An analytical method for assessing recharge using groundwater travel time in Dupuit-Forchheimer aquifers

Chesnaux, R.[a],*, Santoni, S.[b,c], Garel, E. [b,c] and Huneau, F. [b,c]

[a] Université du Québec à Chicoutimi, Centre d'Études sur les Ressources Minérales,
Research Group « Risk Resource Water », 555, Bd de l'Université, Chicoutimi (Quebec),
G7H 2B1, CANADA.
*corresponding author (romain_chesnaux@uqac.ca)

[b]Université de Corse Pascal Paoli, Faculté des Sciences et Techniques, Laboratoire
d'Hydrogéologie, Campus Grimaldi, BP 52, F-20250 Corte, France
[c]CNRS, UMR 6134, SPE, F-20250 Corte, France

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Abstract

An analytical solution to calculate the recharge of unconfined aquifers with Dupuit-Forchheimer type flow conditions is proposed. This solution is derived from an existing closed-form analytical solution initially developed to determine groundwater travel time when the recharge of the aquifer is known. This existing solution has been modified to determine recharge when groundwater travel time is known. An illustration is given with a field case example for the Bonifacio aquifer of the island of Corsica (France), in the Mediterranean. In this aquifer, previously established differences in groundwater residence time between two water samples were determined from anthropogenic atmospheric gas (CFCs and SF_6) measurements. The time difference is entered into the new analytical solution to determine recharge. The calculations yield a value of average recharge that agrees with the results obtained by several other methods that were presented in previous studies to assess the recharge of the Bonifacio aquifer. Also presented in this study is a sensitivity analysis of the new analytical solution, to quantify the influence of different parameters that control recharge: hydraulic conductivity, effective porosity and the groundwater travel time. This study illustrates how geochemical data can be combined with physical models to measure recharge. Such an approach could be adopted in other homogeneous aquifers worldwide that satisfy Dupuit-Forchheimer type flow conditions.

Introduction

Constraining aquifer recharge rates is of primary importance for the sustainable management of water resources, but this calculation has generally shown itself to be complicated and the results imprecise. The difficulty in assessing recharge rates lies in the inherent heterogeneity of aquifers as well as the multiple processes which naturally occur during infiltration (de Vries and Simmers 2002; Scanlon et al. 2002; Meixner et al. 2016; Hartmann et al. 2017). A large panel of methods exists for assessing recharge rates in unconfined aquifers (Lerner et al. 1990; Kinzelbach et al. 2002; Scanlon et al. 2002). A realistic estimation of acceptable accuracy and degree of confidence requires the combined use of several different approaches that complement and support each other (Scanlon et al. 2006; Huet et al. 2016).

Analytical methods can be used to describe groundwater flow conditions and to estimate recharge rates (Chesnaux 2013). This study proposes an analytical approach for assessing recharge which consists of combining two validated techniques, each designed to calculate groundwater travel times in an aquifer. Considering a certain number of assumptions of a steady-state Dupuit-Forchheimer type flow (Dupuit 1863; Forchheimer 1886) in an unconfined aquifer in this study, recharge is estimated by combining the results of two methods for calculating groundwater travel time: a) geochemical age-dating using environmental tracers; and b) a physical method using an analytical solution (Chesnaux et al. 2005). The proposed methodology for calculating recharge based on combining the two methods is illustrated with a field study.

The Bonifacio aquifer located in the southern part of the island of Corsica (France) was selected for the field study illustration because its geology and hydrogeological features are well documented (Orsini et al. 2010; Reynaud et al. 2012; Santoni 2016 [PhD Thesis]; Santoni et al. 2018). Previous studies have made it possible to accurately assess its groundwater residence times using hydrochemistry, anthropogenic atmospheric gases (CFCs and SF_6) and strontium isotopes (Santoni et al. 2016a, b).

