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An Approximate Analytical Method for Vortex-Lift and Center of Pressure on the Slender Wing

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ABSTRACT

This paper presents a simplified approximate analytical method for predicting vortex-lift and its center of pressure on the slender wing at high angle of attack, and proposes two empirical correlations. In comparison with other methods and experimental data, this method provides good accuracy and is suitable for preliminary design.

In recent years, interest in the high angle of attack nonlinear aerodynamics has increased considerably. This interest has led to studies of the separated vortex flow as an important nonlinear effect on flow characteristics: the theory of growth and breakdown of the separation vortex; the influence of vortex separation on aerodynamic characteristics related to aircraft; and how to utilize and control the separation vortex effectively. This is a very important and active research subject in aeronautics.

Currently, many theoretical methods have been developed in calculating the aerodynamic characteristics of leading-edge vortex separation, but they primarily relied on high speed computers. The object of this paper is to seek an approximate analytical method that can provide a fast estimate for various different wing planforms. This method will be extremely useful in the preliminary design stage for designers. Several years ago, the author suggested a simple method for the calculation of tip vortex-lift. Based upon the fundamental work in reference (1) this paper derived a formula for vortex-lift and its center of pressure. For the convenience of engineering estimation, two empirical correlation formulas were given as well. Recently, Purvis[2] derived a similar analytical method. A comparison of references (1) and (2) shows that the results of reference (2) are achieved.

1. ANALYSIS

The Polhamus[3] "leading-edge suction analogy" theory compares well with experimental work and has been widely used in engineering. The following expressions for vortex-lift coefficient come from that theory:

$$C_{L_{V,LE}} = K_{V,T} \sin^2 \alpha \cos \alpha / \cos \Lambda$$

$$K_{V,T} = K_P\left(1 - \frac{K_P}{\pi A}\right) \qquad (1)$$

where K_P, $K_{V,T}$ are the constants for the potential-flow lift coefficient and the leading-edge thrust coefficient, respectively. Λ is the leading-edge swept angle. An important advantage of this theory is that one can study the nonlinear effect by using the potential flow theory. In application, equation (1) proves to be simple and reliable, but it provides only the total lift. If one desires the potential flow of the vortex-lift distribution, then panel or kernel function methods must be used to compute the leading-edge suction distribution. Nevertheless, this is not the intent of this paper. In order to compute the vortex-lift and its center of pressure, we assume, as shown in figure 1, that the vortex-lift in the inboard wing section E_1oE is equivalent to the vortex-lift for the isolated wing section E_1oE. In other words, we consider that the leading-edge-suction analogy is not only applicable to the whole wing planform but is also applicable to each individual inboard wing segment. Therefore,

$$C_{L_{V,LE}}(y) = C'_{L_{V,LE}}(y)S'(y)/S_w \tag{2}$$

$$C'_{L_{V,LE}}(y) = K'_{V,T}(y) \sin^2 \alpha \cos \alpha/\cos \Lambda \tag{3}$$

$$K'_T(y) = K'_P(y)\left(1 - \frac{K'_P(y)}{\pi A'(y)}\right) \tag{4}$$

where the primed variables are for the inboard wing E_1oE. $S'(y)$ and S_w denote the area of inboard wing and the area of whole wing, respectively.

In the calculation of tip-vortex-lift, one can further let the tips AB and A_1B_1 extend outboard an angle ε such that tips AB and A_1B_1 become part of the leading edge. Applying the suction analogy to the whole wing and inboard wing segment separately, one can obtain the total thrust and the thrust of the inboard wing segment. When $\varepsilon \to 0$, it gives the tip-votex-lift (see fig. 1).

$$C_{L_{V,SE}} = K_{V,SE} \sin^2 \alpha \cos \alpha \tag{5}$$

$$K_{V,SE} = \lim_{\varepsilon \to 0}\left(K_{V,T}/\tan \varepsilon\right)_{AB' \ \& \ A_1B'_1} \tag{6}$$

