

An approximate calculation of the effect of opacity in the solar spectral lines of C III

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Summary. In this paper we make use of the fact that the intensity ratio between two spectral lines arising from the same upper level can depend on opacity, to determine the optical thickness of the solar atmosphere to C III radiation. Our analysis is based on observations taken, near the Sun's limb during the *Skylab* mission, with the Naval Research Laboratory's spectrograph. By introducing the measured abundance of carbon and the results of ionization balance calculations an estimate is made of the line-of-sight physical thickness of the regions emitting C III lines at the disc centre.

1 Introduction

In attempting to interpret the intensities of solar spectral lines from ions of charge greater than one and less than about four, a number of authors have found anomalies which they have ascribed in some cases to the effects of opacity (Jordan 1977; Cook & Nicolas 1979; Doschek *et al.* 1976). Although these explanations are often satisfactory in a qualitative way they generally lack quantitative confirmation. In this paper we adopt a method of analysis similar to that first used by Jordan (1967) who pointed out that the intensity ratio between two lines arising from the same upper quantum level is, in optically thin conditions, simply the branching ratio of the relevant radiative transition probabilities but in optically thick conditions can be modified from its optically thin value only by the effect of opacity. Provided the upper level consists of a single quantum state, i.e. it is not degenerate as for example in the case of hydrogen-like ions, no other atomic process can modify the ratio. This is the central idea upon which the present paper is based and it is used to make a quantitative estimate of the effect of opacity.

Here we apply the method to the interpretation of the intensity ratios between pairs of the components of the C III ($2s2p\ ^3P_{0,1,2}^0 - 2p^2\ ^3P_{0,1,2}$) multiplet at about $\lambda 1176\ \text{\AA}$. It is shown that a particular ratio is sensitive to opacity whereas another is not. Thus a failure to find a departure of any particular line pair from the theoretical branching ratio cannot necessarily be taken as evidence for optically thin conditions. However, in C III one of the

ratios is sensitive to opacity and it is possible to calculate the optical thickness in one of the components of the multiplet. To do this we have used the 'escape factor' approximation discussed below. From this calculated value and using a recent calculation of the relative population of the metastable ($2s2p\ ^3P_{0,1,2}^0$) and ground ($2s^2\ ^1S_0$) levels it is shown that estimates may be made of the optical thickness of the other lines of C III. These all turn out to be sufficiently small that opacity may be neglected as it affects the calculation of the population of the excited levels of C III, i.e. the 'effectively optically thin approximation' applies to this ion. Finally in this paper we use the estimated solar abundance of carbon and the calculated ionization balance ratio to determine the physical line-of-sight thickness of the C III 'layer' in the solar atmosphere.

2 Theory

The standard treatment of the effects of opacity in a plasma such as the atmosphere of the Sun requires the simultaneous solution of the equations of collisional excitation and radiative transfer which in turn depend on a knowledge of the temperature, density and radiation field at all points in the plasma, as well as the details of the atomic collision processes, in often complex geometrical models. The difficulties of such treatments are well known and, since major simplifications must be adopted, not justified in circumstances where opacity is relatively modest. Here we analyse the observed variation of the intensities of the C III multiplet at various positions near the solar limb using the 'escape factor' concept for resonance radiation introduced first by Holstein (1947) and treated more recently in detail by Irons (1979). According to Holstein the 'escape factor' may be regarded as the reciprocal of the number of emissions and absorptions of an individual 'unit of atomic excitation' prior to its escape from the plasma. As a method of treating the effects of opacity it is not exact. However, it is capable of reproducing some of the effects, such as those reported here, and it has the virtue of simplicity. The ultimate justification for its use lies in the comparison of its predictions with observations.

