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An Approach to Real-Time Position Estimation at Zero and Low Speed for a PM Motor Based on Saliency

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Abstract—This paper presents a magnetic saliency based position estimation approach for a PM motor fed by a voltage-source PWM inverter. The proposed real-time estimation algorithm detects the motor current harmonics and calculates the inductance matrix including the rotor position information. The position estimation can be performed every period of pulse-width-modulation or carrier cycle. An experimental system using an interior permanent magnet (IPM) synchronous motor has been constructed. Experimental results verifies that the position estimation within 10 degree in electrical angle is obtained at standstill and such an extremely low-speed as 1 r/min by the proposed approach.

I. INTRODUCTION

IN recent years, significant attention has been paid to removing rotor position sensors from ac motor drive systems with a position-closed loop [1]–[8]. The rotor position estimation methods of a Permanent Magnet (PM) synchronous motor can be classified into two groups;

1. based on the back electromotive force produced in stator windings [1][2], and
2. based on magnetic saliency [3]–[6].

Conventional position estimation based on detection of back electromotive force is suitable for middle- and high-speed applications. However, at low speed and standstill, the emf is too small to achieve accurate position estimation. On the other hand, the magnetic saliency based position estimation can be potentially employed at any speed including zero speed.

Lorenz, et al., have proposed a sophisticated approach to estimation of flux, position and speed at zero and low-speed in ac motors [4][5]. The approach is based on tracking the magnetic saliency via inverter-generated high-frequency signal injection with demodulation incorporating heterodyning and closed-loop observer.

This paper presents a magnetic saliency based position estimation approach for a PM motor fed by a voltage-source PWM inverter. The proposed real-time estimation algorithm detects the motor current harmonics and calculates the inductance matrix including the rotor position information. The position estimation can be performed every period of pulse-width-modulation, without any special signal injection.

An experimental system consisting of an Interior Permanent Magnet (IPM) synchronous motor and a voltage-source PWM inverter has been constructed and tested to

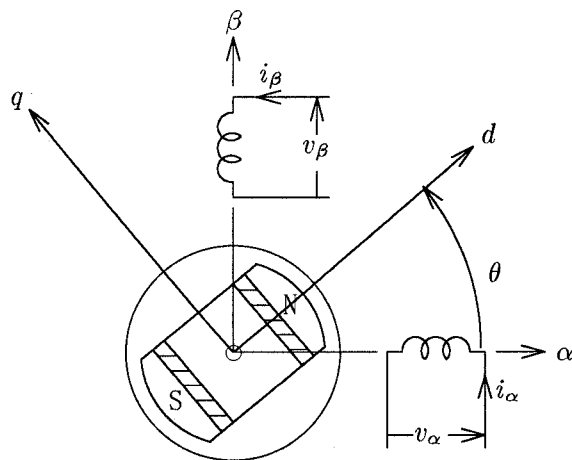


Fig. 1. Model of salient PM motor.

verify the effectiveness and versatility of the approach to position estimation at zero and low-speed. The IPM motor has the magnetic saliency of the q-axis inductance being larger than the d-axis inductance. Experimental results show that the proposed approach has the capability of estimating the rotor position of at standstill and at such an extremely low-speed as 1 r/min.

II. HARMONIC MODEL OF PM MOTOR

Fig.1 shows the simplified model of a PM motor with magnetic saliency. The equations describing the PM motor are given by