Field Site Description

The Bonifacio aquifer is located in the western Mediterranean Sea, in the southern part of the island of Corsica (France). The aquifer consists of a coastal sedimentary Miocene plateau extending over 25 km^2 and lying on a granitic depression (Orsini et al. 2010; Reynaud et al. 2012). There are two aquifer levels separated by an aquitard. Figure 1 presents a simplified map of the hydrogeology of the Bonifacio area. The Bonifacio unconfined aquifer and its properties have been described in detail in Santoni et al. (2016a,b). It is composed of calcareous sandstones and sandy calcarenites representing a very compacted and hard material with a cemented granular structure; despite this, its effective porosity is rather high at 7% (Dörfliger et al. 2002). This unconfined aquifer lies on a 50-m thick silty layer that represents the aquitard level in regards to the unconfined aquifer. The 50 to 100 m thick aquifer is bounded to its sub-horizontal base by the silty aquitard, to the South-West by the Mediterranean Sea (Bonifacio Harbour) and it is elsewhere bounded at its sides by granitic crystalline bedrock outcrops. The hydraulic properties of this aquifer have been characterized during a geophysical survey (Dörfliger et al. 2002); it has been observed that the aquifer can be considered homogeneous. Furthermore, it is favorable to groundwater flow with a hydraulic conductivity comprised between 1.3×10^{-4} m/s and 3×10^{-4} m/s, as determined from pumping tests conducted by Alamy and Chiari (2010).

Finally, Santoni (2016) and Santoni et al. (2018) have shown that groundwater extraction is estimated to comprise only 5% of the aquifer's total annual recharge and that it is limited and localized in the city of Bonifacio. They have also shown that groundwater extraction does not significantly affect groundwater flow at a regional scale in terms of directions and gradients.

Methods

Analytical solution for computing groundwater travel times

The closed-form analytical solution of Chesnaux et al. (2005) is used to calculate advective groundwater travel times in the configuration of a Dupuit-Forchheimer type flow system (Dupuit 1863; Forchheimer 1886; Bear 1972) in an unconfined aquifer (Figure 2).

Equation 1 is the general analytical solution to calculate the travel time of a particle flowing between position x_i at the water table to a downgradient position x within an unconfined, horizontal aquifer under uniform surface recharge.

$$t(x) = n_e \sqrt{\frac{\alpha}{KW}} \left[x\sqrt{\frac{1}{x^2} - \frac{1}{\alpha}} - x_i\sqrt{\frac{1}{x_i^2} - \frac{1}{\alpha}} + \ln\left(\frac{\sqrt{\alpha}/x_i + \sqrt{\alpha/x_i^2 - 1}}{\sqrt{\alpha}/x + \sqrt{\alpha/x^2 - 1}} \right) \right] \qquad (1)$$

with $\alpha = L'^2 + \frac{K h_{L'}^2}{W}$.

where K is the hydraulic conductivity of the aquifer [LT^{-1}], W is the aquifer recharge [LT^{-1}], L' is the length of the aquifer [L], n_e is the effective porosity of the porous medium and $h_{L'}^2$ is the hydraulic head of the constant-head boundary discharge [L].

The analytical solution of Chesnaux et al. (2005) has been verified against numerical modeling. A derived solution for groundwater travel times in a horizontal unconfined aquifer in oceanic islands was established later by Chesnaux and Allen (2008) and has been verified against experimental travel time measurements (Stoeckl and Houben 2012). Equation 1 is now extensively used by scientists and practitioners to calculate groundwater travel time (INOWAS, 2017). More recently, the analytical solution was modified to estimate the changes in groundwater travel time in coastal and island aquifers exposed to sea-level rise due to climate change (Chesnaux 2015; Chesnaux 2016).

CFCs and SF_6 for assessing groundwater travel times

When groundwater residence times are less than 70 years, anthropogenic atmospheric gases such as Chlorofluorocarbons (CFCs) and Sulfur Hexafluoride (SF_6) can be used for groundwater dating (Cook and Solomon 1997; Cartwright et al. 2017).

Groundwater samples from the Bonifacio aquifer have been analyzed for CFCs and SF_6 (sampling wells P1 and P2 in Figure 1). Groundwater residence times have been estimated between 9 and 50 years in the unconfined aquifer (Santoni et al. 2016a). Note that a later approach linking $^{87}Sr/^{86}Sr$ to CFCs and SF_6 made it possible to establish the kinetics of water-rock interactions that support the validity of the obtained groundwater residence times (Santoni et al. 2016b).

Combining analytical and geochemical methods to assess recharge

Knowledge of groundwater travel time makes it possible to constrain recharge. Equation 1 can be used to calculate one of four parameters when the other three are known: effective porosity, hydraulic conductivity, recharge or groundwater travel time. In Equation 1, the term t can be replaced by the known value of groundwater travel time derived from the analysis of CFCs and SF_6. This time value is the essential parameter which makes it possible to adapt the initial Equation 1 by eliminating the need for the original x_i parameter.

