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An analytical study of linear and non-linear convection in Boussinesq–Stokes suspensions

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Abstract

The Rayleigh–Benard situation in Boussinesq–Stokes suspensions is investigated using both linear and non-linear stability analyses. The linear and non-linear analyses are based on a normal mode solution and minimal representation of double Fourier series, respectively. The effect of suspended particles on convection is delineated against the background of the results of the clean fluid. The realm of non-linear convection warrants the quantification of heat transfer and this has been achieved on the Rayleigh–Nusselt plane. Possibility of aperiodic convection is discussed.

Keywords: Rayleigh–Benard convection; Double Fourier series; Suspensions; Boussinesq approximation

1. Introduction

Many non-isothermal problems of practical interest involving fluids as a working medium have attracted engineers, physicists and mathematicians alike. These problems pose either challenging non-linearity in the governing equations, in addition to coupling, or complex boundary conditions. In the presence of micron-sized suspended particles these problems endear themselves all the more to researchers who yearn for challenges. Depending upon the concentration and size of these suspended particles the available literature offers the following choice of mathematical models for the suspensions:

(i) Saffman's dusty gas model (see [1]).
(ii) Non-Newtonian fluid model
  (a) Fluids with stress non-linearly proportional to the symmetric part of the velocity gradient (see [2,3]).
  (b) Fluids with internal angular momentum but the stress being linearly proportional to rate of strain (see [4–6]).

In the first model the suspended particles (dust) are visible and are in low concentrations. In the second model the almost invisible tiny-sized suspended particles which are in high concentration identify themselves with the carrier fluid almost to the point of camouflage. Most of these suspensions are naturally available (see [3]) and some important technologically important ones are synthesized (see [7]). We

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Nomenclature

A_{mn} (t)	amplitudes of streamlines perturbation
B_{mn} (t)	amplitudes of the thermal perturbation
C	couple stress parameter $\left(=\dfrac{\mu'}{\mu d^2}\right)$
d	depth of the fluid layer
$\vec{g}$	acceleration due to gravity
K^2	$(=\pi^2(\alpha^2+1))$
Nu	Nusselt number
$p,\ p_b,\ p'$	actual, basic and perturbation pressures
Pr	Prandtl number $\left(=\dfrac{\gamma}{\chi}\right)$
$\vec{q}, \vec{q}_b, \vec{q}'$	actual, basic and perturbation velocities
$R,\ R_s,\ R_o,\ R_{sc}$	actual, stationary, oscillatory and critical stationary Rayleigh number ($R=\beta g \Delta T d^3/\gamma\chi$)
$t,\ t^*$	actual and non-dimensional time variables
$T,\ T_b,\ T,\ T_0,\ T^*$	actual, basic, perturbation, reference and non-dimensional temperatures
$(x,z)(x^*,\ z^*)$	actual and non-dimensional space variables
$\alpha,\ \alpha_c$	actual and critical wave number
β	co-efficient of thermal expansion
γ	kinematic viscosity $\left(=\dfrac{\mu}{\rho}\right)$
∇	$\hat{i}\dfrac{\partial}{\partial x}+\hat{k}\dfrac{\partial}{\partial z}$ (vector differential operator)
ΔT	temperature difference between the two horizontal plates
η	$(=1+CK^2)$
θ_0	non-dimensional amplitude of temperature perturbation
μ	dynamic viscosity
μ'	couple stress viscosity
$\rho,\ \rho_b,\ \rho',\ \rho_0$	actual, basic, perturbation and reference densities
σ	growth rate
χ	thermal diffusivity
$\psi,\ \psi^*,\ \psi_0$	actual, reference and non-dimensional stream function
ω	frequency

note here that Rosensweig's [7] work concerns magnetorheological fluids.

Rayleigh–Benard convection in fluids without suspended particles is well investigated (see [8,9]). The corresponding problem in Saffman's fluid is also well understood (see [9]), but only to the extent of predicting onset of convection. Recently, Siddheshwar and Chan [10] have made a non-linear study of the problem.