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = r \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} L_0 + L_1 \cos 2\theta & L_1 \sin 2\theta \\ L_1 \sin 2\theta & L_0 - L_1 \cos 2\theta \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d\theta}{dt} \left(2L_1 \begin{bmatrix} -\sin 2\theta & \cos 2\theta \\ \cos 2\theta & \sin 2\theta \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \phi_{mag} \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \right) \quad (1)$$

where,

$$L_0 = \frac{L_d + L_q}{2}, \quad L_1 = \frac{L_d - L_q}{2}. \quad (2)$$

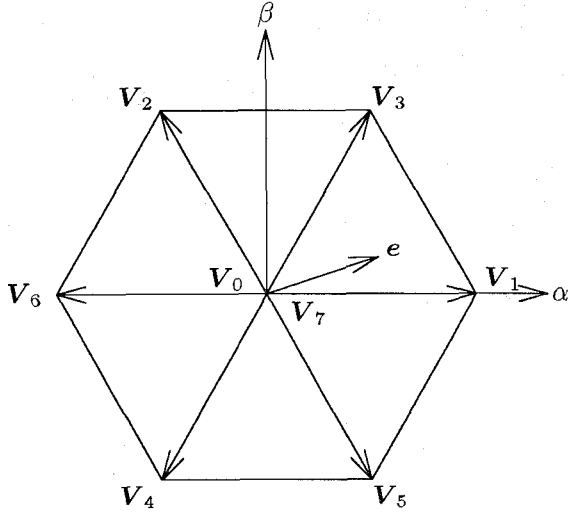


Fig. 2. Voltage vectors of inverter output.

The equations can be simplified by applying the space vector theory.

$$\mathbf{v} = r\mathbf{i} + L \frac{d\mathbf{i}}{dt} + \mathbf{e}_0 \quad (3)$$

Here, the inductance matrix is represented by

$$\mathbf{L} = \begin{bmatrix} L_0 + L_1 \cos 2\theta & L_1 \sin 2\theta \\ L_1 \sin 2\theta & L_0 - L_1 \cos 2\theta \end{bmatrix} \quad (4)$$

and it contains the rotor position information of the PM motor.

When the PM motor is driven by a voltage source PWM inverter, the motor current contains a small amount of harmonic current. The voltage and current vectors can be separated into the fundamental and harmonic components as follows:

$$\mathbf{v} = \bar{\mathbf{v}} + \tilde{\mathbf{v}} \quad (5)$$

$$\mathbf{i} = \bar{\mathbf{i}} + \tilde{\mathbf{i}}. \quad (6)$$

Because the resistive voltage drop can be neglected, the harmonic equivalent circuit can be approximated by the inductance.

$$\tilde{\mathbf{v}} = L \frac{d\tilde{\mathbf{i}}}{dt}. \quad (7)$$

The above equation indicates that the inductance matrix can be calculated from the harmonic components of the motor voltage and current vectors, so that the rotor position can be estimated.

III. HARMONIC VOLTAGE AND CURRENT VECTORS

A three-phase voltage source inverter can output eight discrete voltage vectors including two zero vectors, as shown in Fig.2. The PWM inverter has the capability of producing an output voltage vector by selecting the discrete voltage vectors selectively in a modulation period. The average voltage vector is

$$\mathbf{e} = \sum \zeta_k \mathbf{V}_k \quad (8)$$

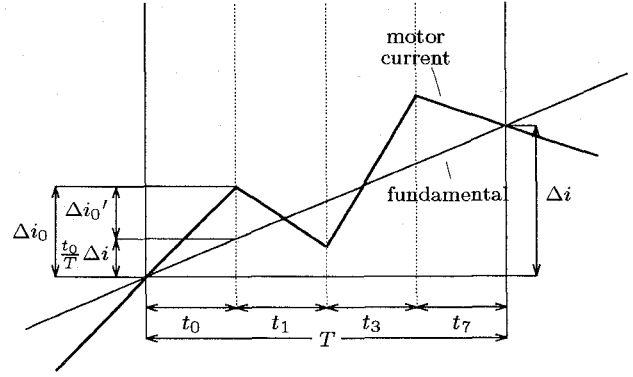


Fig. 3. Variation of motor current.

where, ζ_k means the time ratio of \mathbf{V}_k to the modulation period.

$$\zeta_k = \frac{t_k}{T} \quad (9)$$

Therefore, the harmonic voltage vector is equal to a difference between the inverter output voltage vector and the average output voltage vector.