In this study, the travel time of groundwater was obtained from CFC and SF_6 concentrations from groundwater samples taken at two different locations (x_1 and x_2) along the flowline. The difference in age of water Δt between these two locations represents the travel time; this value is inserted into Equation 2 as a known variable. Equation 2 is derived from Equation 1 to calculate recharge using the difference in age Δt between x_1 and x_2 (Equation 2):

$$W^2 = \frac{WL'^2 + Kh_{L'}^2}{K} \cdot \frac{n_e^2}{(\Delta t)^2} \left[\sqrt{1 - \frac{x_2^2}{L'^2 + \frac{K}{W} h_{L'}^2}} - \sqrt{1 - \frac{x_1^2}{L'^2 + \frac{K}{W} h_{L'}^2}} + \ln\left(\frac{x_2}{x_1} \frac{\sqrt{WL'^2 + Kh_{L'}^2} + \sqrt{WL'^2 + Kh_{L'}^2 - x_1^2 W}}{\sqrt{WL'^2 + Kh_{L'}^2} + \sqrt{WL'^2 + Kh_{L'}^2 - x_2^2 W}} \right) \right]^2 \qquad (2)$$

The recharge W can be calculated by solving Equation 2. W cannot be isolated from Equation 2 and in consequence it cannot be directly extracted and calculated. Solving W from Equation 2 requires the use of numerical methods, for example: solvers built into commercial spreadsheets.

Application to the Bonifacio Aquifer

The configuration of the Bonifacio aquifer makes it appropriate for estimating the groundwater travel time between P1 and P2. P1 and P2 positioned along a flow line are shown on the plan view by the A-A' line in Figure 1. The conceptual cross-section is shown in Figure 3. Groundwater flows from East to West, is recharged by precipitation and discharges to the Mediterranean Sea near the Bonifacio harbour. The aquifer bottom is 100 m below sea level at the shore. The aquifer thickness of 100 m is generally uniform over the extent of the aquifer. The flow in this direction satisfies the Dupuit-Forchheimer flow assumptions where the groundwater flow can be considered steady and unidirectional (1D – flow dimension) within a homogeneous unconfined aquifer bounded by a horizontal substratum. The Eastern boundary of the aquifer consists of a crystalline rock outcrop that is contiguous to the aquifer (Figure 3). This outcrop acts as an impermeable barrier and represents the datum $x = 0$ of the cross-section of the aquifer along A-A'. The discharge into the Mediterranean Sea is located 3,176 m from the outcrop (the discharge is at $x_{L'}$ = 3,176 m on Figure 3). The model considers the assumption that the fresh-saltwater interface has negligible effects on the Dupuit-Forchheimer type flow at a regional scale of several kilometers; therefore, the calculation of groundwater travel times is not significantly affected by saltwater intrusion in this case (Chesnaux and Allen 2008). The two observation piezometers named P1 and P2 and located respectively at x_1 = 1,846 m and x_2 = 3,019 m make it possible to monitor groundwater levels and have been sampled for groundwater residence time evaluation based on analysis of CFCs and SF_6.

The groundwater travel time between P1 and P2 is calculated by first determining the groundwater age of each sample using CFCs and SF_6, and then subtracting one groundwater age from another, as presented in Santoni et al. (2016a). Groundwater ages in this aquifer were calculated by Santoni et al. (2016a) to be t_1 = 9 years and t_2 = 39 years, respectively, at P1 (at distance x_1) and P2 (at distance x_2). The difference of groundwater ages Δt between x_1 and x_2 is therefore of 30 years, which represents the travel time. Considering a mean hydraulic conductivity of the aquifer of $K = 2.5\text{x}10^{-4}$ m/s, an effective porosity of n_e = 7%, $h_{L'}$ = 100 m and computing the positions x_1 and x_2 in Equation 2 with a travel time of 30 years between x_1 and x_2, yields a recharge W = 115 mm/year. This value is close to the value obtained by Santoni et al. (2018) of 132 mm/year using other physical and chemical methods. The total mean annual rainfall is 508 mm (Santoni et al. 2016c; Santoni et al. 2018), which means that in this study, the recharge is calculated to be 23%, whereas it was calculated to be 26% in the study of Santoni et al. (2016c) and Santoni et al. (2018).

