



AN APPROXIMATE RELIABILITY EVALUATION METHOD FOR IMPROVING TRANSPORTATION NETWORK PERFORMANCE

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Abstract. Considering the importance of maintaining network performance at desired levels under uncertainty, network reliability, as a new approach to assessing the performance of degradable urban transportation networks, has become increasingly developed in two recent decades. In this paper, a method for optimizing resource allocation to meet the required levels of transportation network reliability is proposed. The worked out method consists of two stages: at stage one, a method for computing the reliability of network connectivity based on the reliability of computing arc performance with an assumption that capacities are random variables for each arc is presented. These random variables are assumed to be conformed to especial probability density functions which can be modified through investing to improve the performance reliability of the arcs. At stage two, a mixed integer nonlinear programming model is developed to optimize resource allocation in the network. Numerical results are also provided in a simple network to demonstrate the capability of the employed method.

Keywords: uncertainty, optimization, urban transportation networks, connectivity reliability.

1. Introduction

Economic issues of a city or region necessitate a transportation system to be efficient, safe and reliable to provide an accessible network for trip-makers. In order to achieve such expected goals in a network, engineers need powerful scientific tools whereby they can improve network components with appropriate performance (Junevičius and Bogdevičius 2007, 2009; Daunoras *et al.* 2008; Basu and Maitra 2007; Beasley and Christofides 1997, etc.).

Under the present conditions, the level of service (LOS) is the most common measure for assessing the performance of a road segment defined based on the density of vehicles per mile per lane (Highway Capacity Manual 2000). On a large spatial scale, vehicle-miles travelled (VMT), vehicle-hours travelled (VHT) and total delay can be used to evaluate the performance of a segment or an entire freeway system (Chen *et al.* 2003). When designing a network is considered under uncertain conditions, we can hardly use aforementioned performance measures; consequently, network performance reliability has to be evaluated. For example, an earthquake pertaining to its destructive power is a probabilistic event and we are not able to predict if it occurs and how much intensity it will have. Therefore, in degradable

networks dealing with uncertain conditions, it is necessary to employ such effective methods that can consider uncertainty associated with their evaluation parameters.

In this paper, for analyzing network performance reliability, instead of using the presumed arc performance reliability applied in some previous researches, arc capacity functions have been used; therefore, arc performance reliability is computed during network analysis based on a more real assumption. In other words, by means of this method, it would be possible to use the actual quantity of arc performance reliability in any special condition for solving the problem (e. g. considering the expected LOS and arc flow volumes) rather than using the presumed values for arc performance reliability.

This paper addresses the problem of investing in a degradable transportation network in order that when dealing with uncertain conditions, the network operates appropriately and can maintain performance index at a desired level. The proposed method uses the performance functions of arcs that are a probabilistic distribution of capacities. The method also has two main aspects: first, a method for assessing connectivity reliability is presented and second, a mixed-integer nonlinear model is developed for optimizing the allocation of resources, so as network performance reliability can be maximized for different levels of available funds.

Some existing studies on road network reliability analysis along with the definitions of different aspects of transportation network reliability will be presented in the Section 2 of this paper. Section 3 presents a new framework for reliability analysis consisting of arc capacity probability distributions, arc reliability functions, capacity reliability and connectivity reliability. Section 4 discusses investment functions applied to modify arc reliabilities. The optimization programming model for resource allocation is extended in Section 5. Section 6 applies the proposed method to a numerical example. The Section 6 provides a conclusion and identifies directions for future research.

2. Literature Review

The majority of researches have been done on transportation network reliability focusing on reliability assessment. On the contrary, no great attention has been paid to reliability optimization problems encountered in this field.

However, research in the area of transportation hazards generally aids governments in allocating limited resources to the four phases of risk management: mitigation, preparedness, response and recovery (Kutz 2003). The present investigation can be related to those research works in which preparedness or recovery is under consideration.

Reliability is defined as the ability of an item to perform a required function under given environmental and operational conditions and for a stated period of time (Rausand and Høyland 2003). Unfortunately, the reliability analysis of road networks, compared with some other systems (like electric power systems, water distribution systems and communication networks), have received little attention in spite of its importance. However, the existing reliability studies on road networks mainly contain four aspects: travel time reliability, connectivity reliability, capacity reliability and performance reliability.

