as router execution time. Relationships between execution time and performance can be analyzed using the concepts developed below.

This study also attempts to address several other questions: What factors cause a router to be good and how could a routers' performance be increased keeping the same general heuristic; which router in a polyrouter should be used for a particular connection or when should the changeover between routers be made; and finally in what order should the connections be made to give the greatest expected number of completed connections?

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Admittedly none of these questions are answered fully in this paper. However, the general analysis technique may hold the potential for treating such issues. It should also be emphasized that both board environment and router behavior models are statistically based, and therefore do not guarantee that a particular connection can be made between any two points; the approach taken will be to produce, for each connection, a probability that a satisfactory path will be found. No routing functions are performed during the computation of this probability.

## THE BASIC MODELS

The model of a *printed circuit board* is based strictly on the 'density' of a board. Several authors [4,6] have noted that as a board becomes more crowded (additional wires completed) it becomes harder to route wires, as would be expected. At a certain density it becomes nearly impossible to route an additional wire. This idea forms the basis of the problem board model.

The model assumes that each layer of a printed circuit board can be represented as a rectangular *grid of small squares, each of which is either* empty (can be used) or occupied (further use is illegal). The density of occupied squares (or just density) is defined by equation (1) as the ratio of the number of occupied squares to the total number of squares all layers of a board. As density is just the probability of a square being used, equation (2) defines the probability F of a square being free.

$$(1)\quad \text{Density} = D = \frac{\#\ \text{occupied squares}}{\#\ \text{occupied sqs.} + \#\ \text{empty sqs.}}$$

$$(2)\quad \text{Free} = F = 1 - D$$

The initial modeling efforts reported here have been focused on three general classes of boards:

MODEL 1. A board is filled with randomly distributed occupied squares, but no lines or groups of connected grid squares are included. This model might resemble a board containing randomly placed component pads.

MODEL 2. The board has randomly distributed continuous segments of occupied squares, with all wires having both expected width and expected length equal to constant values. This model somewhat resembles a partially routed board, but is more *regular in its layout.*

MODEL 3. The board has randomly distributed contiguous segments of occupied squares, with each segment having a constant expected width and a random length chosen from some given distribution. This model attempts to more accurately represent a partially routed printed circuit board.

Our modeling effort is based on the following argument. If the density of the board is calculated by equation (1), then the probability that a square is occupied is D, and the probability that a square is free is F by equation (2). For a path of length n, the probability that a line can be routed exactly that distance is the probability that n free squares can be found followed by n occupied square, and is given by equation (3). The probability that a line can be routed between two points, however, is not the probability that it can be routed exactly the distance between those points, but rather the probability it can be routed at least that distance, and is given by equation (4), which is simplified to equation (5).

(3) Probability of success for length $n$ =

$$P_x(\text{length}=n) = F^n * D = (1-D)^n * D$$

$$(4)\quad P_x(\text{length} - n) = \sum_{L=n}^{\infty} (1-D)^L * D = 1 - \sum_{L=0}^{n-1} (1-D)^L * D$$

$$P_s(\ell \geq n) = 1 - D - (1-D) * D - (1-D)^2 * D - \ldots\ldots - (1-D)^{n-1} * D$$

$$(1-D)\, P_s(\ell \geq n) = (1-D) - (1-D) * D - (1-D)^2 * D - \ldots\ldots - (1-D)^n * D$$

$$P_s(\ell \geq n)(1-1+D) = (1-D)^n * D$$

$$(5)\quad P_s(\ell \geq n) = (1-D)^n = F^n$$

In a similar manner the expected length for a routed wire can be calculated using the normal forumla for expected value calculations, equation (6):

$$(6)\quad E(\text{length}) = \sum_{\text{length}=0}^{\infty} \text{length} * P_s(\text{length})$$

$$(7)\quad E(\text{length}) = \sum_{n=0} n \cdot (1-D)^n \cdot D$$

$$E(\text{length}) = (1-D) \cdot D + 2(1-D)^2 \cdot D + 3(1-D)^3 \cdot D + \ldots..$$

$$(1-D)\, E(\text{length}) = (1-D)^2 \cdot D + 2(1-D^3) \cdot D + 3(1-D)^4 \cdot D + \ldots..$$

$$E(\text{length}) - (1-D)\, E(\text{length}) = (1-D) \cdot n + (1-D)^2 \cdot D + (1-D)^3 \cdot D + \ldots..$$

$$D \cdot E(\text{length}) = \sum_{n=1}^{\infty} (1-D)^n \cdot D$$

$$(8)\quad E(\text{length}) = \sum_{n=1}^{\infty} (1-D)^n = \frac{1}{1-(1-D)} = \frac{1}{D}$$

The expected length value of equation (8) is of a very simple form, just inversely proportional to the density of the board. Using model 1 a series of experiments were conducted to demonstrate *that this was in fact an accurate model of the* environment. For this experiment (and all others referenced here) the wire width was taken to be one unit. Wires of greater width could be handled mathematically in a very similar manner. The following analysis illustrates the general technique, which uses the unit wire width assumption.

