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An Application of Taguchi's Methods Reconsidered

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This paper was presented at the Annual Meeting of the American Statistical Association in August 1989 in Washington, DC.

The author would like to thank Professor Peter W.M. John for bringing Barker's paper to her attention. This research was completed in 1989 while the author was Professor of Statistics at the University of Texas at San Antonio, which supported the work through a Faculty Research Award.

An Application of Taguchi's Methods Reconsidered

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Abstract

Two aspects of Taguchi's methods for analyzing parameter design experiments that can be improved upon are considered. It is shown how using interaction graphs instead of marginal graphs, and how using the sample variance instead of a signal-to-noise ratio, can lead to product designs that are more robust to variation. The advantages of the alternative analysis will be illustrated by reanalyzing a case study considered by Barker (1986).

Key Words: quality assurance, parameter design, interaction, signal-to-noise ratio, robust

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1 Introduction

Two aspects of Taguchi's methods for analyzing parameter design experiments that can be improved upon are considered: 1) the use of interaction graphs instead of marginal graphs, and 2) the use of the sample variance and the sample mean instead of a signal-to-noise ratio and the sample mean when the objective is to make the response as large as possible with minimum variability. Barker's (1986) case study is reanalyzed using these methods, and parameter settings that should be more robust to variation are obtained. Both topics illustrate applications of important statistical concepts that can be readily understood by quality practitioners without sophisticated statistical backgrounds.

Examination of the alias chains and their sums of squares in an analysis of variance (ANOVA) on the sample mean $\overline{y}$ of Barker's data shows that there is strong evidence of the presence of an interaction. It is shown why the graph of an interaction is more appropriate than the marginal graphs in the interpretation of the problem as well as in the selection of the best levels of the factors. As indicated by Hunter (1985), Ryan (1988), and Gunter (1987), it has been a matter of concern for statisticians that practitioners of the Taguchi methods of quality assurance have often not considered the effects of interactions. Tribus and Szonyi (1989) point out that interactions are to be expected in Barker's experiment, although they do not find them.

A number of statisticians have expressed a preference for the use of the sample mean $\overline{y}$ and the sample variance s^2 instead of the sample mean $\overline{y}$ and a signal-to-noise ratio as proposed by Taguchi for parameter design (see Box, 1988; Leon, Shoemaker and Kackar, 1987; Pignatiello and Ramberg, 1985). It is shown why, in Barker's example, using the variance instead of a signal-to-noise ratio should lead to a more robust product design.

Barker's case study is briefly reviewed in the second section. Section 3 shows that there is strong evidence of the presence of an interaction, and sections 4 and 6 show why an interaction graph is more appropriate than marginal graphs in obtaining information on the problem and in the selection of the best levels of the factors. Section 5 indicates why use of the sample variance is preferable to use of a signal to noise ratio to reduce variability. The results of the analyses of the interactions and of the sample variance are used in section 6 to obtain a product design that should be more robust to variation than Barker's.

2 Barker's Example

Barker (1986) gave an industrial application of the Taguchi methods of parameter design. He examined the butterfly, a small plastic component in the carburetor of a lawnmower engine. The main objective of the study was to obtain the factor settings that produce *consistently* (i.e., with low variability) *large* butterfly breaking strengths, so that the product design is robust to variation.

The six factors in Barker's example will be labeled as follows:

LABEL	FACTOR	LEVELS		
A	Feed rate(Grams/Min)	1000	1200	1400
B	First screw RPM	400	440	480
C	Second screw RPM	850	900	950
D	Gate size (thousands from nominal)	-30	0	+30
E	First temperature($^\circ F$)	280	320	360
F	Second temperature($^\circ F$)	320	360	400

Each one of these factors was considered at three levels, and 27 runs using various combinations of levels of the factors were used. The 27 runs, which are shown in Table 1, correspond to a 3^{6-3}_{III} design using columns 1,2,5,9,10 and 12 (factors A,B,C,D=ABC, E=AB^2C^2 and F=AB^2C respectively) of the $L_{27}(3^{13})$ orthogonal array. For each one of the 27 runs, 18 observations (in an L_{18} orthogonal array) were made, from which the sample mean $\overline{y}$, the sample