2.1. Connectivity Reliability

Possibly, the major impulse for establishing serious researches on road network reliability was natural disasters (e. g. earthquake) that can severely disrupt the network by disconnecting paths. Hence, connectivity reliability was the first measure of performance reliability taken into account to evaluate the performance of degradable transportation networks. Connectivity reliability considers the probability that a pair of nodes in a network remains connected. This measure, in other words, is similar to the reliability measure of communication networks commonly called 'Two-terminal Reliability' (e. g. Grosh 1989). A special case of this index is terminal reliability that is concerned with the existence of at least one path between each origin-destination (OD) pair (Iida and Wakabayashi 1989). Some methods for analyzing the connectivity reliability of transport networks could be found in the previous works like Asakura (1999); Wakabayashi and Iida (1992); Bell and Iida (1997); Asakura *et al.* (2001); Sumalee and Watling

(2003); Du and Nicholson (1997). All of these studies assume links to have a probabilistic and binary state. The state of a link is shown by an integer variable equal to 1 if the link operates normally and 0 when it fails. Note that the states of a link may be considered as different definitions determined by the planner.

As it will be seen, this paper presents a new approach for computing the states of arcs, whereby the performance reliability of an arc is defined considering the probability density function of its capacity and especial v/C ratio as the level of service (LOS). Having the performance reliability of the arcs, the reliability of network connectivity can be computed through a closed formula. Furthermore, since travel time is a function of v/C ratio, there may be an interrelation between connectivity reliability obtained applying this method and travel time reliability.

2.2. Travel Time Reliability

Travel time reliability considers the probability that a trip between a given origin-destination (OD) pair can be completed successfully within a specified time interval (Asakura and Kashiwadani 1991; Bell *et al.* 1999). This measure is useful to evaluate network performance under normal daily flow variations (Chen *et al.* 2002a). One of the important approaches was the definition of travel time reliability regarding the degradation state of a network in which travel time reliability is defined as a function of the ratio of travel times under degraded and non-degraded state (Asakura 1999). Some travel time related aspects of transportation network reliability has recently been introduced taking into account 'travel time budget' like those by Lo *et al.* (2006); Siu and Lo (2008), and 'schedule reliability' by Li and Huang (2005).

2.3. Capacity Reliability

Recently, a capacity-related measure for assessing the performance of degradable road networks has been introduced which is concerned with the probability that a network can accommodate certain traffic demand at a required service level (Chen *et al.* 1999; Yang *et al.* 2000). This measure, capacity reliability, may also be defined as the probability that reserve the capacity (largest multiplier applied to a given basic OD demand matrix that can be allocated to a network without violating arc capacity) of the network is greater than or equal to the required demand for given capacity loss due to degradation. Capacity degradation is subject to various events and can practically be obtained as a random variable. The only method presented in previous researches to compute capacity reliability was a basis for Monte Carlo simulation (Chen *et al.* 2002b).

2.4. Behavioural-Related Reliability

In addition to the three aforementioned types of performance reliability, some other measures were addressed to assess the performance of a transport network. These measures in many cases are related to the behavioural responses of users and are based on different aspects of utility or disutility. Bell (1999, 2000)

defined a performance reliability index as the utility of users when they are extremely pessimistic about the state of the network in which the utility was obtained in mixed-strategy Nash equilibrium. Yin and Ieda (2001) and Yin *et al.* (2004) considered performance reliability as the total disutility of commuters on the basis of minimal expected disutility between each OD pair for different classes of commuters.

2.5. Resource Allocation

Investing in a transportation network may be considered with different points of view, from enhancing the service level of a freeway to adding supplementary links into the existing network. Sánchez-Silva *et al.* (2005) proposed a resource allocation model defining the network reliability index as changes in the accessibility of the network. The model maximized the network reliability index based on a set of possible actions described in terms of failure and repair rates of each link. However, considering the complexity of resource allocation in transportation networks under uncertainty, especially in the case of large networks, an approximation method may provide the most appropriate solution taking account of elapsed time for the analyzing procedure. Unfortunately, very little attempt has been made with respect to the methods of approximation resource allocation in transportation networks. Recently, Mansour-Khaki *et al.* (2006) have presented an approximation method to optimize connectivity reliability on the basis of dynamic programming.

