An Application of Multicriteria Group Decision Making by Soft Covering Based Rough Sets

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Abstract. In this work we define soft covering based rough set and present its related properties. Also we present an example in medicine which aims to find the patients with high prostate cancer risk. Our datas are 56 patients from Selcuk University Meram Medicine Faculty.

1. Introduction

In recent years vague concepts have been used in different areas as medical applications, pharmacology, economics, engineering since the classical mathematics methods are inadequate to solve many complex problems in these areas. Traditionally crisp (well-defined) property $P(x)$ is used in mathematics, i.e., properties that are either true or false and each property defines a set: $\{x : x$ has a property $P\}$ [20].

Researchers have proposed many methods for vague notions. The most successful theoretical approach to the vagueness is undoubtedly fuzzy set theory [34] proposed by Zadeh in 1965. The basic idea of fuzzy set theory hinges on fuzzy membership function, which allows partial membership of elements to a set, i.e., it allows elements to belong to a set to "a degree".

Rough set theory [21] which was proposed by Pawlak in 1982 is another mathematical approach to vagueness to catch the granularity induced by vagueness in information. The advantage of rough set method is that it does not need any additional information about data, like membership in fuzzy set theory. The classical rough set theory is based on equivalence relations and it has been extended to covering based rough set [36–38].

Molodtsov initiated a novel concept of soft set theory [18], which is a completely new approach for modeling vagueness in 1999. A soft set is a collection of approximate descriptions of an object. Molodtsov [18, 19] presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc. He also showed that how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory and etc. Soft systems provide a very general framework with the involvement of parameters. It has been found that fuzzy sets, rough sets and soft sets are closely related [1].

Maji et al. investigated the concept of fuzzy soft set in 2001 [15], a more generalized concept, which is a combination of fuzzy set and soft set and also studied some of its properties. This line of exploration was further investigated by several researchers [16, 31, 32]. Soft set and fuzzy soft set theories have rich potential for applications in several directions.

Application of soft set theory in algebraic structures was initiated by Aktas and Cagman [1]. They introduced the notion of soft groups, extending fuzzy groups. Jun et al. discussed the applications of soft sets to the study of BCK/BCI-algebras [10–13]. Feng et al. [5] investigated soft semirings and soft semiring homomorphism. Atagun and Sezgin [26] introduced the notions of soft near-rings, soft sub-nearrings, soft ideals, idealistic soft near-rings and soft near-ring homomorphisms. Sezgin et al. [26] applied some of the operations to soft near-rings and substructures of near-rings. They also investigated the properties of idealistic soft near-rings with respect to near-ring epimorphisms by corresponding examples. Feng et al. [9] initiated an extension of fuzzy binary relations based on the theory of soft sets, which is called soft binary relation. They considered soft congruence relations over semigroups and they introduced soft homomorphisms and established several isomorphism theorems for soft semigroups using soft congruence relations.

Feng et al. investigated the concept of soft rough set in 2010 [6] which is a combination of soft set and rough set. In [6, 7] basic properties of soft rough approximations were presented and supported by some illustrative examples. In fact, as soft set instead of an equivalence relation was used to granulate the universe of discourse. A new approach was introduced to soft rough sets which is called modified soft rough set (MSR-set) and some basic properties of MSR-sets were investigated in [27].

Feng discussed soft set based group decision making in 2011 [8]. This study can be seen as a first attempt toward the possible application of soft rough approximations in multicriteria group decision making under vagueness.

Prostate cancer is the second most common cause of cancer death among men in most industrialized countries and it depends on various factors as family's cancer history, age, ethnic background, the level of prostate specific antigen (PSA) in the blood. The level of PSA in blood is very important method to an initial diagnosis for patients [3, 28, 30]. However the level of PSA in blood can be increased by inflammation of prostate and benign prostate hyperplasia (BPH). For this reason it is diffucult to differentiate it from benign prostate hyperplasia (BPH). The definitive diagnose of the prostate cancer is possible with prostate biopsy. The results of PSA test, rectal examination, transrectal findings help to the doctor to decide biopsy is necessary or not [17, 20, 25]. However the patients with low cancer risk have to avoid this process due to possible complications and its high cost. Because of this reason before to agree the biopsy the patients with low cancer risk can be determined.

