LBL-25056 c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

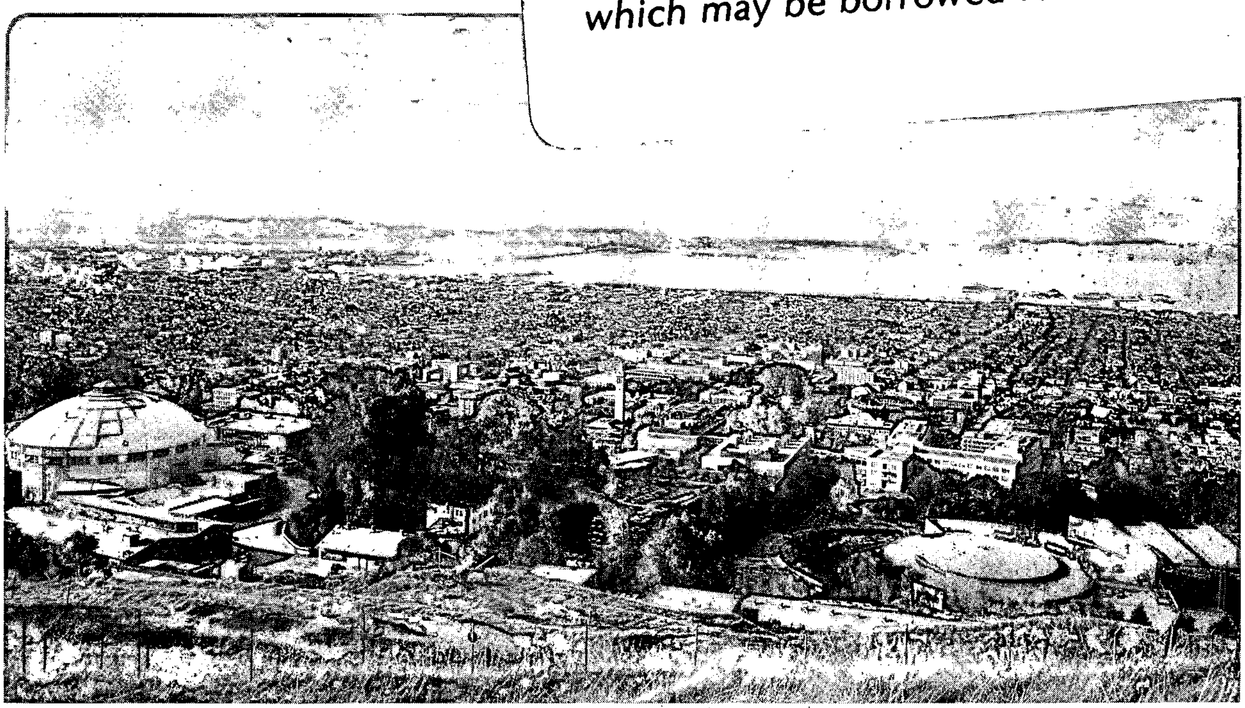
EARTH SCIENCES DIVISION

Presented at the 1988 California Regional Meeting
of the Society of Petroleum Engineers,
Long Beach, CA, March 23–25, 1988

An Analytical Solution for Wellbore Heat Transmission in Layered Formations

Y.-S. Wu and K. Pruess

March 1988



Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

LBL-25056

An Analytical Solution for Wellbore Heat Transmission in Layered Formations

Yu-Shu Wu and Karsten Pruess

Earth Sciences Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, California 94720

March 1988

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Renewable Energy Technologies, Geothermal Technology Division, and by the Director, Office of Energy Research, Office of Basic Energy Sciences, Engineering & Geosciences Division, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

SPE-17497

An Analytical Solution for Wellbore Heat Transmission in Layered Formations

Yu-Shu Wu and Karsten Pruess

Earth Sciences Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, California 94720

Abstract

This paper presents an analytical method for determining wellbore heat transmission during liquid or gas flow along the tubing. The mathematical model describes the heat transfer of the flowing fluid in the wellbore and in the surrounding formation as one whole physical system. The transient heat transfer equations in the two regions with coupling at the sandface are solved simultaneously. Previous treatments of wellbore heat transmission are improved upon in several aspects. Non-homogeneous formations are treated which consist of several layers with different physical properties and arbitrary initial temperature distributions in the vertical direction. Closed form analytical solutions are obtained in real space and in Laplace space, which can be used to calculate the temperature distribution along the wellbore and in the formation, and to evaluate heat transfer rate and cumulative heat exchange between wellbore and formation. A more accurate formula is given for the widely-used transient heat conduction function $f(t_D)$ of thermal resistance. This is shown to differ from Ramey's approximate solution at early time, while approaching it at late time.

Introduction

Heat is transferred to or from the wellbore when there is a difference in temperature between the surrounding formation and the injected (or produced) fluid. In order to evaluate the feasibility of a thermal-recovery project, it is necessary to estimate the heat losses or gains of the flowing fluid in wellbores, the changes in temperature with time and depth, and the heat transfer conditions between wellbore and formation. A quantitative description of heat exchange between a wellbore and surrounding formations is also often required when one attempts to estimate formation temperatures from wellbore measurements.

Studies of wellbore heat transmission during hot or cold fluid injection have appeared in the literature since the 1950's. The techniques available at the present time for dealing with wellbore heat transmission include analytical and numerical methods. Lessem et al.[1] and Squier et al.[3] derived and solved similar systems of differential equations describing the temperature behavior of gas and hot water injection wells. They neglected wellbore thermal resistance and made the following assumptions:

1. There is no conductive heat transfer in the vertical direction of either the flowing fluid or the formation.
2. The mass flow rate of gas or water is constant throughout the injection or production system.
3. The volumetric heat capacities of fluids and formation are constant.
4. The formation is homogeneous and isotropic with constant thermal conductivity.
5. The fluid temperature and the formation temperature at the wellbore surface are equal.

All subsequent work introduced another approximation, namely, that vertical heat transfer in the wellbore was considered steady state.

The classic study by Ramey[4] on wellbore heat transmission improved Moss and White's[2] approach to incorporate an overall heat transfer coefficient. Ramey presented an approximate solution for the temperature of fluids, tubing and casing as a function of time and depth in a well used for hot-fluid injection. Satter[5] suggested a similar method for analyzing wellbore heat loss when taking into account condensing steam flow, and he provided a sample procedure for a given set of reservoir properties. An expression for the overall heat transfer coefficient for any well completion and the early time values of the transient heat conduction function were given by Willhite[6]. The more recent work by Durrant et al.[7] provided an iterative procedure for the wellbore heat transmission problem during flow of steam/water mixtures which includes vertical heat conduction.

The numerical models by Farouq Ali[8] and Wooley[9] were more comprehensive than the analytical models. They include both horizontal and vertical heat conduction in the formation, and different well operation conditions can be dealt with. However, the numerical methods are often too complicated for field application or for reservoir simulation studies since many of the wellbore and formation heat transfer properties needed in modeling are rarely known precisely.

The mathematical model for wellbore heat transmission presented in this paper adopts assumptions similar to those of Lessem, et al. The main differences are that we introduce an overall heat transfer coefficient to consider the wellbore heat resistance and that we allow for non-homogeneous formations. We consider a medium with an arbitrary number of layers with different thermal and physical properties and arbitrary initial temperature distributions (see Figure 1). Both an exact solution and a solution in Laplace space are obtained in this paper for calculations of wellbore heat transmission. The numerical results calculated from the analytical solutions are compared with Ramey's long time approximation. Illustrative applications are given for predicting wellbore heat transmission for engineering designs or reservoir simulation studies in petroleum and geothermal reservoir development.

Mathematical Model

The transient heat transmission problem under consideration is as follows (see Figure 1):

The injection (or production) well is cased to the top of the injection (or production) interval. Heat is transferred along the wellbore solely by convection and then by conduction into formation. The formation consists of N layers with different thermal and physical properties. The system to be modeled is composed of three parts, as shown in Figure 1, (i) fluid flow conduit inside the tubing; (ii) tubing/casing annulus, casing wall and cement; (iii) infinite formation surrounding the casing. The major assumptions and approximations are as follows:

1. well flow rate is constant;
2. fluid flow in the tubing is one-dimensional vertical and steady;
3. the well fluid temperature is lumped radially;
4. the heat conduction in the vertical direction is neglected compared with heat convection by the flowing fluid;
5. radial heat flow between the wellbore and the formation is steady state;
6. in the surrounding earth, the initial geothermal gradient is a known function of depth; and
7. the vertical heat conduction in the formation can be ignored compared with the horizontal.

All the other assumptions are similar to those of the previous work. Therefore, the heat transfer equation in the tubing can be written as:

for liquid flow,

$$\rho_t\, c_t \frac{\partial T_{1j}}{\partial t} + \frac{2}{r_t} q_j'' + \rho_t\, c_t\, V_m \frac{\partial T_{1j}}{\partial z} = 0 \tag{1}$$

$$(j = 1, 2, \ldots N) \quad (z_{j-1} < z < z_j)$$

for gas flow,

$$\rho_t\, c_t \frac{\partial T_{1j}}{\partial t} + \frac{2}{r_t} q_j'' + \rho_t\, c_t\, V_m \left[\frac{\partial T_{1j}}{\partial z} \pm \frac{g}{c_t}\right] = 0 \tag{2}$$

$$(j = 1, 2, \ldots, N)\ (z_{j-1} < z < z_j)$$

where the plus sign on the potential energy term is used for flow down the well and the negative sign is used for flow up the well.[4]

The heat conduction in layer j of the formation is described by

$$\frac{1}{r}\ \frac{2}{\partial r}\ (r\,K_j\ \frac{\partial T_{2j}}{\partial r}) = \rho_j\ c_j\ \frac{\partial T_{2j}}{\partial t} \tag{3}$$

$$(j = 1, 2, \ldots N) \quad (z_{j-1} < z < z_j)$$

The heat flux at tubing surface ($r = r_t$) is:

$$q_j'' = U_j\ \left[T_{1j} - T_{2j}\Big|_{r = r_w}\right] \tag{4}$$

and the overall heat transfer coefficient is defined by:[6]

$$U_j = \left[\frac{1}{h_t} + \frac{r_t \ln\dfrac{r_t + \delta_t}{r_t}}{K_t} + \frac{r_t}{(r_t + \delta_t)\,h_c} + \frac{r_t}{r_c\,h_c} + \right.$$

$$\left. \frac{r_t \ln\dfrac{r_c + \delta_c}{r_c}}{K_c} + \frac{r_t \ln\dfrac{r_w}{r_c + \delta_c}}{K_{cem}}\right]_j^{-1} \tag{5}$$