In this paper, the optimization model reflects how the performance of a network can be maximized taking into account a special level of budget. In other words, by means of this model, it can be determined what links and level of investment must be modified to provide maximum connectivity reliability of a degradable transportation network.

3. Reliability Analysis

As mentioned in the previous section, different works have been done on the reliability of transportation network connectivity in most of which (e. g. Iida and Wakabayashi 1989; Wakabayashi and Iida 1992; Sumalee and Watling 2003) the performance reliability of each arc has been assumed as predefined values. In this paper, the basis of analysis is a random distribution function for the capacity of each arc, which indicates the actual performance of network arcs. Here, the performance reliability of each arc is estimated separately during network analysis with respect to the real condition happening in the network, and then, the reliability of network connectivity is calculated based on the reliability of arc performance.

However, a transportation network is susceptible to a broad range of internal and external incidents that can affect its normal functioning. External incidents mainly include weather conditions and natural phenomena like snowfall, ice formation, hurricane, earthquake and av-

alanche. Internal degradations are due to traffic related factors such as a failure of control systems, accidents as well as maintenance and construction operations. All aforementioned circumstances can severely decrease arc capacities. As a result, the network cannot perform as it would in a normal situation. In such circumstances, it would be important to know about the performance reliability of the network.

In this paper, the effects of all incidents are assumed to be included in the capacity function of the network arcs. Further, it is accepted that these capacity functions are independent functions. Arc capacity functions are discussed in the following part of this section.

3.1. Arc Capacity Functions

An arc capacity function reflects the relation between different levels of arc capacity and their probabilities. In this paper, the capacity function presumed for each arc is a normal probability density function (PDF) for its capacity in which the probability of occurring different capacities is shown as probability density. The purpose of selecting normal distribution is that in some previous works, this type of performance function is used. Such similarity allows us to compare the results obtained from these similar works. A normal PDF for the capacity of arc i , $f_i(c)$ is shown as follows:

$$f_i(c) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(c-\mu_i)^2}{2\sigma_i^2}\right), \quad (1)$$

where: variable c is the value taken by random arc capacity C and μ_i and σ_i are respectively the mean and standard deviations of the capacity of arc i . Note that $f_i(c)$ does not indicate occurrence probability for $C = c$ but shows a special manner of spreading the probability of variable C .

In general, a normal random variable can take every quantity in interval $(-\infty, +\infty)$ while the capacity of arc i is limited between upper bound C_{\max}^i and lower bound C_{\min}^i . An important issue about PDFs is that the integration of the PDF from $-\infty$ to $+\infty$ must be equal to 1. If normal distribution is assumed to have a determined lower and upper bound, then, the sum of areas under the curve of the PDF cannot be equal to 1. In order to overcome this problem, a pseudo-normal PDF is utilized and denoted as $f_i^p(c)$. For this purpose, a constant parameter is defined as follows:

$$C_i^0 = \frac{1 - \int_{C_{\min}^i}^{C_{\max}^i} f_i(c) dc}{C_{\max}^i - C_{\min}^i}. \quad (2)$$

Now, we can define the pseudo-normal PDF as:

$$f_i^p(c) = f_i(c) + C_i^0. \quad (3)$$

Therefore, the aforementioned characteristic is kept because:

$$\int_{C_{\min}^i}^{C_{\max}^i} (f_i(c) + C_i^0) dc = \int_{C_{\min}^i}^{C_{\max}^i} C_i^0 dc + \int_{C_{\min}^i}^{C_{\max}^i} f_i(c) dc =$$

$$C_i^0 (C_{\max}^i - C_{\min}^i) + \int_{C_{\min}^i}^{C_{\max}^i} f_i(c) dc =$$

$$1 - \frac{\int_{C_{\min}^i}^{C_{\max}^i} f_i(c) dc}{C_{\max}^i - C_{\min}^i} (C_{\max}^i - C_{\min}^i) +$$

$$\int_{C_{\min}^i}^{C_{\max}^i} f_i(c) dc = 1. \quad (4)$$

Hence, $f_i^p(c)$ is considered as a probability density function which can hold all characteristics of normal distribution with the exception of infinite bounds. Furthermore, adding constant quantity to a random variable does not affect the spread of distribution (Ross 2009):

$$Var(C_i^0 + C_i) = Var(C_i). \quad (5)$$

It implies that the shape of the second PDF ($f_i^p(c)$) is kept as well as the first one ($f_i(c)$).

