

# An Anisotropic Phong BRDF Model

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## Abstract

We present a BRDF model that combines several advantages of the various empirical models currently in use. In particular, it has intuitive parameters, is anisotropic, conserves energy, is reciprocal, has an appropriate non-Lambertian diffuse term, and is well-suited for use in Monte Carlo renderers.

## 1 Introduction

Physically-based rendering systems describe reflection behavior using the *bidirectional reflectance distribution function* (BRDF). For a detailed discussion of the BRDF and its use in computer graphics see the volumes by Glassner [2]. At a given point on a surface the BRDF is a function of two directions, one toward the light and one toward the viewer. The characteristics of the BRDF will determine what “type” of material the viewer thinks the displayed object is composed of, so the choice of BRDF model and its parameters is important. We present a new BRDF model that is motivated by practical issues. A full rationalization for the model and comparison with previous models is provided in a separate technical report [1].

The BRDF model described in the paper is inspired by the models of Ward [8], Schlick [6], and Neumann and Neumann [5]. However, it has several desirable properties not previously captured by a single model. In particular, it

1. obeys energy conservation and reciprocity laws,
2. allows *anisotropic* reflection, giving the streaky appearance seen on brushed metals,
3. is controlled by intuitive parameters,
4. accounts for *Fresnel behavior*, where specularity increases as the incident angle goes down,
5. has a non-constant diffuse term, so the diffuse component decreases as the incident angle goes down,
6. is well-suited to Monte Carlo methods.

The model is a classical sum of a “diffuse” term and a “specular” term.

$$\rho(\mathbf{k}_1, \mathbf{k}_2) = \rho_s(\mathbf{k}_1, \mathbf{k}_2) + \rho_d(\mathbf{k}_1, \mathbf{k}_2). \quad (1)$$

For metals, the diffuse component  $\rho_d$  is set to zero. For “polished” surfaces, such as smooth plastics, there is both a diffuse and specular appearance and neither term is zero. For purely diffuse surfaces, either the traditional Lambertian (constant) BRDF can be used, or the new model with low specular exponents can be used for slightly more visual realism near grazing angles. The model is controlled by four parameters:

- $R_s$ : a color (spectrum or RGB) that specifies the specular reflectance at normal incidence.

