

# An Approximate Expression for the Calculation of the Electric Field in Vicinity of the Cube Electrode with Rounded Wedges

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**Abstract:** In this paper, an approximate analytical expression for calculation electric field in the centre of the rounded vertex of the metallic cube will be proposed. This expression is determined according to the numerical results obtained using different numerical methods.

**Keywords:** Electric field amplification, numerical methods, approximate expression, cube electrode.

## 1 Introduction

It is well known that electrostatic discharge can occur in vicinity of the wedges of huge metallic bodies. Analysis of such phenomenon is very important especially in cases when it happens in the area where flammable and combustible substances exist, such as petrol stations, where vehicles and other conducting bodies, exposed to the external atmospheric electric field, are located [1]. The charges induced on the surface of the metallic bodies amplify external electric field, so, depending on the intensity of external electric field and radii of the wedge curvature, intensity of perturbed electric field can reach critical value for electric field intensity when breakdown can occur [2]. This can happen even when these objects are grounded [3].

The calculation of the electric field in vicinity of the conducting objects can be done using different numerical techniques. Most popular methods, especially

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in regions where the field is strong, e.g. electric field in vicinity of wedges, are Charge Simulation Method (CSM) [4, 5] and Surface Charge Simulation Method (SCM) [6]. In reference [2] for calculation of electric field in the center of the rounded vertices of neutral conducting cube with rounded wedges, specific Equivalent Source Method (ESM) is used. All these methods have certain advantages and disadvantages but give results with high accuracy.

However, in engineering practice there is need to obtain results faster and in a simple way, without numerical calculation. Because of that, knowledge of approximate analytical expression, which provides results with satisfactory accuracy, would be useful.

In this paper we will determine an approximate expression for electric field intensity at the center of rounded vertices of conducting cube.

## 2 Calculation of the Electric Field of the Cube Electrode using Numerical Methods

In Figure 1 the neutral metallic cube of wedge length  $a$ , immersed in the external electric field is presented. Wedges are rounded having radius of curvature  $r$ . With point  $P$  the center of the rounded vertex is denoted, where we can expect greatest value of the electric field and which is the most critical from the stand point of possible electric field breakdown.

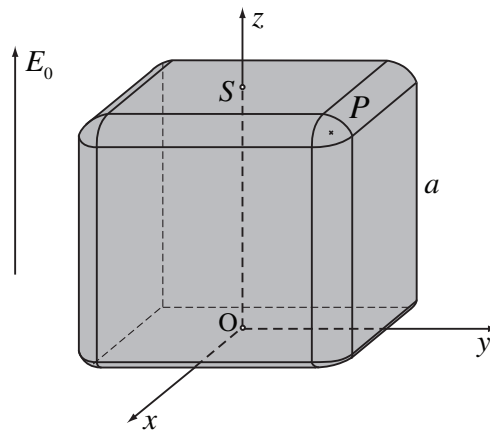


Fig. 1. Conducting cube with rounded wedges.

First the calculation of electric field in the center of the rounded vertex is done using specific Equivalent Source Method (ESM) [2], which can be considered a combination of methods shown in [7, 8]. By applying this method, the unknown electrode surface charges are simulated by equivalent sources whose positions and

types are chosen to achieve high accuracy of electric field calculations in all regions in vicinity of rounded cube including wedges and vertices. Therefore, different types of equivalent sources are placed over the surface of the inner cube of wedge length  $a_1$ ,  $a_1 = a - 2r$ , such as presented in Figure 2.

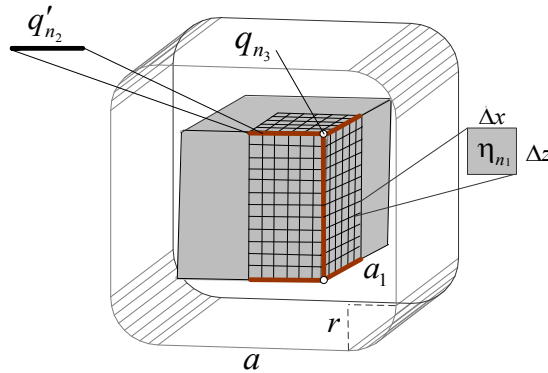


Fig. 2. Distributions of equivalent sources.