Rayleigh–Benard convection in fluids whose stress is non-linearly proportional to the symmetric part of the velocity gradient fairly well investigated (see [11] and references therein). This is true of fluids with internal angular momentum as well (see [12–16] and references therein). In addition to the above one also has the Boussinesq–Stokes suspension model which is essentially Stokes' [6] couple stress model for suspensions along with the Boussinesq approximation. This model constrains the spin of the microelements to match with the vorticity of the carrier fluid. The model was recently considered by Umavathi and Malashetty [17] to investigate an Oberbeck situation in the suspension saturating a porous matrix. In the present paper we address ourselves to the daunting task of quantifying heat transfer in a Rayleigh–Benard situation involving the Boussinesq–Stokes' suspension. The important aspect of the study is that a closed-form solution is obtained for the non-linear, coupled, steady problem. The possibility of aperiodic convection is also investigated.

2. Mathematical formulation

We consider a horizontal layer of thickness 'd' of a Boussinesq–Stokes suspension in a Rayleigh–Benard situation. The Oberbeck–Boussinesq approximation even in the Navier–Stokes case is delicate (see [3]) and hence there is a need to seek the status of the

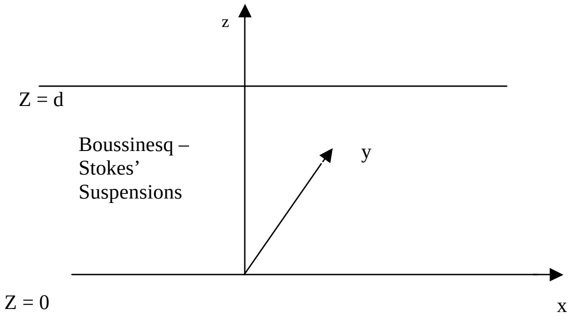

Fig. 1. Schematic diagram of the flow configuration.

approximation in the present model. The consideration of this is by no means straight forward and hence we take an approach that convinces us into assuming the same. The effect of suspended particles is to enhance the viscosity of the carrier fluid and does not in any foreseeable way work to violate the Boussinesq–Stokes approximation. With this argument we accept the approximation for Boussinesq–Stokes suspensions. The schematic of the same is shown in Fig. 1 with a Cartesian co-ordinate system as shown. The suspension is heated from below and cooled from above as in a typical Rayleigh–Benard problem, the temperature difference between the bounding walls being ΔT. The governing equations are (see [6,8]):

$$\nabla \cdot \vec{q} = 0, \tag{2.1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} \right] = -\nabla p + \mu \nabla^2 \vec{q} - \mu' \nabla^4 \vec{q} + \rho \vec{g}, \tag{2.2}$$

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \chi \nabla^2 T, \tag{2.3}$$

$$\rho = \rho_0 [1 - \beta (T - T_0)]. \tag{2.4}$$

The basic state of the fluid is quiescent and is given by

$$0 = -\frac{\mathrm{d} p_b}{\mathrm{d} z} - \rho_b g, \quad \vec{q}_b = (0,0,0), \quad \frac{\mathrm{d}^2 T_b}{\mathrm{d} z^2} = 0,$$

$$\rho = \rho_b [1 - \beta (T_b - T_0)], \tag{2.5}$$

where the subscript 'b' denotes basic state. On this basic state we superpose finite-amplitude perturbations in the form:

$$\vec{q} = \vec{q}_b + \vec{q}', \quad T = T_b(z) + T', \quad p = p_b(z) + p',$$

$$\rho = \rho_b(z) + \rho', \tag{2.6}$$

where primes indicate perturbations. Introducing (2.6) in Eqs. (2.1)–(2.4) and using (2.5), we get

$$\nabla \cdot \vec{q}' = 0, \tag{2.7}$$

$$\rho_0 \left[\frac{\partial \vec{q}'}{\partial t} + (\vec{q}'.\nabla)\vec{q}' \right] = -\nabla p + \mu \nabla^2 \vec{q}' - \mu' \nabla^4 \vec{q}' + \rho' \vec{g}, \tag{2.8}$$

$$\frac{\partial T'}{\partial t} + (\vec{q}'.\nabla) T' = w' + \chi \nabla^2 T', \tag{2.9}$$

$$\rho' = -\beta \rho_0 T'. \tag{2.10}$$

We consider only two-dimensional disturbances and thus restrict ourselves to the xz-plane. We can now introduce a stream function

$$u' = \frac{\partial \psi'}{\partial z}, \quad w' = -\frac{\partial \psi'}{\partial x}, \tag{2.11}$$

which satisfies the continuity equation (2.7).