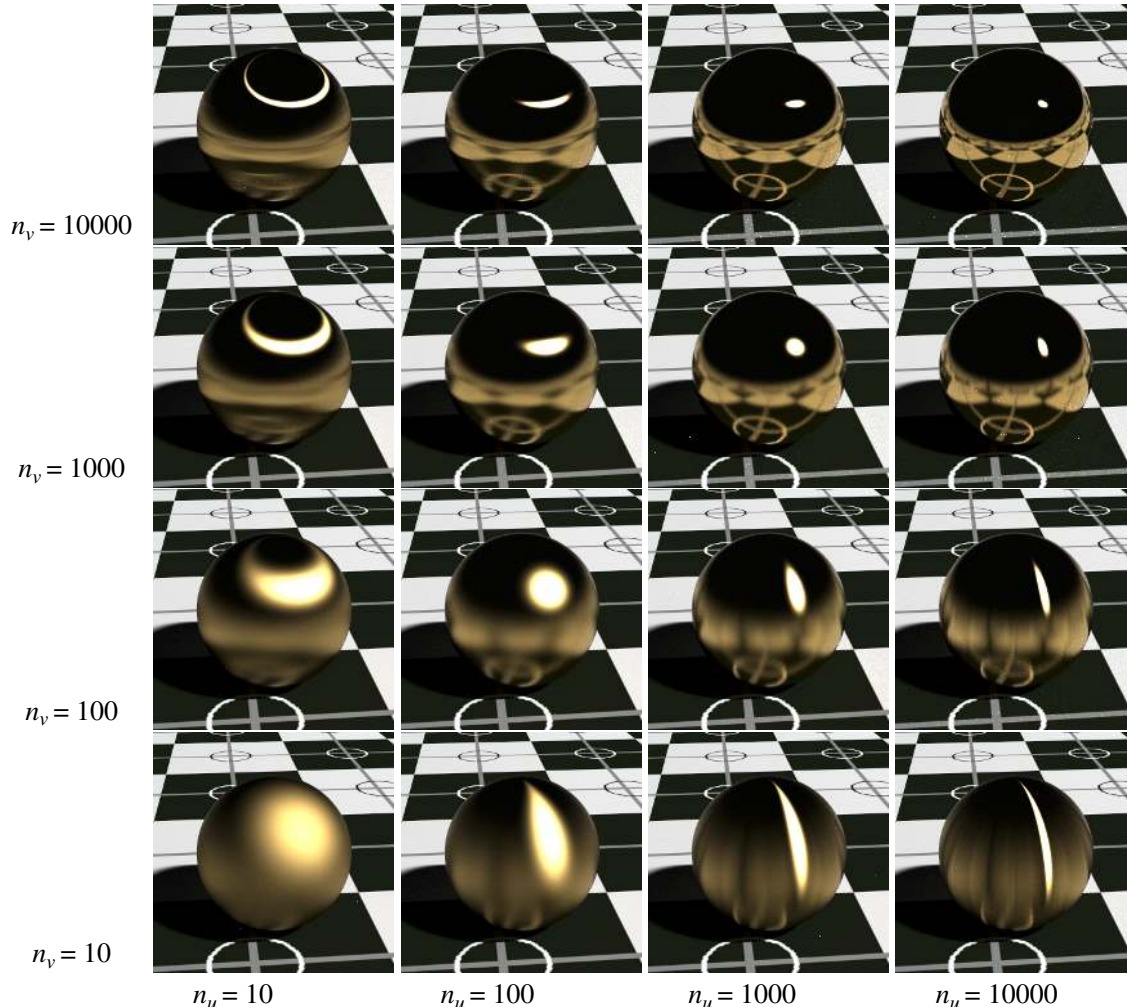


Figure 1: *Metallic spheres for various exponents.*

- $R_d$ : a color (spectrum or RGB) that specifies the diffuse reflectance of the “substrate” under the specular coating.
- $n_u, n_v$ : two phong-like exponents that control the shape of the specular lobe.

We now illustrate the use of the model in several figures. After that, the remainder of the paper deals with specifying and implementing the model. Figure 1 shows spheres with  $R_d = 0$  and varying  $n_u$  and  $n_v$ . The spheres along the diagonal have  $n_u = n_v$  so have a look similar to the traditional Phong model. Figure 2 shows another metallic object. This appearance is achieved by using the “right” mapping of tangent vectors on the surface. Figure 3 shows a “polished” surface with  $R_s = 0.05$ . This means the diffuse component will dominate for near normal viewing angles. However, as the viewing angle becomes oblique the specular component dominates despite its low near-normal value. Figure 4 shows the model for a diffuse surface. Note how the ball on the right has highlights near the edge of the model, and how the constant-BRDF ball on the left is more “flat”. The highlights produced by the new model are present in the measured BRDFs some paints, so are desirable for some applications [4].



Figure 2: A cylinder with the appearance of brushed metal created with the new model with  $R_d = 0, R_s = 0.9, n_u = 10, n_v = 100$ .

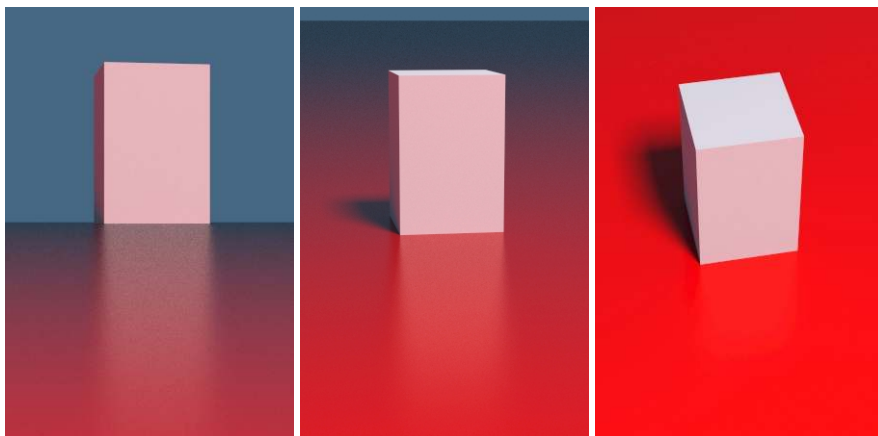


Figure 3: Three views for  $n_u = n_v = 400$  and a red substrate.