$$\mathbf{V}_k' = \mathbf{V}_k - \mathbf{e} \quad (10)$$

On the other hand, applying the approximation of (7) implies the motor current changes linearly as shown in Fig.3. Since the motor current includes both the harmonic and fundamental components, the two components should be separated from each other. The current variation for modulation period is shown by

$$\Delta \mathbf{i} = \sum \Delta \mathbf{i}_k \quad (11)$$

where, $\Delta \mathbf{i}_0, \dots, \Delta \mathbf{i}_7$ are the current variations for the intervals of t_0, \dots, t_7 , respectively. Assuming that the fundamental component changes linearly in a modulation period, the harmonic component of the current variation can be separated by the following equation.

$$\Delta \mathbf{i}_k' = \Delta \mathbf{i}_k - \zeta_k \Delta \mathbf{i} \quad (12)$$

As a result, we can extract only harmonic components of the voltage and current vectors.

IV. ESTIMATION OF INDUCTANCE MATRIX AND ROTOR POSITION

From (7), a relationship between the harmonic components of the voltage vector and the current variation vector is given by

$$L \Delta \mathbf{i}_k' = \mathbf{V}_k' t_k. \quad (13)$$

Lumping the above equations together over a modulation period gives the following equation.

$$L \begin{bmatrix} \Delta \mathbf{i}_0' & \Delta \mathbf{i}_1' & \dots & \Delta \mathbf{i}_7' \end{bmatrix} = \begin{bmatrix} \mathbf{V}_0' t_0 & \mathbf{V}_1' t_1 & \dots & \mathbf{V}_7' t_7 \end{bmatrix} \quad (14)$$

The transposed equation is

$$\begin{bmatrix} \Delta i_0'^T \\ \Delta i_1'^T \\ \vdots \\ \Delta i_7'^T \end{bmatrix} L^T = \begin{bmatrix} V_0'^T t_0 \\ V_1'^T t_1 \\ \vdots \\ V_7'^T t_7 \end{bmatrix}. \quad (15)$$

Therefore, the inductance matrix can be calculated as follows:

$$\begin{aligned} L^T &= \begin{bmatrix} \Delta i_0'^T \\ \Delta i_1'^T \\ \vdots \\ \Delta i_7'^T \end{bmatrix}^{LM} \begin{bmatrix} V_0'^T t_0 \\ V_1'^T t_1 \\ \vdots \\ V_7'^T t_7 \end{bmatrix} \\ &= \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}^T \\ &= \begin{bmatrix} L_0 + L_1 \cos 2\theta & L_1 \sin 2\theta \\ L_1 \sin 2\theta & L_0 - L_1 \cos 2\theta \end{bmatrix}. \quad (16) \end{aligned}$$

Here, the "LM" in the above equation is the *left pseudo inverse operator*[9], and it performs the following calculation

$$H^{LM} = (H^T H)^{-1} H^T. \quad (17)$$

Therefore, the rotor position can be calculated from the inductance matrix.

$$2\theta = \tan^{-1} \frac{L_{12} + L_{21}}{L_{11} - L_{22}} \quad (18)$$

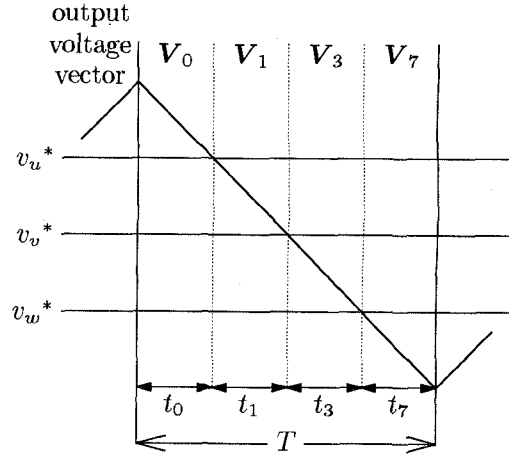
The inverse matrix of $H^T H$ must be existent, otherwise we can not calculate the left pseudo inverse matrix in (16). The matrix of $H^T H$ is calculated as follows:

$$\begin{aligned} &H^T H \\ &= \begin{bmatrix} \Delta i_{0\alpha}' & \Delta i_{1\alpha}' & \cdots & \Delta i_{7\alpha}' \\ \Delta i_{0\beta}' & \Delta i_{1\beta}' & \cdots & \Delta i_{7\beta}' \end{bmatrix} \begin{bmatrix} \Delta i_{0\alpha}' & \Delta i_{0\beta}' \\ \Delta i_{1\alpha}' & \Delta i_{1\beta}' \\ \vdots & \vdots \\ \Delta i_{7\alpha}' & \Delta i_{7\beta}' \end{bmatrix} \\ &= \begin{bmatrix} \sum \Delta i_{k\alpha}'^2 & \sum \Delta i_{k\alpha}' \cdot \Delta i_{k\beta}' \\ \sum \Delta i_{k\alpha}' \cdot \Delta i_{k\beta}' & \sum \Delta i_{k\beta}'^2 \end{bmatrix} \quad (19) \end{aligned}$$

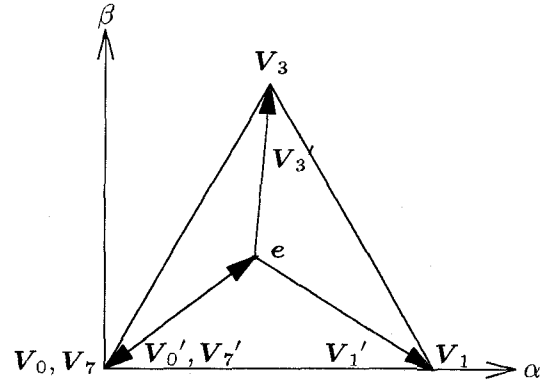
If all α or β components in $\Delta i_k'$ are zero, the determinant of $H^T H$ becomes zero. Similarly, if all of them are linearly dependent, the inverse matrix is not existent because $\det(H^T H) = 0$. Therefore, PWM control should be modified so that all of them are not linearly dependent vectors.

Fig.4(a) shows a timing chart in a conventional sinusoidal PWM scheme. Comparison of the voltage references, v_u^* , v_v^* and v_w^* , with a triangular carrier determines the output voltage vector. In this case, V_0 , V_1 , V_3 and V_7 are output vectors for t_0 , t_1 , t_3 and t_7 , respectively. The following equation expresses the average voltage vector during a modulation period.

$$e = \zeta_0 V_0 + \zeta_1 V_1 + \zeta_3 V_3 + \zeta_7 V_7 \quad (20)$$



(a) comparing voltage references with triangle



(b) voltage vectors

Fig. 4. Conventional sinusoidal PWM scheme.

where,

$$\zeta_0 + \zeta_1 + \zeta_3 + \zeta_7 = 1. \quad (21)$$

For example, when the average voltage vector exists on the α axis, i.e., $v_u^* = v_w^*$ in Fig.4(b), the time ratios are

$$\zeta_0 + \zeta_7 = \zeta_1 = \zeta_3 = \frac{1}{3}. \quad (22)$$

In this case, $\Delta i_0'$, $\Delta i_1'$, $\Delta i_3'$ and $\Delta i_7'$ are linearly independent each other except that $\Delta i_0'$ is linearly dependent on $\Delta i_7'$. As a result, we can calculate the inductance matrix and estimate the rotor position.

However, it is not necessary to output V_3 when the average voltage vector exists on the α axis, i.e., $v_u^* = v_w^*$. In this case, $\Delta i_1'$, $\Delta i_0'$ and $\Delta i_7'$ are linearly dependent each other, because V_1' has the opposite direction to V_0' or V_7' . Due to $\det(H^T H) = 0$, it is impossible to calculate the inductance matrix L .