Sensitivity Analysis

Santoni et al. (2016a) estimated that the degree of uncertainty regarding the determination of groundwater age difference in the Bonifacio aquifer was ± 5 years. Thus the travel time in the Bonifacio aquifer is between 25 and 35 years. Figure 4 presents the results of the sensitivity analysis of the calculation of recharge (in terms of % of precipitation) obtained from Equation 2 as a function of: a- the groundwater age difference Δt measured between x_1 and x_2; b- the hydraulic conductivity of the aquifer; and c- the effective porosity of the aquifer (Figure 4).

Figure 4a shows that when the age of groundwater is entered into the equation as 25 and 35 years, the values obtained for recharge are 27% and 19%, respectively. The sensitivity of Equation 2 to hydraulic conductivity (Figure 4b) when calculating recharge varies between 22% and 25% when the hydraulic conductivity varies from 10^{-5} to $5x10^{-3}$ m/s (almost 3 orders of magnitude). In the case of the effective porosity, the recharge varies between 15% and 30% when the effective porosity varies between 5% and 10% (Figure 4c). Considering all the previously stated uncertainties relating to these parameters, an annual recharge value for the aquifer of Bonifacio can be expected to be comprised between 20% (100 mm) and 30% (150 mm).

Knowledge of the range of mean annual recharge values along the A-A' flow line of the Bonifacio aquifer makes it possible to determine the groundwater travel time from any position x_i along the same flow line towards the aquifer outlet using Equation 1. Figure 5 shows the groundwater travel time along the A-A' flow line from x_i to the outlet as a function of its distance to the outlet. According to this graph, it appears that a water particle requires a maximum of 200 years to travel from the recharge area (the point of contact between the calcarenites and the granitic bedrock, which act as a no-flow boundary condition) to the outlet (the Mediterranean Sea).

Discussions and Conclusion

It is important to note that the calculation of recharge using the proposed approach presents certain limitations, due not only to the uncertainty of the parameters that are entered into Equation 2, but also due to the restrictive assumptions that are associated with a Dupuit-Forchheimer flow type aquifer. In consequence, the proposed approach is only applicable to certain specific aquifers that satisfy these conditions. Santoni et al (2016a) also underline and discuss the limitations inherent to the use of CFCs and SF_6

tracers in the determination of groundwater age. Also, it should be noted that the Dupuit-Forchheimer type flow model considers groundwater flow to be unidimensional, which means that the horizontal component of groundwater velocity is solely considered and the vertical component is neglected. The assumption of a flow that is exclusively horizontal is largely satisfied for laterally extensive, thin aquifers (such as the Bonifacio aquifer), i.e. when $L' >> h$.

The calculation of the recharge of aquifers is strongly dependent on the scale that is considered. Some methods are more appropriate in assessing local recharge, while others are more appropriate in calculating regional recharge. The method proposed in this study makes it possible to calculate an average value for regional recharge, but it must be emphasized that local values of recharge within the aquifer may vary. It should be recalled that the proposed methodology assumes homogenous conditions in any studied aquifer, and therefore the accuracy of the results obtained in the case of the Bonifacio aquifer may be affected.

Data about the age of groundwater can be useful not only to determine recharge rates but also to determine effective porosity when both the hydraulic conductivity and the recharge are known. Note that it is usually easier to estimate the effective porosity and the hydraulic conductivity of an aquifer, as these elements can both be determined by simple experimental measurements. It is more difficult to estimate the groundwater travel times or the recharge, as both of these parameters are difficult to determine accurately and with a good degree of confidence using direct measurements.

Because of their cost-effectiveness, immediacy and ease of use, analytical solutions retain their usefulness despite their reliance on certain limitative assumptions. They can provide relatively accurate estimates of unknown parameters despite the uncertainties associated with their assumptions. The analytical solution developed by Chesnaux et al. (2005), modified to take into account the added parameter of known groundwater age, has shown itself capable of estimating the recharge of unconfined aquifers under certain conditions. The same approach may be applied in other aquifers as long as the same assumptions and conditions are satisfied.

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Figure captions

Figure 1: Simplified map of the hydrogeology of the Bonifacio aquifer (modified from Santoni et al. (2016a)) displaying the selected sampling wells and the cross-section along a flow line in the Dupuit-Forchheimer type flow system.