Table 1. Results of Barker's Experiment

Run	A	B	C	D	E	F	$\bar{y}$	s	SN
1	-1	-1	-1	-1	-1	-1	87.4	31.5	35.9
2	-1	-1	0	0	0	0	115.6	24.6	40.5
3	-1	-1	1	1	1	1	106.2	36.3	36.8
4	-1	0	-1	0	0	1	101.5	34.6	37.3
5	-1	0	0	1	1	-1	117.6	35.4	39.1
6	-1	0	1	-1	-1	0	115.2	24.3	40.5
7	-1	1	-1	1	1	0	131.1	30.7	41.4
8	-1	1	0	-1	-1	1	93.9	35.4	36.8
9	-1	1	1	0	0	-1	134.3	35.5	40.8
10	0	-1	-1	0	1	0	111.6	21.9	40.5
11	0	-1	0	1	-1	1	108.6	30.5	39.2
12	0	-1	1	-1	0	-1	111.9	29.3	40.0
13	0	0	-1	1	-1	-1	105.7	28.0	39.3
14	0	0	0	-1	0	0	118.3	20.1	41.1
15	0	0	1	0	1	1	133.1	34.0	41.5
16	0	1	-1	-1	0	1	104.1	33.2	38.8
17	0	1	0	0	1	-1	144.5	35.4	42.3
18	0	1	1	1	-1	0	133.5	21.4	42.2
19	1	-1	-1	1	0	1	82.5	41.6	33.2
20	1	-1	0	-1	1	-1	85.8	42.0	29.5
21	1	-1	1	0	-1	0	120.4	33.5	40.4
22	1	0	-1	-1	1	0	99.3	36.7	38.0
23	1	0	0	0	-1	1	99.1	41.5	36.2
24	1	0	1	1	0	-1	115.3	41.1	38.8
25	1	1	-1	0	-1	-1	96.2	40.9	34.1
26	1	1	0	1	0	0	121.6	36.7	40.4
27	1	1	1	-1	1	1	120.8	46.4	39.2

standard deviation s, and the "larger the better" signal-to-noise ratio $SN = -10\log[\sum(1/y_i^2)/n]$ of the breaking strength of the butterfly were computed (see Table 1). An analysis of variance (ANOVA) can be performed on each one of the three statistics $\overline{y}, s,$ and SN.

3 Interactions for $\overline{y}$

In reconsidering Barker's example, Tribus and Szonyi (1989) point out that the viscoelastic properties of plastic are very sensitive to temperature, so that one would expect to see an interaction between temperature and feed rate, or between temperature and dimensions (gate size). On the other hand, they state that Barker's use of the L_{27} orthogonal array *guarantees* that interactions will not be observed. As will be shown in this section, there is strong evidence of a $B \times E$ interaction in the data. The analysis that follows illustrates how, even in highly confounded experimental designs with variables at three levels, information on interactions can sometimes still be obtained.

The analysis of variance for $\overline{y}$ taking interactions into account is given in Table 2. The "source" column of Table 2 gives the alias chains, or confounding patterns, for the design. The alias chains are a consequence of Barker's particular choice of six (out of the thirteen) columns in the $L_{27}(3^{13})$ orthogonal array to correspond to the factors. The first six terms correspond to the main effects and the components of two-factor interactions that they are aliased with, and the next seven terms correspond to components of two-factor interactions that are aliased with each other.

In considering an analysis of variance for main effects only (as Barker did in Table VII), the bottom seven sums of squares for interaction given in Table 2 are pooled to give the residual sums of squares. It is apparent from Table 2 that the sums of squares for interaction are vastly different, ranging from $.02/27 = .0007$ to $8268.98/27 = 306.2585$, with six orders of magnitude of difference. It seems unreasonable to pool the interaction sums of squares to estimate a common variance; this inhomogeneity in the estimates of error provided by the interaction terms is evidence that there are interactions present (see Addelman, 1962, p. 34).