Figure 1 shows the results of routing experiments, based on board model 1. A very high degree of agreement between equation 8 and the experimentally determined values was found. The closeness of the agreement is not surprising, because the mathematical model developed is explicitly for the situation that was modeled: single, randomly placed used squares.

When the used squares are not single points (e.g., there are contiguous occupied segments of squares such as wires), then the probability of encountering an occupied square is greatly modified. When modeling a router which places wires in a single direction on each layer of a printed circuit board, the probability of interference is related to the number of distinct segments of squares, rather than the number of occupied squares.

To develop equations for the effective density and expected length for boards with wire segments, the modeling technique shown in Figure 2 is used. Figure 2a shows a line to be run, with one already-routed line segment in the path. The previously routed lines are collapsed into single points (shown in Figure 2b). A correction term is then applied to the expected length calculation to account for the finite length of the previously routed wires: this term is the average segment length divided by 2. Equations 9 and 10 reflect the results of these calculations.

$$(9)\quad D_{effective} = D_e = \frac{\#\text{ distinct segments of sqs.}}{\#\text{ used sqs.} + \#\text{ free sqs.}}$$

$$(10)\quad E(\text{length}) = E(l) = \frac{1}{D_e} - \frac{(\text{average segment lgth})}{2}$$

Running parallel wires has reduced the effective density of a board; intuitively this approach has reduced the conjestion. This suggests the importance of minimizing the number of distinct wire segments that must be used. Figures 3 and 4 summarize experiments for board models 2 and 3. The average segment length of equation 10 requries a priori knowledge of the segment lengths developed during routing. Obviously this value cannot be computed exactly before routing, but it can be approximated before interconnection, or could be accumulated as routing proceeds if it is needed.

Figures 3 and 4 show agreement between the second model and the corresponding experimental values, although with increased error over the values in Figure 2. This increased error is the result of the variance in the average wire lengths, and could have been anticipated. Equations 9 and 10 could be applied to board model 1 as well, since the number of used segments is the number of occupied squares in this case.

Using the expected density values, the probability of success on a path (equations 3 and 4) can be calculated. These probability values are modified because of the finite segment length for the wires that are present, and are given again as equations (3a) and (4a):

$$(3a)\quad P_s(l=n) = (1-D)^{n^*} \text{ (average segment lgth/2)}\cdot D$$

$$(4a)\quad P_s(l=n) = (1-D)^{n^*} \text{ (average segment lgth/2)}$$

Note that using equations 9 and 10 the expected number of successful paths on a complete route may be calculated, as the number of paths tried times the probability of success for each path. The variance of equation (10) can also be calculated to give a confidence factor for routing. It is also of interest, though, to obtain equations for the probability of success $P_s$ for several routers and attempt comparisons using this type of analysis.

MODELING EXAMPLES

A modification of Lee's algorithm is the 'router-in-a-box' version [7]. As shown in Figure 5 the two points to be interconnected form the corners of a two-dimensional rectangle, and Lee's algorithm is applied within that rectangle. Intuitively this restriction appears justifiable, since the remaining three quadrants not used for routing generally lead away from the destination point, and are therefore likely to be unproductive.

Figure 6 shows an enlargement of one area of Figure 5, with a square at location (x,y) in the box, and the four adjacent neighboring cells. The routing procedure is assumed to be up and to the left; the problem is to find the probability of routing from point (x,y) to point (1,1). This is a function of the probabilities of success from two neighbor points only (because of the algorithm used), and is given by equation (11):

$$\begin{aligned}(11)\quad P_s(x,y \text{ to } 1,1) = {} & F * P_s(x-1,y) * F * P_s(x,y-1) \\ & + (1-F) * F * P_s(x-1,y) \\ & + (1-F) * F * P_s(x,y-1) \\ & + F * P_s(x-1,y) * F * \\ & (1-P_s(x,y-1)) \\ & + F * P_s(x,y-1) * F * \\ & (1-P_s(x-1,y))\end{aligned}$$

Equation (11) is an iterative equation, but is easily tabulated becuase the boundary conditions are known. Figure 7 shows the boundary conditions for the two box sides, and the remainder of the values may be tabulated by a straightforward application of (11) along either the rows or the columns.