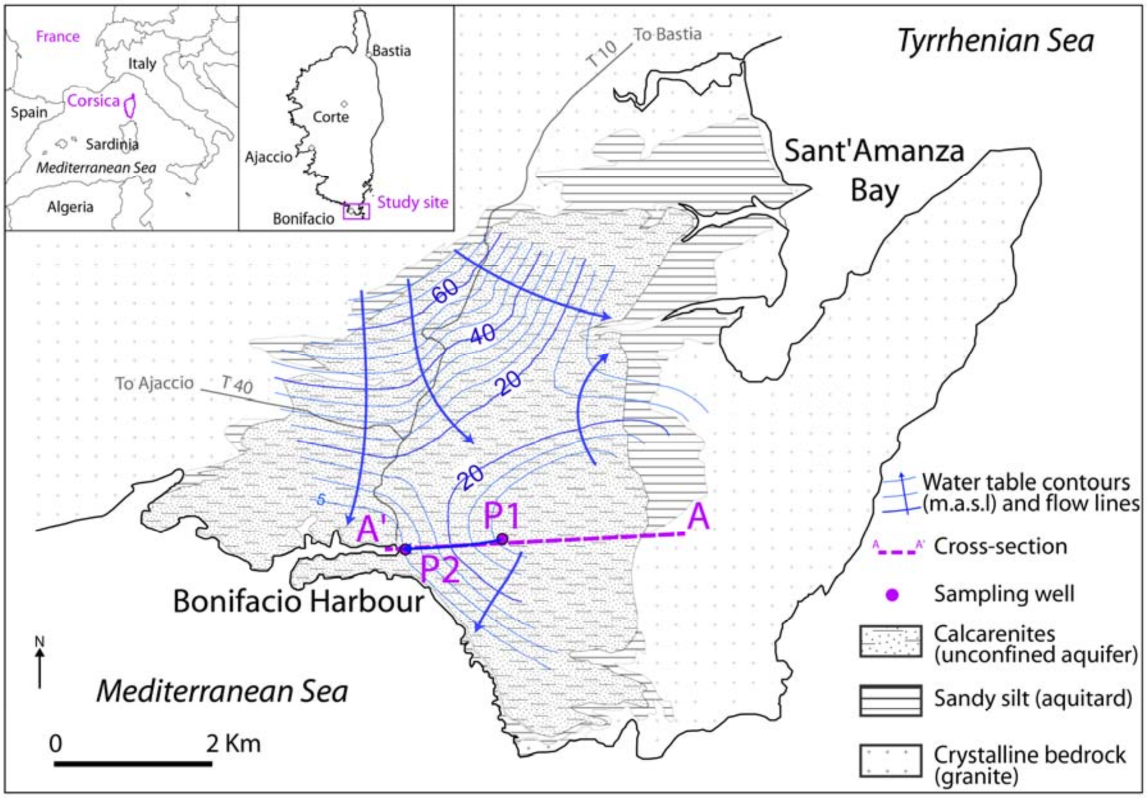

Figure 2: The simplified Dupuit-Forchheimer flow-type system used for developing the general analytical solution of Chesnaux et al. (2005). The left-hand boundary is impermeable (zero q_x flux) and flow discharges through the right-hand fixed-head boundary. The travel time is calculated between the two arbitrary points: x_i (the departure point at the water table) and x (see Equation 1).