The initial conditions are:

in the well,

$$T_{1j}\ (z,\ t = 0) = G_j(z)\ \text{(known functions)} \tag{6}$$

$$(j = 1, 2, \ldots N)\ (z_{j-1} < z \le z_j)$$

and in the formation,

$$T_{2j}\ (r, z,\ t = 0) = G_j(z) \tag{7}$$

$$(j = 1, 2, \ldots N)\ (z_{j-1} \le z \le z_j)$$

It is required in (6) and (7),

$$G_1(0) = T_{air}\ \text{(constant)} \tag{8}$$

The boundary conditions are:

$$T_{11}\ (z = 0,\ t) = T_{inj} \tag{9}$$

and

$$\lim_{r\to\infty} T_{2j}(r, z, t) = G_j(z) \tag{10}$$

$$(j = 1, 2, \ldots N)$$

Analytical Solution

Define the following dimensionless parameters for radial distance, time and depth:

$$r_D = \frac{r}{r_w} \tag{11}$$

$$t_D = \frac{tV_m}{H} \tag{12}$$

$$z_D = \frac{z}{H} \tag{13}$$

The dimensionless temperatures in wellbore and formation are:

$$\theta_j(z_D, t_D) = \frac{T_{1j}(z,t) - G_j(z)}{T_{inj} - T_{air}} \tag{14}$$

$$(j = 1, 2, \ldots N)$$

$$\phi_j(r_D, z_D, t_D) = \frac{T_{2j}(r, z, t) - G_j(z)}{T_{inj} - T_{air}} \tag{15}$$

$$(j = 1, 2, \ldots N)$$

The unsteady-state solution of this system in Laplace space becomes (see Appendix A)

$$\bar{\theta}_j(z_D, s) = C_j(s) \exp\left[-\left[s + \beta_j - D_j(s)\right] z_D\right] + Y_j(z_D, s) \tag{16}$$

$$(j = 1, 2, \ldots N)$$

where

$$C_1(s) = \frac{1}{s} + \frac{\delta_1(z_D)}{s(s + \beta_1 - D_1(s))} \tag{17}$$

$$C_j(s) = \left[\bar{\theta}_{j-1}(z_{D_{j-1}}, s) + \frac{\delta_j(z_D)}{s(s + \beta_j - D_j(s))}\right] \exp\left[\left[s + \beta_j - D_j(s)\right] z_{D_{j-1}}\right] \tag{18}$$

$$(j = 2, 3, \ldots N)$$

and

$$\bar{\phi}_j\,(r_D,\,z_D,\,s) = \frac{\omega_j\,\bar{\theta}_j(z_D,\,s)\,K_o(\sqrt{\sigma_j s}\;r_D)}{\omega_j\,K_o(\sqrt{\sigma_j s}) + \sqrt{\sigma_j s}\;K_1(\sqrt{\sigma_j s})} \tag{19}$$

$$(j = 1,\,2,\,\ldots\,N)$$

The functions Y_j (z, s), $D_j(s)$, and δ_j (z_D) and the parameters β_j, ω_j and σ_j are defined in Appendix A. The temperature function $\bar{\theta}_j$ in Laplace space can be determined recursively from layer j = N to N+1 since it is assumed that there is no vertical heat conduction both in the wellbore and in the formation. Therefore, downstream wellbore fluid or formation temperatures have no effect on upstream ones.

Another important variable of interest for wellbore heat transmission is the heat flow rate transferred into (or from) the formation. For the case of a linear initial temperature distribution in each layer of the formation:

$$G_j\,(z) = T_{cj} + \gamma_j\,z \tag{20}$$

where T_{cj} are constant, continuity at the interfaces of layers requires

$$T_{c1} = T_{air} \tag{21a}$$

and

$$T_{cj} + \gamma_j\,z_j = T_{cj+1} + \gamma_{j+1}\,z_j$$

$$(j = 1,\,2,\,\ldots\,N\!-\!1) \tag{21b}$$

Then

$$G_j'(z) = \gamma_j \qquad \text{(constant)} \tag{22}$$

$$Y_j\,(z_D,\,s) = -\,\frac{\delta_j}{s(s + \beta_j - D_j(s))} \qquad \text{(constant)} \tag{23}$$

For the heat flux into (or from) the formation we have the following expression in Laplace space:

$$\bar{q}_j''(z,\,s) = -\,U_j\,(T_{inj} - T_{air})\left[\bar{\phi}_j\,(1,\,z_D,\,s) - \bar{\theta}_j\,(z_D,\,s)\right] \tag{24}$$

$$(j = 1,\,2,\,\ldots\,N)$$

The cumulative heat flow rate is

$$\bar{Q}_c(s) = \frac{2\pi\, r_t H^2\, (T_{inj} - T_{air})}{V_m} \sum_{j=1}^{N} \frac{U_j}{s} \left[1 - \frac{D_j(s)}{\beta_j}\right] \cdot$$

$$\cdot \left\{ \left[\bar{\theta}_{j-1}\left[z_{Dj-1}, s\right] + \frac{\delta_j}{s\left[s + \beta_j - D_j(s)\right]} \right] \left[1 - \exp(-(s + \beta_j - D_j(s))(z_{Dj} - z_{Dj-1})\right] \right.$$

$$\left. /(s + \beta_j - D_j(s)) - \frac{\delta_j}{s\left[s + \beta_j - D_j(s)\right]} (z_{Dj} - z_{Dj-1}) \right\} \qquad (25)$$

The above solutions in Laplace space can be evaluated by numerical inversion techniques[10]. Analytical solutions in real space are desirable for validating the numerical inversion results and for predicting the early-time transient behavior of the system since the numerical Laplace transform cannot be expected to give accurate results for early time. We have obtained solutions in real space for the case of a linear initial temperature distribution in each layer in the formation in Appendix B. For layer 1 ($0 \le z_D \le z_{D1}$) or for a homogeneous formation we have,

$$\theta_1\,(z_D, t_D) = \begin{cases} I_1 & (t_D \le z_D) \\ I_1 + I_2 + I_3 & (t_D > z_D) \end{cases} \qquad (26)$$

where

$$I_1 = \frac{4\delta_1}{\pi^2} \int_0^\infty \frac{D_1^*(u)}{u} \frac{\left[\exp(-\frac{u^2}{\sigma_1} t_D) - 1\right]}{\left\{\left[D_1^*(u)\,(\beta_1 - \frac{u^2}{\sigma_1}) - R_1(u)\right]^2 + \frac{4}{\pi^2}\right\}} du \qquad (27)$$

$$I_2 = \frac{2\, e^{-\beta_1 z_D}}{\pi} \int_0^\infty \frac{\left\{1 - \exp\left[-\frac{u^2}{\sigma_1}(t_D - z_D)\right]\right\}}{u}$$

$$\cdot \exp\left[z_D\, R_1(u)/D_1^*(u)\right] \sin\left[\frac{2 z_D}{\pi D_1^*(u)}\right] du \qquad (28)$$

$$I_3 = \frac{2\delta_1 e^{-\beta_1 z_D}}{\pi} \int_0^\infty \frac{D_1^*(u) \left\{1 - \exp\left(-\frac{u^2}{\sigma_1}(t_D - z_D)\right)\right\}}{u \left\{\left[D_1^*(u)\left(\beta_1 - \frac{u^2}{\sigma_1} - R(u)\right]^2 + \frac{4}{\pi^2}\right\}}$$

$$\cdot \exp\left[z_D R_1(u)/D_1^*(u)\right] \left\{\frac{2}{\pi} \cos\left[\frac{2z_D}{\pi D_1^*(u)}\right] + \sin\left[\frac{2z_D}{\pi D_1^*(u)}\right]\right.$$

$$\left.\cdot \left[D_1^*(u)\left(\beta_1 - \frac{u^2}{\sigma_1}\right) - R_1(u)\right]\right\} du \qquad (29)$$

For layer j = 2, 3, ... N, the dimensionless wellbore temperatures are:

$$\theta_j(z_D, t_D) = A_j(t_D) + \int_0^{t_D} \left[\theta_{j-1}(z_{Dj-1}, \tau)\right.$$

$$\left. - A_j(\tau)\right] B_j(z_D, t_D - \tau) d\tau \qquad (30)$$

where

$$A_j(t_D) = \frac{4\delta_j}{\pi^2} \int_0^\infty \frac{D_j^*(u)}{u} \frac{\left\{\exp\left(-\frac{u}{\sigma_1} t_D\right) - 1\right\}}{\left\{\left[D_j^*(u)\left(\beta_j - \frac{u^2}{\sigma_j}\right) - R_j(u)\right]^2 + \frac{4}{\pi^2}\right\}} du \qquad (31)$$

$$B_j(z_D, t_D) = \begin{cases} 0 & (t_D \le z_D - z_{Dj-1}) \\ \frac{2}{\pi\sigma_j} \exp[-\beta_j (z_D - z_{Dj-1})] \int_0^\infty u \sin\left[\frac{2(z_D - z_{Dj-1})}{\pi D_j^*(u)}\right] & \end{cases}$$

$$\cdot \exp\left[(z_D - z_{Dj-1}) R_j(u)/D_j^*(u) - \frac{u^2}{\sigma_j}(t_D - z_D + z_{Dj-1})\right] du \qquad (32)$$

$$(t_D > z_D - z_{Dj-1})$$

In Equations (27)-(32), $R_j(u)$ and $D_j^*(u)$ are defined in Appendix B. The dimensionless temperature function in the formation layer j is given by

$$\phi_j(r_D, z_D, t_D) = \int_0^{t_D} \theta_j(z_D, \tau)\, g_j(r_D, t_D - \tau)\, d\tau \qquad (33)$$

where

$$g_j\,(r_D,\,t_D) = \frac{2}{\pi\sigma_j\beta_j}\int_0^{\infty}\frac{u}{D_j^*(u)}\exp\left(-\frac{u^2}{\sigma_j}t_D\right)$$

$$\cdot\left\{Y_o\,(ur_D)\left[\omega_j\,J_o(u)+uJ_1(u)\right]-J_0(ur_D)\left[uY_1(u)+\omega_j\,Y_0(u)\right]\right\}du \qquad (34)$$

Discussion

To validate the analytical solutions, a series of tests have been run. The numerical inversion results of the Laplace transformed solution of Equation (16) have been compared with the numerical integration of the exact solution of Equation (26) and also with Ramey's long-time solution. The integrals appearing in Equation (26) were calculated with the numerical integral evaluation routine from the NAG fortran Library,[11] on a CRAY computer. Convergence was very rapid and smooth.

The example problem is a hot-water injection at a constant rate. The fluid and formation data for the calculation is given in Table 1. As shown in Figure 2, the numerical Laplace inversion results are in perfect agreement with the exact solution, and at long times, both the solutions and Ramey's solution converge to the same curve.

