

An Analytical Assessment of the Critical Speed Formula

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ABSTRACT

The Critical Speed Formula is used in the field of accident reconstruction for the estimation of the speed of a vehicle that has been given a sudden unidirectional steer maneuver by the driver and when the tires develop a high enough sideslip to leave curved visible marks on the pavement. This and other uses of the formula are investigated in this paper. Reconstructions are done using computerized dynamic simulations of a turn maneuver for 3 different, driver forward control modes: braking, coasting and accelerating. The experimental results of Shelton (*Accident Reconstruction Journal*, 1995) are analyzed statistically and are compared to the results of the simulations. Results show that the Critical Speed Formula can give reasonably accurate results but that the accuracy varies with several factors. One is where along the trajectory measurements are made to estimate the tire mark curvature. Another factor is the forward control mode; the accuracy is the highest when the vehicle accelerates through the turn and is the lowest for braking. The experimental data is also used to determine the statistical uncertainty of speed estimation.

INTRODUCTION

Controversy has arisen over the use in accident reconstruction of a formula from particle mechanics called a Critical Speed Formula (CSF), $v_c = \sqrt{f g R}$.

For a radial frictional force with coefficient f and path with radius R , the CSF gives the constant speed that must be exceeded to cause the mass to slide outward from the circular path. The CSF is used to estimate the initial speed of a vehicle that leaves tire marks on the pavement while the vehicle's center of gravity follows a curved path as the result of a sudden steering change such as an emergency avoidance maneuver. An in-depth derivation of the CSF (including the effect of banked roadways) with a discussion of its applications is given by Fricke (1990). Lambourn (1989) presents results that illustrate the use of the CSF. Experimental data and its application to the formula has been presented by Shelton (1995) with claims of good agreement. On the other hand, on the basis of different experimental data, Dickerson et al. (1995) conclude that the CSF is inaccurate for most reconstruction purposes. In this paper, the CSF is derived, the assumptions that apply to its derivation and applications are discussed, the CSF is examined using the results of a computerized dynamical simulation and the utility and accuracy of the formula are discussed. Shelton's data, taken over the course of 5 years at California Highway Patrol (CHP) facilities, is analyzed and are compared to the results of the simulation.

The CSF is also used to estimate velocities for circumstances different from the sudden steer maneuver discussed above. An example is for the design of superelevation and speed limits on roadway curves. This and other applications including some that are inappropriate are discussed.

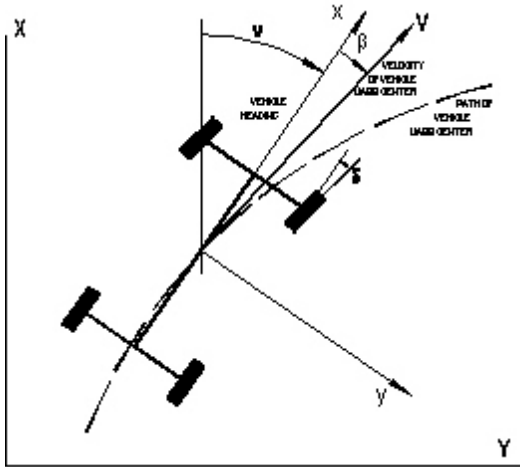


Figure 1. Terminology and symbols from SAE Recommended Practice J670

TERMINOLOGY

The terminology and the coordinate system of the SAE Vehicle Dynamics Recommended Practice J670 is followed here. Figure 1 shows the x-y-z coordinate system attached to the vehicle where the z coordinate points into the plane of the paper. The velocity, V , of the mass center of a turning vehicle with front wheel steer angle δ has a vector direction that does not align with the vehicle heading axis (x axis in Fig 1). This causes a velocity component (not shown) perpendicular the x axis, called the sideslip velocity, and the angle β called the sideslip angle. As the vehicle travels over a curved path, the heading axis changes relative to the fixed (X,Y,Z) reference; this is the course angle, ν , and is often referred to as the vehicle's yaw (or yaw angle). The rate of change of this angle is the yaw rate or yaw velocity. A sideslip angle exists not only at the mass center of the vehicle but differing values also exist at each wheel. It is the presence of each tire's sideslip that generates a mark on the road surface in severe turning maneuvers. Since the wheels' sideslips differ from front to rear in a turn, the sideslip is accompanied by a yaw rate and so the tire marks are usually referred to as *yaw marks*; the term *scuff marks* is also used. The intensity of the tire marks is related to the downward, or normal, force at each wheel. In a high speed turn, the downward

force differs significantly from wheel to wheel due to vehicle inertial load transfer. The left-front wheel (for the right turn illustrated in Fig 1) has the highest combination of normal force and sideslip and is referred to as the *leading front tire* in this paper.