In addition, by the definition of vortex-lift distribution

$$C_{L_{V,LE}}(y) = \frac{2}{S_w}\int_0^y c(y)c_{l_V}(y)\, dy \tag{7}$$

where $c(y)$ is the local chord length and $c_{l_V}(y)$ is the sectional vortex-lift coefficient. Substituting equations (2) and (3) into equation (7) yields

$$c(y)c_{l_V}(y) = \frac{1}{2}\frac{d}{dy}\left[K'_{V,T}(y)S'(y)/\cos \Lambda\right] \sin^2 \alpha \cos \alpha \tag{8}$$

Inserting equations (5) and (6), one obtains the tip-vortex-lift

$$C_{L_{V,SE}} = \lim_{\varepsilon\to 0}\left\{\frac{1}{S_w^*}\int_{b/2}^{\frac{b}{2}+c_t\varepsilon}\frac{d}{dy}\left[K'_{V,T}(y)S'(y)/\tan\varepsilon\right]dy\,\sin^{2^*}\alpha\cos\alpha\right\} \tag{9}$$

or

$$C_{L_{V,SE}} = \left[\frac{dK'_{V,T}}{d\varepsilon} + \frac{K'_{V,T}}{S_w}\frac{dS'}{d\varepsilon}\right]_{\varepsilon=0}\sin^2\alpha\cos\alpha \tag{10}$$

where b is span and c_t is wing tip chord length. Equation (10) has been derived in reference (2). Similarly, one can obtain the center of pressure of the leading edge and the tip-vortex-lift as

$$\frac{x_{V,LE}}{c_r} = \frac{\cos\Lambda}{S_w c_r K_{V,T}}\int_0^{b/2} x_{LE}\frac{d}{dy}[K'_T(y)S'(y)/\cos\Lambda]\,dy \tag{11}$$

$$\frac{x_{V,SE}}{c_t} = \frac{1}{S_w c_t K_{V,SE}}\lim_{\varepsilon\to 0}\int_{b/2}^{\frac{b}{2}+c_t\varepsilon} x'_{LE}\frac{d}{dy}[K'_T(y)S'(y)/\tan\varepsilon]\,dy \tag{12}$$

respectively, where c_r is wing root chord length; x_{LE} is the chordwise distance measured from the leading edge; $x'_{LE} = x - x_{LE,t}$ is the chordwise distance measured from the wing tip leading edge. Equations (9) to (12) are the general expressions for the tip-vortex-lift and its center of pressure. From the expression, one can see that they are functions of potential-flow lift and the geometric configuration.

For slender wing, according to Jones' slender body theory, $K_P = \frac{\pi}{2}A$, $K_{V,T} = \frac{\pi}{4}A$. Substituting into equation (8) yields equation (13):

$$c(y)c_{l_V}(y) = \pi y/\cos\Lambda\,\sin^2\alpha\cos\alpha \tag{13}$$

From the above equation, vortex-lift distribution obtained from the slender-body theory is a linear distribution; it agrees with conic flow assumptions. Figure 2 presents a comparison of the sectional vortex-lift and inboard-wing-section vortex-lift with reference (2). It shows very good agreement.

Substituting K_P and $K_{V,T}$ into equation (9) or (10), one obtains

$$\begin{aligned} C_{L_{V,SE}} &= \pi A\left(\frac{c_t}{b}\right)\sin^2\alpha\cos\alpha \\ C_{L_{V,SE}} &= 4\left(\frac{c_t}{b}\right)\frac{K_P^2}{\pi A}\sin^2\alpha\cos\alpha \end{aligned} \tag{14}$$

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*Translator's note: error in original text.

Equation (14) is obtained under the slender-body theory; one can get better results by using more accurate values of K_P. Following the above equation $K_{V,SE} = \pi$ for a rectangular wing with a small aspect ratio, which agrees with Lamar's results in reference (4). Figure 3 compares the results obtained from equation (14) with Lamar's (ref. (4)) and Purvis' (ref. (2)) results at $K_P = 2\pi A \Big/ \left(\sqrt{A^2 + 4} + 2\right)$. From the figures, it is shown that our results agree quite well with Lamar's. Purvis' results seem too high.