The results from the numerical Laplace inversion by the Stehfest algorithm generally need checking against some other solution, in particular for early times. The comparison of the numerical Laplace inversion with the exact solution of Equation (26) and Ramey's approximate solution is given in Figure 3. It is obvious that the numerical inversion gives very poor results for $t_D \leq z_D$. This probably occurs because of the rapidly changing condition at the sandface until the entire wellbore is full of injected water when $t_D > 1$. When the time is a little longer, the numerical inversion will give very accurate results. Instead of the analytical solutions in the Laplace space, the exact solution in the real space would be used for applications in which the very early time transient behavior is important, such as in temperature well logging analysis.[12]

As in most studies on wellbore heat transfer, the vertical heat conduction is ignored here, in comparison with horizontal flow. We examine this approximation by comparing the horizontal and vertical temperature gradients in the formation derived from the solutions obtained above. As shown in Figure 4, the ratio of vertical and horizontal temperature gradients is always smaller than 1%, and reaches its maximum around the temperature penetration fronts. A larger vertical heat flow may occur on the interface of formation layers with different properties, where the temperatures obtained by neglecting vertical flow are vertically discontinuous. Figure 5 shows that despite rather different thermal diffusivities ($K/\rho c$ = 1.72 E-6 m^2/s for sandstone, 1.17 E-6 m^2/s for clay), the difference in temperatures is very small on the interface of sandstone and clay whose properties are given in Table 1. These results should be conservative because vertical temperature differences are overestimated by neglecting the vertical flow. Therefore, the assumption that the vertical heat flow in the formation is negligible is probably acceptable for most engineering calculations. The approximation of neglecting vertical heat flow in the formation will break down at some distance from the well after long times for non-homogeneous reservoirs. Quantitative estimates of the distance and time limits for its applicability can only be made from numerical models.

A steady-state approximation for vertical heat transfer in the wellbore has been made in almost all previous wellbore heat transfer models. This approximation is not resorted to here, and it can be tested by comparison of the results from the transient and the steady-state solutions obtained in this paper. The temperature distributions from the two solutions are given in Figure 6 for sandstone data as given Table 1. It is obvious that the steady-state solution overestimates the temperature increase at early time but the differences disappear at long times.

The transient heat conduction function $f(t_D)$, discussed in detail by Ramey[4] and Willhite[6] is widely used for wellbore heat transfer calculations. However, it lacks a theoretical basis, except in the long-time limit of the line source equation given by Ramey. We obtain an accurate formula for $f(t_D)$ as a special case for a uniform and homogeneous formation (subscripts omitted):

$$f(t_D) = \frac{2\pi K(T_2|_{r=r_w} - G(z))}{\frac{dq''}{dz}} = \frac{\phi(1, z_D, t_D)}{\omega\left[\theta(z_D, t_D) - \phi(1, z_D, t_D)\right]} \tag{35}$$

It is interesting to note that in a more rigorous formulation, $f(t_D)$ is a function not only of dimensionless time t_D, but also of dimensionless depth z_D. This can be seen explicitly from Figure 7, in which $f(t_D)$ from Equation (35) is plotted for different depths z_D. Only after $t_D \geq 500$ (2500 hrs for this case), does $f(t_D)$ become independent of z_D. This means that use of an $f(t_D)$ independent of $f(z_D)$ will not give accurate results during the early transient time for wellbore heat transfer problems.

Heat loss (or gain) from wells is important for evaluating a thermal recovery project. The behavior of heat flux and cumulative heat transfer into the surrounding formation are given in Figures 8 and 9 for hot-water injection into a well in a homogeneous sandstone formation. The calculation parameters are in Table 1. It is obvious from Figure 9 that the heat losses from the well never reach a steady state since the formation is modeled as an infinite radial system.

In an actual reservoir, formations are neither uniform nor homogeneous, and layered formations may be a realistic approximation. In order to take into account effects of formation heterogeneity on wellbore heat transfer, the temperature distribution along the wellbore was calculated for hot-water injection into a formation consisting of two layers. The upper 500 meters is sandstone, and the lower 500 meters is clay. Problem parameters are given in Table 1. As shown in Figure 10, if only sandstone properties are used, well temperatures are underestimated since thermal diffusivity in sandstone is larger than that in clay. Figure 10 suggests that the assumption of constant formation properties introduces errors for non-homogeneous reservoirs.

Conclusions

An exact analytical solution for determining wellbore heat transfer has been developed. The analytical solution is applicable to field predictions and reservoir simulation studies of wellbore heat transmission in uniform and layered formations. Illustrative examples were given for temperature distributions along the wellbore and in the formation, and for heat transfer rates and cumulative heat loss (or gain) between wellbore and formation.

Analysis of the calculated results leads to the following conclusions:

1. Vertical heat conduction in the formation may be ignored for engineering applications.
2. The approximation of using a depth-independent heat conduction function $f(t_D)$ will give large errors for early times.
3. Effects of formation heterogeneity should be included for more accurate predictions of wellbore heat transmission in nonhomogeneous formations.

Acknowledgement

The authors are indebted to Paul Witherspoon and Jahan Noorishad for a careful review of the manuscript. This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Renewable Energy Technologies, Geothermal Technology Division, and by the Director, Office of Energy Research, Office of Basic Energy Sciences, Engineering & Geosciences Division, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

References

[1] Lessem, L. B., Greytok, F., Marotta, F., and Mcketta, J. J., Jr., ''A Method of Calculating the Distribution of Temperature in Flowing in Gas Wells,'' Trans., AIME (1957) 210, 169.

[2] Moss, J. T. and White, P. D., ''How to Calculate Temperature Profiles in a Water-Injection Well,'' Oil and Gas Jour. (March 9, 1959) 57, No. 11, 174.

[3] Squier, D. P., Smith, D. D. and Dougherty, E. L., ''Calculated Temperature Behavior of Hot-Water Injection Wells,'' J. Pet. Tech. (April 1962), 436-440.

[4] Ramey, H. J. Jr., ''Wellbore Heat Transmission,'' J. Pet. Tech. (April 1962) 427-435.

[5] Satter, A., ''Heat Losses During Flow of Steam Down a Wellbore,'' J. Pet. Tech. (July 1965) 845-851.

[6] Willhite, G. P., ''Over-All Heat Transfer Coefficients in Steam and Hot Water Injection Wells,'' J. Pet. Tech. (May 1967) 607-615.

[7] Durrant, A. J. and Thambynayagam, R. K. M., ''Wellbore Heat Transmission and Pressure Drop for Steam/Water Injection and Geothermal Production: A Simple Solution Technique,'' SPE Reservoir Eng. (March 1986), 148-157.

[8] Farouq Ali, S. M., ''A Comprehensive Wellbore Steam/Water Flow Model for Steam Injection and Geothermal Applications'' SPEJ (October 1981) 527-534.

[9] Wooley, G. R., ''Computing Downhole Temperatures in Circulation, Injection and Production Wells,'' J. Pet. Tech. (Sept. 1980) 1509-1522.

[10] Stehfest, H., ''Numerical Inversion of Laplace Transforms,'' Communications, ACM (January 1970) 13, No. 1, 47-49.

[11] Numerical Algorithms Group: NAG FORTRAN Library Manual, Mark 11, Vol. 1, Chapter D01, 1984.

[12] Barelli, A. and Palama, A., ''A New Method for Evaluating Formation Equilibrium Temperature in Holes During Drilling,'' Geothermics, (1981) 10, No. 2, 95-102.

[13] Carslaw, H. S. and Jaeger, J. C., ''Conduction of Heat in Solids,'' Second edition, Oxford University Press, Longon (1959).

[14] Hildebrand, F. B., ''Advanced Calculus for Applications,'' Second edition, Prentice-Hall, Inc., Englewood Cliffs (1976).

Nomenclature

$A_j(t_D)$	defined in (31)
$\bar{A}_j(s)$	defined in (B.2)
$B_j(z_D, t_D)$	defined in (32)
$\bar{B}_j(z_D, s)$	defined in (B.3)
c_j	formation specific heat of layers j [J/kg °C]
C_j	constant in (A.23, A.24)
$D_j(s)$	defined in (A.25)
$D_j{}^*(u)$	defined in (B.15)
$f(t_D)$	transient heat conduction function, defined in (35)
$f_j(t_D)$	defined in (B.7)
$\bar{f}_j(s)$	defined in (B.2)
g	gravity acceleration [m/sec^2]
$g_j(r_D, t_D)$	defined in (B.12)
$\bar{g}_j(r_D, s)$	defined in (B.6)
$G_j(z)$	geothermal gradient (°C/m)
H	depth to top of permeable interval (m)
h_c	heat transfer coefficient of casing (W/m^2°C)
h_t	heat transfer coefficient of tubing (W/m^2°C)
I_j	defined in (27) - (29) (j = 1, 2, 3)
J_o	zero-order Bessel function of the first kind
J_1	first-order Bessel function of the first kind
K_c	thermal conductivity of casing wall [W/m°C]
K_{cem}	thermal conductivity of cement [W/m°C]
K_j	thermal conductivity of layer j formation [W/m°C]
K_o	zero-order modified Bessel function of the second kind
K_1	first-order modified Bessel function of the second order

N	total formation layer number with different physical properties
q_j''	heat flux between tubing and sandface [W/m^2]
$\bar{q}_j''(z,s)$	defined in (24)
Q	injection rate [m^3/sec]
Q_c	accumulate heat rate between well and formation [J]
$\bar{Q}_c(s)$	defined in (25)
r	radius [m]
r_D	dimensionless radius (11)
r_w	outside radius of cement zone [m]
r_t	inside radius of tubing [m]
r_c	inside radius of casing [m]
$R_j(u)$	defined in (B.14)
s	Laplace transform variable
t	time [sec]
T	temperature [°C]
T_{air}	surface temperature [°C]
T_{cj}	constant temperature in G_j (z) [°C]
t_D	dimensionless time (12)
T_{inj}	surface injection fluid temperature [°C]
$T_{1j}(z,t)$	temperature along tubing [°C]
$T_{2j}(r,z,t)$	temperature in formation [°C]
U_j	overall heat transfer coefficient [W/ m^2°C]
V_m	mean flow speed inside tubing [m/sec]
Y_o	zero-order Bessel function of the second kind
Y_1	first-order Bessel function of the second kind
z	vertical coordinate [m]
z_D	dimensionless vertical coordinate (13)

z_j	depth of bottom of layer; ($j = 1, 2 \ldots N$, $z_o = 0$)
z_{Dj}	z_j/H, dimensionless of depth of layer, ($j = 1, 2, \ldots, N$)
β_j	dimensionless constant (A-3)
γ_j	geothermal gradient of layer j [°C/m]
δ_c	thickness of casing wall [m]
$\delta_j(z_D)$	defined in (A.4)
δ_t	thickness of tubing wall [m]
$\theta_j(z_D, t_D)$	dimensionless temperature functions of wellbore
$\xi_j(z_D)$	dimensionless function (A.5)
ρ	density [kg/m^3]
σ_j	dimensionless constant (A.6)
$\phi_j(r_D, z_D, t_D)$	dimensionless temperature function of formation