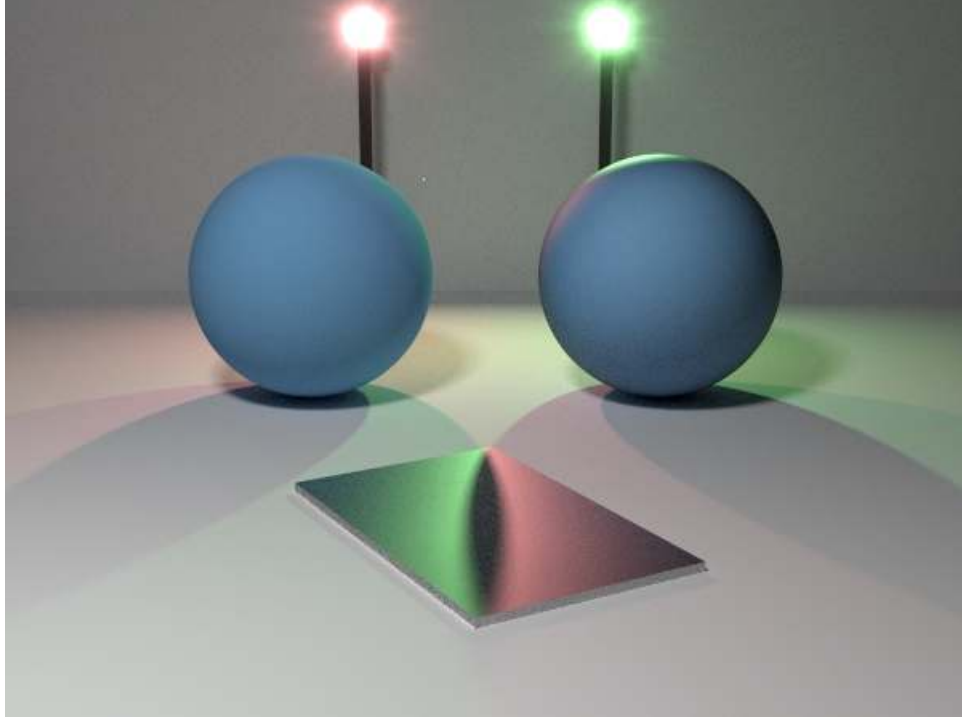


Figure 4: An image with a Lambertian sphere (left) and a sphere with  $n_u = n_v = 5$ . After a figure from Lafortune et al. [4].

## 2 The Model

The specular component  $\rho_s$  of the BRDF is:

$$\rho_s(\mathbf{k}_1, \mathbf{k}_2) = \frac{\sqrt{(n_u + 1)(n_v + 1)}}{8\pi} \frac{(\mathbf{n} \cdot \mathbf{h})^{n_u \cos^2 \phi + n_v \sin^2 \phi}}{(\mathbf{h} \cdot \mathbf{k}) \max((\mathbf{n} \cdot \mathbf{k}_1), (\mathbf{n} \cdot \mathbf{k}_2))} F((\mathbf{k} \cdot \mathbf{h})) \quad (2)$$

In our implementation we use Schlick's approximation to Fresnel fraction [6]:

$$F((\mathbf{k} \cdot \mathbf{h})) = R_s + (1 - R_s)(1 - (\mathbf{k} \cdot \mathbf{h}))^5, \quad (3)$$