Operating curl on Eq. (2.8), to eliminate pressure, introducing the stream function ψ and non-dimensionalizing the resulting equation as well as Eq. (2.9) using the following definition:

$$(x^*, z^*) = \left(\frac{x}{d}, \frac{z}{d} \right), \quad t^* = \frac{t}{d^2/\chi}, \quad \psi^* = \frac{\psi'}{\chi/d}$$

$$T^* = \frac{T'}{\Delta T} \tag{2.12}$$

we get the dimensionless equations in the form

$$\frac{1}{Pr} \frac{\partial}{\partial t} (\nabla^2 \psi) = -R \frac{\partial T}{\partial x} + \nabla^4 \psi - C \nabla^6 \psi + \frac{1}{Pr} \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x,z)}, \tag{2.13}$$

$$\frac{\partial T}{\partial t} = -\frac{\partial \psi}{\partial x} + \nabla^2 T + \frac{\partial(\psi, T)}{\partial(x,z)}, \tag{2.14}$$

where the asterisks have been dropped for simplicity. In arriving at Eq. (2.13) use has been made of Eq. (2.10).

Eqs. (2.13)–(2.14) are solved for stress-free, isothermal, vanishing couple–stress boundaries and hence we have

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^4 \psi}{\partial z^4} = T = 0 \quad \text{at } z = 0, 1. \tag{2.15}$$

3. Linear stability theory

In this section, we discuss the linear stability analysis which is of great utility in the local non-linear stability analysis discussed in the next section. To make this study we neglect the Jacobians in Eqs. (2.13) and (2.14) and assume the solutions to be periodic waves (see [8]) of the form

$$\begin{bmatrix} \psi \\ T \end{bmatrix} = e^{\sigma t} \begin{bmatrix} \psi_0 \sin(\pi\alpha x) \\ \theta_0 \cos(\pi\alpha x) \end{bmatrix} \sin(\pi z). \tag{3.1}$$

Substituting Eq. (3.1) in the linearized version of Eqs. (2.13) and (2.14), we get

$$\left(\frac{\sigma}{Pr} + \eta K^2\right) K^2 \psi_0 = -R\pi\alpha\theta_0, \tag{3.2}$$

$$(\sigma + K^2)\theta_0 = -\pi\alpha\psi_0, \tag{3.3}$$

where $\eta = 1 + CK^2$ and $K^2 = \pi^2(\alpha^2 + 1)$. η is representative of the viscosity of the fluid. In the case of fluids having no suspended particles (i.e. Newtonian fluids) we have $\eta = 1$. Analysing the expression for η it is obvious that the suspended particles add to the viscosity in conformity with Einstein's observation. Since suspended particles enhance viscosity of the fluid it means that the enforcement of Bousinesq approximation in suspensions is better than in the Newtonian fluid case. This is a sort of justification for the assumption of Boussinesq approximation in suspensions. For a non-trivial solution of ψ_0 and θ_0, we require

$$R = \frac{(\sigma + K^2)[\sigma/Pr + \eta K^2]K^2}{\pi^2\alpha^2}. \tag{3.4}$$

3.1. Marginal state

If σ is real, then marginal stability occurs when $\sigma = 0$. This gives the stationary Rayleigh number R_s in the form

$$R_s = \frac{\eta K^6}{\pi^2\alpha^2}. \tag{3.5}$$

The critical wave number a_c satisfies the equation

$$3C\pi^2(\alpha_c^2)^2 + 2(1 + C\pi^2)\alpha_c^2 - (1 + C\pi^2) = 0. \tag{3.6}$$

Clearly the critical wave number α_c depends on the couple stress parameter C. In the absence of couple stress i.e., $C = 0$, we get the classical result $\alpha_c^2 = 0.5$ and $R_{sc} = 657.5$ for clean fluids (see [8]).

3.2. Oscillatory motions

We put $\sigma = i\omega$ (ω: real) in Eq. (3.4) and rearranging we get the oscillatory Rayleigh number R_o in the form

$$R_o = \frac{\eta K^6 - \omega^2 K^2/Pr}{\pi^2\alpha^2} + i\omega\frac{K^4(\eta + 1/Pr)}{\pi^2\alpha^2}. \tag{3.7}$$

Since R_o is real, either $\omega = 0$ (marginal state) or $\eta + 1/Pr = 0$ ($\omega \neq 0$: oscillatory). The latter cannot be true as $C \geqslant 0$ and hence overstable motions are not possible in Boussinesq–Stokes suspensions. This points to the fact that the principal of exchange of stability is valid in this case.