Subscripts

c	casing
cem	cement
D	dimensionless
j	formation layer index ($j = 1, 2, \ldots, N$)
t	tubing
1	in tubing
2	in formation

Appendix A. Analytical Solution in Laplace Space

In terms of the dimensionless variables defined in Eq. (11) - (15), the problem becomes

$$\frac{\partial \theta_j}{\partial t_D} + \frac{\partial \theta_j}{\partial z_D} + \beta_j \left(\theta_j - \phi_j\Big|_{r_D=1}\right) + \delta_j(z_D) = 0 \tag{A.1}$$

$$\frac{\partial^2 \phi_j}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial \phi_j}{\partial r_D} = \sigma_j \frac{\partial \phi_j}{\partial t_D} \tag{A.2}$$

where j = 1, 2, . . . N; and

$$\beta_j = \frac{2U_jH}{\rho_t\, c_t\, r_t\, V_m} \tag{A.3}$$

$$\delta_j(z_D) = \begin{cases} \xi_j(z_D) & \text{for liquidflow} \\ \xi_j(z_D) \pm \dfrac{Hg}{(T_{inj} - T_{air})\, c_t} & \text{for gas flow} \end{cases} \tag{A.4}$$

$$\xi_j(z_D) = \frac{HG_j'(z)}{T_{inj} - T_{air}} \tag{A.5}$$

$$\sigma_j = \frac{\rho_t\, c_j\, V_m\, r_w^2}{H\, K_j} \tag{A.6}$$

with initial conditions:

$$\theta_j\,(z_D,\, t_D = 0) = 0 \tag{A.7}$$

$$\phi_j\,(r_D,\, z_D,\, t_D = 0) = 0 \tag{A.8}$$

and boundary conditions:

$$\theta_1\,(z_D = 0,\, t_D) = 1 \tag{A.9}$$

$$\theta_j\,(z_{Dj-1},\, t_D) = \theta_{j-1}\,(z_{Dj-1},\, t_D) \tag{A.10}$$

$$(j = 2, 3, \ldots N)$$

$$\frac{\partial \phi_j}{\partial r_D}\Bigg|_{r_D=1} = \omega_j \left[\phi_j\Big|_{r_D=1} - \theta_j\right] \tag{A.11}$$

$$\lim_{r_D \to \infty} \phi_j \left(r_D, z_D, t_D\right) = 0 \tag{A.12}$$

In (A.11)

$$\omega_j = \frac{U_j \, r_t}{K_j} \tag{A.13}$$

The Laplace transforms of $\theta_j(z_D, t_D)$ and $\phi_j \ (r_D, z_D, t_D)$ are defined as follows:[13]

$$\overline{\theta}_j(z_D, s) = \int_0^\infty \theta(z_D, t_D) e^{-t_D s} \, dt_D \tag{A.14}$$

$$(j = 1, 2, \ldots N)$$

and

$$\overline{\phi}_j(r_{d,} \; z_D, s) = \int_0^\infty \phi_j(r_D, z_D, t_D) e^{-t_D s} \, dt_D \tag{A.15}$$

$$(j = 1, 2, \ldots N)$$

Application of the Laplace transformation to the partial differential equations (A.1), (A.2) and the boundary conditions (A.9) - (A.12) with incorporating the initial conditions (A.7), (A.8) yields

$$\frac{d\overline{\theta}_j}{dz_D} + (s + \beta_j) \, \overline{\theta}_j - \beta_j \, \overline{\phi}_j \Big|_{r_D = 1} + \frac{\delta_j(z_D)}{s} = 0 \tag{A.16}$$

$$\frac{d^2\overline{\phi}_j}{dr_D^2} + \frac{1}{r_D} \frac{d\overline{\phi}_j}{dr_D} - \sigma_j \, s \, \overline{\phi}_j = 0 \tag{A.17}$$

$$\frac{d\overline{\phi}_j}{dr_D}\Bigg|_{r_D=1} = \omega_j \left[\overline{\phi}_j \Big|_{r_D=1} - \overline{\theta}_j \right] \tag{A.18}$$

$$\overline{\theta}_1 \; (z_D = 0, s) = \frac{1}{s} \tag{A.19}$$

$$\overline{\theta}_j \; (z_{Dj-1}, s) = \overline{\theta}_{j-1} \; (z_{Dj-1}, s) \tag{A.20}$$

where $j = 1, 2, \ldots N$. The solutions of (A.16), (A.17) in Laplace space, satisfying the boundary condition (A.18) - (A.20), are:

$$\bar{\theta}_j(z_D, s) = C_j(s) \exp \left\{ - \left[s + \beta_j - D_j(s) \right] z_D \right\} + Y_j(z_D, s) \tag{A.21}$$

$$\bar{\phi}_j(r_D, z_D, s) = \frac{\omega_j \bar{\theta}_j(z_D, s) K_o(\sqrt{\sigma_j s}\, r_D)}{\omega_j K_o(\sqrt{\sigma_j s}) + \sqrt{\sigma_j s}\, K_1(\sqrt{\sigma_j s})} \tag{A.22}$$

where $Y_j(z_D, s)$ are the particular solutions of (A.16), which depend on the initial temperature profile.

$$C_1 = \frac{1}{s} - Y_1(z_D = 0, s) \tag{A.23}$$

$$C_j = \left[\bar{\theta}_{j-1}(z_{Dj-1}, s) - Y_j(z_{Dj-1}, s) \right] \exp \left\{ \left[s + \beta_j - D_j(s) \right] z_{Dj-1} \right\} \tag{A.24}$$

$$(j = 2, 3, \ldots N)$$

and

$$D_j(s) = \frac{\beta_j \omega_j K_o(\sqrt{\sigma_j s})}{\omega_j K_o(\sqrt{\sigma_j s}) + \sqrt{\sigma_j s}\, K_1(\sqrt{\sigma_j s})} \tag{A.25}$$

Heat flux into (or from) the formation in layer j is defined as

$$2\pi r_t\, q_j''(z, t) = -2\pi r_w K_j \left. \frac{\partial T_{2j}}{\partial r} \right|_{r=r_w} \tag{A.26}$$

so that

$$q_j''(z, t) = \frac{-K_j (T_{inj} - T_{air})}{r_t} \left. \frac{\partial \phi_j}{\partial r_D} \right|_{r_D=1}$$

$$= -U_j (T_{inj} - T_{air}) \left[\phi_j(1, z_D, t_D) - \theta_j(z_D, t_D) \right] \tag{A.27}$$

Total cumulative heat transfer is

$$Q_c(t) = \int_0^t \left[\sum_{j=1}^{N} \int_{z_{j-1}}^{z_j} 2\pi r_t\, q_j''(z, \tau)\, dz \right] d\tau \tag{A.28}$$

For linear vertical initial temperature distributions in each layer of the formation, we can obtain the explicit form of the particular solution $Y_j(z, s)$, in Eq. (23). Then the expressions for heat flux and total cumulative heat transfer in the Laplace transformed space can be derived as given in Eq. (24), (25), respectively.

Appendix B. Analytical Solutions in Real Space

For the case of linear initial temperature distributions in each layer of the formation, Eq. (A.21) can be written as:

$$\bar{\theta}_j(z_D, s) = \bar{A}_j(s) + \left[\bar{\theta}_{j-1}(z_{D_{j-1}}, s) - \bar{A}_j(s)\right] \bar{B}_j(z_D, s) \tag{B.1}$$

$$(j = 1, 2, \ldots N)\ (z_{Dj-1}, \leq z_D \leq z_{Dj})$$

where

$$\bar{A}_j(s) = -\frac{\delta_j}{s(s + \beta_j - D_j(s))} = \frac{1}{s}\bar{f}_j(s) \quad (\bar{f}_j(s) = s\bar{A}_j(s)) \tag{B.2}$$

$$\bar{B}_j(z_D, s) = \exp\left[-\beta_j(z_D - z_{Dj-1})\right] \cdot \exp\left[-s\,(z_D - z_{Dj-1})\right] \cdot \exp\left[D_j(s)(z_D - z_{Dj-1})\right] \tag{B.3}$$

and

$$\bar{\theta}_o\,(z_{D0}, s) = \frac{1}{s} \tag{B.4}$$

the dimensionless temperature function in the formation,

$$\bar{\phi}_j\,(r_D, z_D, t_D) = \bar{\theta}_j\,(z_D, s)\,\bar{g}_j\,(r_D, s) \tag{B.5}$$

$$(j = 1, 2, \ldots N)$$

where

$$\bar{g}_j\,(r_D, s) = \frac{\omega_j\, K_o\,(\sqrt{\sigma_j s}\, r_D)}{\omega_j\, K_o\,(\sqrt{\sigma_j s}) + \sqrt{\sigma_j s}\, K_1\,(\sqrt{\sigma_j s})} \tag{B.6}$$

Since the functions $\bar{A}_j(s)$, $\bar{B}_j(z_D, s)$ and $\bar{g}_j(r_D, s)$ have a branch point at the origin, we have to use the inversion theorem for Laplace transformations by evaluating the contour integral[13]. The following inversion can be proven after some algebraic operations[14].