The total number of equivalent sources is

$$N_u = N_1 + N_2 + N_3 = 3N^2/2 + 3N + 2,$$

where:

$N_1$  is the number of equivalent sources in form of charges uniformly distributed over square subsurfaces,  $\Delta x = \Delta y = \Delta z = a_1/N$ ;

$N_2$  is the number of equivalent sources in form of line charges which are placed along wedges of the inner cube; and

$N_3$  is the number of equivalent sources in form of point charges placed in vertices of the inner cube.

The unknown weight of these equivalent sources are determined by solving system of linear equation, which is formed by satisfying boundary condition for the potential at properly adopted points on the cube surface. After that electric field in vicinity of the cube wedges is easily determined.

The convergence of the results for the electric field in the center of the rounded vertex for different number of equivalent sources are presented in Table 1.

Presented Method can be also used for determining electric field strength in vicinity of the grounded cube, Figure 3.

The intensity of the electric field in the center of the rounded vertex is also determined using software package *WIPL-D Pro v6.1*.

Table 1. Convergence of the results of the electric field strength of the cube in point  $P$ , for different ratio  $r/a$ .

$N$	$E/E_0$			
	$r/a = 0.1$	$r/a = 0.09$	$r/a = 0.08$	$r/a = 0.07$
10	4.336717	4.544749	4.785361	5.068996
20	4.563315	4.804222	5.082650	5.411113
30	4.633830	4.899724	5.204450	5.561894
40	4.634345	4.924193	5.251959	5.634817
50	4.635001	4.904250	5.255324	5.662390
$N$	$E/E_0$			
	$r/a = 0.06$	$r/a = 0.05$	$r/a = 0.045$	$r/a = 0.04$
10	5.412173	5.843578	6.107993	6.419282
20	5.808551	6.305353	6.606204	6.954964
30	5.992949	6.531556	6.858242	7.237551
40	6.093667	6.664376	7.009863	7.410920
50	6.146839	6.746229	7.107801	7.526802
$N$	$E/E_0$			
	$r/a = 0.035$	$r/a = 0.03$	$r/a = 0.025$	$r/a = 0.02$
10	6.795703	7.297202	7.886452	8.755585
20	7.367138	7.867311	8.498072	9.342160
30	7.686196	8.229561	8.909221	9.801508
40	7.885693	8.461661	9.183118	10.128359
50	8.022321	8.623504	9.377510	10.367122
$N$	$E/E_0$			
	$r/a = 0.0175$	$r/a = 0.015$	$r/a = 0.0125$	$r/a = 0.01$
10	9.345742	10.103686	11.122890	12.584888
20	9.894750	10.587021	11.495794	12.769931
30	10.372503	11.073209	11.971599	13.200219
40	10.729123	11.459143	12.381076	13.616308
50	10.996018	11.758181	12.714240	13.978905
$N$	$E/E_0$			
	$r/a = 0.0075$	$r/a = 0.005$	$r/a = 0.0025$	$r/a = 0.001$
10	14.901248	19.262151	31.341548	64.598187
20	14.743242	18.368906	28.109352	54.092578
30	15.057437	18.392685	27.136626	49.900849
40	15.438993	18.633397	26.814655	47.676967
50	15.808547	18.938323	26.276399	46.345598