DERIVATION, CRITICAL SPEED FORMULA

Figure 2 shows a particle with mass m (represented as a dark circle) moving along a circular path of radius R with radial and tangential coordinates (r, t). To remain on a circular path it is necessary for a radial force F_r be applied; in practice this is supplied by friction. The following assumptions are now made.

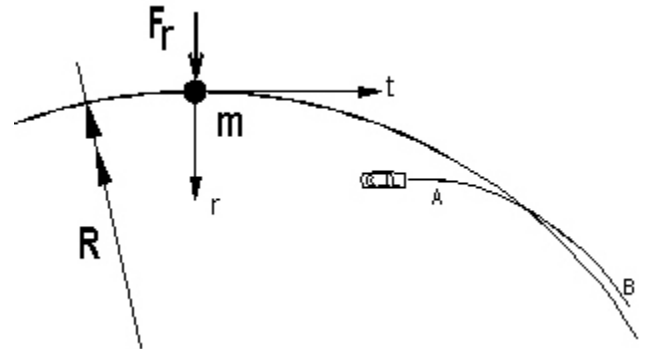


Figure 2. Free body diagram of a particle of mass m traversing a circular path of radius R . In addition, a vehicle is shown in a sharp right turn from A to B over a nearly circular trajectory.

1. m is a point mass with weight $W = mg$ (g is the gravitational constant),
2. it follows a circular path with radius R ,
3. the velocity v points in the tangential direction t and is constant,
4. the resultant tangential force is zero.

Under these conditions, the tangential acceleration a_t is zero and the radial acceleration a_r is centripetal and equal to

$$a_r = v^2/R = R\omega^2 \quad (1)$$

Here $v = v_t$ is the tangential velocity and ω is the angular velocity of the line from the center of the circle to the mass. Application of Newton's Second

Law in the tangential and radial directions, respectively, gives:

$$m a_t = \Sigma F_t = 0 \quad (2)$$

$$m a_r = m v^2/R = \Sigma F_r = F_r \quad (3)$$

Since the acceleration a_t is zero, the tangential speed is constant and $v = R\omega$. If the force F_r that keeps the mass on the circular path is supplied by Coulomb friction with coefficient f and with a normal force $W = mg$ acting in the z direction, then $F_r \leq f mg$ and Eq 3 can be solved for v to give:

$$v \leq \sqrt{f g R} \quad (4)$$

For low speeds, $F_r < f m g$. When the radial force reaches its maximum value permitted by friction sliding occurs and the above equations no longer apply. When sliding just begins the equality holds and the speed is referred to as the critical speed v_c . Equation 4 with an equal sign is often referred to as the Critical Speed Formula as used in the field accident reconstruction and written as:

$$v_c = \sqrt{f g R} = \sqrt{15fR} \quad (5)$$

The first form of Eq 5 is valid for any consistent set of units; the second gives a speed of miles per hour if R is in feet.

CRITICAL SPEED FORMULA USAGE

Equation 5 is used to estimate vehicle speeds in several applications, some appropriate and some not. To use Eq 5 appropriately, the assumptions listed above largely must be satisfied. A vehicle must be at or near the friction limit of all of its tires. The tires may or may not be leaving visible curved marks on the road surface depending on the surface conditions (for example, marks may not appear under wet or icy conditions). The mass center of the vehicle must be sweeping out a circular arc (or a curve near to a circular arc) and the speed of the vehicle must be (approximately) constant. The resultant tangential force must be near zero. Of course, the existence of

curved tire marks on the pavement is necessary for measurements to estimate R . The presence of marks also insures that the tires have reached their friction limit so the equality, Eq 5, is applicable. A nearly constant speed implies that wheels must not be locked by braking or spinning under power. The process of estimating the radius R from the tire marks is described by Fricke (1990) and Lambourn (1989) and summarized by Shelton (1995). To use the CSF the friction coefficient must also be known and is usually established by independent means. This is not discussed here.