$$f_j(t_D) = L^{-1}\left[\bar{f}_j(s)\right] = \frac{\delta_j}{2\pi i}\int_0^{\infty} e^{-t_D\lambda}\left[\bar{f}_j(\lambda e^{-i\pi}) - \bar{f}_j(\lambda\, e^{i\pi})\right] d\lambda$$

$$= -\frac{4\delta_j}{\pi^2\sigma_j}\int_0^\infty e^{-\frac{u^2}{\sigma_j}t_D}\frac{uD_j^*(u)}{\left[D^*(u)\left[\beta_j - \frac{u^2}{\sigma_j}\right]R_j(u)\right]^2 + \frac{4}{\pi^2}}du \qquad (B.7)$$

$$L^{-1}\left\{\exp\left[D_j(s)\,(z_D - z_{Dj-1})\right]\right\} = \frac{1}{2\pi i}\int_0^\infty e^{-t_D\lambda}\left\{\exp\left[D_j(\lambda e^{-i\pi})(z_D - z_{Dj-1})\right]\right.$$

$$\left. - \exp\left[D_j(\lambda e^{i\pi})(z_D - z_{Dj-1})\right]\right\}d\lambda = \frac{2}{\lambda\sigma_j}\int_0^\infty u\sin\left[\frac{2(z_D - z_{Dj-1})}{\pi D_j^*(u)}\right]$$

$$\cdot\exp\left[(z_D - z_{Dj-1})\,R_j(u)/D_j^*(u) - \frac{u^2}{\sigma_j}t_D\right]du \qquad (B.8)$$

then

$$A_j(t_D) = L^{-1}\left\{\frac{1}{s}\bar{f}_j(s)\right\} = \int_0^{t_D} f_j\,(\tau)d\tau = \frac{4\delta_j}{\pi^2}\int_0^\infty \frac{D^*(u)}{u}$$

$$\cdot\frac{\left[e^{-\frac{u^2}{\sigma_j}t_D} - 1\right]}{\left[D_j^*(u)\left[\beta_j - \frac{u^2}{\sigma_j}\right] - R_j(u)\right]^2 + \frac{4}{\pi^2}} \qquad (B.9)$$

$$B_j(z_D, t_D) = L^{-1}\left\{\bar{B}_j(z_D, s)\right\} = \exp\left[-\beta_j(z_D - z_{Dj-1})\right]L^{-1}\left\{\exp\left[-s\,(z_D - z_{Dj-1})\right]\right.$$

$$\left.\cdot\exp\left[D_j,(s)(z_D - z_{Dj-1})\right]\right\}$$

$$= \begin{cases} 0 \qquad (t_D \le z_D - z_{Dj-1}) \\ \frac{2}{\pi\sigma_j}\exp\left[-\beta_j\,(z_D - z_{Dj-1})\right]\int_0^\infty u\sin\left[\frac{2(z_D - z_{Dj-1})}{\pi D_j^*(u)}\right] \\ \cdot\exp\left[(z_D - z_{Dj-1})\,R_j\,(u)/D_j^*(u) - \frac{u^2}{\sigma_j}(t_D - z_D + z_{Dj-1})\right]du \\ \qquad (t_D > z_D - z_{Dj-1}) \end{cases} \qquad (B.10)$$

Taking the inverse Laplace transform of (B.1) and using (B.9), (B.10) and the convolution property of the Laplace transform, we have

$$\theta_j(z_D, t_D) = L^{-1}\left\{\bar{\theta}_j(z_D, s)\right\}$$

$$= A_j(t_D) + \int_0^{t_D} \left[\theta_{j-1}(z_{Dj-1}, \tau) - A_j(\tau)\right] B_j\,(z_D, t_D - \tau)\,d\tau \qquad \text{(B.11)}$$

and

$$g_j(r_D, t_D) = L^{-1}\left\{\bar{g}_j(r_D, s)\right\} = \frac{1}{2\pi i}\int_0^{\infty} e^{-t_D\lambda}\left[\bar{g}_j(r_D, \lambda e^{-i\pi}) - \bar{g}_j\,(r_D, \lambda e^{i\pi})\right] d\lambda$$

$$= \frac{2}{\pi\sigma_j\beta_j}\int_0^{\infty} \frac{u e^{-\frac{u^2}{\sigma_j}t_D}}{D^*(u)}$$

$$\cdot \left\{Y_0\,(ur_D)\left[\omega_j J_0(u) + uJ_1(u)\right] - J_0\,(ur_D)\left[uY_1(u) + \omega_j Y_0(u)\right]\right\} du \qquad \text{(B.12).}$$

Then,

$$\phi_j\,(r_D, z_D, t_D) = \int_0^{t_D} \theta_j(z_D, \tau)\, g_j(r_D, t_D - \tau)\, d\tau \qquad \text{(B.13)}$$

In the above solutions

$$R_j(u) = J_0\,(u)\left[\omega_j J_0(u) + uJ_1(u)\right] + Y_0(u)\left[\omega_j Y_0(u) + uY_1(u)\right] \qquad \text{(B.14)}$$

$$D_j^*(u) = \left\{\left[\omega_j Y_o(u) + uY_1(u)\right]^2 + \left[\omega_j J_o(u) + uJ_1(u)\right]^2\right\}/\beta_j\omega_j \qquad \text{(B.15)}$$

For layer 1; the solution (B.11) is simplified as Eq. (26).

Table 1. Calculation Data

$\gamma = .03°C/m$	$T_{air} = 20°C$
$T_{inj} = 100°C$	$H = 1000$ m
$Q = 100\ m^3/day$	$r_w = 0.08$ m (no tubing) (6.33 in ID)
$\rho_w = 958\ kg/m^3$	$c_w = 4196$ J/kg · °C
$U = 978\ W/m^2°C$	
Sandstone	
$\rho = 2200\ kg/m^3$	K = 2.8 W/m °C
c = 740 J/kg °C	
Clay	
$\rho = 1500\ kg/m^3$	K = 1.4 W/m °C
c = 800 J/kg °C	

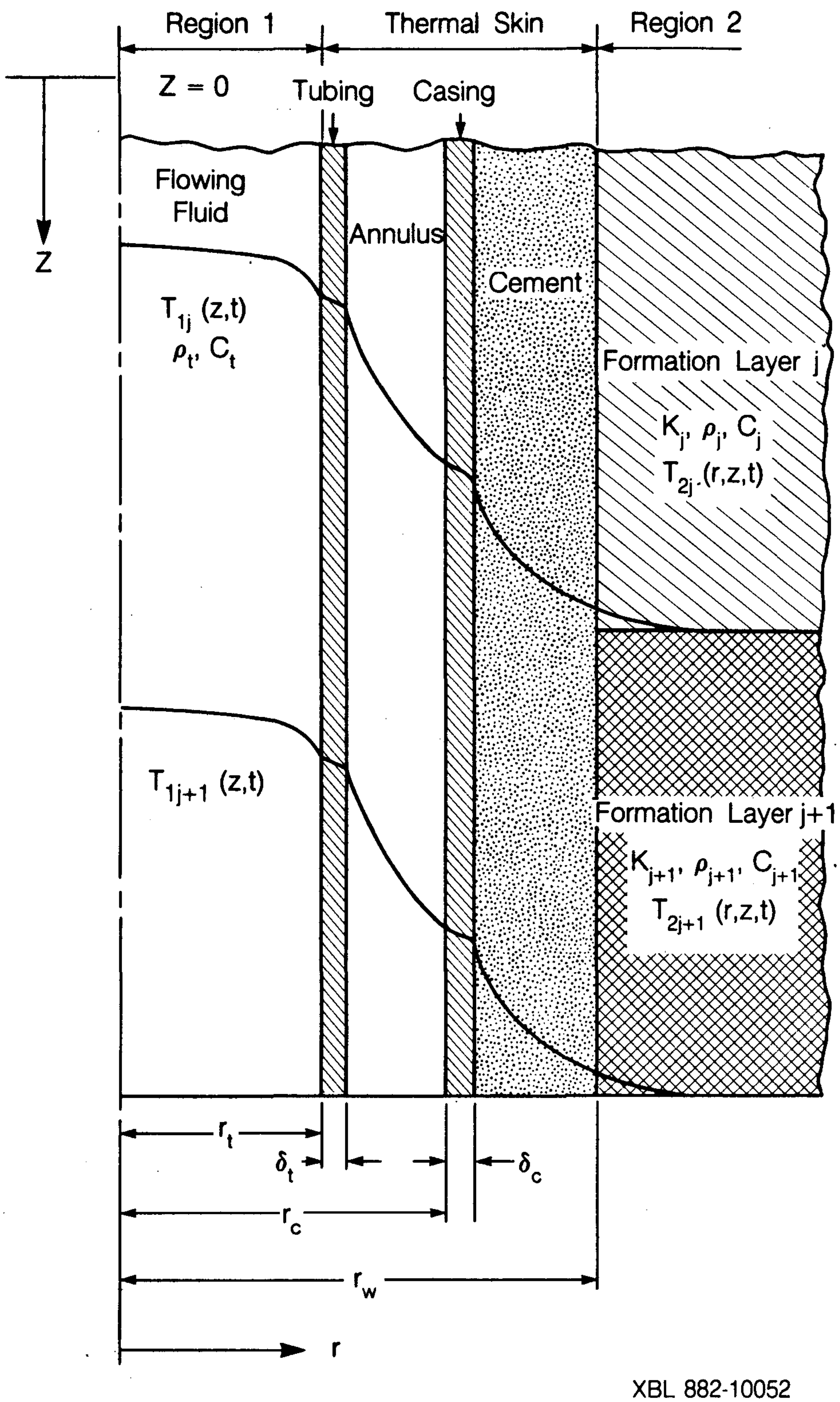


Figure 1. Schematic of wellbore and formation system.

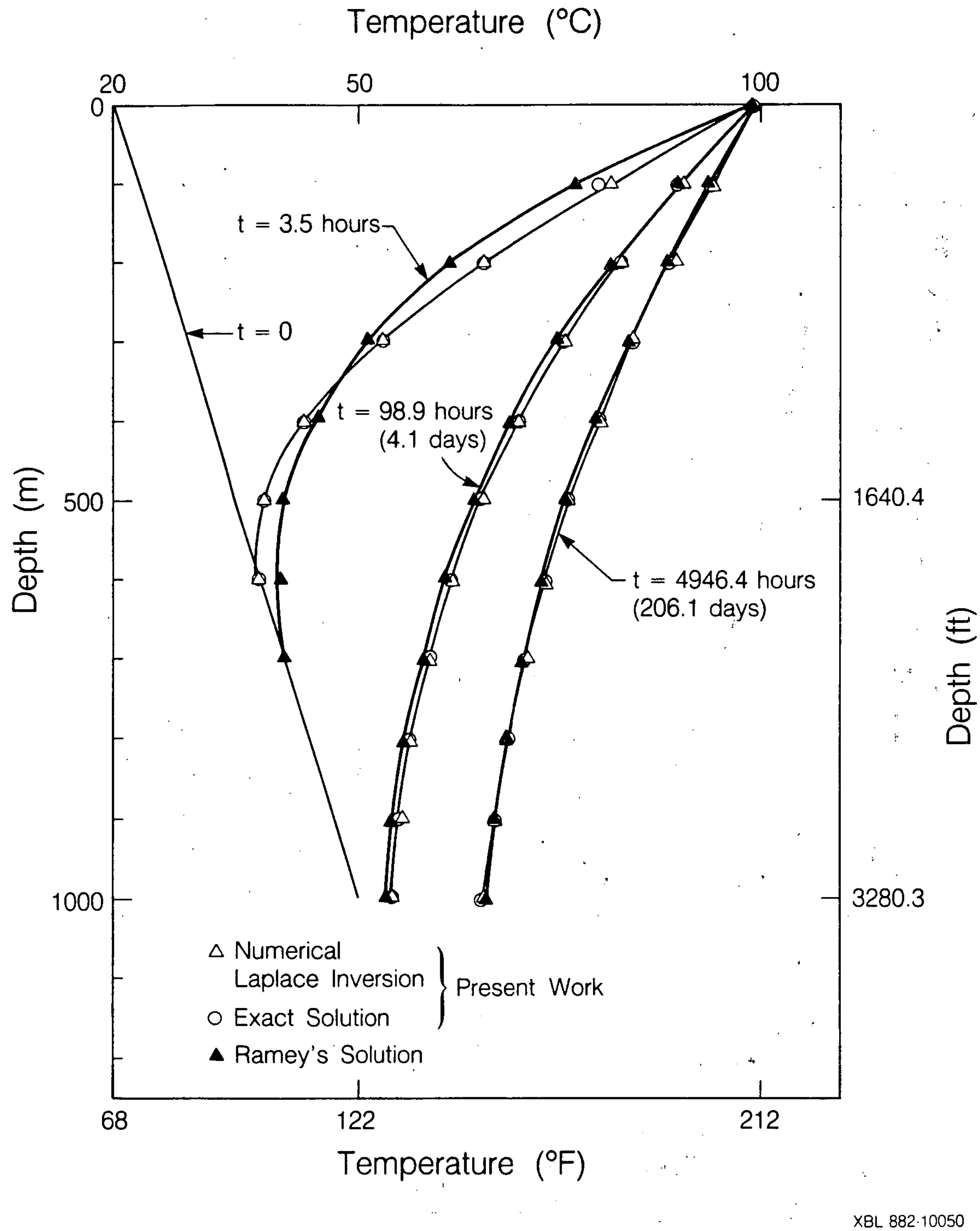


Figure 2. Comparison of numerical inversion of Laplace transformation with exact solution and Ramey's solution.