*WIPL-D Pro v6.1* is powerful 3D software for electromagnetic analysis of arbitrary structures made of part by part homogeneous, isotropic and linear media in frequency domain. *WIPL-D* code is based on the Method of Moments (MoM). Metallic and dielectric surfaces are modelled by bilinear surfaces and for basic functions polynomial type expansions are used that automatically satisfy continuity of currents of arbitrary metallic and/or dielectric junctions and metallic ends. To obtain unknown coefficient of these expansions Galerkin testing procedure is applied. Although this software is made for analysis of sinusoidal time-varying

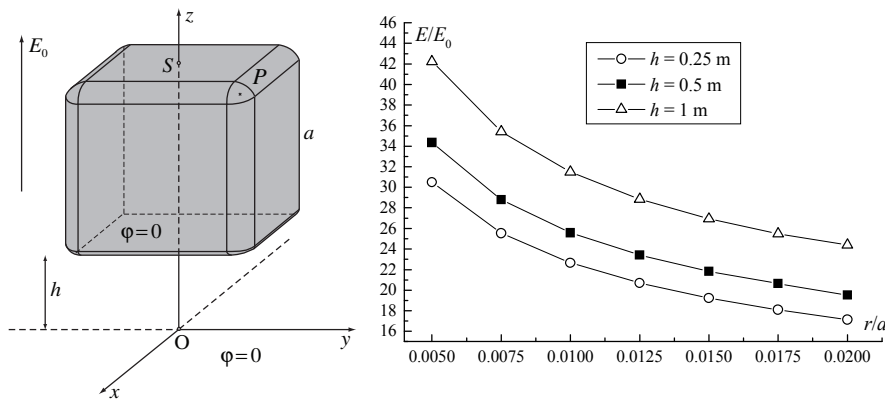


Fig. 3. Electric field strength of grounded cube electrode in point *P*.

fields it can be successfully used for analysis of arbitrary structures in electrostatic and stationary electromagnetic field by choosing frequency for which the structure is much smaller than wavelength. Putting such structure in field of plane wave, problem of structure in homogeneous electric field can be simulated.

The convergence of the results obtained by software package *WIPL-D*, for different number of segments *N*, are presented in Figure 5. (Wedge of cube is modelled in shape of quarter of the cylinder along circumference and *N* segments at length this quarter of the cylinder) Figure 4.

The results obtained by software package *WIPL-D Pro v6.1* are compared with results obtained by *ESM* in Figure 6.

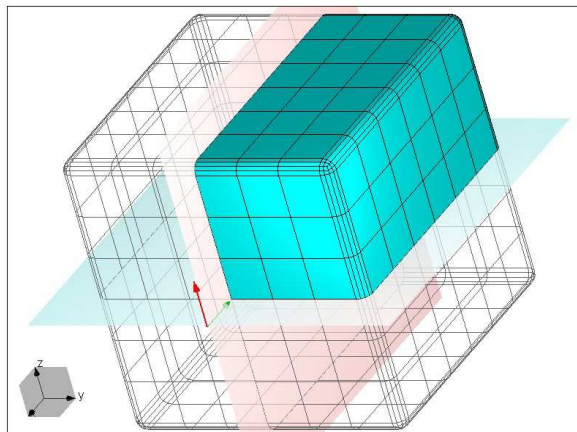


Fig. 4. Cube electrode modelled in *WIPL-D*.

Agreement of the results is very good for ratio  $0.01 \leq r/a \leq 0.1$ , but for ratio