In the next section we perform a non-linear stability analysis and quantify the heat transfer by conduction and convection and see the effect of suspended particles, through C, on it.

4. Non-linear theory

The finite-amplitude analysis can be carried out via a double Fourier series representation for the stream function ψ and temperature T in the form

$$\psi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}(t)\sin(m\pi\alpha x)\sin(n\pi z), \tag{4.1}$$

$$T = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} B_{mn}(t)\cos(m\pi\alpha x)\sin(n\pi z). \tag{4.2}$$

Substituting Eqs. (4.1)–(4.2) into the set of two coupled non-linear partial differential equations (2.13)–(2.14), we get a system of coupled, non-linear ordinary differential equations. It is however logical to use the observed fact that laboratory systems and practical situations involving suspensions often exhibit flows dominated by a few spatial harmonics. This allows one to choose a minimal representation from the above

Fourier series. Further, these serve as starting values for solving a more general non-linear convection problem.

The first effect of non-linearity is to distort the temperature field through the interaction of ψ and T. The distortion of the temperature field will correspond to a change in the horizontal mean, i.e., a component of the form $\sin(2\pi z)$ will be generated. Thus a minimal double Fourier series which describes the finite-amplitude free convection is given by

$$\psi = A(t)\sin(\pi\alpha x)\sin(\pi z), \tag{4.3}$$

$$T = B(t)\cos(\pi\alpha x)\sin(\pi z) + D(t)\sin(2\pi z), \tag{4.4}$$

where the amplitudes A, B and D are to be determined from the dynamics of the system.

Substituting Eqs. (4.3)–(4.4) into Eqs. (2.13)–(2.14) and equating the coefficients of like terms we obtain the following non-linear autonomous system (generalized Lorenz model) of differential equations:

$$\dot{A} = -\frac{R\pi\alpha Pr}{K^2}B - \eta\, Pr\, K^2 A, \tag{4.5}$$

$$\dot{B} = \pi\alpha A - K^2 B - \pi^2\alpha AD, \tag{4.6}$$

$$\dot{D} = -4\pi^2 D + \pi^2\alpha\frac{AB}{2}, \tag{4.7}$$

where the overdot denotes the time derivative.

The non-linear system of autonomous differential equations is not amenable to analytical treatment for the general time-dependent variable and we have to solve it using a numerical method. However, one can make qualitative predictions as discussed below.

The generalized Lorenz [18] model (4.5)–(4.7) is uniformly bounded in time and possesses many properties of the full problem. Also the phase-space volume contracts at a uniform rate given by

$$\frac{\partial\dot{A}}{\partial A} + \frac{\partial\dot{B}}{\partial B} + \frac{\partial\dot{D}}{\partial D} = -[\eta\, Pr\, K^2 + K^2 + 4\pi^2], \tag{4.8}$$

which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase-space; in particular they may be attracted to a fixed point, a limit cycle or perhaps, a strange attractor.

5. Linear autonomous system

Before solving the non-linear system of equation, we consider the linear system of autonomous system and analyse the critical points. The nature of the critical points obtained from the linear system reveals information about the trajectories in the phase plane. The nature of these trajectories is retained by the non-linear system but with distortions dictated by the non-linear terms.

The linearized system is

$$\dot{A} = -\frac{R\pi\alpha\, Pr}{K^2}B - \eta\, Pr\, K^2 A, \tag{5.1}$$

$$\dot{B} = -\pi\alpha A - K^2 B, \tag{5.2}$$

$$\dot{D} = -4\pi^2 D. \tag{5.3}$$

To find the critical points of the above linear autonomous system of equations, we follow Simmons [19] and accordingly the auxiliary equation is obtained from Eqs. (5.1)–(5.3).

$$\begin{vmatrix} -\eta\, Pr\, K^2 - \lambda & -\dfrac{R\pi\alpha\, Pr}{K^2} & 0 \\ -\pi\alpha & -K^2 - \lambda & 0 \\ 0 & 0 & -4\pi^2 - \lambda \end{vmatrix} = 0.$$

On simplification, we get

$$\lambda^2 + (\eta\, Pr\, K^2 + K^2)\lambda + \left(\eta\, Pr\, K^4 - \frac{R\pi^2\alpha^2\, Pr}{K^2}\right) = 0. \tag{5.4}$$

Let λ_1 and λ_2 be the roots of Eq. (5.4). We now discuss three cases based on the nature of the roots of Eq. (5.4).