To test for inhomogeneity in the pooled components of the residual sum of squares, Bartlett's criterion can be used (see Davies, 1956, pp. 288-289). The test statistic is 23.73, which is highly significant when compared to $F_{.01,6,\infty} = 2.8$ at the $\alpha = 0.01$ level of significance. This indicates that there is strong evidence of the presence of interactions.

Table 2. ANOVA for $\bar{y}$, with factors and interactions

Source	d.f.	Sum of Squares
A=DE²	2	25,489.58/27
B=DF²	2	33,750.02/27
C=EF	2	44,015.54/27
D=AE² =BF²	2	22,766.96/27
E=AD=CF	2	12,150.02/27
F=BD=CE	2	20,643.86/27
AB=CD²	2	.02/27
AB²=CE²=CF²=EF²	2	.08/27
AC=BD²=BF=DF	2	.02/27
AC²=BE²	2	8,268.98/27
AD²=BC=AE=DE	2	.08/27
AF=CD=BE	2	8,217.62/27
AF²=BC²	2	.02/27
Residual	0	0
Total	26	175,302.80/27

On the other hand, the two large sums of squares for interaction in Table 2 correspond to the terms $AC^2 = BE^2$ and $AF = CD = BE$, which contain the two components BE^2 and BE of the $B \times E$ interaction. In order to test the hypothesis that it is the $B \times E$ interaction that accounts for the large sums of squares, the linear $\times$ linear component $B_L \times E_L$ of the $B \times E$ interaction is given in Table 3. The residual sum of squares is found by pooling the remaining components of the sums of squares corresponding to two-factor interactions. The variance σ^2 can be estimated by the residual mean square, whose extremely small value (compared to the other mean squares) gives unusually large values for the F-statistics. These indicate that all main effects and $B_L \times E_L$ are highly significant.

Table 3. ANOVA for $\overline{y}$, with factors and $B_L \times E_L$

Source	df	Sum of squares	Mean square	F-statistic
A	2	25,489.58/27	12,744.79/27	637,239.5**
B	2	33,750.02/27	16,875.01/27	843,750.5**
C	2	44,015.54/27	22,007.77/27	1,100,388.5**
D	2	22,766.96/27	11,383,48/27	569,174.0**
E	2	12,150.02/27	6,075.01/27	303,750.0**
F	2	20,643.86/27	10,321.93/27	516,096.5**
$B_L \times E_L$	1	16,486.56/27	16,486.56/27	824,328.0**
Residual	13	.26/27	.02/27	
Total	26	175,302.80/27		

**significant at $\alpha = .01$

The model corresponding to the ANOVA in Table 3 can be written in response surface notation (see Box and Draper, 1986) as

$$\overline{y}_i = \beta_0 + \sum_{j=1}^{6} \beta_j x_{ji} + \sum_{j=1}^{6} \beta_{jj} x_{ji}^2 + \beta_{25} x_{2i} x_{5i} + \epsilon_i \tag{1}$$

where x_{ji} is the i^{th} value of the j^{th} factor. In this notation, Factor A becomes Factor 1, Factor B becomes Factor 2, etc. Factor A in Table 3 combines the linear term $\beta_1 x_{1i}$ and the quadratic term $\beta_{11} x_{1i}^2$. The linear $\times$ linear component $B_L \times E_L$ of the $B \times E$ interaction corresponds to the term $\beta_{25} x_{2i} x_{5i}$. The least squares estimates of the β parameters and the corresponding t-statistics are shown in Table 4.