$\mathbf{a} \cdot \mathbf{b}$	scalar (dot) product of vectors $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{k}_1$	normalized vector to light
$\mathbf{k}_2$	normalized vector to viewer
$\mathbf{n}$	surface normal
$\mathbf{u}, \mathbf{v}$	tangent vectors that form an orthonormal basis along with $\mathbf{n}$ .
$\rho(\mathbf{k}_1, \mathbf{k}_2)$	BRDF
$\mathbf{h}$	normalized half-vector between $\mathbf{k}_1$ and $\mathbf{k}_2$
$p(\mathbf{k})$	probability density function for reflection sampling rays
$F(\cos \theta)$	Fresnel reflectance for incident angle $\theta$

Table 1: Important terms used in the paper

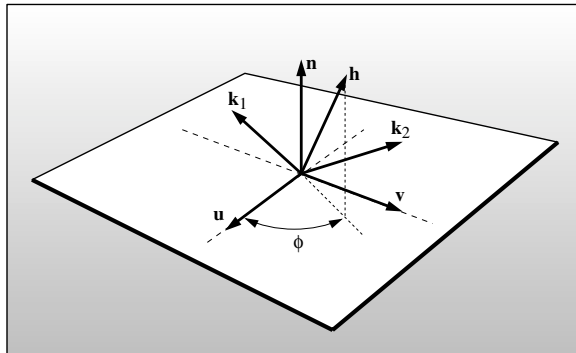


Figure 5: *Geometry of reflection. Note that  $\mathbf{k}_1$ ,  $\mathbf{k}_2$ , and  $\mathbf{h}$  share a plane, which usually does not include  $\mathbf{n}$ .*

where  $R_s$  is material's reflectance for the normal incidence. It is not necessary to call trigonometric functions to compute the exponent in Equation 2, so the specular BRDF can be written:

$$\rho_s(\mathbf{k}_1, \mathbf{k}_2) = \frac{\sqrt{(n_u + 1)(n_v + 1)}}{8\pi} \frac{(\mathbf{n} \cdot \mathbf{h})^{\frac{(n_u(\mathbf{h}\mathbf{u})^2 + n_v(\mathbf{h}\mathbf{v})^2)}{(1 - (\mathbf{h}\mathbf{n})^2)}}}{(\mathbf{h} \cdot \mathbf{k}) \max((\mathbf{n} \cdot \mathbf{k}_1), (\mathbf{n} \cdot \mathbf{k}_2))} F((\mathbf{k} \cdot \mathbf{h})). \quad (4)$$

It is possible to use a Lambertian BRDF together with our specular term in a way similar to that which is done for most models [6, 8]. However, we use a simple angle-dependent form of the diffuse component which accounts for the fact that the amount of energy available for diffuse scattering varies due to the dependence of specular term's total reflectance on the incident angle. In particular, diffuse color of a surface disappears near the grazing angle because the total specular reflectance is close to one in this case. This well-known effect cannot be reproduced with a Lambertian diffuse term and is therefore missed by most reflection models. Another, perhaps more important, limitation of the Lambertian diffuse term is that it must be set to zero to ensure energy conservation in the presence of a Fresnel-weighted term.

The diffuse term is:

$$\rho_d(\mathbf{k}_1, \mathbf{k}_2) = \frac{28R_d}{23\pi} (1 - R_s) \left( 1 - \left( 1 - \frac{(\mathbf{n} \cdot \mathbf{k}_1)}{2} \right)^5 \right) \left( 1 - \left( 1 - \frac{(\mathbf{n} \cdot \mathbf{k}_2)}{2} \right)^5 \right) \quad (5)$$

Note that our diffuse BRDF does not depend on  $n_u$  and  $n_v$ . The somewhat unusual leading constant is designed to ensure energy conservation.

### 3 Using the BRDF model in a Monte Carlo setting

In a Monte Carlo setting we are interested in the following problem: given  $\mathbf{k}_1$ , generate samples of  $\mathbf{k}_2$  with a distribution which shape is similar to the cosine weighted BRDF. This distribution should be a *probability density function* (pdf), and we should be able to evaluate it for a given randomly generated  $\mathbf{k}_2$ . The key part of our thinking on this is inspired by discussion by Zimmerman [9] and by Lafortune [3] who point out that greatly undersampling a large value of the integrand is a serious error while greatly oversampling a small value is acceptable in practice. The reader can verify that the densities suggested below have this property.