In its present application the 'escape factor' treatment is physically equivalent to assuming that the only effect of opacity is to cause photons to be scattered out of the line-of-sight. This is a reasonable approximation in circumstances where, for example, the spectral line is optically thin at the disc centre but thick at the limb. The intensity of such a line may then be written:

$$I(q-p) = \frac{1}{4\pi} g\{\tau_0(q-p)\} A(q-p) \int n(p) dx \text{ photon cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}, \quad (1)$$

where $n(p)$: upper level population density (cm^{-3}), $A(q-p)$: transition probability (s^{-1}), and $g\{\tau_0(q-p)\}$ is the 'escape factor' and is a function of τ_0 the optical thickness at the centre of, in this case, a Doppler broadened line. The evaluation of the function $g(\tau_0)$ is discussed by Irons (1979). Here we use the following expression for $g(\tau_0)$:

$$g(\tau_0) = 1 + \sum_{n=1}^{\infty} (-1)^n \tau_0^n / \sqrt{n+1} n!).$$

It follows that the intensity ratio of two lines arising from a common upper level is given by:

$$\frac{I(q-p)}{I(r-p)} = \frac{g\{\tau_0(q-p)\} A(q-p)}{g\{\tau_0(r-p)\} A(r-p)}. \quad (2)$$

Where the plasma is optically thin, the 'escape factors' are unity and the intensity ratio is the ratio of the radiative transition probabilities.

3 Results

During the *Skylab* mission the NRLS082B UV slit spectrograph was used to record the spectrum of the quiet Sun at a number of positions near the limb. A description of these observations is given by Mariska, Feldman & Doschek (1978) who, however, do not discuss the C III multiplet at λ 1176 Å which was recorded on the same photographs. Fig. 1 shows a microdensitometer trace corrected for the plate response, of the spectral region around the C III multiplet when the entrance slit (projected area on the Sun: 2×60 arcsec) was imaged at a position 2 arcsec above, and tangential to, the visible limb. It may be seen that some of the components are resolved but that the components arising from the (2–2) and (1–1) transitions are completely blended. (The individual components will be referred to by the values of the J quantum numbers of their lower and upper levels, i.e. $2s2p^3P_1 - 2p^2^3P_0$ by 1–0 etc.) The instrumental resolution was 0.06 Å (Doschek *et al.* 1976).

In order to estimate the relative intensities of the six individual components a Gaussian fitting procedure was used to match the microdensitometer traces. The fit was of the form

$$S(\lambda) = \sum_{k=1}^6 S(k) \exp - \left\{ \frac{\lambda - \lambda(k)}{\Delta\lambda(k)} \right\}^2, \quad (3)$$

where $S(\lambda)$ is the intensity per wavelength interval in arbitrary units at wavelength λ , $S(k)$ is the peak intensity in the same units for the component k where the correspondence between k and $(J' - J)$ is given in Table 1, $\lambda(k)$ is the rest wavelength of component k and $\Delta\lambda(k)$ is related to the full width at half maximum of component k by $\text{FWHM} = 2\sqrt{\ln 2} \Delta\lambda(k)$. The last was taken to be the same for all six components. Wavelength values were taken from the very careful work of Bockasten (1955) who claims that they 'should be reliable to a few units in the third decimal place'. An iteration procedure was used to obtain the best fits to the observed profiles. This gave rise to the values of $S(k)$ listed in Table 1. The quality of the fit is illustrated in Fig. 1 for the position 2 arcsec above the limb, where the dashed line

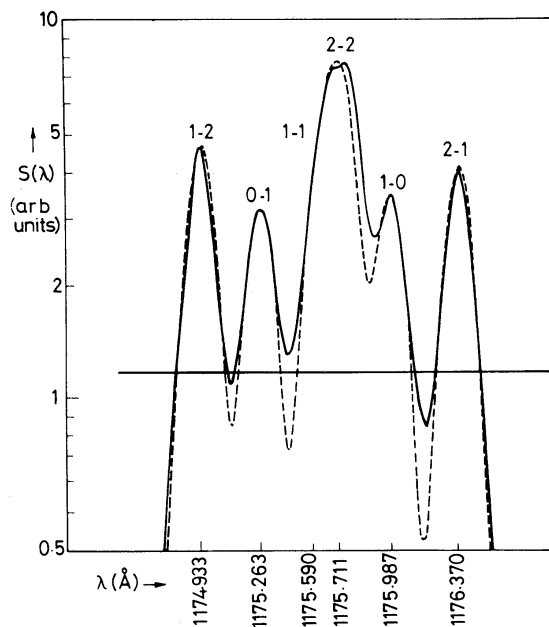


Figure 1. The observed profile of the C III ($2s2p^3P - 2p^2^3P$) multiplet as recorded by the NRL spectrometer on *Skylab* (solid line) with the Gaussian fitted curve superimposed (dashed line). These observations were taken with the spectrometer pointing 2 arcsec above the limb. Data below the straight line is less reliable since it lies in the 'toe' of the H–D curve.