TABLE 4. Response surface parameter estimates and t-statistics

Variable	Parameter Estimate	t-statistic
Intercept	131.58	6967.4
x_1	-3.43	-535.2
x_2	8.33	1299.0
x_3	9.52	1483.5
x_4	4.74	439.6
x_5	5.00	779.4
x_6	-2.72	-423.5
x_1^2	-11.04	-994.0
x_2^2	-0.01	-1.0
x_3^2	0.006	0.5
x_4^2	-8.54	-769.0
x_5^2	-0.01	-1.0
x_6^2	-10.26	-923.5
x_2x_5	7.13	907.9

The model in equation (1) with the parameter estimates of Table 4 (excluding the terms in x_2^2, x_3^2 and x_5^2) reproduce exactly Barker's values of the mean response $\overline{y}$ given in Table 1. This explains the small value of the residual sum of squares in Table 3. The mean square for the linear $\times$ linear component of the $B \times E$ interaction shown in Table 3 is larger than the mean squares of all factors except B and C (and is comparable to that of B), so that the $B \times E$ interaction is as important as most of the factors themselves.

Barker mentions that a three-level experiment should be performed if the possibility of a second order (curved) relationship among the factors is anticipated. This line of reasoning leads to the consideration of interactions, since a second order model includes not only the quadratic terms x_i^2 and x_j^2 for factors i and j, but also the interaction term x_ix_j.

4 Graphs

This section illustrates how more information on a problem can be obtained from interaction graphs than from marginal graphs.

Figure 1a for the $B \times E$ interaction shows the average of the values of $\overline{y}$, the mean breaking strength of the butterfly, at the various combinations of B (First Screw RPM) and E (First Temperature) levels. Figures 1b and c give the marginal graphs of the average values of $\overline{y}$ at the three levels of B and E respectively.

Several practical conclusions may be drawn from the $B \times E$ interaction graph of Figure 1a, namely:

1) The breaking strength $\overline{y}$ of the butterfly diminishes with increasing temperature E if the first screw is at 400 RPM ($B = -1$), while $\overline{y}$ increases with E (and increases sharply with E) if the first screw is at 440 or 480 RPM (B=0 or 1). The butterfly breaking strength is fairly insensitive to changes in temperature E when $B = -1$, but is very sensitive when $B = 1$.

2) For each First Temperature level E, the butterfly breaking strength $\overline{y}$ increases with B (number of RPM's). However, at the low temperature $E = -1$ the breaking strength is fairly insensitive to changes in B, while at the high temperature $E = 1$ the breaking strength is very sensitive to changes in B.

These practical implications of the $B \times E$ interaction graph of Figure 1a are not even suspected from the B and E marginal graphs of Figures 1b and 1c, which are the only ones considered by Barker.

As shown in Figure 1a, there is a large difference in the average response $\overline{y}$ at levels 0 and 1 of E, but these differences average out to give the small differences in response of the marginal graph for E (Figure 1c). This explains why the $B \times E$ interaction has a larger mean square than factor E itself in the ANOVA of Table 3.