Case (i): For two real and equal roots i.e., $\lambda_1 = \lambda_2$ In this case

$$(\eta\, Pr + 1)^2 K^4 = 4\left(\eta\, Pr\, K^4 - \frac{R\pi^2\alpha^2\, Pr}{K^2}\right),$$

i.e.,

$$R = \frac{[4\eta\, Pr - (\eta\, Pr + 1)^2]K^6}{4\pi^2\alpha^2\, Pr}. \tag{5.5}$$

For the above value of R critical point is a node and system becomes stable as the path approach and enter the critical point.

Case (ii): For two real and distinct roots, i.e., $\lambda_1 \neq \lambda_2$

In this case

$$(\eta\,Pr+1)^2K^4 > 4\left(\eta\,Pr\,K^4 - \frac{R\pi^2\alpha^2\,Pr}{K^2}\right),$$

i.e.,

$$R > \frac{[4\eta\,Pr-(\eta\,Pr+1)^2]K^6}{4\pi^2\alpha^2\,Pr}. \tag{5.6}$$

For this value of R the critical point is a saddle point and system becomes unstable as paths never approach the critical points.

Case (iii): For two imaginary roots, i.e., $\lambda_1 \neq \lambda_2$

In this case

$$(\eta\,Pr+1)^2K^4 < 4\left(\eta\,Pr\,K^4 - \frac{R\pi^2\alpha^2\,Pr}{K^2}\right),$$

i.e.,

$$R < \frac{[4\eta\,Pr-(\eta\,Pr+1)^2]K^6}{4\pi^2\alpha^2\,Pr}. \tag{5.7}$$

For this range of value of R the critical point is a spiral and system is asymptotically stable if paths approach the critical point as $t \to -\infty$ and system becomes unstable for $t \to \infty$ if path spirals out.

From qualitative predictions we now look into the possibility of an analytical solution. In the case of steady motions, Eqs. (4.5)–(4.7) can be solved in closed form. Setting the left-hand sides of Eqs. (4.5)–(4.7) equal to zero, we get

$$R\pi\alpha B + \eta K^4 A = 0, \tag{5.8}$$

$$\pi\alpha A + K^2 B + \pi^2\alpha A D = 0, \tag{5.9}$$

$$8D - \alpha A B = 0. \tag{5.10}$$

Writing B and D in terms of A using Eqs. (5.8) and (5.10) and substituting these in Eq. (4.10), we get

$$A\left\{\pi\alpha - \frac{\eta K^6}{R\pi\alpha} - \frac{\pi\eta K^4 A^2}{8R}\right\} = 0. \tag{5.11}$$

The solution $A = 0$ corresponds to pure conduction which we know to be a possible solution though it is unstable when R is sufficiently large. The remaining solutions are given by

$$\frac{A^2}{8} = \frac{R\pi^2\alpha^2 - \eta K^6}{\pi^2\alpha^2\eta K^4}. \tag{5.12}$$

6. Heat transport

In the study of convection in Boussinesq–Stokes suspensions, the quantification of heat transport is important. This is because the onset of convection, as Rayleigh number is increased, is more readily detected by its effect on the heat transport. In the basic state, heat transport is by conduction alone.

If H is the rate of heat transport/unit area, then

$$H = -\chi\left\langle\frac{\partial T_{\text{total}}}{\partial z}\right\rangle_{z=0}, \tag{6.1}$$

where the angular bracket corresponds to a horizontal average and

$$T_{\text{total}} = T_0 - \Delta T\,\frac{z}{d} + T(x,z,t). \tag{6.2}$$

Substituting Eq. (4.4) in (6.2) and using the resultant equation in Eq. (6.1), we get

$$H = \frac{\chi\Delta T}{d} - \frac{\chi\Delta T}{d}\,2\pi D. \tag{6.3}$$

The Nusselt number Nu is defined by

$$Nu = \frac{H}{\chi\Delta T/d} = 1 - 2\pi D. \tag{6.4}$$

Writing D in terms of A, using Eqs. (5.8)–(5.10), and substituting in Eq. (6.4), we get

$$Nu = 1 + \frac{2\eta K^4}{R}\left(\frac{A^2}{8}\right). \tag{6.5}$$

Substituting Eq. (5.12) in Eq. (6.5), we get

$$Nu = 1 + \frac{2}{\pi^2\alpha R}(R\pi^2\alpha^2 - \eta K^6). \tag{6.6}$$

The second term on the right-hand side of Eq. (6.6) represents the convective contribution to heat transport.