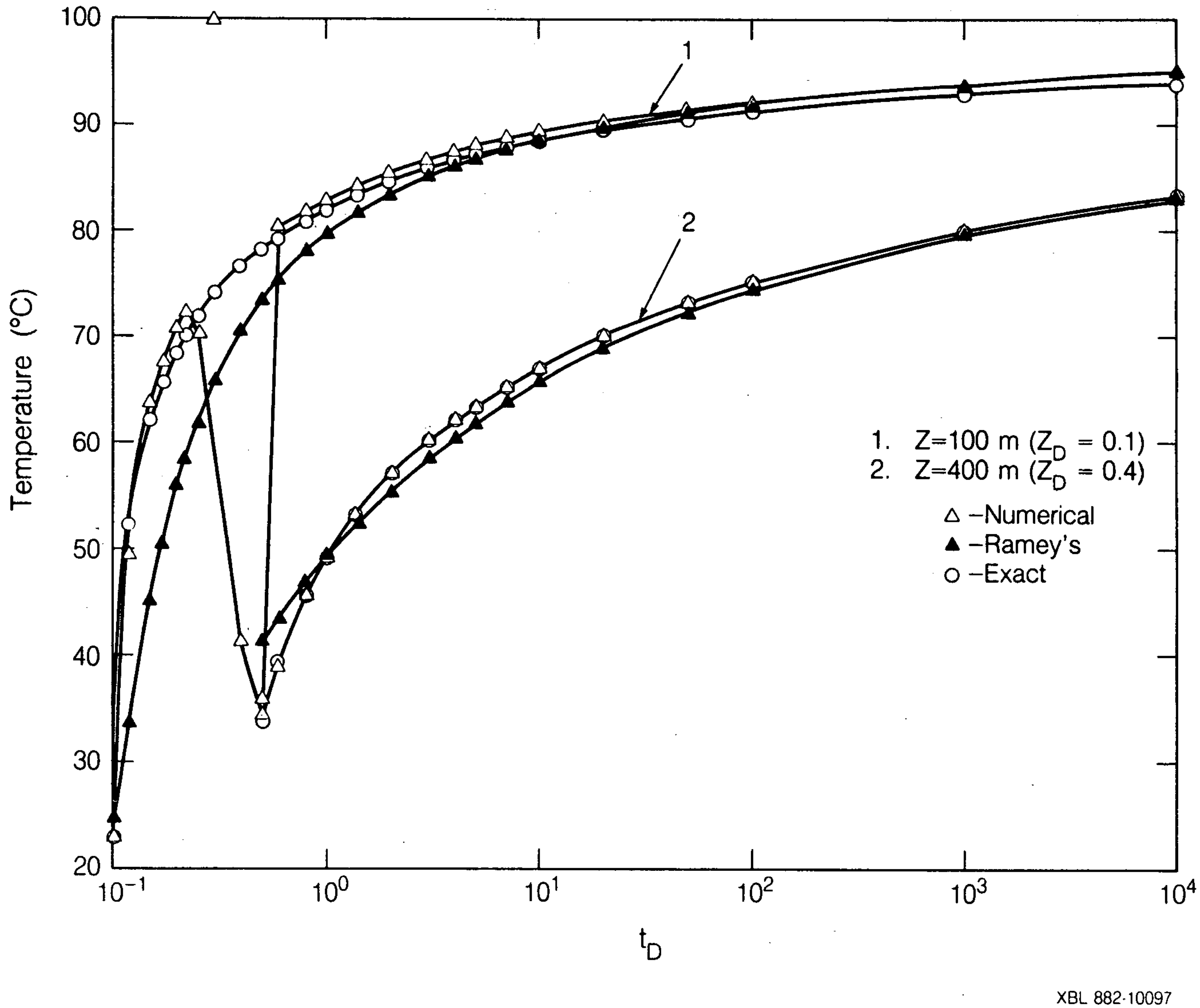


Figure 3. Check on numerical inversion of Laplace transformation.

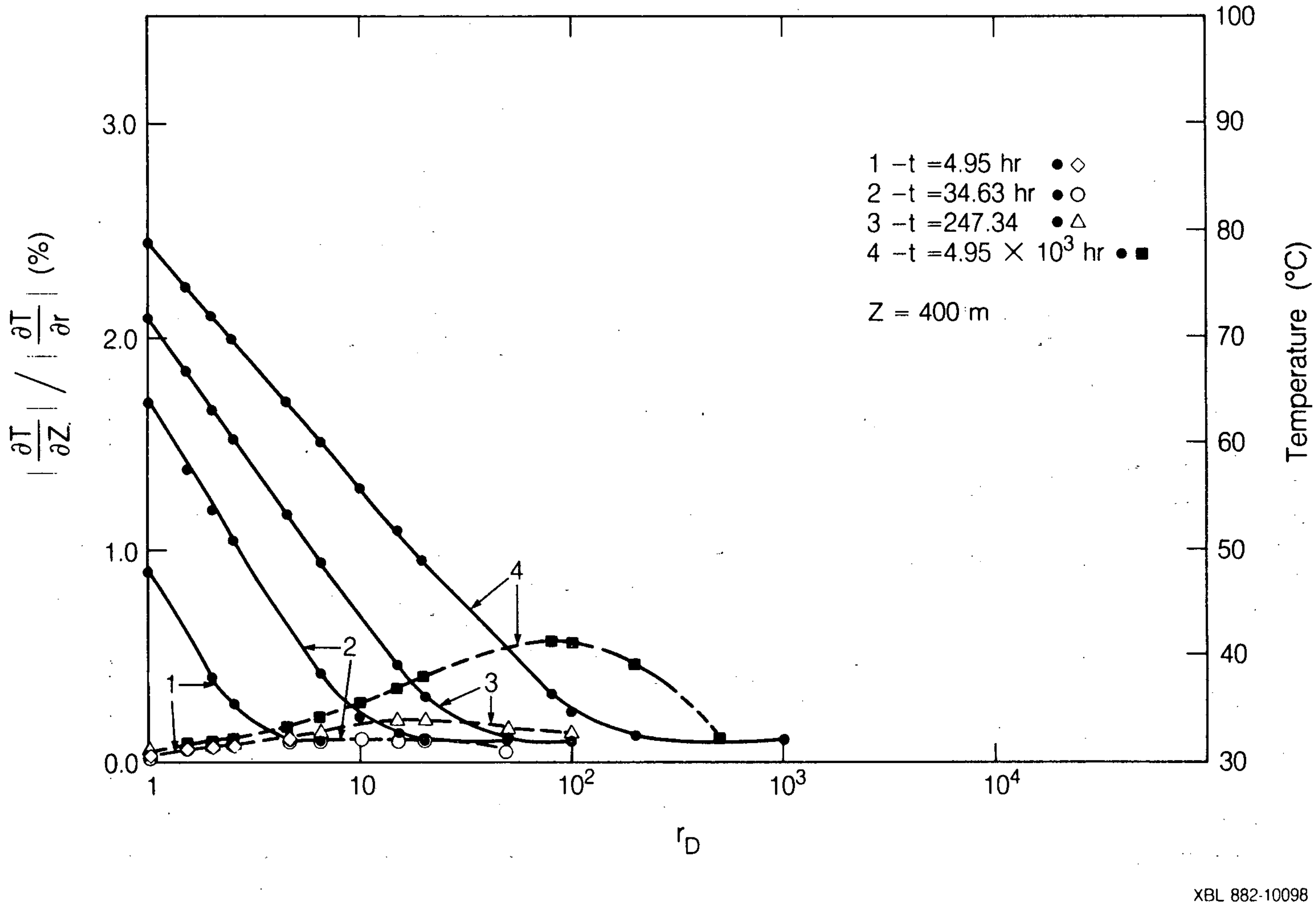
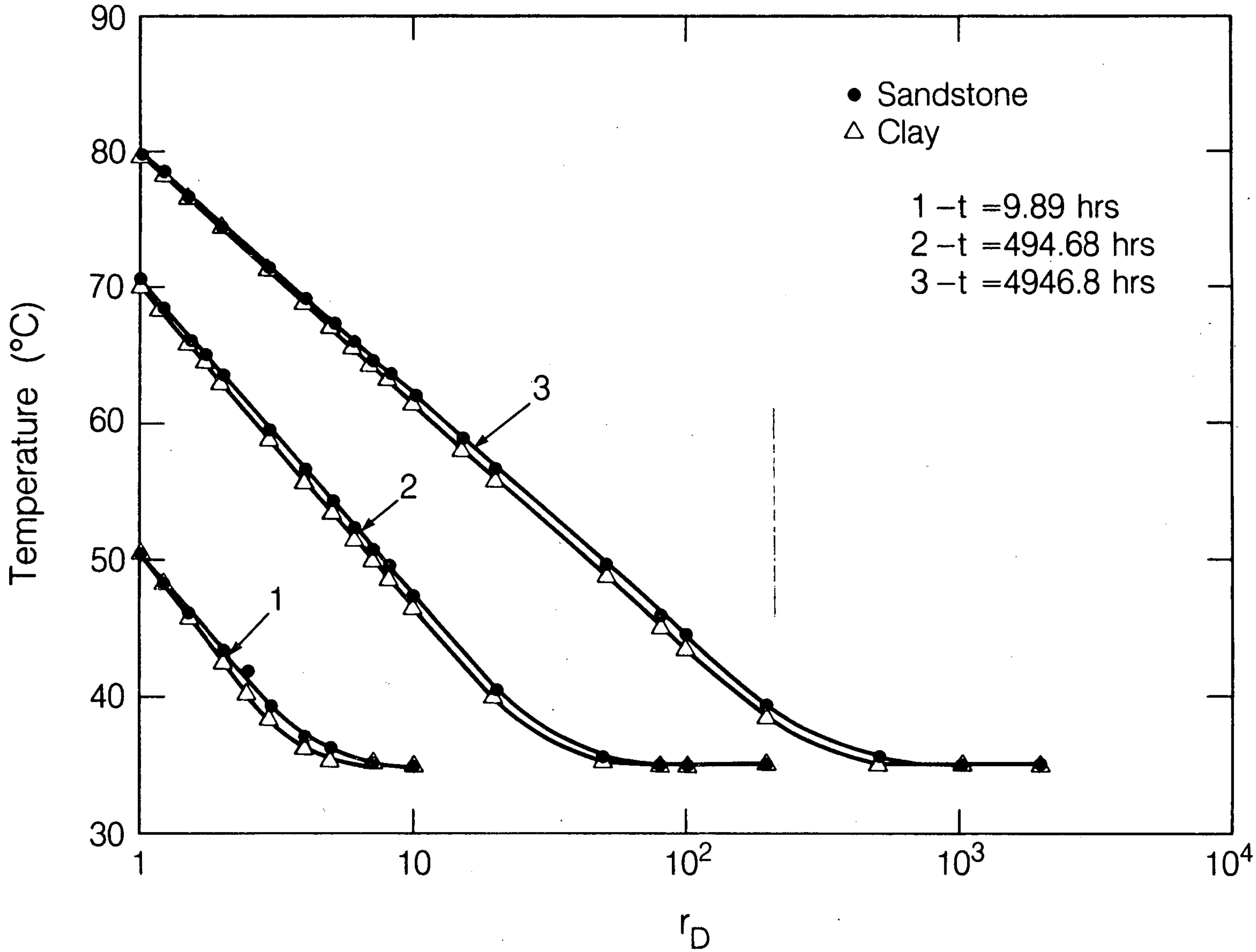