There are several researches in the area of the prostate cancer prognosis or diagnosis. One of them is FES which is a rule-based fuzzy expert system using the laboratory datas PSA, PV and age of the patient and aim to help to an expert-doctor to determine the necesssity of biopsy and the risk factor [23]. Benecchi [2] developed a neuro-fuzzy system by using both serum data (total prostate specific antigen and free prostate specific antigen) and clinical data (age of patients) to enhance the performance of tPSA (total prostate specific antigen) to distinguish prostate cancer. Keles et al. [14] built a neuro-fuzzy classifier to be used in the diagnosis of prostate cancer and BPH diseases. Since the symptoms of these two illness are very close to each other the differentiation between them is an important problem. Saritas et al. [24] have devised an artifical neural network that provides a prognostic result indicating whether patients have cancer or not by using their free prostate specific antigen, total prostate specific antigen and age data. Yuksel et al. [33] devised a prediction system named soft expert system (SES) by using the prostate specific antigen (PSA), prostate volume (PV) and age factors of patients based on fuzzy sets and soft sets.

In this paper, we study a new concept called as soft covering based rough set which is a combination of covering soft set and rough set. We define soft covering approximations and investigate their basic properties. Then we give counterexamples for unsatisfied properties. Also, we are inspired of the method given by Feng in [8] and applied this method by using soft covering based rough sets to a medicine problem calculating the risk of prostate cancer. As stated in the above the definitive diagnosis of prostate cancer is possible with biopsy process. However this process can lead some complications in patients and it has high cost. For these reasons decreasing the number of the biopsy process is an important problem. We aim

to make a prediction system by using upper and lower approximations of soft covering based rough sets generated with the values PSA, PV and age of each patient and help to the doctor . There are many research papers and methods about prediction systems in medicine [2, 14, 23, 24, 33]. Our work is a starting point using soft rough sets and approximations for a prediction system in medicine.

2. Preliminaries

In this section, we introduce the fundamental ideas behind rough sets, soft sets, soft rough sets and fuzzy soft sets.

Definition 2.1. *[21] Let U be a finite set and R be an equivalence relation on U. Then the pair (U, R) is called a Pawlak approximation space. R generates a partition $U/R = \{Y_1, Y_2, ..., Y_m\}$ on U where $Y_1, Y_2, ..., Y_m$ are the*

equivalence classes generated by the equivalence relation R. In the rough set theory, these are also called elementary sets of R. For any $X \subseteq U$, we can describe X by the elementary sets of R and the two sets:

$$R_-(X) \quad = \quad \cup\{Y_i \in U/R : Y_i \subseteq X\},$$
$$R^-(X) \quad = \quad \cup\{Y_i \in U/R : Y_i \cap X \neq \emptyset\}$$

which are called the lower and the upper approximation of X, respectively.

[22] Let (U, R) be a Pawlak approximation space. A subset $X \subseteq U$ is called definable (crisp) if $R_-(X) = R^-(X)$; in the opposite case, i.e., if $R_-(X) \neq R^-(X)$, X is said to be rough(or inexact).

Let U be an initial universe set and E be the set of all possible parameters with respect to U. Usually, parameters are attributes, characteristics or properties of the objects in U. The notion of a soft set is defined as follows:

Definition 2.2. *[18] A pair $G = (F, A)$ is called a soft set over U, where $A \subseteq E$ and $F : A \longrightarrow P(U)$ is a set-valued mapping.*

Example 2.3. *[35] Let X be a set and $F : X \mapsto P(X)$ defined by $F(x) = \{x\}$ for any $x \in X$. Then the soft set (F, X) is called a simple soft set generated by X.*

Theorem 2.4. *[1] Every rough set may be considered as a soft set.*

The following result indicates that soft sets and binary relations are closely related.

Theorem 2.5. *[6] Let $G = (F, A)$ be a soft set over U. Then G induces a binary relation $R_G \subseteq A \times U$, which is defined by*

$$(x, y) \in R_G \iff y \in F(x)$$

where $x \in A$, $y \in U$.

Conversely, assume that R is a binary relation from A to U. Define a set valued mapping $F_R : A \longrightarrow P(U)$ by

$$F_R(x) = \{y \in U : (x, y) \in R\}$$

where $x \in A$. Then $G_R = (F_R, A)$ is a soft set over U. Moreover, it is seen that $G_{R_G} = G$ and $R_{G_R} = R$.

As pointed out by several researchers, information systems and soft sets are closely related [4, 39]. Let $G = (F, A)$ be a soft set over U. If U and A are both nonempty finite sets, then G could induce an information system in a natural way. In fact, for any attribute $a \in A$, one can define a function $a : U \longrightarrow V_a = \{0, 1\}$ by

$$a(x) = \begin{cases} 1 & , if \ x \in F(a) \\ 0 & , otherwise \end{cases}$$

Therefore, every soft set may be considered as an information system. This justifies the tabular representation of soft sets used widely in the literature.