For certain points in Figure 5 an interesting phenomena occurs: The probability of success may be uniform in a region of the board. That is, equation (11) yields

$$\begin{aligned}(12)\quad P_s(x,y \text{ to } 1,1) = x = {} & F * x * F * x \\ & + (1-F) * F * x * 2 \\ & + F * x * F * (1-x) * 2\end{aligned}$$

which can be simplified to

(13) $P_s(x,y \text{ to } 1,1) = x = \frac{2 * F - 1}{F^2}$

This equation is graphed as Figure 8.

This is an unexpected and surprising result: The probability of completing a path between two points in the uniform region is independent of the path length between those points, and dependent only upon the board density. This result was suggested as possible by Leonard [5], but is not obvious. Clearly, however, it is a very desirable characteristic for any router! Figure 8 also shows a relatively sharp cut-off point for routing at a board density of about 35 , a figure which has been observed [4] in other work.

Path length independence is not valid over the entire 'box' of Figure 5. The boundary conditions cause certain regions to have different (and generally much lower) $P_s$ values; these regions are shown as the shaded area of Figure 9. The extent of this region is dependent only on the board density. The method used to calculate these regions is by tabulation of the $P_s$ value for the entire board, using equation 11.

The path length independence formulas developed above do not hold for the original Lee algorithm. In this latter case the equation corresponding to equation (13) is a cubic in $P_s$, implying that there are (potentially) three distinct regions of stable probability. Although the original Lee algorithm gives a higher probability of successful routing than the heuristic described above, the analysis is also much more difficult. Equation (13) might well be used as a more directly obtainable lower bound on performance for the Lee algorithm.

Consider next a quite different routing heuristic used previously by the authors [6]. It can be described as a one-turn line probe router which attempts paths such as those shown in Figure 10a and 10b, with a parameter k designating how many lines are attempted in each direction. The equation for the $P_s$ (interconnection of two points) is developed below, culminating in equation (14f)

(14a) Probability of failing to route at least one line, $\ell$ units long:

$$\prod_{K=-k}^{k} (1 - F^{K} \cdot F^{\ell})$$

(14b) Probability of success for previous (14a):

$$1 - \prod_{K=-k}^{k} (1 - F^{|K|} \cdot F^{\ell})$$

(14c) Probability of a successful interconnection over one cross of Figure 9:

$$(1 - \prod_{K=-k}^{k} (1 - F^{|K|} \cdot F^{\ell})) \cdot (1 - \prod_{K=-k}^{k} (1 - F^{|K|}F^{m}))$$

(14d) Probability of failure of (14c):

$$1 - (1 - \prod_{K=-k}^{k} (1 - F^{|K|}F^{\ell})) \cdot (1 - \prod_{K=-k}^{k} (1 - F^{|K|}F^{m}))$$

(14e) Probability of failure, on both crosses, of a route:

$$(1 - (1 - \prod_{K=-k}^{k} (1 - F^{|K|}F^{\ell}) \cdot (1 - \prod_{K=-k}^{k} (1 - F^{|K|}F^{m})))^2$$

(14f) $P_s$ for a route:

$$1 - (1 - (1 - \prod_{K=-k}^{k} (1 - F^{|K|}F^{\ell})) \cdot (1 - \prod_{K=-k}^{k} (1 - F^{|K|}F^{m})))^2$$

This equation is of limited use; however reasonable approximations can be made. First, assume that $k \ll \ell$ and $k \ll m$, to yield (15a). Then, assume $\ell = m$ (meaning the two points to be interconnected determine a square, to yield (15b), (15c), and (15d)

(15a) $$1 - (1 - (1 - \prod_{p=-k}^{k} (1 - F^{\ell})) \cdot (1 - \prod_{p=-k}^{k} (1 - F^{m})))^2$$

(15b) $$1 - (1 - (1 - \prod_{p=-k}^{k} (1 - F^{\ell})) \cdot (1 - \prod_{p=-k}^{k} (1 - F\ )))^2$$

(15c) $$1 - (1 - (1 - \prod_{p=-k}^{k} (1 - F^{\ell}))^2)^2$$

(15d) $$1 - (1 - (1 - (1 - F^{\ell})^{2K+1})^2)^2$$

Equation (14f) has the property that is maximized if the two points being interconnected are located at the corners of a square, rather than at the corners of an elongated rectangle, and the value of the equation drops off rather sharply if the two points form a very elongated rectangle. Equation (15) assumes (optimistically) that the point pairs always determine a square, rather than a rectangle, and thus (15) represents a maximum potential performance figure for the router. The length of the path being routed does clearly effect the $P_s$ value for this router, because of the $k^{6}$ term in the equation. Table 1 shows the $P_s$ value for a range of effective board densities. Experience [6] with this router has indicated that it functions reasonably well until the effective density reaches between 4 and 5%, which is verfied by Table 1.