3.2. Arc Performance Reliability

In this paper, the performance reliability of arc i is defined as the probability that the capacity of arc i is greater than or equal to its flow volume. In other words, the performance reliability of arc i can be formed as follows:

$$R_i = P(C_i \geq v_i) = \begin{cases} a, & v_i \geq C_{\min}^i; \\ 0, & C_{\max}^i < v_i; \end{cases} \quad (6)$$

$$a = \int_{v_i}^{C_{\max}^i} f_i^p(x) dx = \int_{v_i}^{C_{\max}^i} \left(f_i(x) + \frac{1 - \int_{C_{\min}^i}^{C_{\max}^i} f_i(c) dc}{C_{\max}^i - C_{\min}^i} \right) dx,$$

where: v_i and C_i are flow volume and the random capacity of arc i . In fact, when arc flow volume is lower than the upper bound of capacity, the integration of $f_i(c)dc$ from v_i to C_{\max}^i expresses arc reliability; otherwise, arc reliability is 0; namely, there is no chance for arc capacity to be greater than or equal to flow volume assigned to the arc.

3.3. A Challenge on Equilibrium Flow Consideration in Networks with Stochastic Supply

In congested urban transportation road networks, arc flow volumes are computed with respect to the relation between travel cost (i.e. travel time) and volume on arcs, like:

$$TC_i = g_i(v_i, c_i), \quad (7)$$

where: TC_i is travel cost on arc i . Assuming that all driver (user) travelling in the network from origin O to destination D (O and D are specific nodes of the networks) are familiar with the whole network and know

well the expected travel cost for traversing each arc based on Wardrop's user equilibrium principle (Sheffi 1985), the following mathematical programming can be applied to assign Origin-Destination (OD) travel demands for the network and to compute volumes on arcs v_i :

$$\text{Minimize } Z = \sum_{i \in I} \int_0^{v_i} TC_i(x, c_i) dx. \quad (8)$$

Subject to:

$$\sum_{r \in R_w} f_r = q_w, \quad \forall w \in W \quad (9)$$

$$v_i = \sum_{r \in R} f_r X_i^r, \quad \forall i \in I \quad (10)$$

$$f_r \geq 0, \quad \forall r \in R \quad (11)$$

where: R – the set of network paths, W – the set of OD pairs, R_w – the set of paths, q_w – flow between OD pair $w \in W$, f_r – user equilibrium flow on path $r \in R$ and X_i^r is 1 if arc i belongs to path r ; otherwise 0.

The above problem has been known as user equilibrium (UE) assignment. In many previous related literature (e. g. Chen *et al.* 1999, 2002a, 2002b; Yang *et al.* 2000), with simplification assumptions, professionals have tend to consider the UE with respect to stochastic arc capacities, i. e. all users are assumed to be aware of variations in arc capacities or exact arc travel times. Hence, the UE is converted to stochastic user equilibrium (SUE) with the following objective:

$$\text{Minimize } Z = \sum_{i \in I} \int_0^{v_i} TC_i(x, C_i) dx. \quad (12)$$

Note that C_i is a random variable used in equation 12 rather than c_i used in equation 8 that is non-random capacity of arc i . Although we can rely on such assumption where variations are due to special incidents (like accident, control device failure, etc.), talking about day-to-day capacity variations, this assumption would not be realistic. Therefore, in this paper, we assume that users are ordinarily aware of average travel times (or arcs capacities) at a specific hour of workdays, and therefore, for the sake of assigning the existing traffic to the network arcs, the expected values of random arc capacities in this particular case accepted as the average capacity values μ_i , will be considered. Thus, the objective function of the UE problem is defined as:

$$\text{Minimize } Z = \sum_{i \in I} \int_0^{v_i} TC_i(x, \mu_i) dx. \quad (13)$$

When a real network is under consideration, we try to design arcs with such properties that they can operate at a desired LOS in the network. Each level of service (from A to E) takes a special amount of $(v/C)_i$ (or α_i), which is considered as the LOS index of arc i ; therefore, the reliability of arc performance can be considered as the probability that the capacity of arc i is greater than or equal to v_i/α_i . This definition may help us to eval-

uate the reliability of network performance at a special LOS. Hence, relation 6 can be rewritten as following:

$$R_i = P(C_i \geq v_i / \alpha_i) = \begin{cases} \int_{v_i/\alpha_i}^{C_{\max}^i} f_i^p(c) dc, & v_i/\alpha_i \geq C_{\min}^i; \\ 0, & C_{\max}^i < v_i/\alpha_i. \end{cases} \quad (14)$$

It is to say, $\alpha_i = 1$ indicates that all arcs of the network can operate with their maximum capacities.