Definition 2.6. *[15] Let $A \subseteq E$. (f_A, E) is defined to be a fuzzy soft set on (U, E) if $f_A : E \to I^U$ is a mapping defined by $f_A(e) = \mu_{f_A}^e$ where $\mu_{f_A}^e = \bar{O}$ if $e \in E - A$ and $\mu_{f_A}^e \neq \bar{O}$ if $e \in A$, where $\bar{O}(u) = 0$ for each $u \in U$.*

Example 2.7. *[29] Miss X and Mr. Y are going to marry and they want to hire a wedding room. The fuzzy soft set (f_A, E) describes the "capacity of the wedding room". Let $U = \{a, b, c, d, e\}$ be the wedding rooms under consideration, $E = \{big = e_1, central = e_2, cheap = e_3, expensive = e_4, elegant = e_5, quality = e_6, good \ serving = e_7\}$ be the parameter set and $A = \{e_2, e_5, e_6\}$ be a subset of E. $(f_A, E) = \{e_2 = \{a_{0.3}, b_{0.5}, c_{0.9}, d_{0.8}, e_{0.6}\}, e_5 = \{a_{0.8}, b_{0.6}, c_{0.2}, d_{0.1}, e_{0.5}\}, e_6 = \{a_{0.7}, b_{0.5}, c_{0.3}, d_{0.2}, e_{0.4}\}\}$ is a fuzzy soft set on (U, E).*

Definition 2.8. *[6] Let $G = (F, A)$ be a soft set over U. Then the pair $P = (U, G)$ is called a soft approximation space. The lower and upper soft rough approximations of any set $X \subseteq U$ is defined as follows, respectively:*

$$\underline{apr}_P (X) = \bigcup_{a \in A} \{F(a) : F(a) \subseteq X\}$$
$$\overline{apr}^P (X) = \bigcup_{a \in A} \{F(a) : F(a) \cap X \neq \emptyset\}$$

If $\overline{apr}^P (X) = \underline{apr}_P (X)$, X is said to be soft $P-$definable; otherwise X is called a soft $P-$rough set.

3. Soft Covering Based Rough Sets

From the concept of soft set, we know that a soft set is determined by the set-valued mapping from a set of parameters to the powerset of the universe. In this section, we will use a special kind of soft set and with this soft set, we will establish a soft covering approximation space.

Definition 3.1. *[6] A soft set $G = (F, A)$ over U is called a full soft set if $\bigcup_{a \in A} F(a) = U$.*

Definition 3.2. *[6] A full soft set $G = (F, A)$ over U is called a covering soft set if $F(e) \neq \emptyset$, $\forall e \in A$.*

Definition 3.3. *Let $G = (F, A)$ be a covering soft set over U. We call the ordered pair $S = (U, C_G)$ a soft covering approximation space.*

Definition 3.4. *Let $S = (U, C_G)$ be a soft covering approximation space, $x \in U$, the soft minimal description of x is defined as*

$$Md_S(x) = \{F(e) : e \in A \wedge x \in F(e) \wedge (\forall a \in A \wedge x \in F(a) \subseteq F(e) \implies F(a) = F(e))\}.$$

Definition 3.5. *Let $S = (U, C_G)$ be a soft covering approximation space. For a set $X \subseteq U$, the soft covering lower and upper approximations are respectively defined as*

$$S_-(X) = \cup \{F(e) : e \in A \wedge F(e) \subseteq X\}$$
$$S^-(X) = S_-(X) \cup \{Md_S(x) : x \in X - S_-(X)\}.$$

In addition, $POS_S(X) = S_-(X), NEG_S(X) = U - S^-(X), BND_S(X) = S^-(X) - S_-(X)$ are called the soft covering positive, negative and boundary regions of X, respectively.

It is easy to see from the definitions that soft lower approximation in [6, 7] is the same as that soft covering lower approximation. On the other hand, we find that soft covering upper approximation is smaller than soft upper approximatin in [6, 7]. This is an advantage since soft covering boundary region becomes smaller.