Table 1

| Effective Density % | $P_s$ % |
|---|---|
| 2 % | 100 |
| 4% | 98 |
| 6% | 73 |
| 8% | 29 |
| 10% | 12 |

If the primary rectangle shape is an elongated rectangle, equation (15) is not valid. Since (15) represents a maximum performance figure, however, it should influence the objective function used by any placement method: obviously minimizing the distance between modules does not necessarily maximize the routability of a board! Equation (14) is not easily reduced. It is possible to assume an 'average' rectangle, but this is not mathematically justfiable. Additional work needs to be done to determine how the shape of the rectangle formed by the two points to be interconnected effects routability.

The experimental results described in the previous sections used statistical simulation models rather than production routers and production circuit boards. Verification of the techniques suggested in this paper requires that production environments be used. We are presently in the preliminary stages of experimental verification of the models presented here, using two routers. A Lee's router with limited turn ability (all turns cause layer changes giving parallel lines on each layer), and a many-turn line probe router are presently nearing completion for production use in the Design Automation System at Lawrence Livermore Laboratory. A series of experiments using production boards and constrained versions of the routing programs is being planned and conducted. Using well understood routers and known module placement and interconnection requirements, the experiments are intended to verify or contradict the models presented here.

Experimental results will be reported in a later version of this paper.

## ORDERING

Given a set of point-to-point interconnections, the ordering problem attempts to determine the optimal sequence of interconnections to maximize the expected number of completed wires. Previous work [4] indicates no clear significance for any particular ordering: although in each case a particular ordering may be best post priori, no ordering scheme seems clearly superior for the general case.

Our model gives one solution to this problem, based on the $P_s$ figure. $P_s$ is a function of D, the path length, and the router. This function can be determined, as illustrated above. To properly order the wires for routing it is necessary to maximize the sum of the $P_s$ values for each wire, to obtain the expected number of wires that will be completed (equation (16)).

$$(16) \qquad \text{maximize} \sum_{m=1}^{\#\,\text{wires}} P_s \text{ (interconnection m)}$$

For example, the Lee heuristic above has $P_s$ given by (13), where D increases proportionately to the length of each routed wire. The longer the wire that is completed the more the board density is increased for the unrouted wires. To maximize (16) it is clear that short wires should be routed first.

For a more general case, such as the second router heuristic discussed, the situation is far more complex. D increases with long wires faster than with short ones, but $P_s$ decreases both with increasing length of the path and with increasing D. The conclusion about ordering here is very dependent upon the router heuristic, the $P_s$ equation, and the distribution of wires that must be completed.

Assuming the function for F shown in Figure 11, the desired ordering again is shortest wire first to maximize (16). Note that this is wire-length distribution dependent, in general, by the argument in the preceeding paragraph. It should also be noted that differences in the expected values for different orderings are sometimes very small in a numerical sense, which may be the reason that previous experimental results were inconclusive: this figure is also subject to statistical variance related to the from-to length distribution. We have yet to find a practical router where shortest wire first does not maximize the expected completion value provided by the mechanism presented above.

## MODEL LIMITATIONS

It is not clear a priori that this model will be accurate. The model assumes wires randomly distributed over the board area, while the wires usually are clustered towards the center of a normal board. Further, the model implicitly assumes an infinite board, but there are noticable edge effects on finite boards. These edge effects may be compensated for at some expense in model complexity. Perhaps the strongest limitation of the model is that it is derived from our board model 1, while board model 3 is much more realistic. The corrections that must be made to obtain model 3 predictions are not mathematically justifiable in an absolute sense, but are closer to 'educated' guesses that could be experimentally verified.

## CONCLUSIONS

A model has been presented for printed circuit board routing. The model is statistically based, and is predictive in that statistical sense. The model is supported by limited experimental evidence and verification, and conforms to several interesting and diverse observable features of the routing process.

From our calculations several interesting ideas have been developed; some have yet to be verified or disproven. For both the Lee "router-in-a-box" heuristic and the line probe router the best $P_s$ values are found when the points to be interconnected are located on the corners of a square, and not necessarily when they are closer together or located on the corners of a rectangle. This suggests that doing placement of components should attempt to minimize an objective function which considers primary rectangle shape factors as well as minimum distance.

For the line probe router the parameter k (number of wires to run in parallel) could be chosen, using equation (15d), to achieve a certain $P_s$ value (related to confidence required) prior to routing. This would give the router a degree of adpative behavior depending upon its environment, and could potentially be quite effective in reducing computation time.