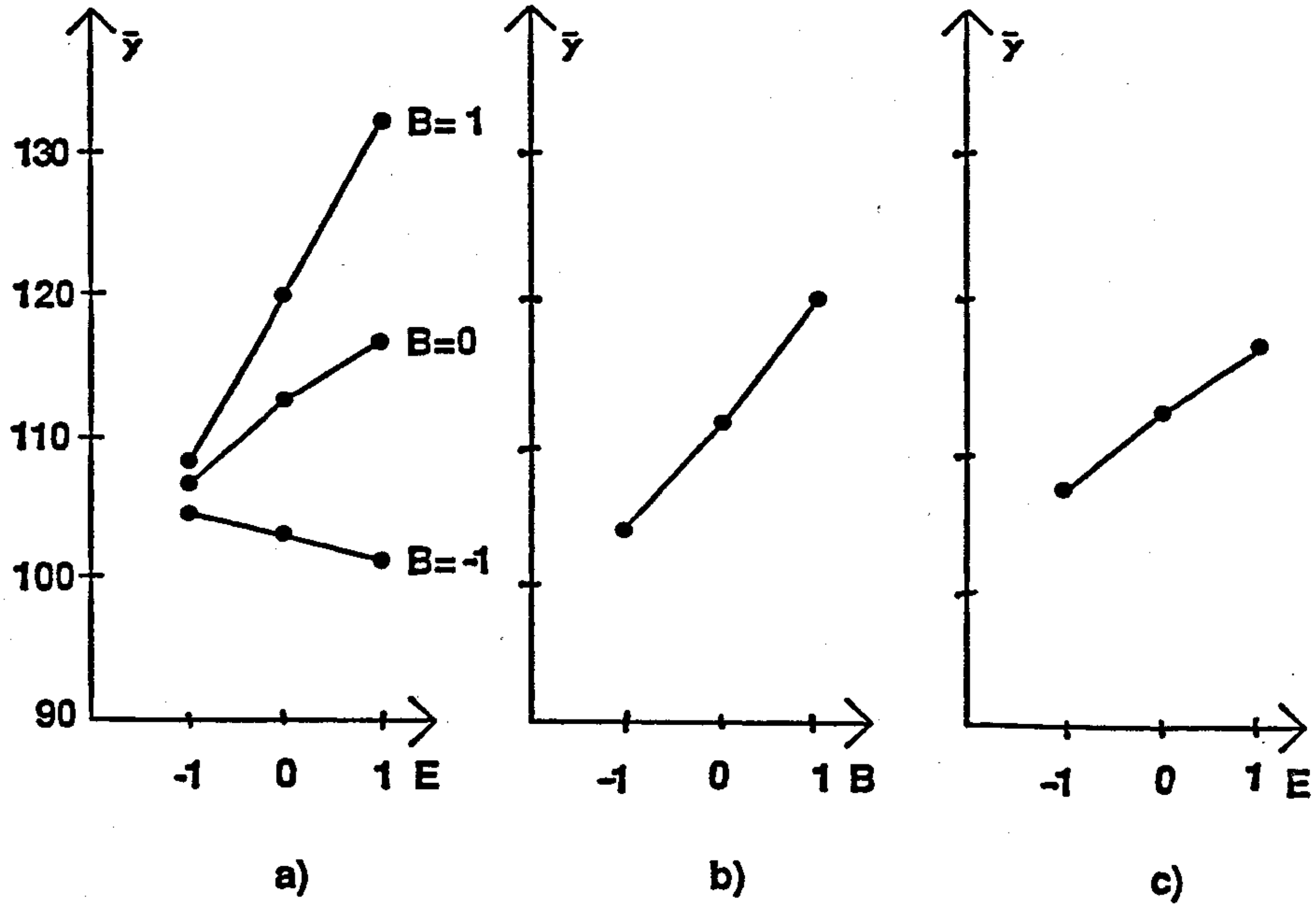

Figure 1. B X E interaction and B and E marginals for $\bar{y}$

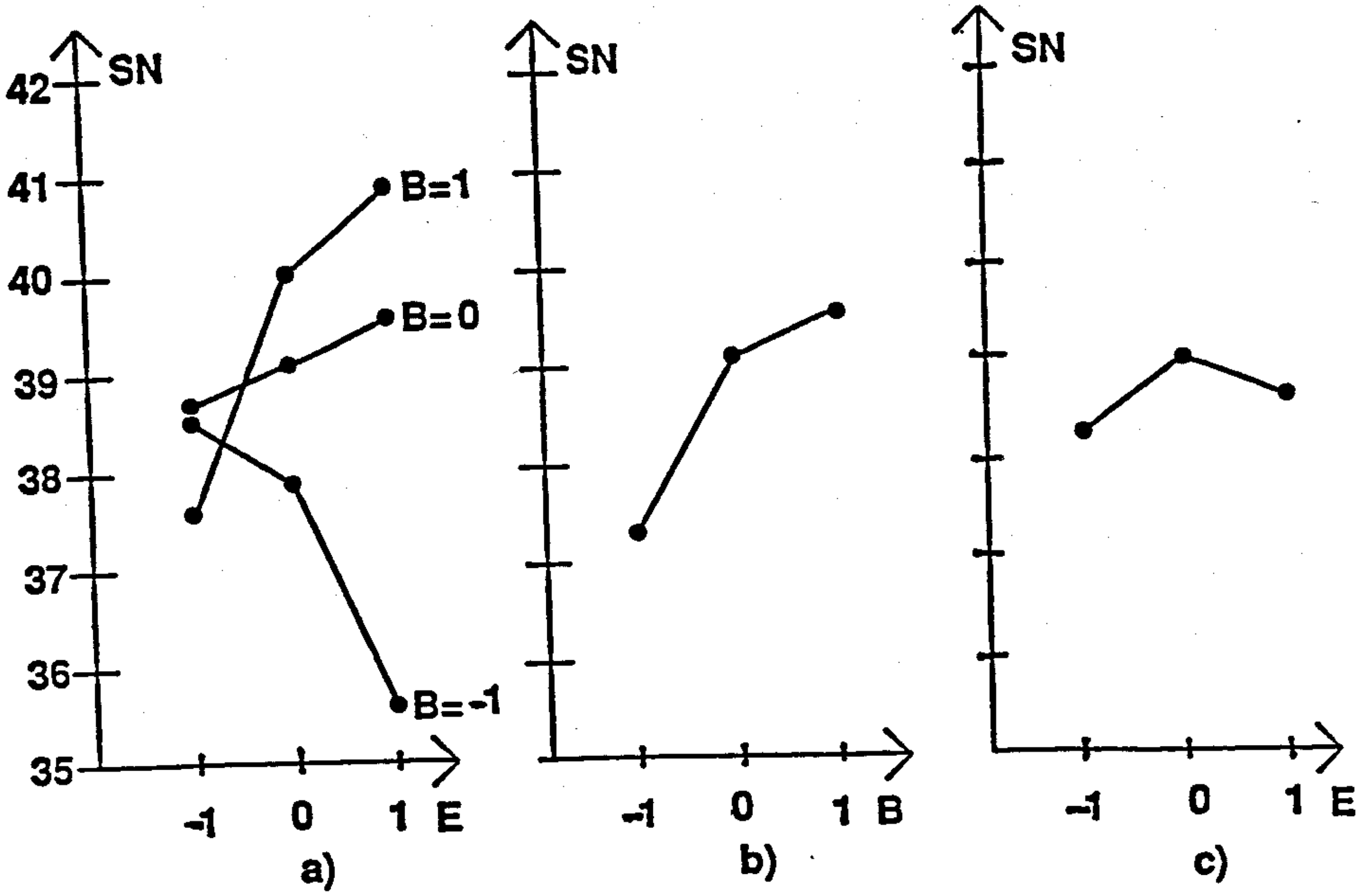

Figure 2. B X E interaction, and B and E marginals for SN

For the signal-to-noise ratio SN, Figure 2a shows the $B \times E$ interaction graph and Figures 2b and 2c show the B and E marginal graphs respectively.

5 Why s^2 instead of SN

As mentioned by Hunter (1985), Taguchi's method of parameter design for finding the best combination of levels of the factors consists of the following steps: 1) Find those factors that maximize the signal-to-noise ratio SN as a measure of variability. 2) Find those factors (when they exist) that do not have an effect on SN but that do affect the mean $\overline{y}$, and use them to adjust the response to target. These two steps are used to obtain consistently (i.e., with low variability) high (as the target) butterfly breaking strengths, so that the product design is robust to variation. It will be shown that the signal-to-noise ratio is not an appropriate measure of variability in Barker's example, so that the variance (or a function of it such as $\log s$) is more suitable for step 1. The factors affecting the sample mean $\overline{y}$ as a measure of location can then be used to adjust the response to target.