Substituting K_P and $K_{V,T}$ into equations (11) and (12), yields

$$\frac{x_{V,LE}}{c_r} = \frac{2}{3} \qquad \text{(Triangular wing/delta wing)} \tag{15}$$

$$\frac{x_{V,SE}}{c'_r} = \frac{2}{3} \qquad \text{(Trapezoidal wing)} \tag{16}$$

$$\frac{x'_{V,SE}}{c_t} = \frac{1}{2} \tag{17}$$

where c'_r is the wing root chord length corresponding to the trapezoidal wing leading edge, $c'_r = \frac{b}{2} \tan \Lambda$.

From equations (15), (16), and (17), one can see that center of pressure of the leading-edge vortex-lift and tip-vortex-lift obtained from the slender-body theory agrees with the existing results in the literature. Furthermore, one discovers that the center of pressure of the leading-edge vortex-lift and the center of pressure of tip-vortex-lift for a trapezoidal wing can be obtained separately. This gives the theoretical basis for the following empirical corrections.

2. EMPIRICAL CORRECTIONS

Experiments proved that the "suction analogy" theory is only applicable for a certain range of angle of attack and aspect ratio. In order to extend the applicability of this theory to a wider range, many researchers have introduced different empirical or semiempirical correction factors; as examples, the correction factor $K^*_{V,LE}$ for the leading-edge vortex-lift for a delta wing in reference (5), and the improvement factor $K_{V,SE}$ of tip-vortex-lift in reference (6), etc. This paper also provides the correction formula.

1. Center of pressure correction.

From equation (15), under the conic flow assumption, the center of pressure of tip-vortex-lift is located at 2/3 root chord. However, the physical vorticity strength does not quite have a conic distribution, especially near the trailing edge. As angle of attack increases, the vortex core moves inboard. Following the experimental data analysis, the center of pressure for a delta wing is

$$\frac{x_{V,LE}}{c_r} = \frac{2}{3} - 0.3729/\tan \Lambda \tag{18}$$

If one further considers the effect of angle of attack on the movement of center of pressure, one should use

$$\frac{x_{V,LE}}{c_r} = \frac{x_p}{c_r} - \left[\left(\frac{x_p}{c_r}\right) - \left(\frac{2}{3} - 0.3729/\tan \Lambda\right)\right] \sin (3.6\alpha) \tag{19}$$

where x_p/c_r is the corresponding center of pressure for potential flow. The above equation is good for $\alpha \leqslant 25°$.

2. Vortex-lift increment.

Experiments proved, due to the effects of the leading-edge vortex on the tip and rear half of the wing planform, that the vortex-lift for a delta wing obtained from experiments is higher than the calculated value. Reference (7) gives a correction factor.

$$k_V = 1.77\left\{1 - \sin \frac{\pi\lambda}{2}\right\} \sin \frac{\pi\lambda}{2} + 1 \tag{20}$$

to represent the lift increment due to vortex interaction. Including the effect of trailing-edge sweep back angle, the above equation becomes

$$k_V = \left\{1.77\left(1 - \sin \frac{\pi\lambda'}{2}\right) \sin \frac{\pi\lambda'}{2} + 1\right\} \frac{\lambda'}{\lambda} \tag{21}$$

where λ is taper ratio, $\lambda = c_t/c_r$, $\lambda' = \dfrac{c_t}{c_t + b \tan \Lambda}$ is the taper ratio for a reference tapered delta wing. The above equations (19) and (20) are plotted in figure 4 and figure 5, respectively.

Based on the above analysis, for a trapezoidal wing, total vortex-lift and its center of pressure can be expressed as

$$C_{L_V} = K_{V,tot} \sin^2 \alpha \cos \alpha \tag{22}$$

$$K_{V,tot} = k_V\left[K^*_{V,LE}K_{V,T}/\cos \Lambda + K_{V,SE}\right] \tag{23}$$

$$\left(\frac{x}{c_r}\right)_V = k_V\left\{K^*_{V,LE}K_{V,T}/\cos \Lambda\left(\frac{x_{V,LE}}{c'_r}\right)\left(\frac{c'_r}{c_r}\right) + K_{V,SE}\left(\frac{x_{后面心}}{c_r}\right)\right\} \Big/ K_{V,tot} \tag{24}$$

where $x_{后面心}$ represents the geometric center of the trapezoid. According to equation (17), the center of pressure of tip-vortex-lift is located near the center of the tip. By considering the effect of trailing-edge sweep back angle, one can approximately consider that center of pressure of the tip (vortex) is located at the geometric center of the trailing wing planform.