Definition 3.6. *Let $S = (U, C_G)$ be a soft covering approximation space. A subset $X \subseteq U$ is called soft covering based definable if $S_-(X) = S^-(X)$; in the opposite case, i.e., if $S_-(X) \neq S^-(X)$, X is said to be a soft covering based rough set.*

Example 3.7. *Let* $S = (U, C_G)$ *be a soft covering approximation space, where* $U = \{a, b, c, d, e, f, g, h\}$, $A = \{e_1, e_2, e_3, e_4, e_5\} \subseteq E$, $F(e_1) = \{a, b\}$, $F(e_2) = \{b, c, d\}$, $F(e_3) = \{e, f\}$, $F(e_4) = \{g\}$ *and* $F(e_5) = \{g, h\}$. *For* $X_1 = \{a, b, c\} \subseteq U$, *we have*

$$S_-(X_1) = \{F(e) : e \in A \wedge F(e) \subseteq X_1\} = \{a, b\}$$
$$S^-(X_1) = S_-(X_1) \cup \{Md_S(x) : x \in X_1 - S_-(X_1)\} = \{a, b, c, d\}$$

Thus, $S_-(X_1) \neq S^-(X_1)$ *and* X_1 *is a soft covering based rough set. For* $X_2 = \{e, f, g\} \subseteq U$, *we have*

$$S_-(X_2) = \{F(e) : e \in A \wedge F(e) \subseteq X_2\} = \{e, f, g\}$$
$$S^-(X_2) = S_-(X_2) \cup \{Md_S(x) : x \in X_2 - S_-(X_2)\} = \{e, f, g\}$$

Thus, $S_-(X_2) = S^-(X_2)$ *and* X_2 *is a soft covering based definable set.*

Theorem 3.8. *Let* $G = (F, A)$ *be a soft set over* U, $S = (U, C_G)$ *be a soft covering approximation space and* $X, Y \subseteq U$. *Then the soft covering lower and upper approximations have the following properties:*

1. $S_-(U) = S^-(U) = U$
2. $S_-(\emptyset) = S^-(\emptyset) = \emptyset$
3. $S_-(X) \subseteq X \subseteq S^-(X)$
4. $X \subseteq Y \Longrightarrow S_-(X) \subseteq S_-(Y)$
5. $S_-(S_-(X)) = S_-(X)$
6. $S^-(S^-(X)) = S^-(X)$
7. $\forall e \in A, \ S_-(F(e)) = F(e)$
8. $\forall e \in A, \ S^-(F(e)) = F(e)$

Proof. From Definition 3.4 and Definition 3.5, we can easily prove that properties 1, 2, 3, 7 and 8.

4. Since $X \subseteq Y$, $\forall x \in S_-(X)$, $\exists e \in A$ such that $x \in F(e)$ and $F(e) \subseteq X \subseteq Y$. According to Definition 3.5, $F(e) \subseteq S_-(Y)$, so $x \in S_-(Y)$. Hence $S_-(X) \subseteq S_-(Y)$.

5. According to property 3, $S_-(S_-(X)) \subseteq S_-(X)$. $\forall x \in S_-(X)$, $\exists e \in A$ such that $x \in F(e)$ and $F(e) \subseteq X$. According to property 6, $S_-(F(e)) \subseteq S_-(X)$. Since $S_-(F(e)) = F(e)$, $F(e) \subseteq S_-(X)$, we have $x \in S_-(S_-(X))$. Therefore, $S_-(X) \subseteq S_-(S_-(X))$. Thus $S_-(S_-(X)) = S_-(X)$.

6. According to property 3, $S^-(X) \subseteq S^-(S^-(X))$. $\forall x \in S^-(X)$, $\exists e \in A$ such that $x \in F(e)$ and $F(e) \subseteq S^-(X)$. Therefore, $S_-(S^-(X)) = S^-(X)$. According to property 3, $S^-(S^-(X)) = S_-(S^-(X))$. So $\forall x \in S^-(S^-(X))$, $x \in S^-(X)$. Thus $S^-(S^-(X)) = S^-(X)$. □

Theorem 3.9. *Let* $G = (F, A)$ *be a soft set over* U, $S = (U, C_G)$ *be a soft covering approximation space and* $X, Y \subseteq U$. *Then the soft covering lower and upper approximations do not have the following properties:*

1. $S_-(X \cap Y) = S_-(X) \cap S_-(Y)$
2. $S^-(X \cup Y) = S^-(X) \cup S^-(Y)$
3. $X \subseteq Y \Longrightarrow S^-(X) \subseteq S^-(Y)$
4. $S_-(X) = -(S^-(-X))$
5. $S^-(X) = -(S_-(-X))$
6. $S_-(-S_-(X)) = -S_-(X)$
7. $S^-(-S^-(X)) = -S^-(X)$

The symbol "$-$" denotes the complement of the set. The following examples show that the equalities mentioned above do not hold.