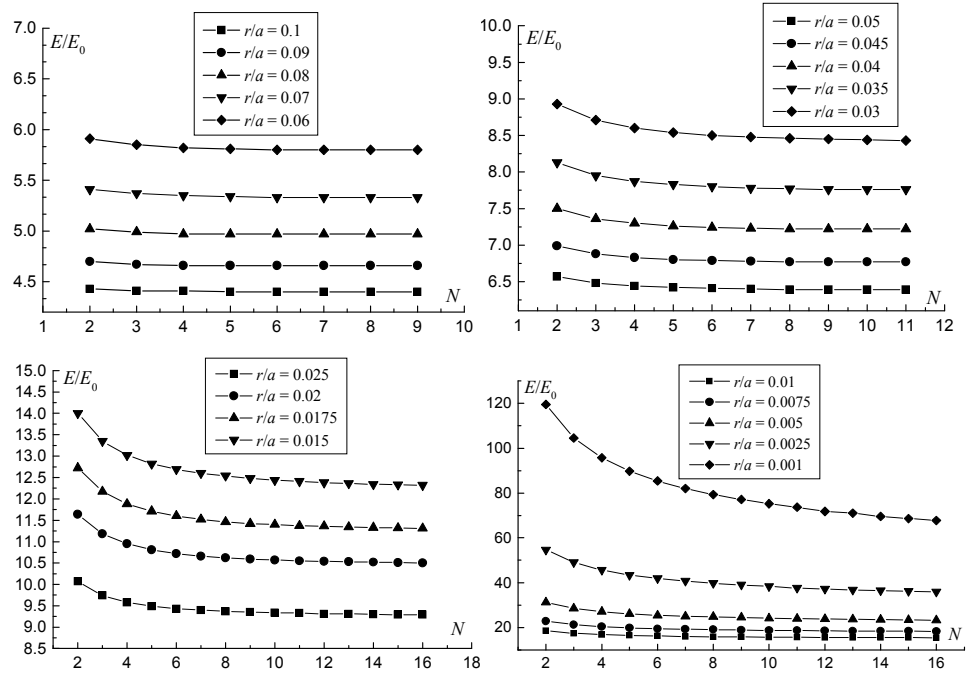


Fig. 5. The convergence of the results obtained by WIPL-D, for ratio  $0.001 \leq r/a \leq 0.1$ .

$r/a < 0.01$  some disagreements exist. For determining results with high accuracy by applying specific Equivalent Source Method in ratio  $r/a \leq 0.01$  more unknowns (equivalent sources) were needed. In that case, calculations with PC took too much time and that was a limiting factor.

### 3 Approximate Analytical Expression

Electric field amplification,  $k$  is defined as ratio of resulting electric field and electric field of unperturbed external electric field,

$$k = \frac{E}{E_0}.$$

In the centre of the rounded vertices it depends on ratio of the wedge length and radius of wedge curvature,  $r/a$ .

According to the presented results, for calculation of  $k$  in the centre of the rounded vertices a simple analytical expression can be formed

$$\frac{E}{E_0} = k' \left( \frac{a}{r} \right)^p, \quad (1)$$

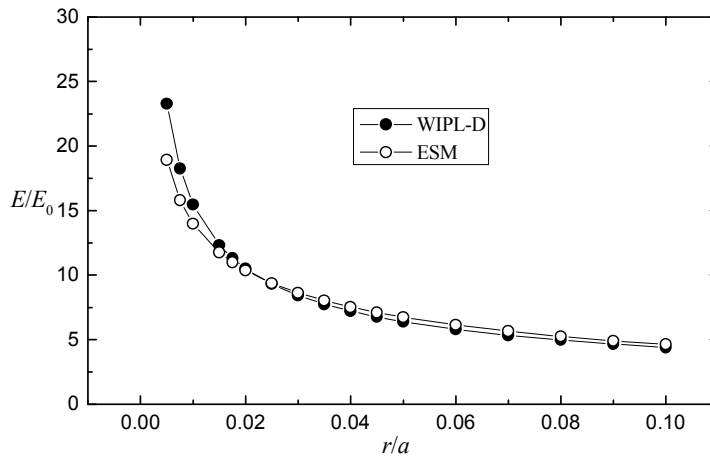


Fig. 6. Comparison of the results for the electric field strength in point *P*, for different ratio *r/a*.

where *k'* and *p* can be easily determined using simple mathematical procedure. The results for the values of *k'* and *p* for different ratio *r/a* are presented in Figure 7.