7. Results and discussion

A linear and non-linear stability analyses of convection in Stokes' couple stress fluid is performed

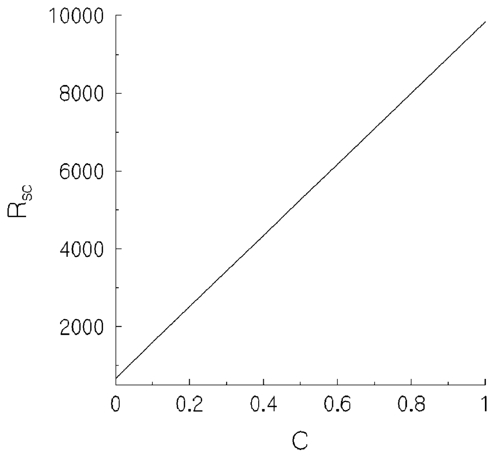

Fig. 2. Plot of critical Rayleigh number R_{sc} versus couple stress parameter C.

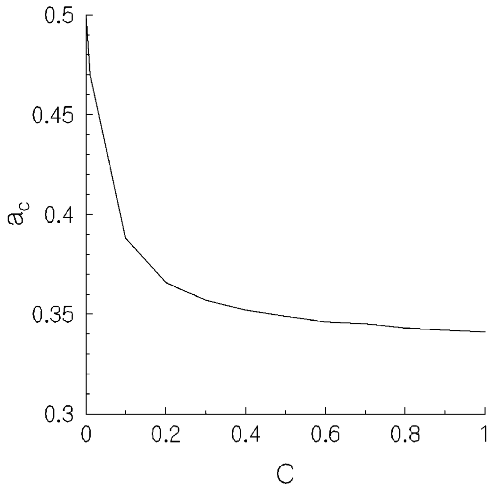

Fig. 3. Plot of critical wave number a_c versus C.

resulting in autonomous systems. The linear theory gives us the critical Rayleigh number R_{sc} for the onset of convection. In the present problem oscillatory convection is discounted and the stationary Rayleigh number is applicable. *A* plot of R_{sc} versus the couple stress parameters C is made in Fig. 2. We find that R_{sc} increases linearly with increase in C and the slope is equal to $k^8/\pi^2\alpha^2$ (see Eq. (3.5)). For $C = 0$ the value of R_{sc} is 657.5 classical (Rayleigh–Benard result).

Fig. 3 is the plot of the critical wave number a_c versus C. We find from the figure that a_c decreases

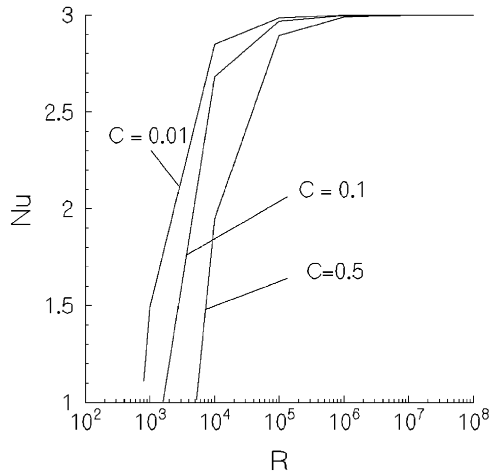

Fig. 4. Plot of Rayleigh number R versus Nusselt number Nu for different values C.

with increase in C. C is indicative of the concentration of the suspended particles and the Fig. 3 thus implies that the cell size increases with increase in C. This can be anticipated because R_{sc} increases with increase in C as shown in Fig. 2.

The realm of non-linear convection warrants the quantification of heat transfer. This is depicted in the Nusselt–Rayleigh plane in Fig. 4. We find from this figure that Nu increases with increase in R and decreases with increase in C. This can be looked at in conjunction with the results of the Figs. 2 and 3.

A phase-space analysis is done on the autonomous system, and this indicates the possibility of chaotic motion. The same may be realized by numerically solving the time-dependent non-linear autonomous system of Eqs. (4.5)–(4.7). The condition under which a saddle point, node or spiral may be obtained for the time dependent system has been discussed in Section 5.

It is clear from the above results that suspended particles whose spin matches with the vorticity of the fluid make the system stable.

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