The model allows comparison of expected router performance as a function of the density of a printed circuit board and the routing heuristic, and thus allows comparison of routers and their cost/performance tradeoffs. The model also gives some insight into the ordering problem, and supports the intuitively reasonable influence of ordering on performance of a particular router.

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[7]

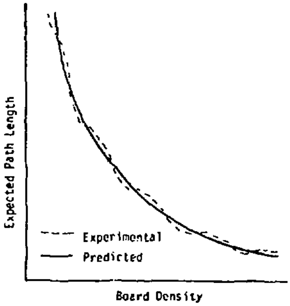
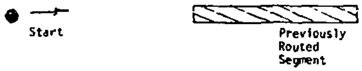
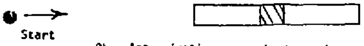




Figure 1. Model 1 Experimental Verification (1000 paths per density).




2a. Actual Situation




2b. Approximation corrects segment length by (average segment length)/2.

Figure 2. Model 2 Development

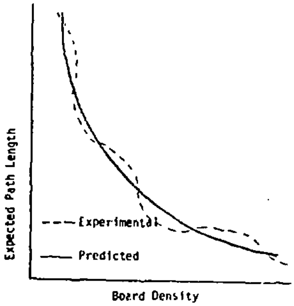



Figure 3. Board Model 2 Experimental Verification.

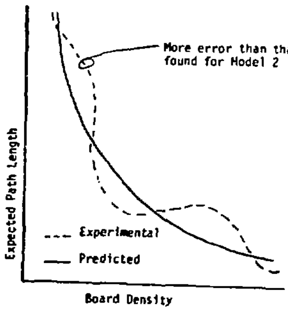



Figure 4. Board Model 3 Experimental Verification.

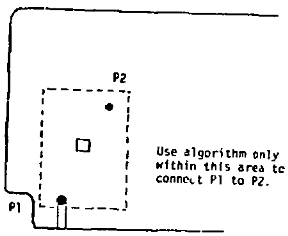



Figure 5. Constrained Lee Algorithm.

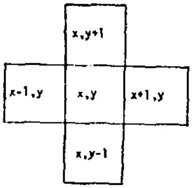



Figure 6. The Adjacent Square Relationships.

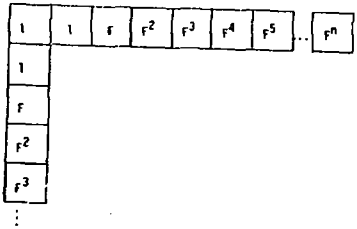



Figure 7. Boundary Conditions Imposed on Constrained Lee Router.

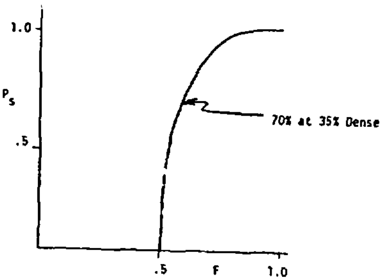



Figure 8. Probability of a Successful Route as a Function of F.

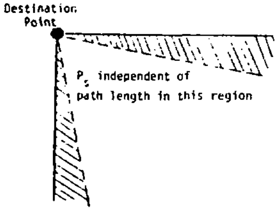



Figure 9. Edge Effects and Path Independence.

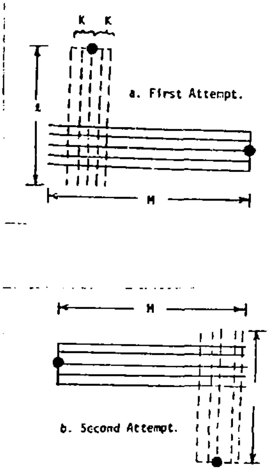
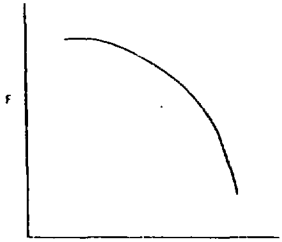
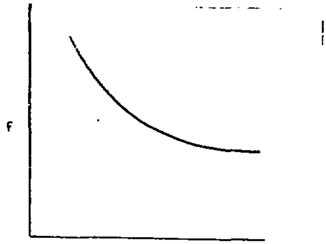



Figure 10. A Simple One-Bend Routing Algorithm.




11a. Connections Routed Shortest First [Maximizes (16) if Router is Modeled by (15f)]




11b. Longest First Routing. [Minimizes (16) if Router is Modeled by (15f)]

Figure 11. Distribution of Interconnection Outcomes.