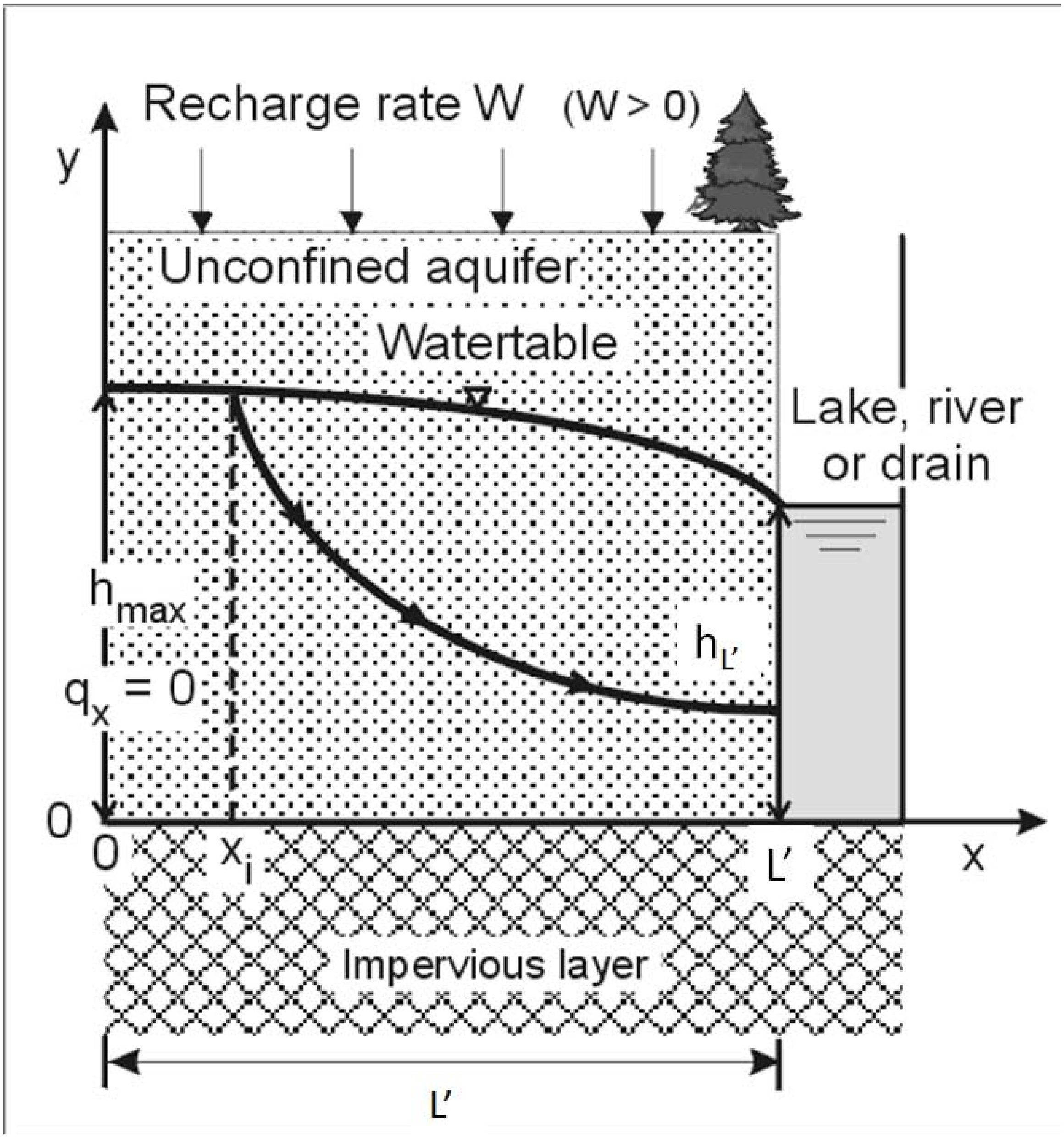

Figure 3: Conceptual hydro-stratigraphic model along the flow line A-A' represented in Figure 1 (modified from Santoni et al. (2016a)).

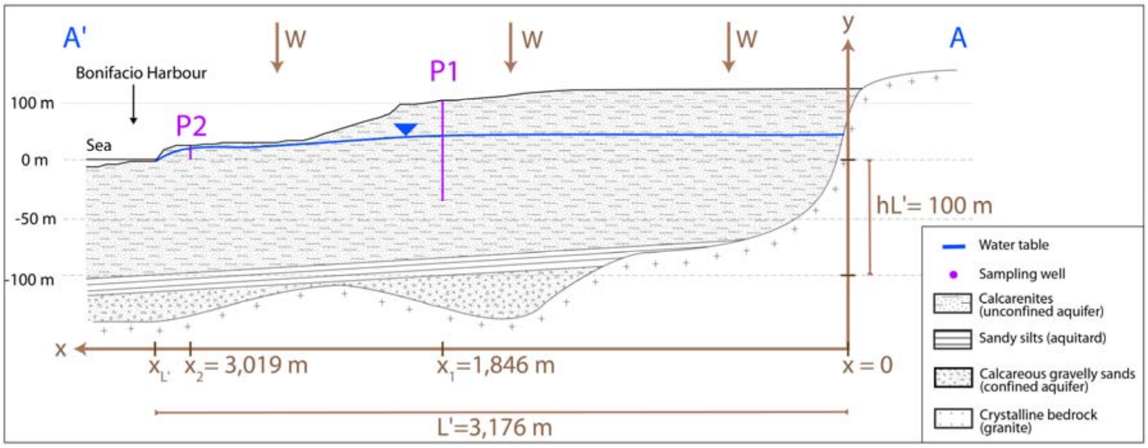

Figure 4: Sensitivity analysis of % recharge as a function of a) Δt, b) K and c) n_e.