In addition, $\zeta_0 + \zeta_7$ is close to unity, and ζ_1 and ζ_3 are very small, when the average voltage vector e exists near the origin in the α - β coordinates. In this area, the position estimation can not be performed with enough high

accuracy, because all components in $\Delta i_k'$ have zero or a small value. The fact mentioned above shows that the conventional sinusoidal PWM scheme is not applicable to the proposed position estimation.

V. MODIFICATION OF PWM CONTROL

In order to perform precise position estimation, the authors propose a novel PWM scheme using redundant voltage vectors. The average voltage vector during a modulation period can be expressed by

$$e = \sum s_k \zeta_k V_k \quad (23)$$

$$\sum s_k \zeta_k = 1 \quad (24)$$

Here, s_k , which takes a value of 1 or 0 during the modulation period, indicates whether V_k is selected as an output vector or not, respectively. Lumping the equations together gives the following equation.

$$\begin{bmatrix} e_\alpha \\ e_\beta \\ 1 \end{bmatrix} = \begin{bmatrix} s_0 V_{0\alpha} & s_1 V_{1\alpha} & \cdots & s_7 V_{7\alpha} \\ s_0 V_{0\beta} & s_1 V_{1\beta} & \cdots & s_7 V_{7\beta} \\ s_0 & s_1 & \cdots & s_7 \end{bmatrix} \begin{bmatrix} \zeta_0 \\ \zeta_1 \\ \vdots \\ \zeta_7 \end{bmatrix} \quad (25)$$

In the above equation, ζ_k has to be decided so that the PWM inverter outputs the average voltage vector e during the modulation period. No general solution to (25) exists because the number of unknown variables is more than the number of equations. However, introduction of a *right pseudo inverse matrix*[9] enables to solve ζ_k as follows:

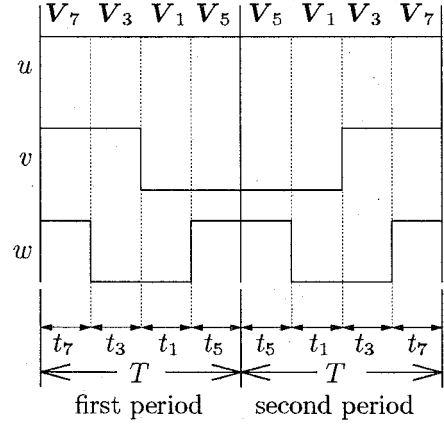
$$\begin{bmatrix} \zeta_0 \\ \zeta_1 \\ \vdots \\ \zeta_7 \end{bmatrix} = \begin{bmatrix} s_0 V_{0\alpha} & s_1 V_{1\alpha} & \cdots & s_7 V_{7\alpha} \\ s_0 V_{0\beta} & s_1 V_{1\beta} & \cdots & s_7 V_{7\beta} \\ s_0 & s_1 & \cdots & s_7 \end{bmatrix}^{\text{RM}} \begin{bmatrix} e_\alpha \\ e_\beta \\ 1 \end{bmatrix} \quad (26)$$

where,

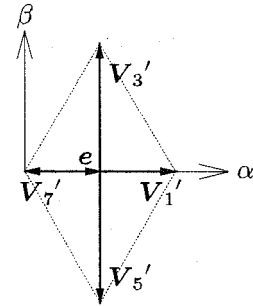
$$F^{\text{RM}} = F^T (FF^T)^{-1} \quad (27)$$

As the result of the calculation, one can decide a suitable PWM pattern under any combination of V_k during the modulation period.