A direct and appropriate application of the CSF is when a vehicle is traveling on a curved road at a reasonably steady speed and steer angle and the conditions of the speed, friction and road curvature are sought at which the vehicle begins to sideslip radially. This is a common application for highway curve design, superelevation and speed-limit determinations. Another common application is to estimate the speed of a vehicle that is given a sudden change in steer angle at a relatively high speed, often as an emergency avoidance maneuver. This is illustrated in Fig 2 by a vehicle that makes a sudden high speed turn near A and traverses a near circular path to point B. It is desired to estimate the speed at the beginning of the maneuver using tire mark evidence and the frictional condition of the tire-road interface. This is discussed in detail later in this paper.

There are frequently encountered situations in accident reconstruction where the CSF, though appealing, should not be used. These case are when the vehicle develops a yaw rate from some cause other than a sudden steer input by the driver. In cases such as this, a tire mark curvature may be observed but the vehicle mass center is not following a circular path. This happens when the curvature of the tire marks is due to a yaw rate developed from external influences. Examples are when the brakes of a forward moving vehicle are locked with its left and right wheels on different coefficient surfaces (such as a shoulder and road or ice and no ice) and when a vehicle loses directional control and begins to yaw (with or without braking)

on a banked or crowned icy road surface. Another is when a vehicle is moving with high yaw rate as the result of a collision. It is never appropriate to use Eq 5 to estimate the speed of the vehicle in these cases because the assumptions made in deriving Eq 5 are not satisfied sufficiently well. In these cases, the center of gravity of the vehicle travels nearly in a straight line rather than a circular arc. When braking, the resultant tangential force (in the direction of the path) is not (near) zero. When steering control is temporarily lost (over an icy area) and tire marks later appear on a higher friction surface, the force tangent to the path due to the sideslip of the vehicle's already established yaw typically is not small. What often happens is that the CSF gives an unrealistically high value of speed that doesn't fit other circumstances of an accident. Or, if the yaw rate is high (such as after a collision), the measured radius is low and the speed estimate is unrealistically low. Cases in between can be most deceptive since the calculated velocity may appear to have a realistic value but actually is meaningless.

SIMULATION, SUDDEN TURN MANEUVER

Sudden Steer Maneuver Simulations were done of an automobile making a sudden right turn at speeds of 64.4 km/hr (40 mph), 80.5 km/hr (50 mph) and 96.6 km/hr (60 mph) for a frictional drag coefficient $f = 0.65$ and are presented in this section. The simulation equations and computer program were developed by Brach (1991) for the planar motion of a four-wheeled vehicle with a rigid suspension on a flat surface. The properties of the vehicle and other conditions of the simulation are listed in detail in the Appendix to this paper. In summary, the vehicle makes a right turn as a result of a 10° step increase in the front wheel steer angle δ under three modes of driver forward control. The first, referred to as BRAKE, includes a moderate amount of braking (equal to a 20% drag by weight at each wheel for straight ahead braking). The second, referred to as COAST, corresponds to coasting through the maneuver with no braking or forward traction at the wheels. The last, ACCEL, is with forward traction at the front wheels (that provides an

acceleration of 0.1g to the vehicle in straight-ahead motion). This is done for each of the 3 initial speeds.

Each condition is simulated for a time period of 3 seconds. At an initial speed of 80.5 km/hr (50 mph), the vehicle travels roughly 20 m (66 ft) in each of the three-second intervals. A circular arc is fit to the center of gravity positions over the first one-second interval (actually, after about 0.1 seconds into the turn, to permit large tire forces to develop) of each trajectory to compute the radius R. The CSF is used to calculate the corresponding critical speed using the same friction coefficient as in the simulation. For comparison the same is done using the trajectory of the last of the 3 one-second intervals. For further comparison, the same procedure is repeated using the coordinates of the leading front tire instead of the center of gravity to estimate the radius R. (This is a common practice in the accident investigation field.) Note that the simulation cannot indicate when tire marks begin to appear (that is, at what percent wheel-slip and pavement conditions the threshold of visible tire marks occurs). Note that the speed of the vehicle changes over the entire trajectory, the greatest in the presence of braking. No sensitivity studies were attempted to investigate the effect of changes in vehicle properties on the use of the CSF. Although vehicle inertial properties are not expected to play an important role in frictional phenomena, additional study is necessary to confirm this.