Figure 4. Ratio of vertical and horizontal temperature gradients and temperature distribution in formation.



XBL 882-10202

Figure 5. Discontinuity of vertical temperature on sandstone-clay interface.

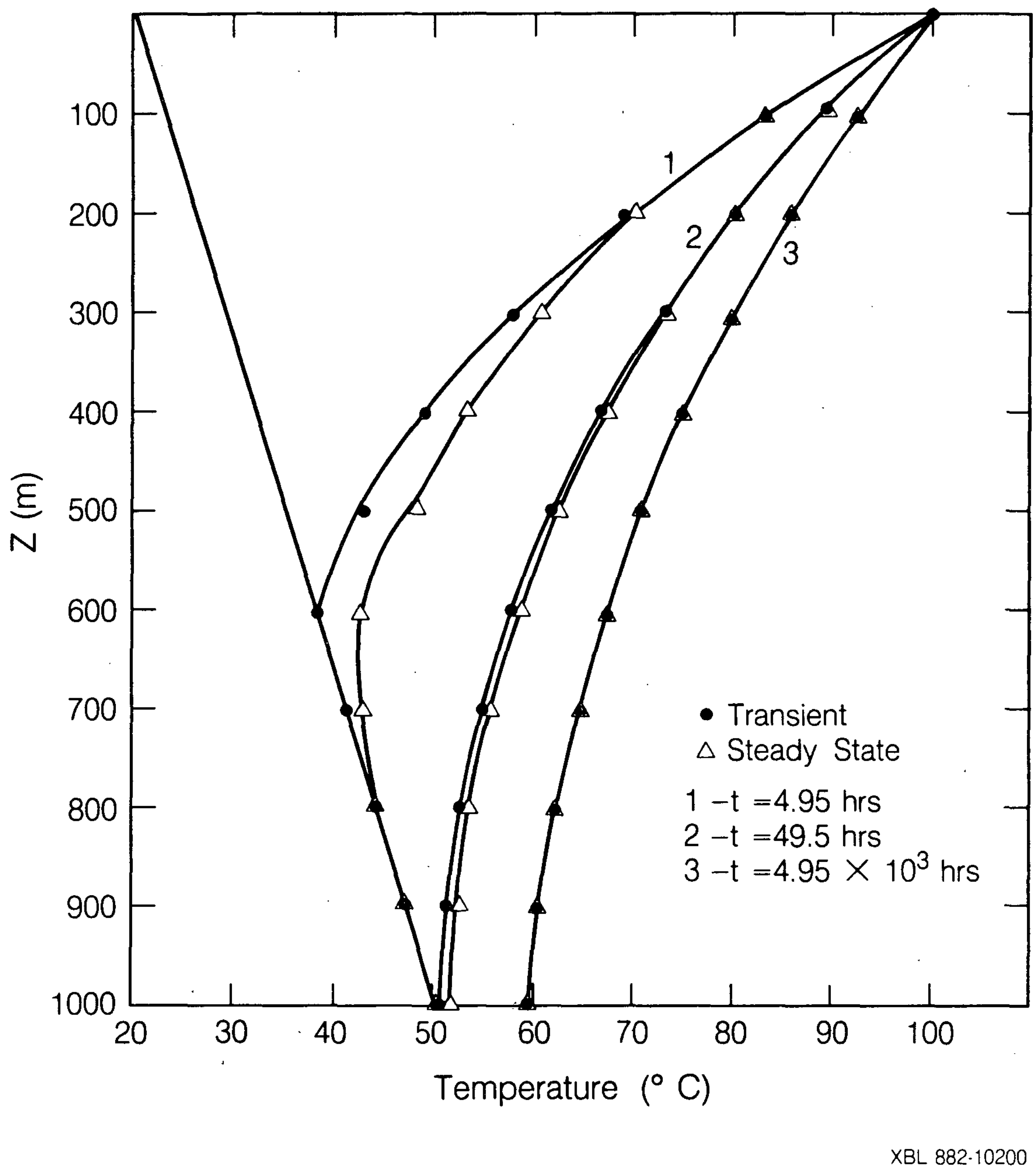
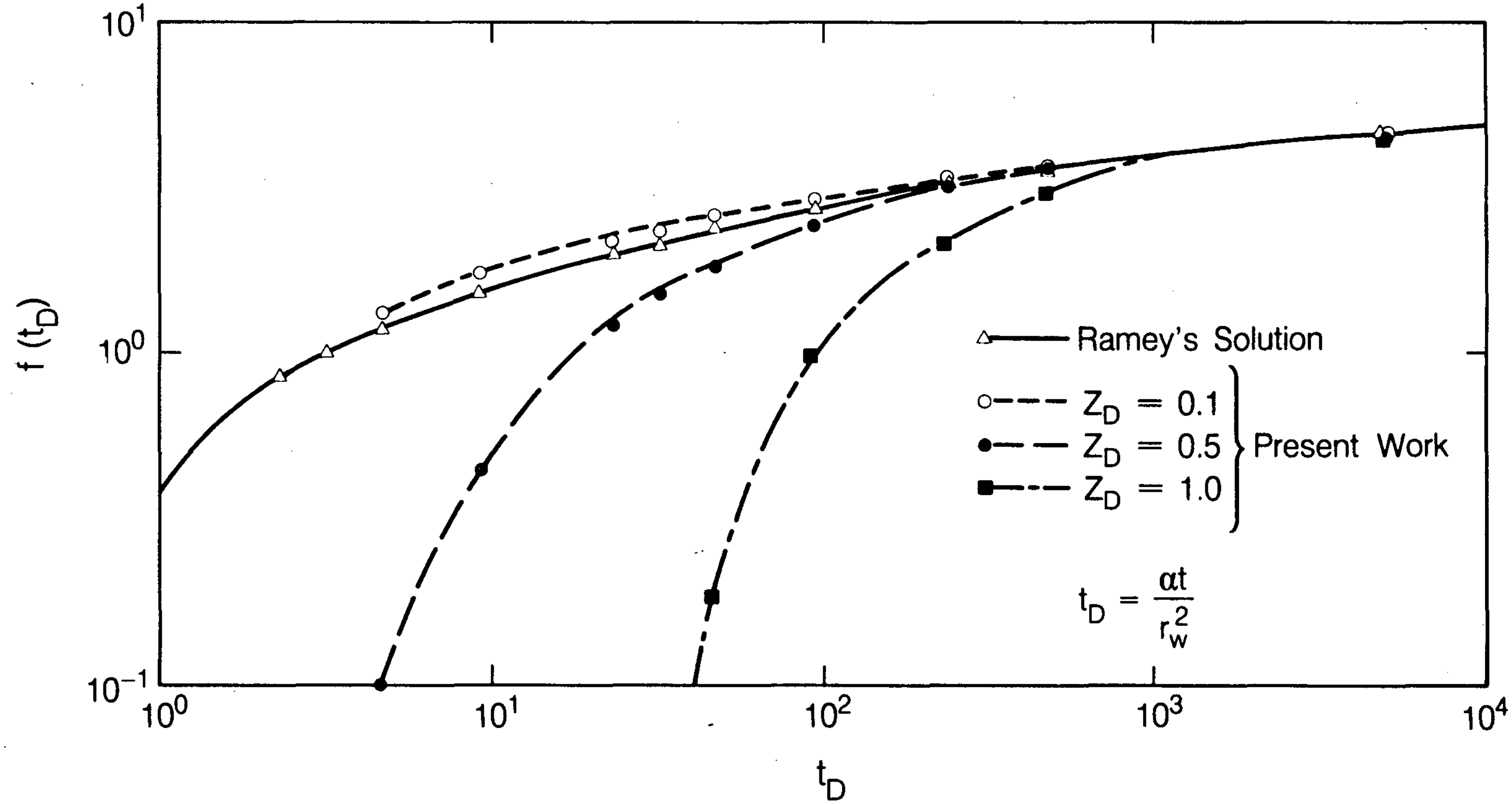
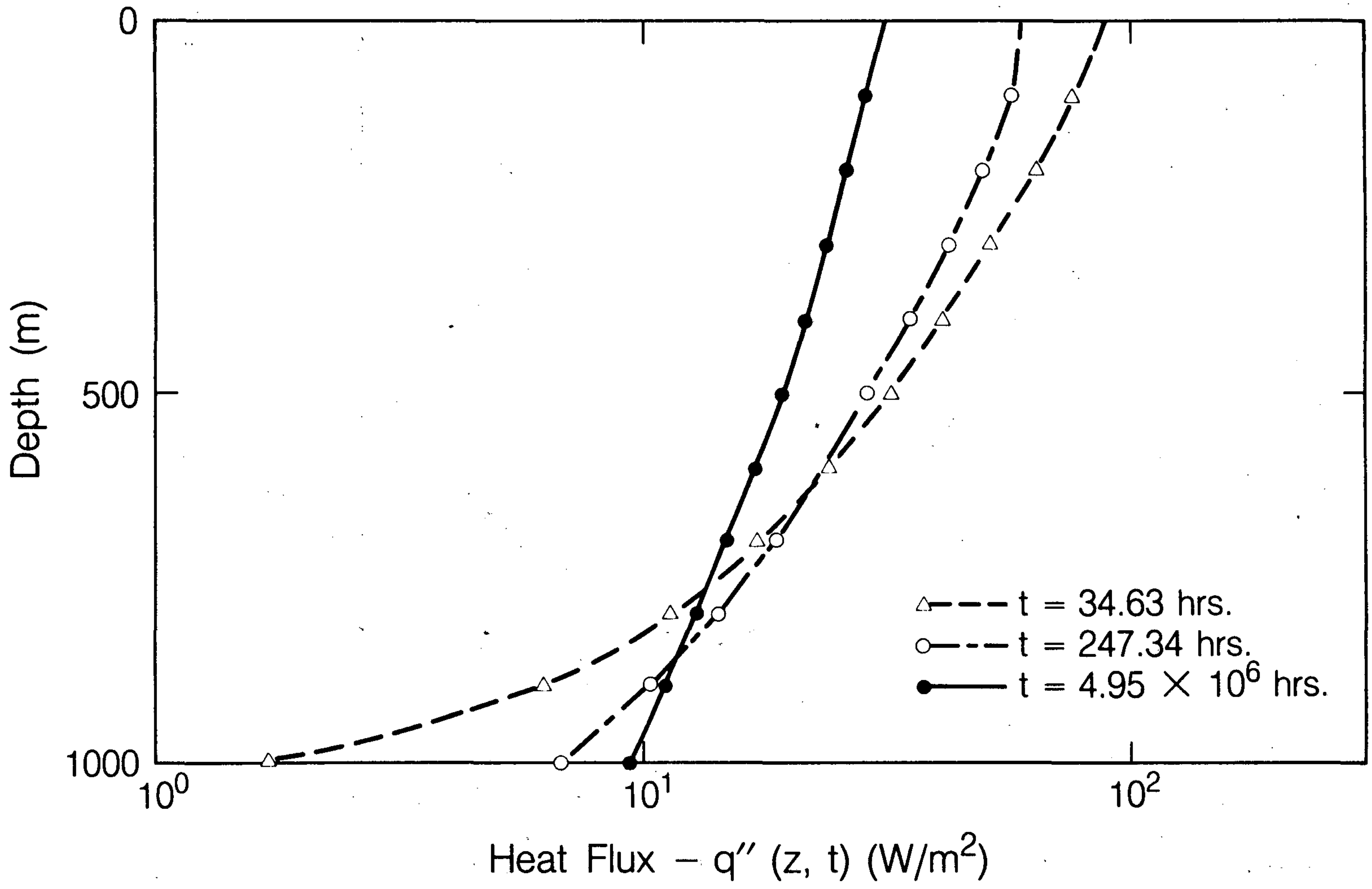