Example 3.10. *Let* $S = (U, C_G)$ *be a soft covering approximation space, where* $U = \{a, b, c, d, e, f, g\}$, $A = \{e_1, e_2, e_3, e_4\} \subseteq E$, $F(e_1) = \{a, b, c\}$, $F(e_2) = \{b, c, d\}$, $F(e_3) = \{d, e\}$ *and* $F(e_4) = \{f, g\}$. *Suppose that* $X = \{a, b, c, d\} \subseteq U$ *and* $Y = \{d, e\} \subseteq U$.

1. $S_-(X) = \{a, b, c, d\}$, $S_-(Y) = \{d, e\}$, $S_-(X) \cap S_-(Y) = \{d\}$ and $S_-(X \cap Y) = \emptyset$. This shows that $S_-(X \cap Y) \neq S_-(X) \cap S_-(Y)$.

4. $-(S^-(-X)) = \{a, b, c\}$, $S_-(X) = \{a, b, c, d\}$. This shows that $-(S^-(-X)) \neq S_-(X)$.

5. $-(S_-(-X)) = \{a, b, c, d, e\}$, $S^-(X) = \{a, b, c, d\}$. This shows that $-(S_-(-X)) \neq S^-(X)$.

6. $S_-(-S_-(X)) = \{f, g\}$, $-S_-(X) = \{e, f, g\}$. This shows that $S_-(-S_-(X)) \neq -S_-(X)$.

7. $S^-(-S^-(X)) = \{d, e, f, g\}$, $-S^-(X) = \{e, f, g\}$. This shows that $S^-(-S^-(X)) \neq -S^-(X)$.

Example 3.11. *Let $S = (U, C_G)$ be a soft covering approximation space and (F, A) be a soft set given in the above example. Suppose that $X = \{a, b\} \subseteq U$ and $Y = \{c, d\} \subseteq U$.*

2. $S^-(X) = \{a, b, c, d\}$, $S^-(Y) = \{a, b, c, d, e\}$, $S^-(X) \cup S^-(Y) = \{a, b, c, d, e\}$ and $S^-(X \cup Y) = \{a, b, c, d\}$. This shows that $S^-(X \cup Y) \neq S^-(X) \cup S^-(Y)$.

Example 3.12. *Let $S = (U, C_G)$ be a soft covering approximation space and (F, A) be a soft set given in the above example. Suppose that $X = \{d\} \subseteq U$ and $Y = \{b, c, d\} \subseteq U$.*

3. $S^-(X) = \{d, e\}$ and $S^-(Y) = \{b, c, d\}$. This shows that $S^-(X) \nsubseteq S^-(Y)$.

In this paper we study a new concept called as soft covering based rough set and its basic properties and also present an example in medicine which states the prostate cancer risk. This new concept is a special example of soft rough sets. We need to define soft covering based rough set since it is more useful for our medicine application.

4. Multicriteria Group Decision Making

Feng [8] applied soft rough sets to multicriteria group decision making problem. The soft rough set based decision making method in [8] can be summarized as follows:

1. Step: Input the original description soft set $G = (F, A)$.

2. Step: Construct the evaluation soft set $G_1 = (V, T)$ using the primary evaluation results of the expert group T.

3. Step: Compute soft rough approximations and then obtain the soft sets $G_{1_-} = (V_-, T)$ and $G_1^- = (V^-, T)$.

4. Step: Compute the corresponding fuzzy sets μ_{G_1}, $\mu_{G_{1_-}}$ and $\mu_{G_1^-}$ of the soft sets $G_1 = (V, T)$, $G_{1_-} = (V_-, T)$ and $G_1^- = (V^-, T)$.

5. Step: Construct the fuzzy soft set $G_F = (\alpha, C)$ using the fuzzy soft sets $\mu_{G_{1_-}}$, μ_{G_1} and $\mu_{G_1^-}$.

6. Step: Input the weighting vector R and compute the weighted evaluation values $v(u_k)$ of each alternative $u_k \in U$. Then rank all the alternatives according to their weighted evaluation values; one can select any of the objects with the largest weighted evaluation value as the most preferred alternative.

We use this method to help to doctors for diagnosing the prostate cancer risk. In our work we use soft covering approximations instead of soft rough approximations in 3. Step. We may expect to gain much more useful information with the help of soft covering approximations.