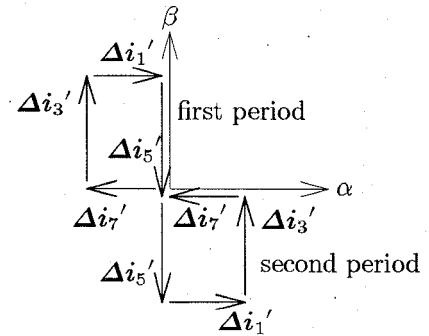
Fig.5(a) shows a PWM pattern in the case that the average voltage vector e exists on the α axis. Note that V_5 is selected during an interval of time, different from V_0 being selected during the corresponding interval in the conventional sinusoidal PWM scheme. As shown in Fig.5(b), V_3' and V_5' produce β -axis components in the harmonic current vectors while V_1' and V_7' produce α -axis components. Fig.5(c) shows the trajectory of the harmonic current vector. The trajectory starts from the origin toward the left at the beginning of the first modulation period. Since the harmonic current vector changes toward the direction near that of the corresponding harmonic voltage vector, the trajectory draws a rectangle and returns to the origin at the end of the first period. All of $\Delta i_7'$, $\Delta i_3'$, $\Delta i_1'$ and $\Delta i_5'$ are not linearly dependent vectors in the first period. The



(a) PWM pattern



(b) harmonic voltage vectors



(c) trajectory of harmonic current vector

Fig. 5. PWM scheme using redundant voltage vectors. The average voltage vector exists on the α axis.

TABLE I
IPM MOTOR PARAMETERS.

number of poles		4
resistance	r	15 Ω
d -axis inductance	L_d	125 mH
q -axis inductance	L_q	206 mH
rated power		100 W
rated speed		1500 rpm
modulation period	T	333 μ s
dc voltage	E_d	280 V

rotor position, therefore, can be estimated by the proposed algorithm, based on the harmonic component of the current variation, independent of the location of the average voltage vector. Note that the sum of the harmonic current vectors in the first modulation period shown in Fig.5 is not equal to zero vector. In order to compensate for the deviation, the sequence of selecting the voltage vectors should be modified as shown in the second modulation period.

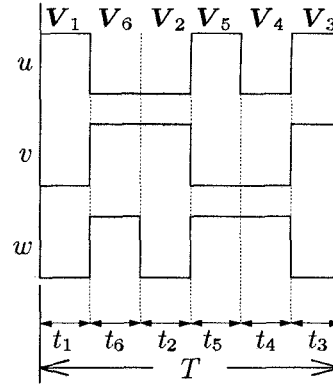
Fig.6 shows a PWM scheme, where the average voltage vector exists on the origin of the α - β coordinate. This situation corresponds to the case of position estimation at standstill. Each voltage vector is output for $1/6$ of the modulation period. Note that the inverter selects neither V_1 nor V_7 . The sequence is scheduled so that the trajectory of the harmonic current vectors starts from the origin at the beginning of the modulation period and returns to the origin at the end. Since all of the harmonic current vectors are not linearly dependent, the position estimation can be performed even at a standstill.

VI. EXPERIMENTAL RESULT

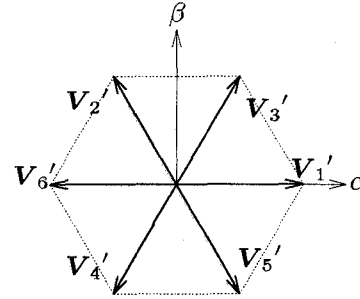
Fig.7 shows the configuration of the experimental system consisting of a V40 CPU and a digital signal processor (ADSP-2101). The V40 CPU supervises the whole operation of the system, and the DSP calculates the PWM pattern, the inductance matrix and the estimated rotor position.

The counter board is capable of synthesizing the PWM signal of the inverter, following the PWM pattern calculated by the DSP. The PWM signal in a modulation period consists of six voltage vectors with any order of them. The detected motor currents are transformed from three-phase to α - β components. The counter board also generates the sample/hold signals by which the transformed motor currents are held just before every switching so that the switching noises do not affect the signals being held. Each current variation in (11) is applied to an A/D converter through a differential amplifier. The detection circuit consisting of the sample/hold amplifiers and the differential amplifiers makes a significant contribution to precisely detecting each current variation.