Figure 3 summarizes the simulation results for the initial speed of 80.5 km/hr (50 mph). Trajectory data for the COAST case are listed in the Appendix. In all cases, using the early part of the trajectory to determine the radius R leads to a critical speed higher than the true value but all within 5%. Using the later part of the path to determine the radius R in all cases leads to a critical speed below the true value i.e., underestimation of the speed. In the case of braking during the turn, it is almost 20% low. In all cases, using the leading front tire (LFT) path to estimate R gives a critical speed value lower than when the center of gravity path is used. For early path estimates this improves the accuracy, but has the opposite effect for later path estimates. The ACCEL mode shows the least sensitivity to which

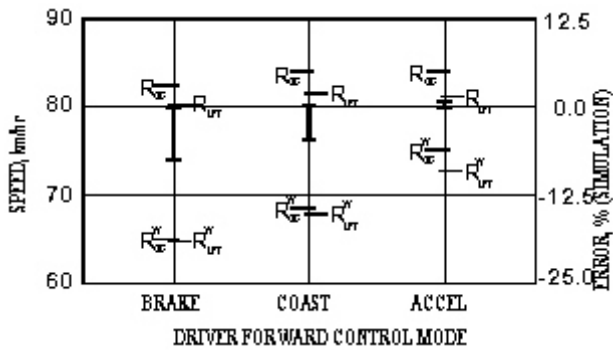


Figure 3. Vertical bars indicate the speed range over the first one second of each simulation, for each mode of driver control from a speed of 80.5 km/hr (50 mph). R_{CG} indicates a critical speed calculated from the radius of the center of gravity of the vehicle. R_{LFT} is the critical speed using the radius of the leading front tire. Values of R without an asterisk are from the first third of the 3-second trajectory; R with an asterisk uses the last third.

part of the trajectory is used to establish R . This is likely because the forward acceleration in the presence of sideslipping tires maintains conditions closer to a constant velocity. In contrast, braking causes a more rapid change in velocity and a less constant radius of curvature, so its critical velocity shows the greatest sensitivity. It also has the largest error.

Figure 4 shows the accuracy over all three driver control modes combined for initial speeds of 64.4 km/hr (40 mph), 80.5 km/hr (50 mph) and 96.6 km/hr (60 mph). It shows that the accuracy remains relatively high when R is estimated from the early part of the tire marks. It shows that the accuracy of the CSF varies inversely with the initial speed of the vehicle.

Road Edge Reentry A situation frequently arises in reconstructions when the wheels of one side of a car leave the paved portion of a roadway and a steering reentry is attempted in the presence of a significant rise of the roadway edge height (edge drop off). Forced steering of the front wheels, often attempted to initiate reentry over the edge, can cause an interaction with the road edge that introduces a significant yaw rate to occur as the tire lifts onto the pavement. Since a steer angle δ already exists as the

car enters the roadway, behavior is similar to a sudden steer maneuver as analyzed above. Simulations show that as long as the initial yaw rate is not high enough to cause the vehicle to spin more than about 90° the CSF gives an estimate with an accuracy similar to what was seen above. This seems to be the case even when the steer angle is given a sudden reversal after reentry to the roadway to simulate a correctional response of the driver. The steer reversal has little effect because by the time it is introduced, steering control is already lost.

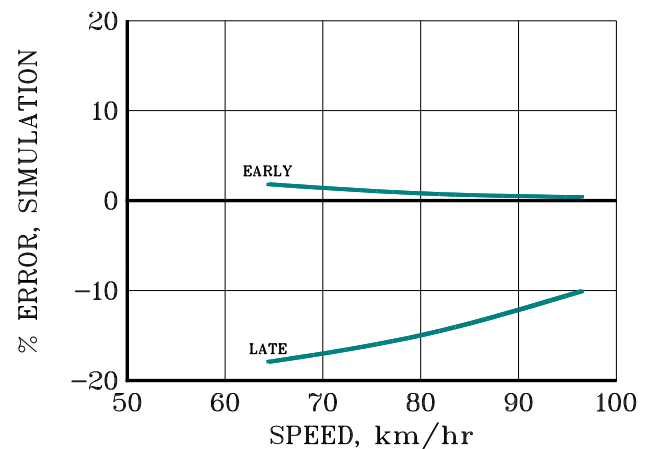


Figure 4. Difference between the initial speed and the speed from the Critical Speed Formula using 3 points from the simulated trajectory. *Early* uses the first one-second time interval of the trajectory; *Late* uses the third one-second time interval. Both curves are averages over all three driver control modes.