5. An application of soft covering approximations to diagnose the prostate cancer risk

Feng [8] gave an application of soft rough approximations in multicriteria group decision making problems and his method enables us to select the optimal object in more reliable manner. In this work, we used soft covering approximations at Feng's method and aim to obtain the optimal choice for applying biopsy to the patients with prostate cancer risk by using the PSA, PV and age data of patients. We determine the risk of prostate cancer. Our aim is to help the doctor to determine whether the patient needs biopsy or not.

We choose 56 patients from Selcuk University Meram Medicine Faculty with prostate complaint as the data.

U	PSA	PV	Age
u_5	100	44	58
u_{10}	7,9	41	54
u_{15}	38	23	59
u_{20}	90	55	67
u_{25}	100	62	60
u_{30}	50	40	80
u_{35}	6,04	33	58
u_{40}	100	47	75
u_{45}	11,5	50	65
u_{50}	100	46	83
u_{55}	15	55	71

Table 1: The input PSA, PV and Age values of several patients

1. Step Let $U = \{u_k : u_1 = 1, u_2 = 2, \ldots, u_{56} = 56, k = 1, \ldots, 56\}$ be the universe and $A = \{PSA, Age, PV\}$ be the parameter set. Now we obtain parametrized subsets of the universe. The patients whose PSA in blood is 38 and higher than 38, age is 54 and older than 54 and PV is 20 and bigger than 20 are chosen with doctor's suggestion. We generate the soft set $G = (F, A)$ which is based on PSA, age and PV values of patients over U. Since $G = (F, A)$ is a covering soft set, $S = (U, C_G)$ is the soft covering approximation space. Consider the following:

$$F(PSA) = \{4, 5, 8, 12, 13, 15, 16, 19, 20, 23, 25, 27, 28, 30, 31, 33\} \cup$$

$$\{37, 38, 40, 43, 46, 50, 52, 54, 56\}$$

$$F(Age) = \{1, \ldots, 20, 22, \ldots, 40, 43, \ldots, 56\}$$

$$F(PV) = \{1, \ldots, 56\}$$

u_k	PSA	PV	Age
u_5	1	1	1
u_{10}	0	1	1
u_{15}	1	1	1
u_{20}	1	1	1
u_{25}	1	1	1
u_{30}	1	1	1
u_{35}	0	1	1
u_{40}	1	1	1
u_{45}	0	1	1
u_{50}	1	1	1
u_{55}	0	1	1

Table 2: Tabular Presentation of the Soft Set

2. Step Let $T = \{T_{d_1}, T_{d_2}, T_{d_3}\}$ be the specialist doctors group who evaluate the patients with respect to the parameters PSA, PV and age. Now we generate the soft set $G_1 = (V, T)$ over U by using the first evaluation of the results of specialist doctors group T. Each specialist need to examine all the objects in $U = \{u_k : u_1 = 1, u_2 = 2, \ldots, u_{56} = 56, k = 1, \ldots, 56\}$ and will be requested to only point out "the optimal alternatives" as his/her evaluation result. Hence each specialist's primary evaluation results are subsets of 56 patients from Selcuk University Meram Medicine Faculty with prostate complaint

as the data. For simplicity, we assume that the evaluations of these specialists in $T = \{T_{d_1}, T_{d_2}, T_{d_3}\}$ are of the same importance.

$$X_{d_1} = V(T_{d_1}) = \{4, 5, 8, 12, 13, 15, 16, 19, 20, 23, 25, 27, 28, \ldots, 56\}$$

$$X_{d_2} = V(T_{d_2}) = \{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, \ldots, 56\}$$

$$X_{d_3} = V(T_{d_3}) = \{1, \ldots, 8, 11, \ldots, 16, 19, 20, 22, 23, 24, 25, \ldots, 56\}$$

3. Step Now, we show how to use soft covering based rough sets to support this group decision making process. We consider the soft rough approximations of the specialist T_{d_i}'s primary evaluation result X_{d_i} with respect to the soft approximation space. Let us choose $S = (U, C_G)$ as the soft covering approximation space. By using the soft covering approximations, we obtain two other soft sets $G_{1_-} = (V_-, T)$ and $G_1^- = (V^-, T)$ over U, where

$$V_- : T \rightarrow P(U), V_-(T_{d_i}) = S_-(X_{d_i}), i = 1, 2, 3$$

$$V^- : T \rightarrow P(U), V^-(T_{d_i}) = S^-(X_{d_i}), i = 1, 2, 3$$

The soft set G_1^- can be seen as the evaluation result of the specialist doctor group T with low confidence while the soft set G_{1_-} represents the evaluation result of the specialist doctor group T with high confidence.