Table 1. Values of $S(k)$ which are taken to be equal to $I(J'-J)$.

Positions with respect to visible limb (arcsec)	k	1	2	3	4	5	6
	$J'-J$	1-2	0-1	1-1	2-2	1-0	2-1
-12		1.15	0.68	0.51	2.73	0.77	0.96
-4		1.03	0.84	0.52	2.37	0.80	0.97
-2		1.40	1.20	0.72	3.51	1.24	1.34
0		2.62	2.00	1.45	5.02	2.04	2.24
+2		4.50	3.10	2.79	6.50	3.38	4.00
+4		3.06	2.25	1.76	5.50	2.20	2.70
+6		1.24	1.00	0.71	2.98	1.01	1.17
+8		0.41	0.31	0.21	1.23	0.31	0.37

is the Gaussian fit. The disparities in the troughs are probably due to the adoption of Gaussian rather than Voigt profiles and are accentuated by the logarithmic intensity scale. The error carried over to the final results is negligible from this cause. The disparity between the observed and fitted widths in the region of the (2-2) component is of greater significance. It is present only for positions within a few arcsec of the limb i.e. when opacity is greatest, and cannot therefore be due to an error in the laboratory wavelengths. It probably is due to the broadening of the (2-2) component due to opacity. However, we do not attempt to treat this problem quantitatively in this paper.

The total intensity of a component line is the integral under the profile. Since we take all the profiles to have the same Gaussian width $\Delta\lambda(k)$ the intensity is proportional to the value of $S(k)$ and since the intensities are in arbitrary units, for simplicity, we put $I(J'-J) = S(k)$ numerically.

In Fig. 2, plots of the intensity ratios $I(2-2)/I(1-2)$ and $I(0-1)/I(2-1)$ against position with respect to the limb, show that the first ratio departs significantly from the optically thin

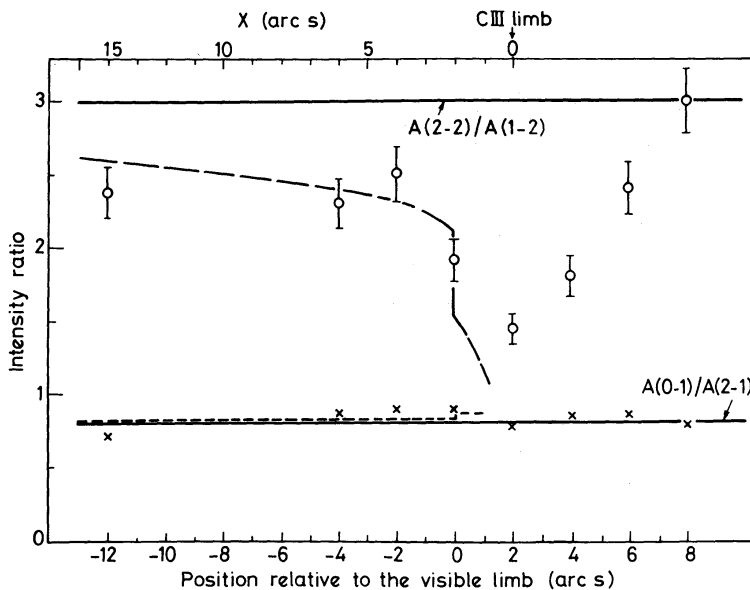


Figure 2. The observed values of the intensity ratio $I(2-2)/I(1-2)$ and $I(0-1)/I(2-1)$ at different positions relative to the limb are plotted as points. The dashed curves are the best fits to the observed points based on the geometry illustrated in Fig. 3.

Table 2. f -values (Hibbert 1974) and relative optical thicknesses for the components of the $2s2p^3P-2p^2^3P$ multiplet at an electron temperature (equal to the ion temperature) of 7.5×10^4 K and an electron density of 8.0×10^9 cm $^{-3}$.

$J'-J$	1-2	0-1	1-1	2-2	1-0	2-1
f	0.115	0.275	0.069	0.206	0.092	0.069
$\tau_0(\text{rel})$	4.48	3.98	2.69	15.00	3.58	5.01

value of $A(2-2)/A(1-2) = 3.0$ whereas the second does not ($A(0-1)/A(2-1) = 0.8$). It is shown later that the theory predicts negligible departure of the second ratio so that the scatter of the observed ratios about the mean may be taken as a measure of the statistical scatter of the observational data. This comes out to be about 7.5 per cent and is the basis used to put error bars on the points in Fig. 2. The uncertainties in the values of the ratios of the A -values is probably negligible in comparison.