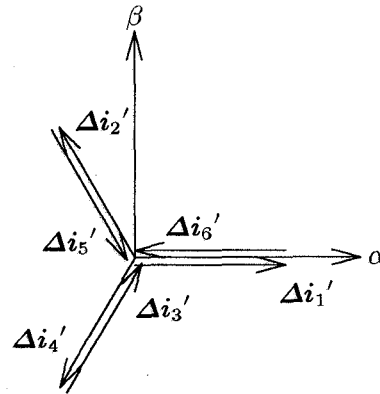
Table I shows the parameters of the PM motor used in the experimental system. Since the tested motor is an interior permanent magnet (IPM) motor, it has the saliency



(a) PWM pattern



(b) Trajectory of harmonic current vectors



(c) Trajectory of harmonic current vectors

Fig. 6. PWM scheme using redundant voltage vectors. The average voltage vector exists on the origin of the α - β coordinate.

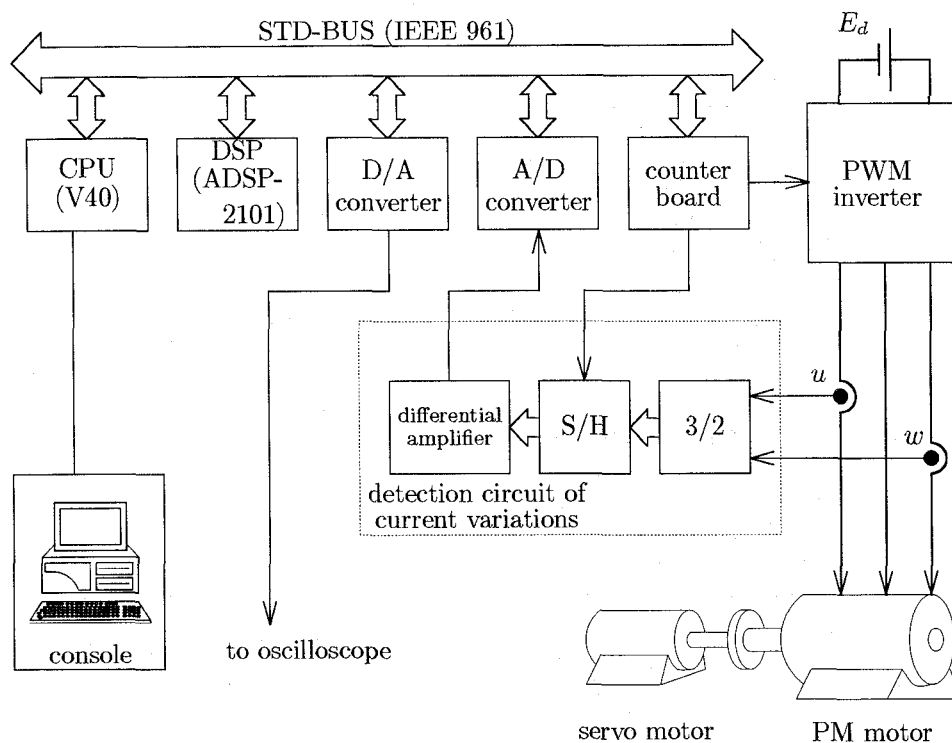


Fig. 7. Configuration of experimental system.

of the q -axis inductance being larger than d -axis inductance. The DSP can perform all the calculations for the inductance matrix and rotor position, and the PWM control within a modulation period of $333\mu\text{s}$. In the following experimental results, the PWM pattern shown in Fig.6 is used to achieve the rotor position estimation at standstill and in the low-speed range.