Now we obtain the soft covering upper and lower approximations of three specialist doctors first eveluation results to get the soft sets G_1^- and G_{1_-}. Consider

$$V_-(T_{d_1}) = S_-(X_{d_1}) = F(PSA)$$

$$V_-(T_{d_2}) = S_-(X_{d_2}) = \emptyset$$

$$V_-(T_{d_3}) = S_-(X_{d_3}) = F(PSA)$$

$$V^-(T_{d_1}) = S^-(X_{d_1}) = F(PSA) \cup F(age) = F(age)$$

$$V^-(T_{d_2}) = S^-(X_{d_2}) = F(PSA) \cup F(age) \cup F(PV) = F(PV) = U$$

$$V^-(T_{d_3}) = S^-(X_{d_3}) = F(PSA) \cup F(age) = F(age)$$

4. Step The results of the specialist three doctors evaluation can be formulized in terms of fuzzy sets. For $X \subseteq U$, the characteristic function of X is denoted by χ_X. Based on the soft set $G_1 = (V, T)$, we can define fuzzy set μ_{G_1} in U by

$$\mu_{G_1} : U \rightarrow [0, 1], \ u_k \rightarrow \mu_{G_1}(u_k) = \frac{1}{3} \sum_{i=1}^{3} \chi_{V(T_{d_i})}(u_k)$$

In a similar way, we can get the fuzzy sets $\mu_{G_{1_-}}$ and $\mu_{G_1^-}$ as follows:

$$\mu_{G_{1_-}} : U \rightarrow [0, 1], u_k \rightarrow \mu_{G_{1_-}}(u_k) = \frac{1}{3} \sum_{i=1}^{3} \chi_{V_-(T_{d_i})}(u_k)$$

and

$$\mu_{G_1^-} : U \rightarrow [0, 1], u_k \rightarrow \mu_{G_1^-}(u_k) = \frac{1}{3} \sum_{i=1}^{3} \chi_{V^-(T_{d_i})}(u_k)$$

where $V_-(T_{d_i}) = S_-(X_{d_i})$, $V^-(T_{d_i}) = S^-(X_{d_i})$ and $k = 1, \ldots, 56, i = 1, 2, 3$.

From $G_{1_-} \subseteq G_1 \subseteq G_1^-$, it is easy to see that $\mu_{G_{1_-}} \subseteq \mu_{G_1} \subseteq \mu_{G_1^-}$. These fuzzy sets $\mu_{G_{1_-}}, \mu_{G_1}, \mu_{G_1^-}$ can be interpreted as some vague concepts like "the patients under high risk" , "the patients under middle risk" and "the patients under low risk" respectively.

By this way we obtain the fuzzy sets $\mu_{G_1}, \mu_{G_{1_-}}, \mu_{G_1^-}$ by the memberships we get above. For example, we obtain these fuzzy sets for first patient,

$$\mu_{G_1}(1) = \frac{1}{3}, \ \mu_{G_{1_-}}(1) = 0, \ \mu_{G_1^-}(1) = 1$$

u_k	$\mu_{G_1^-}$	μ_{G_1}	$\mu_{G_{1_-}}$
u_5	1	1	$\frac{2}{3}$
u_{10}	1	$\frac{1}{3}$	0
u_{15}	1	1	$\frac{2}{3}$
u_{20}	1	1	$\frac{2}{3}$
u_{25}	1	1	$\frac{2}{3}$
u_{30}	1	1	$\frac{2}{3}$
u_{35}	1	$\frac{1}{3}$	0
u_{40}	1	1	$\frac{2}{3}$
u_{45}	1	$\frac{2}{3}$	0
u_{50}	1	1	$\frac{2}{3}$
u_{55}	1	$\frac{2}{3}$	0

Table 3: Tabular presentation of the membership of several patients

5. Step Let $C = \{L, M, H\}$ be a set of parameters where L, M and H denote "under low risk", "under middle risk" and "under high risk" respectively. Now we can define a fuzzy soft set $G_F = (\alpha, C)$ over U, where $\alpha : C \to I^U$ is given by $\alpha(L) = \mu_{G_1^-}$, $\alpha(M) = \mu_{G_1}$ and $\alpha(H) = \mu_{G_{1_-}}$.