Figure 6. Comparison of steady-state and transient heat transfer in wellbore.



XBL 882-10049

Figure 7. Effect of depth on $f(t_D)$.



XBL 882-10201

Figure 8. Heat flux into formation.

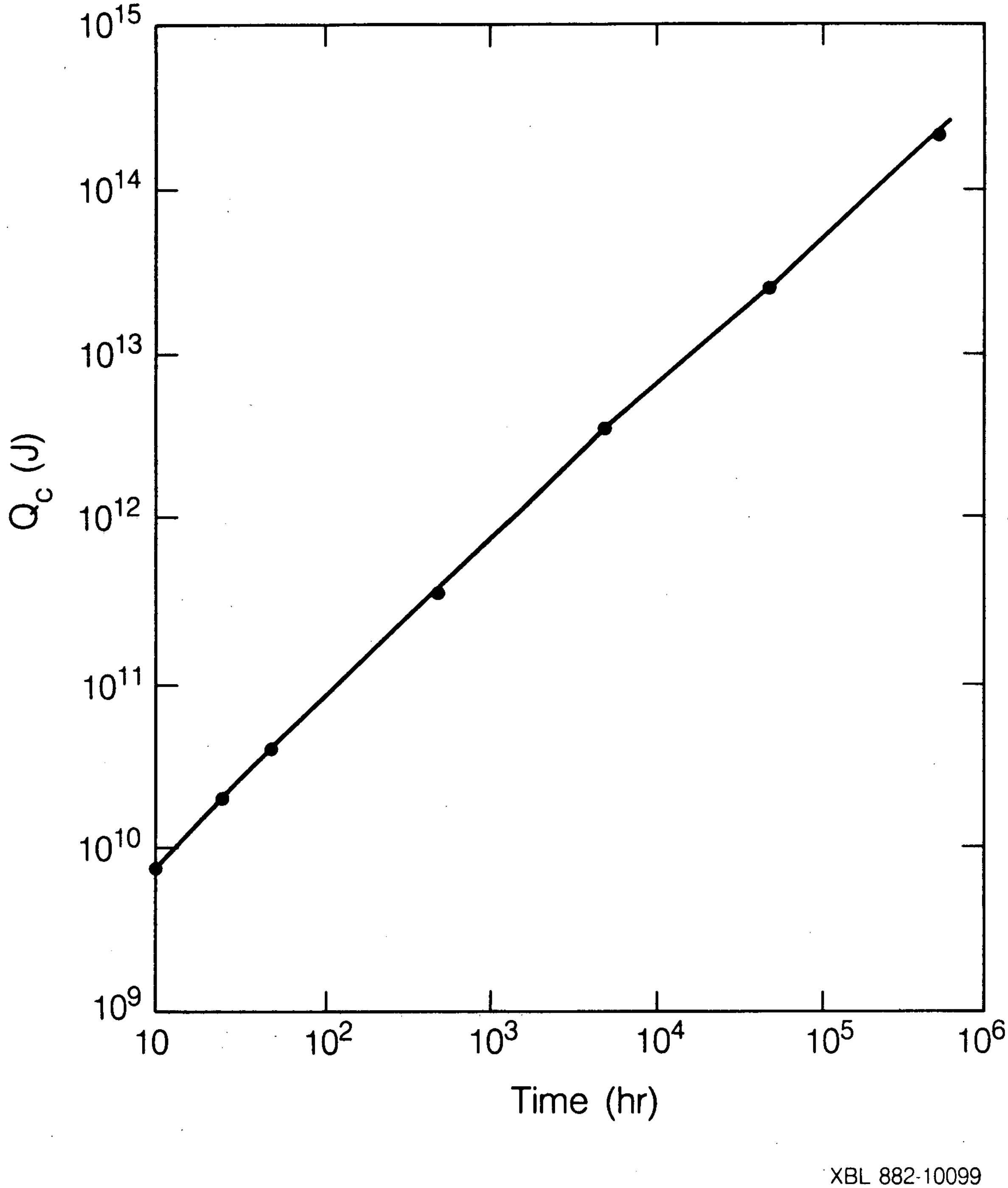


Figure 9. Cumulative heat transfer into formation.

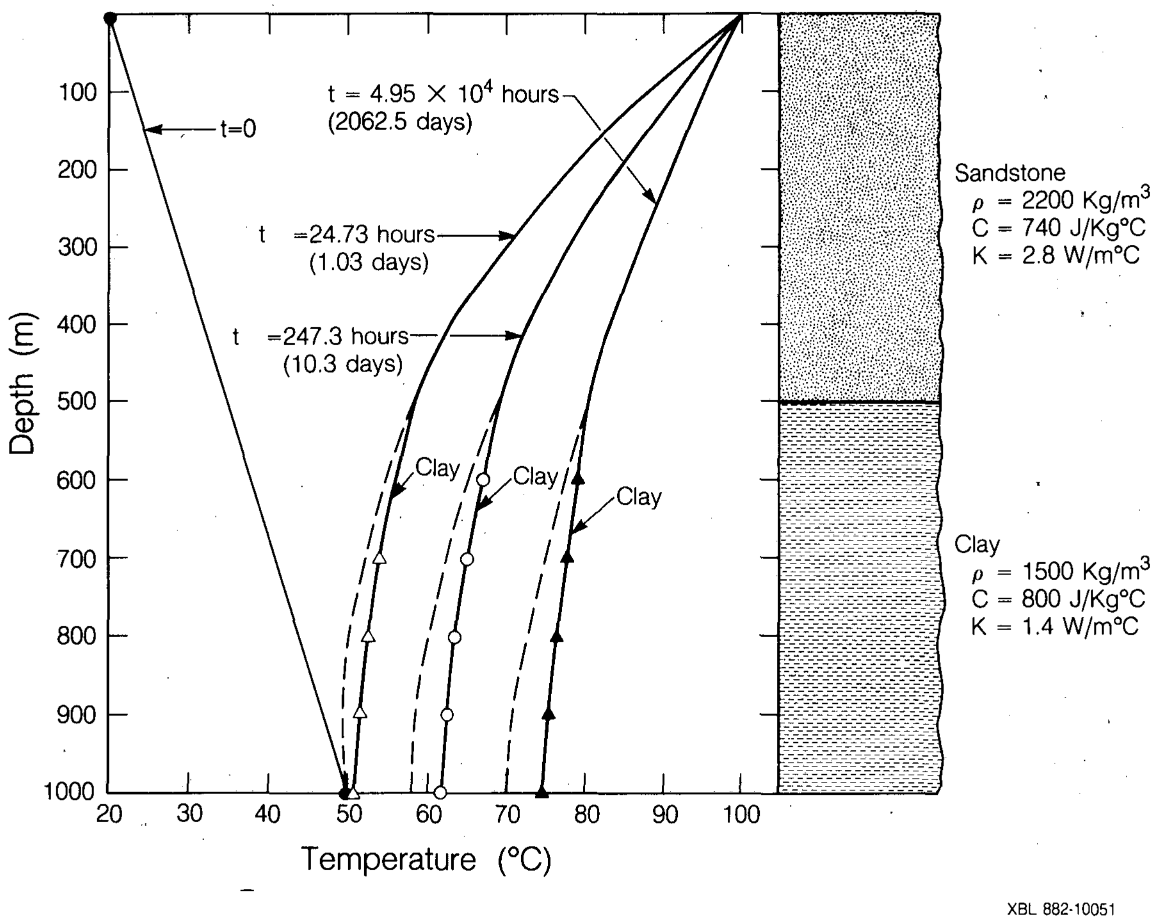


Figure 10. Wellbore temperature distribution of sandstone-clay 2-layer formation.

LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720