We first generate a half-vector  $\mathbf{h}$  using the following pdf:

$$p_h(\mathbf{h}) = \frac{\sqrt{(n_u + 1)(n_v + 1)}}{2\pi} (\mathbf{n} \cdot \mathbf{h})^{n_u \cos^2 \phi + n_v \sin^2 \phi}, \quad (6)$$



Figure 6: A closeup of the model implemented in a path tracer with 9, 26, and 100 samples.

To evaluate the rendering equation we need both a reflected vector  $\mathbf{k}_2$  and a probability density function  $p(\mathbf{k}_2)$ . Note that if you generate  $\mathbf{h}$  according to  $p_h(\mathbf{h})$  and then transform to the resulting  $\mathbf{k}_2$ :

$$\mathbf{k}_2 = -\mathbf{k}_1 + 2(\mathbf{k}_1 \cdot \mathbf{h})\mathbf{h}, \quad (7)$$

the density of the resulting  $\mathbf{k}_2$  is **not**  $p_h(\mathbf{k}_2)$ . This is because of the difference in measures in  $\mathbf{h}$  and  $\mathbf{v}_2$  space. So the actual density  $p(\mathbf{k}_2)$  is:

$$p(\mathbf{k}_2) = \frac{p_h(\mathbf{h})}{4(\mathbf{k}_1 \cdot \mathbf{h})}. \quad (8)$$

Monte Carlo renderers that use this method converge reasonably quickly (Figure 6). In an implementation where the BRDF is known to be this model, the estimate of the rendering equation is quite simple as many terms cancel out.

Note that it is possible to generate an  $\mathbf{h}$  vector whose corresponding vector  $\mathbf{k}_2$  will point inside the surface, i.e.  $(\mathbf{k}_2 \cdot \mathbf{n}) < 0$ . The weight of such a sample should be set to zero. This situation corresponds to the specular lobe going below the horizon and is the main source of energy loss in the model. This problem becomes progressively less severe as  $n_u, n_v$  become larger.

The only thing left now is to describe how to generate  $\mathbf{h}$  vectors with pdf of Equation 7. We start by generating  $\mathbf{h}$  with its spherical angles in the range  $(\theta, \phi) \in [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$ . Note that this is only the first quadrant of the hemisphere. Given two random numbers  $(\xi_1, \xi_2)$  uniformly distributed in  $[0, 1]$ , we can choose

$$\phi = \arctan \left( \sqrt{\frac{n_u + 1}{n_v + 1}} \tan \left( \frac{\pi \xi_1}{2} \right) \right), \quad (9)$$

and then use this value of  $\phi$  to obtain  $\theta$  according to

$$\cos \theta = (1 - \xi_2)^{\frac{1}{n_u \cos^2 \phi + n_v \sin^2 \phi + 1}}. \quad (10)$$

To sample the entire hemisphere, the standard manipulation where  $\xi_1$  is mapped to one of four possible functions depending on whether it is in  $[0, 0.25)$ ,  $[0.25, 0.5)$ ,  $[0.5, 0.75)$ , or  $[0.75, 1.0)$ . For example for  $\xi_1 \in [0.25, 0.5)$ , find  $\phi(1 - 4(0.5 - \xi_1))$  via Equation 9, and then “flip” it about the  $\phi = \pi/2$  axis. This ensures full coverage and stratification.

For the diffuse term it would be possible to do importance sample with a density close to cosine-weighted BRDF 5 in a way similar to that described by Shirley et al [7], but we use a simpler approach and generate samples according to cosine distribution. This is sufficiently close to the complete diffuse BRDF to substantially reduce variance of the Monte Carlo estimation. To generate samples for the entire BRDF, simply use a weighted average of 1 and  $p(\mathbf{k}_2)$ .

## References

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