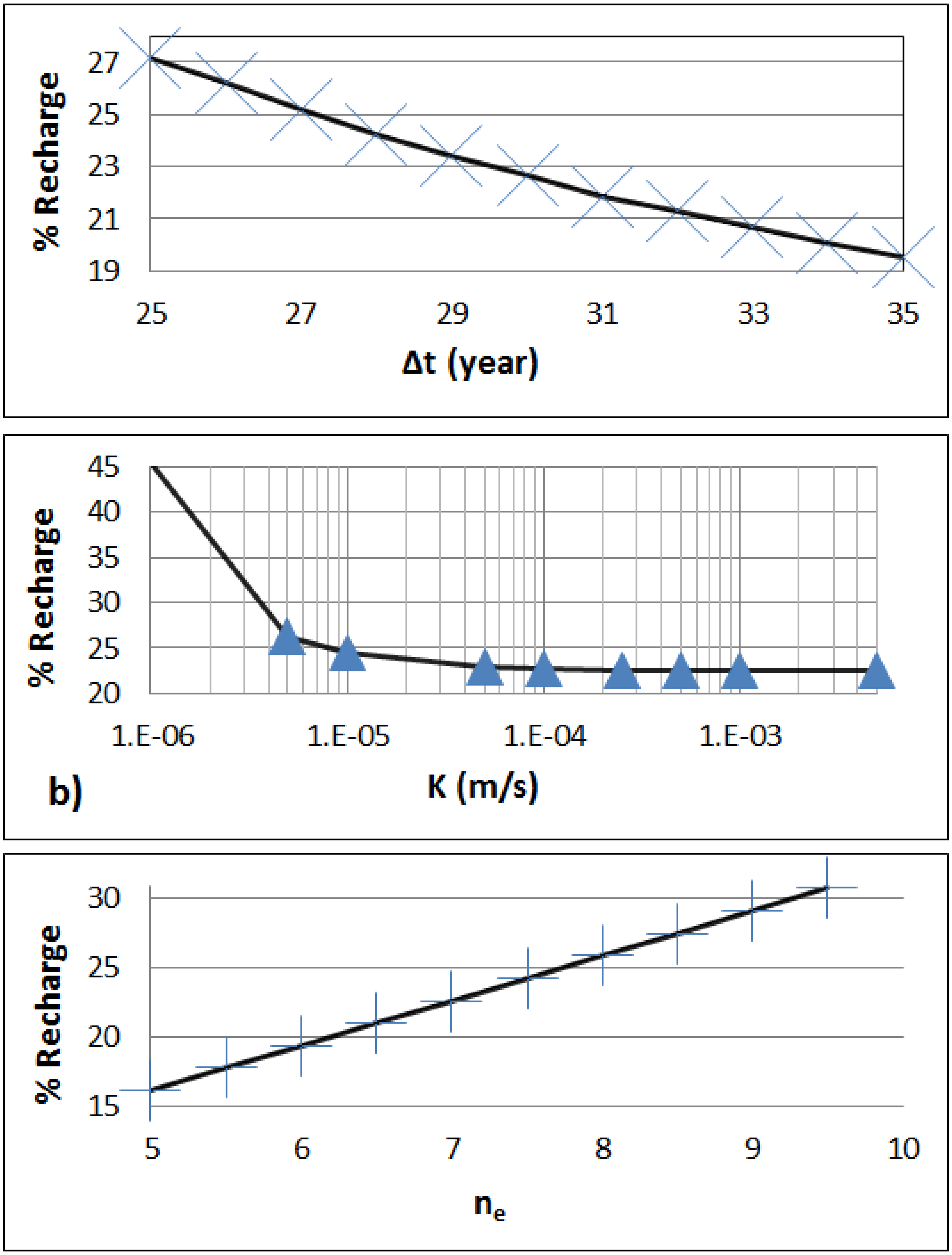

Figure 5: Groundwater travel times from the position of the departure point of the water particle to its discharge into the Mediterranean Sea, using Equation 1.