Road edge reentry was simulated by adding an initial yaw rate to a step increase in the steer angle. Actual reentry may be more complicated than this, but little information on the details of the dynamics of road edge reentry is available. There also seems to be little experimental data to verify the use of the CSF for the road edge reentry maneuver. For these reasons, a speed calculated from the CSF should be verified independently whenever possible.

EXPERIMENTAL RESULTS; BRAKING, COASTING & ACCELERATING

Shelton (1995) reports on 94 experiments conducted

by the California Highway Patrol (CHP) where the speed was measured by radar and also computed using the CSF from measured skid mark radii and measured frictional drag coefficients. (The reader is encouraged to refer to that paper for a full account of the experiments, techniques and circumstances under which the data were collected.) A large majority (70) of these were carried out under coasting conditions, 8 were done with the car being braked and 16 with the vehicle accelerating in the turn. In this section, the CHP data is analyzed and compared to the theoretical results presented above. The data of primary interest is the error of the critical speed estimation process. This is assessed using the difference between the calculated and measured critical speeds as a percentage of the measured speed. One question that arises is if the data indicates a significant difference between the three modes of driver control, BRAKE, COAST and ACCEL. Table 1 shows the results from a one-way Analysis of Variance (ANOVA) applied to the CHP data where the factors are the modes of driver control and the response is the percentage error.

Table 1. Analysis of Variance

Factor	Sum of Squares	Degrees of Freedom	Mean Square	Ratio
Between				
Driving Modes	821.03	2	410.5	13.7
Within				
Driving Modes	2734.3	91	30.05	3.10*

* Critical Value from the Snedecor-F Distribution

The driving mode factor has a mean square ratio of 13.7 which is considerably greater than the Critical Value of 3.10 at the 5% level of significance. This indicates a conclusion that the accuracy of estimation of the critical speed between the BRAKE, COAST and ACCEL experimental runs differs

Table 2. Average Differences, %, between the Speed Calculated by the CSF and the Measured Speed (Shelton, 1995)

Mode	average error %
Braking	-13.5 %
Coasting	-4.6 %
Accelerating	-1.2 %

significantly. In short, this means that the accuracy depends on whether the driver brakes, coasts or accelerates in the turn. The corresponding experimental values are listed in Table 2. For all driver control modes, the experimentally estimated speeds from the CSF are less than the measured speeds, with the highest accuracy for ACCEL through COAST to the lowest for BRAKE. This trend is identical to that found earlier from the simulation for values of R computed from the later part of the simulated trajectories. Note that the experimental accuracies listed in Table 2 are consistently higher than the values of Fig 3 using the late part of the simulated trajectories. This implies that the experimental measurements were made relatively early in the tire marks (which is a part of the stated method of the CHP). The range of CHP speeds is from 53.1 km/hr (33 mph) to 154.5 km/hr (96 mph). The accuracy of the measurements for such a wide range is quite good in light of what is shown in Fig 3 and 4 from the simulation.

Since the simulation indicates a speed dependence of the critical speed (see Fig 4) the CHP data should be examined to see if there is a correspondence. Figure 5 shows the measured error of the COAST data plotted as a function of speed. While the very high speed data shows a small error there does not seem to be a clear indication one way or the other from the data if accuracy increases with speed.

UNCERTAINTY IN SPEED ESTIMATION

Relative Uncertainty The relative uncertainty $\delta v_c / v_c$ of the critical speed due to small variations δf and δR in the friction coefficient, f , and the radius, R , can be expressed in the following way (Brach, 1994):

$$\frac{\delta v_c}{v_c} = \frac{1}{2} \left[\left(\frac{\delta f}{f} \right) + \left(\frac{\delta R}{R} \right) \right] \quad (6)$$

One feature of this expression is that it illustrates the relative significance of measurement variations in the friction coefficient and the radius. For