The variation of the line intensity ratio shown in Fig. 2 is interpreted as being due to opacity primarily in the (2-2) component. The alternative explanation that it is due to an unidentified line blending with the (1-2) component appears to be unlikely since it (the unidentified line) would have to exhibit unreasonably strong limb brightening, i.e. on the disc its intensity would require to be very much less than that of (1-2) and yet be stronger than (1-2) at the limb.

The optical thickness τ_0 is defined as the product of the absorption coefficient at the line centre (k_0 in cm $^{-1}$) and the physical thickness of the absorbing plasma along the line-of-sight (l in cm). Mitchell & Zemansky (1961) give an expression for k_0 so that:

$$\tau_0 = k_0 l = 1.16 \times 10^{-6} \sqrt{\frac{M}{T}} \lambda_0 n f l, \quad (4)$$

where T : ion temperature (K), M : atomic weight, λ_0 : the rest wavelength (cm), n : lower level population density (cm $^{-3}$) and f : absorption oscillator strength. The values of M , T , λ_0 and l are effectively the same for all components so that their relative optical thicknesses are proportional to $n f$. Relative values of n have been calculated in the optically thin approximation by using the atomic rates given by Dufton *et al.* (1978) and the f -values calculated by Hibbert (1974) given in Table 2. The inclusion of proton rates between either the $2s2p^3P$ or $2p^2^3P$ levels has negligible effect on the values derived for the level populations of C III (Doyle *et al.* 1979). Relative populations were calculated for $n_e = 8.0 \times 10^9$ cm $^{-3}$ and $T_e = 7.5 \times 10^4$ K (peak populations of C III in ionization balance. Taking $T_e = 5 \times 10^4$ K, would only change the final values by about 2 per cent). The corresponding relative optical thicknesses for the components of the C III multiplet are given in Table 2. In particular $\tau_0(2-2)/\tau_0(1-2) = 3.35$. By dividing the observed ratio plotted in Fig. 2 by the ratio of the transition probabilities, the corresponding values of $g\{\tau_0(2-2)\}/g\{\tau_0(1-2)\}$ were found and are given in Table 3 for various positions x with respect to the C III limb – taken to be the position marked on Fig. 2. Values of $\tau_0(2-2)$ were then found by plotting $g(\tau_0)/g(\tau_0/3.35)$ against τ_0 and reading off the corresponding values of τ_0 for the ratio $g\{\tau_0(2-2)\}/g\{\tau_0(1-2)\}$.

If we had taken instead the limb at that position corresponding to the visible limb then our estimates for τ_0 at 2 and 4 arcsec into the disc are reduced by 34 and 28 per cent respectively. For positions further into the disc it does not matter which choice is taken.

In calculating the variation of intensity ratios in the vicinity of the limb a model atmosphere, as illustrated in Fig. 3, was taken. This is not intended to mean that the

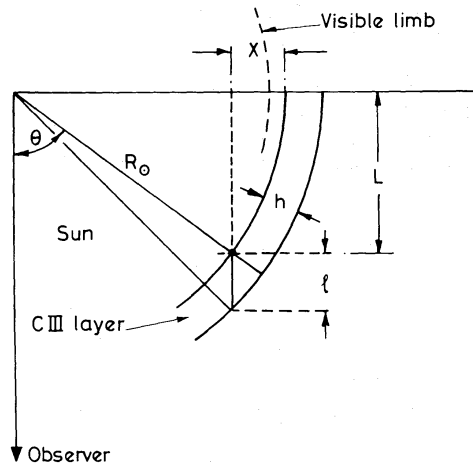
Table 3. Values of the optical thickness in the (2-2) component for different positions near the limb calculated from the observed intensity ratios of $I(2-2)/I(1-2)$ and from the assumed geometry scaled for the best average fit. Note that the values of l/h are insensitive to the value of h for h less than about 4 arcsec.