Fig.8 shows the experimental result of the position estimation proposed in this paper at standstill. A circle "o" and the corresponding vertical bar mean the average value and existing range of estimated position obtained from twenty-times' trial at a rotor position, respectively. The experimental result shows that the real-time position estimation is achieved at standstill with such a satisfactory accuracy as to add a position-closed loop to a PM motor drive system. Note that the vertical axis is not θ but 2θ . It would be possible to distinguish the polarity of the rotor magnetic pole by means of another technique based on magnetic saturation [3].

Fig.9 shows another experimental result at 1 r/min. The conventional position estimation based on back electromotive force would not be applicable at such a low speed. In this experiment, the inverter is controlled to continue to select the zero average voltage vector, and the rotor of the IPM motor is rotated by the servo motor which is directly connected to the shaft. These experimental results show the effectiveness and versatility of the position estimation proposed in this paper.

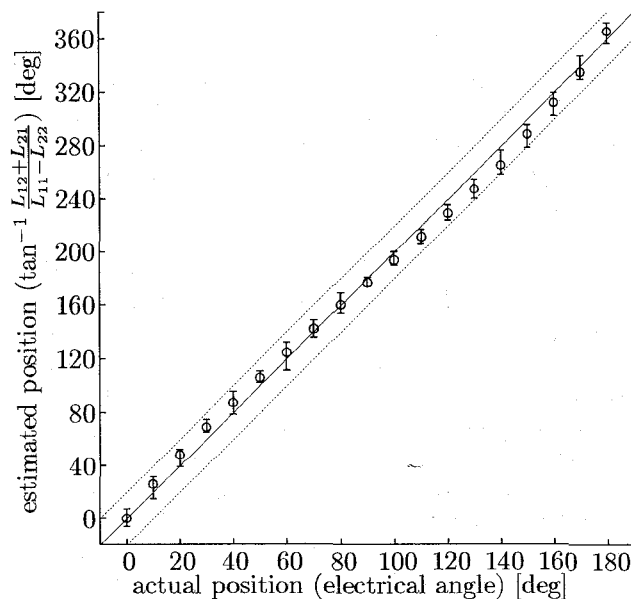


Fig. 8. Experimental result at standstill.

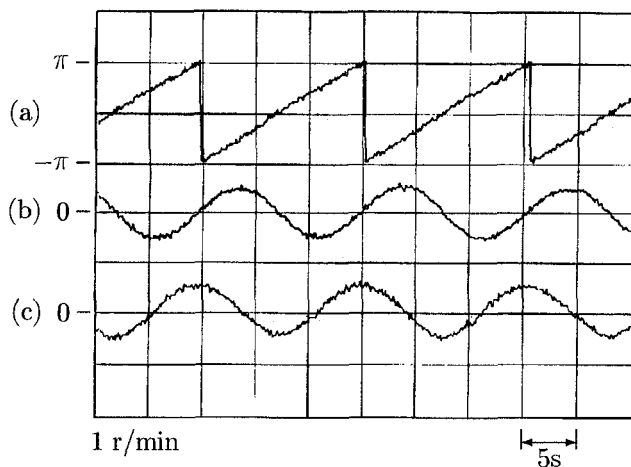


Fig. 9. Experimental result at 1 r/min. (a) estimated position $\tan^{-1} \frac{L_{12}+L_{21}}{L_{11}-L_{22}}$. (b) $L_{12} + L_{21}$. (c) $L_{11} - L_{22}$.

VII. CONCLUSION

This paper has proposed an approach to real-time position estimation based on magnetic saliency. A novel PWM scheme using redundant voltage vectors has been proposed to achieve the position estimation. Experimental results have demonstrated that the proposed approach has capacity of estimating the rotor position with an error of less than 10 degrees in electrical angle at standstill and such an extremely low-speed as 1 r/min. The proposed approach is applicable to other ac motors with magnetic saliency such as synchronous reluctance motors. In addition, it enables to identify inductance parameters in real-time for motor control, without requiring user input of machine constants. The authors are due to construct a position-sensorless IPM motor systems with a position-closed loop based on the proposed approach in the next stage.

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