When Barker ran the confirmation experiment at the selected optimal levels, he found that "the variation in this verification experiment is still greater than we can live with" (p. 41); he then proceeded to tighten tolerances to reduce variation. Figure 2 shows the $B \times E$ interaction graph and the B and E marginal graphs for the "larger the better" signal-to-noise ratio SN. Note that they are just distortions of the corresponding graphs for $\overline{y}$ given in Figure 1. It can be shown that, for the other variables, the graphs of SN are also distortions of the graphs of $\overline{y}$. These pictorial results indicate that, in Barker's example, the signal-to-noise ratio SN is driven mainly by the "signal" $\overline{y}$, which is only slightly contaminated by noise. In fact, the correlation coefficient between $\overline{y}$ and SN is $r = 0.86$, which is highly significant ($p < 0.001$). This means that SN and $\overline{y}$ are both measures of location, and that the "larger the better" signal-to-noise ratio SN is not an appropriate measure of variability in Barker's example.

Barker's objective was not just to produce high butterfly breaking strengths, but to make them *consistently* high. The engineering goal of consistency (i.e., low variability, or robustness) can be more efficiently approached by using the variance, instead of a signal-to-noise ratio, as a statistical measure of variability. The statistic $\log s$ can be used as a measure of variability instead of the sample variance s^2 (or the standard deviation s) because the logarithmic transformation gives improved statistical properties (see Kackar 1985, p. 184).

6 Suggested Levels

The best levels of the factors should be found from the interaction graphs (instead of from the marginal graphs, which "average out" differences) when an interaction is significant, and from the marginal graphs when the corresponding interactions are not significant. Figure 2 illustrates the fact that the best factor levels from an interaction graph and from marginal graphs do not always coincide: from Figure 2a both B and E should be at level 1 to maximize SN, while Figures 2b and 2c indicate that B and E should be at levels 1 and 0 respectively. From Figure 1a, the third levels of B and E maximize $\overline{y}$ for the significant $B \times E$ interaction.

The numbers in Table 5 summarize the best levels of the six factors for $\overline{y}$ and $\log_{10} s$. The significance of the factors at the $\alpha = 0.05$ and $\alpha = 0.01$ levels in the corresponding analyses of variance are also indicated. Factors A and F should both be at level 0. Factors B, C and D are highly significant in the ANOVA for $\overline{y}$ but not significant in that of $\log s$, so that they should be at levels 1, 1 and 0 respectively so as to maximize $\overline{y}$. To try to reduce variability it seems reasonable to set factor E at its low level of -1, which is highly significant in the ANOVA for s and moderately so for that of $\log s$.

Table 5. Best levels and significance of factors for $\overline{y}$ and $\log s$.

	$\overline{y}$	$\log s$
A	0**	0**
B	1**	-1
C	1**	-1
D	0**	-1
E	1**	-1*
F	0**	0**
$B \times E$	**	

$*\alpha = 0.05$

$**\alpha = 0.01$

The best levels selected by Barker using $\overline{y}$ and SN and those suggested here considering $\overline{y}$ and $\log s$ differ only in the setting of factor E (at level 1 in Barker with level -1 being proposed). An additional experiment should be performed at the levels 0, 1, 1, 0, -1 and 0 for factors A, B, C, D, E and F respectively, to confirm that the variability is reduced.

7 Summary

Two aspects of Taguchi's methods of parameter design in quality assurance that can be improved upon were considered. It was shown how use of interaction graphs instead of marginal graphs, and use of the variance instead of a signal-to-noise ratio, can lead to product designs that are more robust to variation. These methods were illustrated using Barker's case study.

There has been concern in the statistical community that users of the Taguchi methods of quality assurance have often not taken interactions into account. If factors are at three levels to be able to detect curved relationships, then inclusion of all second order terms implies consideration of interactions. Pignatiello and Ramberg's (1985) application of the Taguchi methods illustrates how, if interactions are present but are not taken into account, inaccurate conclusions may be drawn. When an interaction is significant, the graph of the interaction should be used instead of the marginal graphs to gain a better understanding of the problem as well as for determining the best levels of the factors in parameter design.

Barker's objective was to produce consistently high butterfly breaking strengths. In the pursuit of this engineering goal, it was shown that statistical considerations lead to the use of the sample variance instead of a signal-to-noise ratio as a measure of spread to achieve consistency (with the sample mean as a measure of location). It was pointed out that the "larger the better" signal-to-noise ratio is not appropriate because it measures location instead of spread.

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