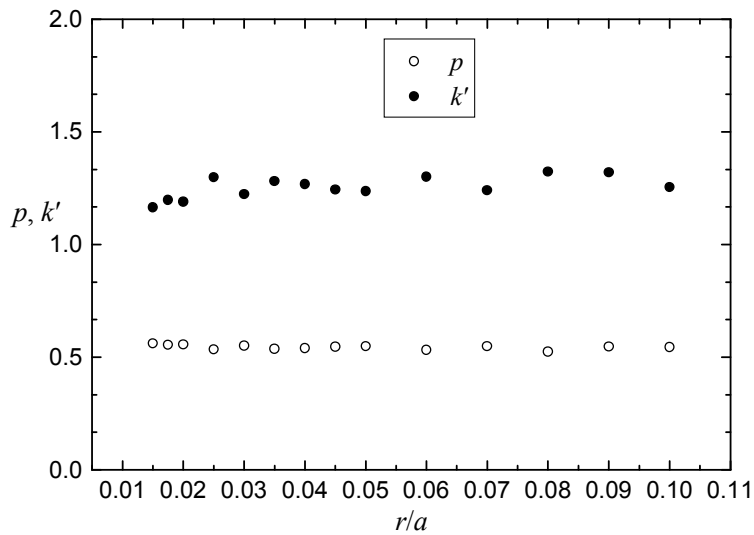


Fig. 7. The value of constants *k'* and *p*.

If the middle values for these constants are chosen equation (1) becomes

$$\frac{E}{E_0} \approx 1.25 \left(\frac{a}{r}\right)^{0.546} \tag{2}$$

Simple expression (2) although gives approximate results that are satisfactory

for every day engineering practice.

The comparison of the results for the electric field amplification obtained with equation (2) and results obtained by software package WIPL-D, for different ratio  $r/a$  are presented in Figure 8.

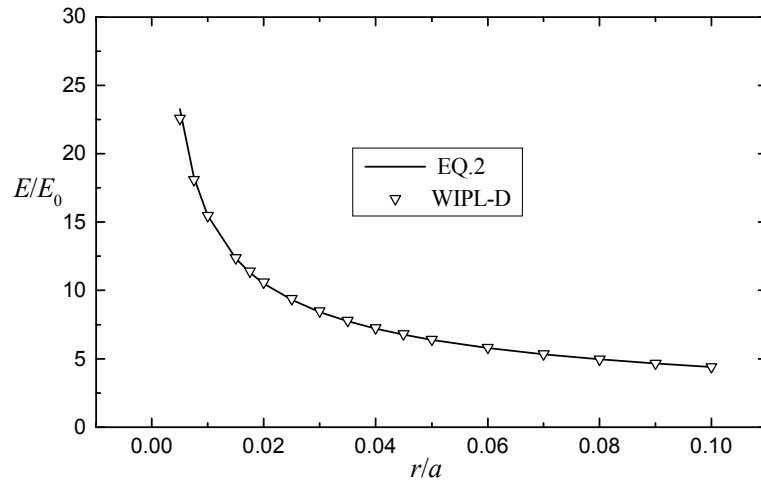


Fig. 8. Comparison of the results for the electric field amplification in point  $P$ , for different ratio  $r/a$ .

#### 4 Conclusion

In this paper intensity of resulting electric field in the center of the rounded vertices of neutral conducting cube is determined by using different numerical methods.

According to the obtained numerical results simple approximate analytical expression for calculation of electric field in the centre of the rounded vertices of cube is proposed, equation (2).

Using this expression, electric field strength in the centre of the rounded vertices can be easily determined with satisfying accuracy for all ratios of wedge curvature and wedge length  $r/a$ , e.g. for ratio

$$r/a = 0.001, \quad E/E_0 \approx 54.31.$$

Results presented in this paper can be applied to determinate the maximum value of electric field strength in vicinity of the huge conducting objects with rounded wedges immersed to the homogenous electric field, e.g. vehicles, equipments etc.



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