Position relative to visible limb (arcsec)	Observed Intensity Ratio $I(2-2)/I(1-2)$	$\frac{g\{\tau_0(2-2)\}}{g\{\tau_0(1-2)\}}$	$\tau_0(2-2)$	x (arcsec)	l/h ($h = 1$ arcsec)	Predicted $\tau_0(2-2)$
-12	2.38	0.785	0.50	14	5.78	0.40
-4	2.30	0.759	0.59	6	8.61	0.59
-2	2.50	0.825	0.40	4	10.36	0.71
0	1.92	0.634	1.03	2	13.94	0.96

atmosphere is necessarily conceived of as a uniform shell-like structure but may be composed of blobs of plasma (spicules) distributed around the surface. The figure shows the geometrical relationship between the length l of the absorbing path and the thickness of the shell h as a function of x , the distance from the limb. Since the variation in l is the only quantity that causes the opacity to vary as the distance to the limb varies, the variation in this ratio should match that of the optical thickness: i.e. the value of l/h given in Table 3 should be proportional to the corresponding values of $\tau_0(2-2)$. The last column shows the best fit to the τ_0 values. The dashed curve in Fig. 2 shows the corresponding values of the intensity ratio. The reasonably good agreement is satisfactory confirmation that the 'escape factor' technique is able to reproduce the effect of opacity.

At disc centre $l/h = 1$ and the opacity can be calculated using the same constant of proportionality as used to derive the values in the last column of Table 3. This gives $\tau_0(2-2) = 0.068$ as the best estimate of the opacity at disc centre.

A similar method of analysis may be used for the intensity ratio between the (0-1) and (2-1) components. However, in this case the ratio of the optical thickness is 0.79



$$\frac{l}{h} = \frac{\delta R_{\odot}}{h} \left\{ \sqrt{(1+h/R_{\odot})^2 - (1-x/R_{\odot})^2} - \sqrt{1-(1-x/R_{\odot})^2} \right\}$$

$$\approx \delta / \cos \theta \quad (\text{for } h \rightarrow 0)$$

$\delta = 1$ inside the 'visible limb' and 2 outside

Figure 3. The assumed model for the atmosphere showing the relationship between the line-of-sight thickness (l) and the radial thickness (h) of the emitting 'layer'.

as compared with 3.35 in the other case. This means that the intensity ratio is relatively insensitive to opacity. By calculating values of $\tau_0(0-1)$ and $\tau_0(2-1)$ from the value of $\tau_0(2-2)$ as described below the intensity ratio of $I(0-1)/I(2-1)$ was found as a function of limb position and is shown as a dashed line in Fig. 2. It may be seen to be within the statistical scatter of the observed data thus providing further confirmation of the method of analysis.

By substituting appropriate values into equation (4) and taking the ion temperature T to be the same as the electron temperature T_e i.e. 7.5×10^4 K, a value for the product nh may be derived. Thus

$$nh = 1.7 \times 10^{12} \text{ cm}^{-2},$$

where n is in this case the population density of the $2s2p^3P$ ($J=2$) level.

The assumption of equal ion and electron temperature requires to be examined. It is well known that the ion temperatures derived from the Doppler broadening of spectral lines emitted from the solar atmosphere are larger than the values derived for the electron temperatures on the basis of calculations of the ionization balance (see Boland *et al.* 1975; Doschek *et al.* 1976). Thus the measured FWHM of the components of the C III multiplet studied here give an apparent ion temperature of 4.9×10^5 K. This may be interpreted as arising from the bulk motion of large regions of the solar atmosphere (the macroturbulent alternative) or as fine scale motions akin to thermal motions as far as radiative transfer is concerned (the microturbulent alternative). These alternatives lead to different effects arising from opacity. In the macroturbulent case the effect of turbulence is to broaden the line as it escapes from a plasma having equal electron and ion temperatures, i.e. the case treated above. If the microturbulent alternative is assumed then the ion temperature is taken to be the apparent value derived from the observed line broadening. Substituting this value into equation (4) we derive for the microturbulent solution

$$nh = 4.3 \times 10^{12} \text{ cm}^{-2},$$

where again n is the population density of the $2s2p^3P$ ($J=2$) level.

There is no reason to exclude any solution lying between the extremes represented by the microturbulent and macroturbulent solutions. To this uncertainty must be added those arising from the observations and the calculations of the populations.