3. CONCLUDING REMARKS

Applying current methods to a set of trapezoid wings, the results compare quite well with experiments as shown in figure (6).

This method can be extended to the calculation of vortex-lift and its center of pressure for a wing-body combination. It is also not difficult to apply to a wing with a cranked leading edge such as a straked wing.

The compressibility effect can be taken care of by the Prandlt-Gothert law.

From the equations obtained in this paper, one can find a best combination of sweep back angle, taper ratio, and aspect ratio to achieve maximum vortex-lift. This method is suitable for preliminary design.

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November 6, 1981

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*Editorial note: Titles have been added for references (3) through (6) and refer-ence (8), which was used on a figure, has been added to list.

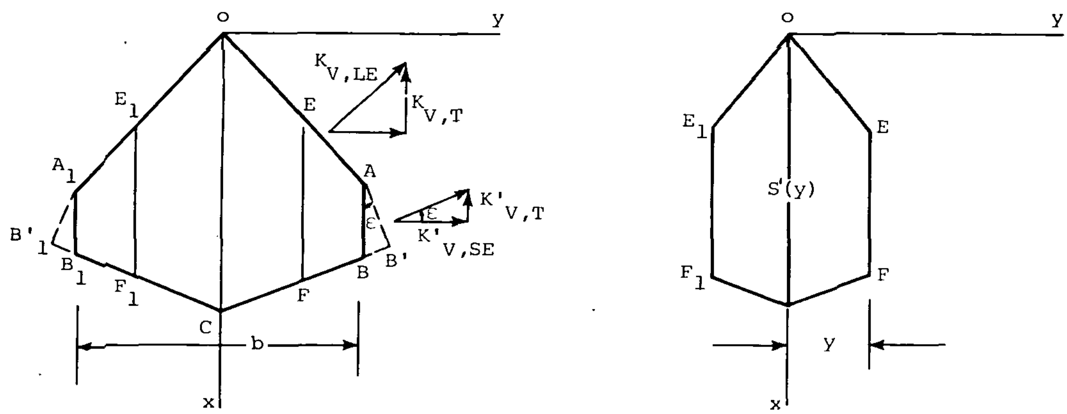

Figure 1.- Geometric relationship and configuration for tip-vortex-lift calculation.

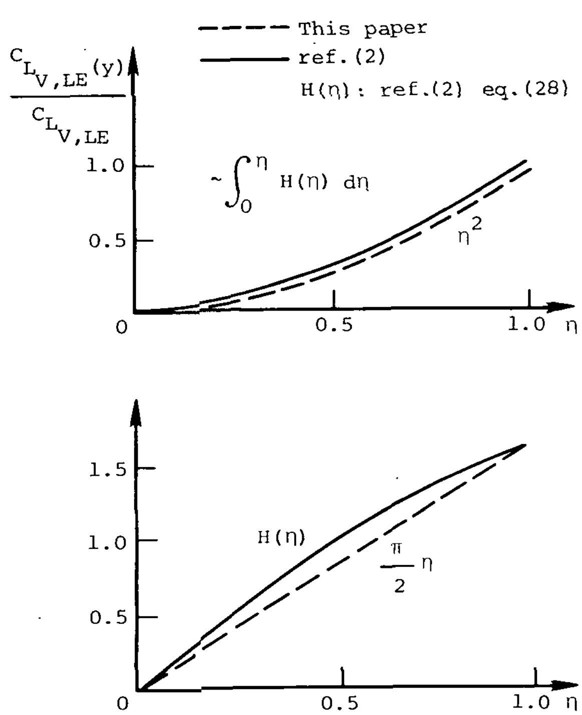

Figure 2.- Vortex-lift distribution: A comparison between this paper and ref. (2).