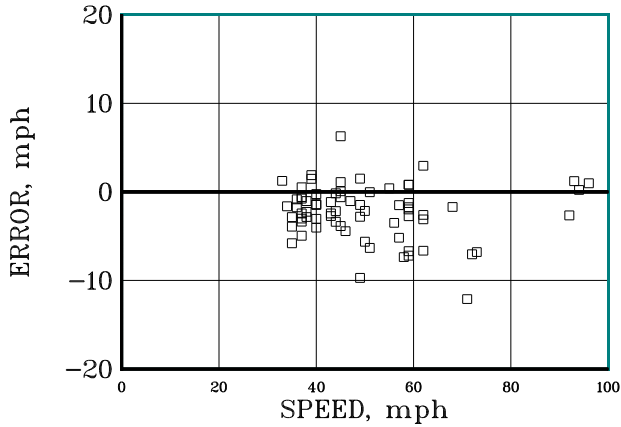


Figure 5. Differences between the measured vehicle speed and the value calculated using the CSF. Data from Shelton (1995) .

comparative purposes, the ratios, $\delta f/f$ and $\delta R/R$ can be roughly estimated as $0.1/0.5 = 0.2$ and $2/100 = 0.02$. (Note that the CSF is typically not used with low values of friction coefficient f because tire marks usually are not evident when pavement is wet or snow or ice covered.) These values indicate that determination of the critical speed v_c is more sensitive to frictional measurement variations. Keep in mind that the CSF (as reflected in Eq 6) does not explicitly contain quantities such as changes in steer angle, suspension effects, temperature effects, etc., so other means must be used to assess variations of such implicit quantities. The CHP data can provide some of this information by examining statistical variations.

Statistical Variations The critical speed as estimated from the CSF can be expected to possess a statistical distribution. That of the CHP data is shown in Fig 6. Typically, the variance of a statistical distribution is determined by repeated measurements under identical conditions. In some ways the CHP data is remarkable in that it provides a direct indication of the amount and variation of the error that might be expected in field use of the CSF. The mean values are given in Table 2 whereas the variance of the data shown in Fig 6 is $s_v = 5.75\%$. Speculation may exist that the CHP data would have more variation than field data because it was developed, at least in part, from training sessions. It

might also be said that the variation should be smaller than field use because experimental conditions were fairly well controlled. In the end, it provides more information than is usually available for reconstruction methods.

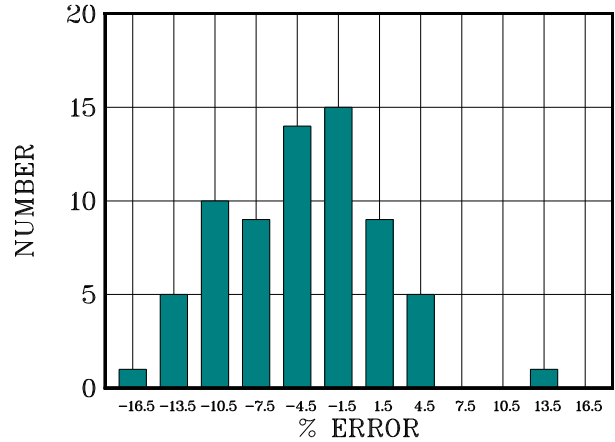


Figure 6. Distribution of 70 values (Shelton, 1995) of the difference between measured and calculated speeds for COAST driver control mode. The mean value is -4.6%.

CONCLUSIONS

On the basis of a computerized dynamical simulation combined with experimental results, the main conclusion from this study is that under the appropriate circumstances it is feasible to accurately determine the initial speed of a vehicle using the Critical Speed Formula. The accuracy can be reasonable but is influenced by forward acceleration control of the driver. The simulation shows that if the very beginning of the path into the turn is used to estimate the radius of the path, the estimate will be somewhat high. The experiments, which use the curvature of the early path typically underestimate the speed. This could indicate that tire sideslip marks do not show on the roadway as early as the high tire forces develop in the simulation. It could also be a result of the lack of suspension system compliance in the simulation. Whatever the reason, both theory and experiment indicate that the radius should be established by the earliest possible part of the tire marks as possible. In theory, the curvature of the path of the mass center should be used, however both theory and experiment indicate that the

curvature traced out by the leading front tire can be used and, in fact, may even give more accurate results. Whether the driver accelerates, coasts or brakes through the sudden turn does not affect the applicability of the CSF to estimate speed but rather influences the bias of the estimate. In the presence of acceleration, the experimental results indicate that on average the estimate will about 1% to 2% below the actual initial speed. When coasting, the estimate will on average be about 5% low and with braking, it will be around 13.5% low. So it pays to investigate the driver's forward motion control mode. Although not discussed here, the nature of any striations observed in the tire marks may help establish the mode; see Fricke (1990).