Since h may be presumed to be the same for all spectral lines of C III it is possible to estimate the optical thickness of any line of C III for which the population density of the lower level relative to $n(2s2p^3P_2)$ can be calculated. Thus it is possible to determine the optical thickness at line centre of the main resonance line ($2s^2^1S - 2s2p^1P$, $\lambda 977 \text{ \AA}$) at any position on the disc. Substituting appropriate values in equation (4) yields for disc centre:

$$\tau_0(2s^2^1S - 2s2p^1P) = 0.49.$$

The corresponding value of the escape factor is 0.70 which means that it is a fair approximation to assume the $\lambda 977 \text{ \AA}$ line to be 'effectively optically thin' at disc centre and that it is unnecessary to take opacity into account in calculating the relative populations of the various levels of C III.

The physical thickness of the C III 'layer' h may be estimated if the absolute population density of the $2s^2^1S$ ground level of C III (C^{2+}) can be calculated. Now

$$n(2s^2^1S) = \frac{n(2s^2^1S)}{n(C^{2+})} \frac{n(C^{2+})}{n(C)} \frac{n(C)}{n(H)} \frac{n(H)}{n_e} n_e, \quad (5)$$

where the meanings of the symbols are obvious.

The value of $n(2s^2\ ^1S)/n(C^{2+}) = 0.511$ may be taken from the calculations of Dufton *et al.* (1978) already discussed. The value of $n(C^{2+})/n(C) = 0.501$, i.e. the ionization balance ratio of the C III population to total carbon is taken from the tables of Summers (1979) or of Jordan (1969) – it makes negligible difference to this calculation which is used. Note that an average value is taken for the temperature range over which C III has a population exceeding, say, 30 per cent of its peak value. This incidentally defines the limits of the physical thickness of the C III layer. The abundance of carbon $n(C)/n(H) = 5.75 \times 10^{-4}$ is taken from the paper by Withbroe (1978) and except possibly for the value to be taken for the electron density n_e is the least well established factor in equation (5). $n(H)/n_e$ is taken to be 0.87 to account for the electrons arising from the ionization of elements heavier than hydrogen. For n_e we have chosen the value $8.0 \times 10^9\text{ cm}^{-3}$ since this gives for $n_e T_e$ a value of $6 \times 10^{14}\text{ cm}^{-3}\text{ K}$ consistent with the commonly adopted value for the transition region ($T_e = 7.5 \times 10^4\text{ K}$ is the peak of the C III population according to Summers 1979). This density value is also consistent with the value derived from the intensity ratio of the (1–0) component to any other (Dufton *et al.* 1978). Substitution of these values leads to values of h in the range $50\text{ km} \pm 65$ per cent where the uncertainty arises from the range of possible values of $n h$ and the uncertainty in the carbon abundance.

Doschek & Feldman (1977) published a study of the C III and Si III spectrum near the solar limb including the $\lambda 1176\text{ \AA}$ and $\lambda 1909\text{ \AA}$ lines. They report finding (1) a variation of the intensity ratio of 1176 to 1909 with height above the limb and (2) a difference in the random velocities needed to explain the Doppler broadening of the 1176 and 1909 lines. It appears that the present analysis could probably explain the first of these as due to opacity but leaves the second and more intriguing observation, unresolved.

Conclusion

We have shown that near or at the solar limb, there is measurable optical thickness in the $\lambda 1176\text{ \AA}$ multiplet, primarily in the (2–2) component, e.g. at 1 arcsec inside the limb $\tau_0(2-2) = 1.21$ whereas at disc centre $\tau_0(2-2) = 0.068$. At disc centre the $\lambda 1176\text{ \AA}$ multiplet is optically thin but the main resonance line $\lambda 977\text{ \AA}$, has an optical thickness of 0.49 for disc centre data in quiet regions and can be assumed to be ‘effectively optically thin’. We have also shown how it is possible to estimate the physical line-of-sight thickness of the C III ‘layer’ h and have derived a value in the range $50\text{ km} \pm 65$ per cent.

This method may be readily applied to other ions, e.g. Si III, a Mg-like ion, which has a similarly term structure as C III except with principal quantum number = 3. It may also be used to derive an estimate of the physical line-of-sight thickness of line emitting layers in specific solar features and it may be possible to extend the analysis of the present data above the limb to derive a model for the temperature/density structure of the transition zone.

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