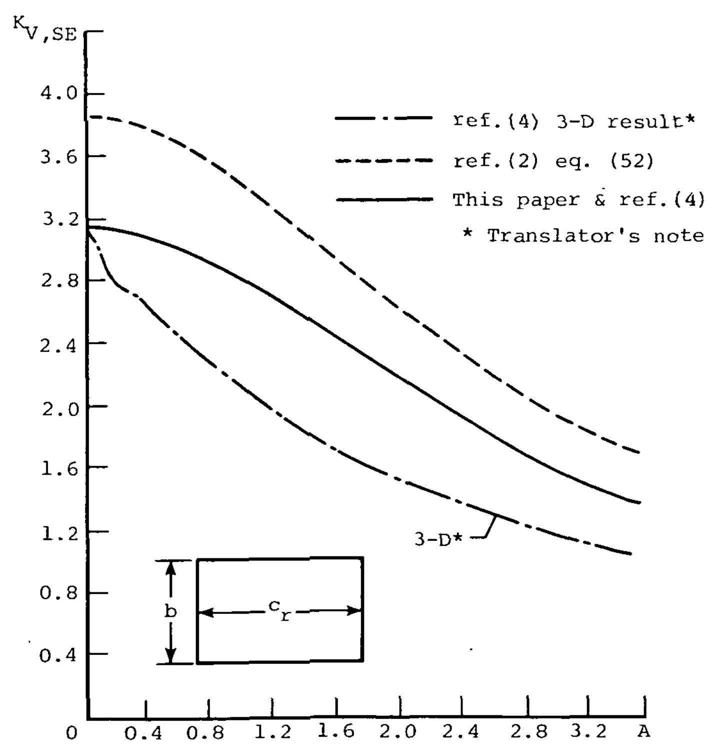

Figure 3.- Comparison of $K_{V,SE}$ for rectangular wing.

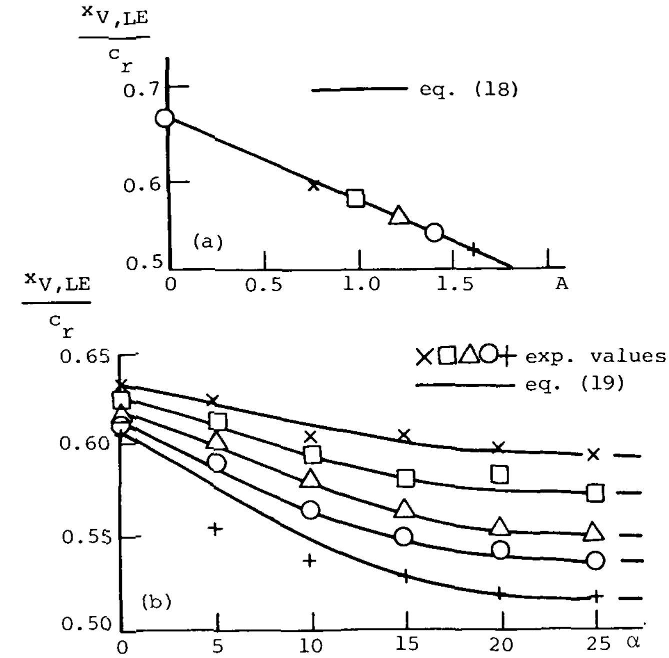

Figure 4.- Variation of center of pressure for vortex-lift as a function of aspect ratio and angle of attack, delta wing.

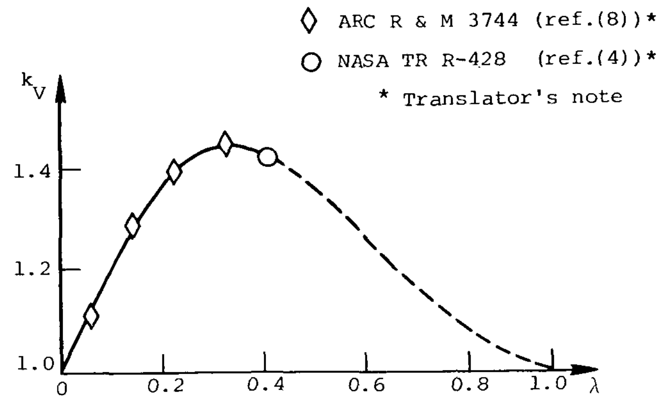

Figure 5.- Vortex-lift increment for a cropped delta wing.