Providing that the tire marks are visible and long enough, it appears that the CSF can be used to estimate the speed of a vehicle that loses steering control on sudden roadway reentry over a pavement edge, ie, an edge drop off. The method should not be used wherever yaw marks are not initiated by a sudden steer maneuver. This includes when a vehicle develops a high yaw rate from a collision, braking on a dual coefficient surface or other loss of control due from uneven or nonuniform pavement traction.

The experimental data from the CHP indicates that the error of estimation of initial speed using the CSF has a near normal distribution with a standard deviation of 5.75%. If the CHP experience is assumed to be what is encountered in typical accident investigation practice, this means that a critical speed established by this procedure will be within $\pm 2(5.75\%)$ of the average error values listed above roughly 95% of the time.

ACKNOWLEDGMENT

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APPENDIX: SIMULATION

Some of the pertinent features of the computer simulation, the input and sample output are given in this Appendix. The tire side force and wheel slip used in this simulation both are modeled as exponential curves. A rigid suspension system is used with instantaneous acceleration weight transfer.

Specific values of all input variable are as follows:

Initial forward speeds: 64.37 km/hr (40 mph), 80.47 km/hr (50 mph) and 96.56 km/hr (60 mph)

Number of wheels: 4

tire-road friction coefficient: 0.65

steer angle: 10°, instantaneous rise time

wheel side force coefficients: 9375 lb/radian front wheels & 10228 lb/rad, rear wheels

braking wheel slip coefficients: 10000 lb/slip (initial slope)

distance; cg to front wheels: 4.3 ft

distance; cg to rear wheels: 5.2 ft

front and rear track: 5.0 ft

static wheel normal forces: front wheels, 985.3 lb; rear wheels, 814.7 lb

weight & yaw moment of inertia: 3600 lb, 2600 ft-lb-sec²

center of gravity height: 1.2 ft

integration interval: 0.005 sec

positions and velocities are measured with respect to a fixed, earth based coordinate system (such as X and Y in Fig 1)

Sample Output: Initial Speed, 80.47 km/hr (50 mph), COAST conditions:

time sec	x position ft	x position ft	velocity ft/sec	course angle deg	yaw rate deg/sec	radius ft
0.00	0.00	0.00	73.30	0.00	0.00	—
0.10	7.32	0.05	73.12	0.53	10.31	468.04
0.20	14.62	0.22	72.94	1.97	17.98	388.61
0.30	21.90	0.53	72.75	4.03	22.89	334.58
0.40	29.15	0.99	72.51	6.48	25.83	302.34
0.50	36.36	1.63	72.24	9.16	27.52	283.05
0.60	43.52	2.44	71.91	11.96	28.46	270.99
0.70	50.62	3.44	71.53	14.84	28.95	262.69
0.80	57.65	4.63	71.10	17.74	29.18	256.83
0.90	64.61	6.01	70.62	20.67	29.25	252.75
1.00	71.47	7.56	70.09	23.59	29.23	248.06
1.10	78.23	9.30	69.52	26.51	29.15	244.66
1.20	84.88	11.21	68.90	29.42	29.05	240.90
1.30	91.41	13.29	68.23	32.32	28.92	237.63
1.40	97.82	15.53	67.52	35.20	28.78	235.22
1.50	104.09	17.93	66.77	38.08	28.64	230.77
1.60	110.22	20.48	65.97	40.93	28.50	228.20
1.70	116.19	23.18	65.14	43.77	28.35	222.59
1.80	122.01	26.01	64.27	46.60	28.20	219.35
1.90	127.67	28.96	63.35	49.41	28.05	217.10
2.00	133.15	32.04	62.41	52.21	27.90	211.52
2.10	138.46	35.23	61.42	55.00	27.76	204.72
2.20	143.59	38.52	60.41	57.76	27.61	199.47
2.30	148.53	41.90	59.36	60.52	27.46	193.53
2.40	153.28	45.37	58.28	63.25	27.31	190.80
2.50	157.83	48.92	57.16	65.98	27.16	187.67
2.60	162.19	52.53	56.02	68.69	27.02	182.37
2.70	166.35	56.19	54.85	71.38	26.87	180.12
2.80	170.30	59.91	53.65	74.06	26.72	173.53
2.90	174.05	63.66	52.43	76.73	26.58	163.01
3.00	177.60	67.44	51.18	79.38	26.43	158.45