6. Step Given a weighting vector $R = (r_L, r_M, r_H)$ such that $r_L + r_M + r_H = 1$,

$$v(u_k) = r_L.\alpha(L)(u_k) + r_M.\alpha(M)(u_k) + r_H.\alpha(H)(u_k)$$

is called the weighted evaluation value of the alternative $u_k \in U$, $k = 1, \ldots, 56$. Assume that the weighting vector $R = (0.25, 0.5, 0.25)$.

Finally, we can select the object u_p such that $v(u_p) = max\{v(u_k) : k = 1, \ldots, 56\}$ as the patient with the highest cancer risk. When we rank all the alternatives according to their weighted evaluation values, we can select any of the objects with the largest weighted evaluation value as the highest cancer risk.

The results are as follow:

$5 \approx 8 \approx 12 \approx 13 \approx 15 \approx 16 \approx 19 \approx 20 \approx 23 \approx 25 \approx 27 \approx 28 \approx 30$

$\approx 31 \approx 33 \approx 37 \approx 38 \approx 40 \approx 43 \approx 46 \approx 50 \approx 52 \approx 54 = 0,91 >$

$4 \approx 29 \approx 56 = 0.75 > 2 \approx 3 \approx 6 \approx 7 \approx 11 \approx 14 \approx 22 \approx 24 \approx 34$

$\approx 36 \approx 39 \approx 44 \approx 45 \approx 47 \approx 48 \approx 51 \approx 55 = 0,58 > 1 \approx 9 \approx 10 \approx 18$

$\approx 26 \approx 35 \approx 49 \approx 53 = 0.41 > 17 \approx 21 \approx 32 \approx 41 \approx 42 = 0,25$

In 6th Step, we obtained the set of weighted evaluation values $\{0.91, 0.75, 0.58, 0.41, 0.25\}$ for all patients. By using these values and in the light of expert doctor's suggestions we get rules as follow.

RULE-1: If a patient has 0.91 as a weighted evaluation value, then this patient is under very high degree cancer risk.

RULE-2: If a patient has 0.75 as a weighted evaluation value, then this patient is under high degree cancer risk.

RULE-3: If a patient has 0.58 as a weighted evaluation value, then this patient is under middle degree cancer risk.

RULE-4: If a patient has 0.41 as a weighted evaluation value, then this patient is under low degree cancer risk.

RULE-5: If a patient has 0.25 as a weighted evaluation value, then this patient is under very low degree cancer risk. Now we can give the rule sets:

$R_1 = \{5, 8, 12, 13, 15, 16, 19, 20, 23, 25, 27, 28, 30, 31, 33, 37, 38, 40, 43, 46, 50, 52, 54\}$

$R_2 = \{4, 29, 56\}$

$R_3 = \{2, 3, 6, 7, 11, 14, 22, 24, 34, 36, 39, 44, 45, 47, 48, 51, 55\}$

$R_4 = \{1, 9, 10, 18, 26, 35, 49, 53\}$

$R_5 = \{17, 21, 32, 41, 42\}$

u_k	L	M	H	$v(u_k)$
u_5	1	1	$\frac{2}{3}$	0,91
u_{10}	1	$\frac{1}{3}$	0	0,41
u_{15}	1	1	$\frac{2}{3}$	0,91
u_{20}	1	1	$\frac{2}{3}$	0,91
u_{25}	1	1	$\frac{2}{3}$	0,91
u_{30}	1	1	$\frac{2}{3}$	0,91
u_{35}	1	$\frac{1}{3}$	0	0,41
u_{40}	1	1	$\frac{2}{3}$	0,91
u_{45}	1	$\frac{2}{3}$	0	0,58
u_{50}	1	1	$\frac{2}{3}$	0,91
u_{55}	1	$\frac{2}{3}$	0	0,58

Table 4: The weighted evaluation value of some patients

6. Conclusion

In this work we get inspired from Feng's multicriteria group decision making problem and devised a prediction system to diagnose the prostate cancer risk. In this method the biopsy must be applied to the patients under very high and high degree cancer risk. The biopsy is unnecessary for the patients under middle degree cancer risk but these patients should be followed by the expert doctor. Either biopsy or doctor's follow are unnecessary for the patients under low and very low degree prostate cancer risk. Therefore the biopsy is necessary for the patients who are in the set of R_1 and R_2. That is in our system the biopsy should be applied to 26 patients. But, in medicine faculty the biopsy is applied to all 56 patients and it is seen that only 23 of them are cancer. Our aim is to help the doctor to decide whether the patient needs biopsy or not.

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