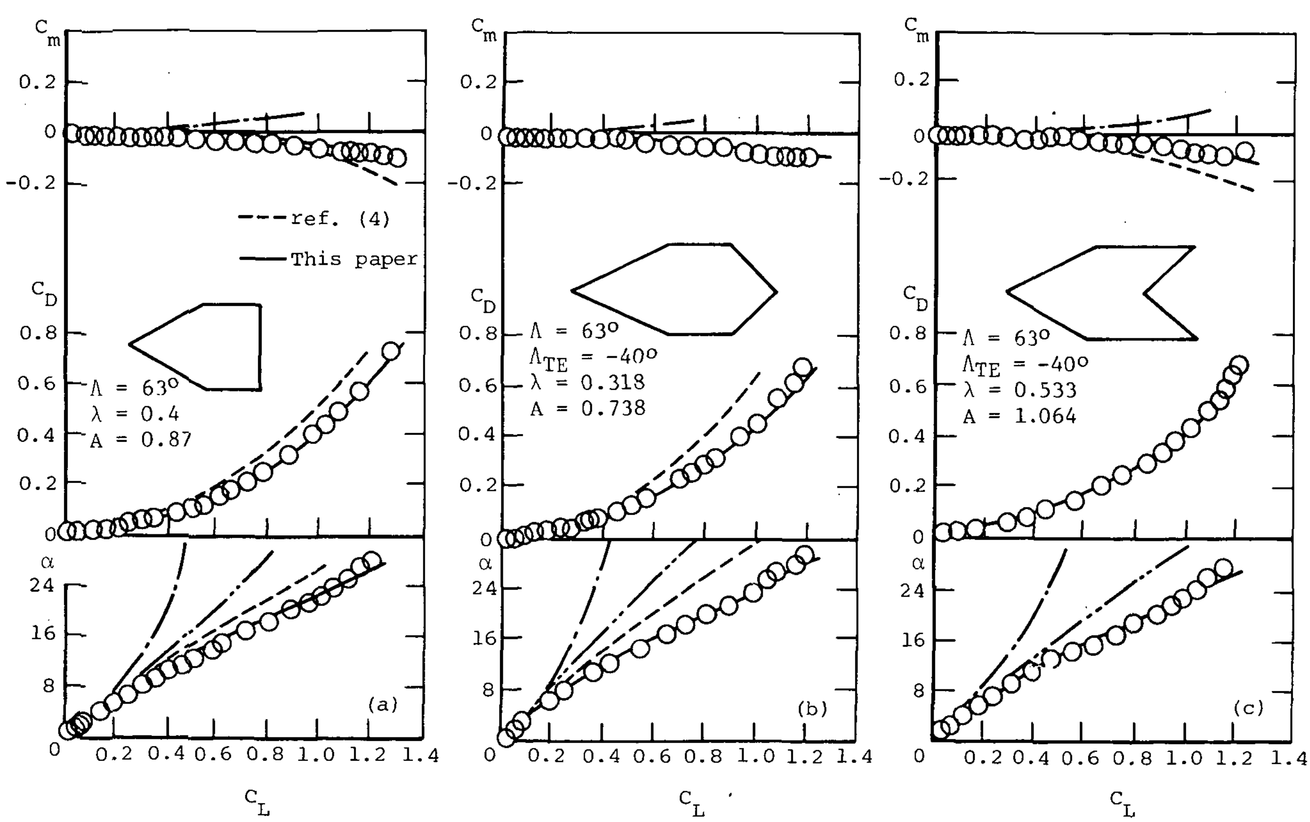

Figure 6.- Comparison of aerodynamic characteristics for a set of tapered trapezoidal wings.

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15. Supplementary Notes

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16. Abstract

A simplified approximate analytical method for predicting vortex-lift and center of pressure on slender wings is presented. Two empirical corrections are proposed. The method is compared with experimental data and is shown to provide accurate estimates. (Translator)

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