3.4. Connectivity Reliability

This section discusses how the connectivity reliability of a network (or OD pairs) can be calculated based on the reliability of arc performance and considering a required service level in the form of v/C ratio. On the other hand, since the travel time of an arc is subject to its flow volume to capacity ratio, there may be an interrelation between travel time reliability and connectivity reliability presented in this paper.

There exist various ways to compute the exact connectivity reliability for a special OD pair in a network. Generally, due to a huge computation volume arisen from the exact solution procedures in large networks, these methods cannot be used in a realistic scale network. A relevant approximate method is to compute an upper and lower bound of reliability using minimal cut and path sets (precise information about minimal cut and path sets can be found in reliability related texts, e. g. Bedford and Cooke 2001; Grosh 1989). Using this approximation method, connectivity reliability between OD pair w can be computed as follows:

$$R_w^{connectivity} = (R_w^{\min} + R_w^{\max}) / 2, \quad (15)$$

where: $R_w^{connectivity}$, R_w^{\max} and R_w^{\min} are respectively average, lower bound, and upper bound of connectivity reliability between OD pair w . To evaluate the connectivity reliability of all OD pairs as a unique index, the weighted average of OD pair connectivity reliability may be utilized with respect to the ratio of the required demand between OD pairs to the whole required demand:

$$R_{net}^{connectivity} = \sum_{w \in W} \left(\frac{R_w^{connectivity} q_w}{\sum_{w \in W} q_w} \right), \quad (16)$$

where: q_w is the required demand between OD pair w . Hence, the reliability of network connectivity is a function of arc reliability:

$$R_{net}^{connectivity} = f(R_1, R_2, \dots, R_i, \dots, R_n). \quad (17)$$

This method provides a closed formula to evaluate the reliability of network connectivity in the calculation procedure of which not only service level of each arc but also random arc capacities are taken into account. The most important advantages of employing the closed formula to evaluate network performance reliability are simple to use in optimization techniques and have very little elapsed time for the computation procedure com-

pared with simulation based techniques such as Monte Carlo and Latin Hypercube Sampling (LHS).

3.5. Investment Functions

An investment function shows the amount of investment (or investment) needed to promote the reliability of a link from a base level to a higher level and/or from the higher level to an even higher one and so forth. Pertaining to the structure of arcs in a transportation network, it is assumed that investment functions can take only especial levels of investments, and therefore are discrete functions. We further have assumed that the investments have assigned to transform arc capacity PDFs to modify their performances in a manner that each investment level provides a special modified PDF for the capacity of each arc. E_{ij} represents the quantity of the investment function of arc i for investment level j and shows how much investment it needs to meet level j of reliability, R_{ij} when the performance reliability of arc i is at its base level, i.e. E_{ij} will result in R_{ij} . Recall that we have assumed that investment E_{ij} alters the form of base arc capacity PDF, $f_i(c)$, by changing σ_i and μ_i to σ_{ij} and μ_{ij} that are a standard deviation and mean of the probability density function of arc i at investment level j , $f_{ij}(c)$. Now, using base expected arc capacities μ_i and then calculating flow volumes v_i , the reliability of arc performance at level j can be computed as:

$$R_{ij} = P(C_{ij} \geq v_i / \alpha_i) = \begin{cases} \int_{v_i/\alpha_i}^{C_{\max}^{ij}} f_{ij}^p(x) dx, & v_i/\alpha_i \geq C_{\min}^{ij}; \\ 0, & C_{\max}^{ij} < v_i/\alpha_i, \end{cases} \quad (18)$$

where: C_{ij} is the random capacity of arc i at investment level j and C_{\max}^{ij} and C_{\min}^{ij} are respectively upper and lower bounds for capacity of arc i at investment level j .

3.6. Resource Allocation Model

This section discusses the objective and constraints of the resource allocation problem. The objective is to maximize the reliability of network connectivity $R_{net}^{connectivity}$:

$$\text{Maximize } R_{net}^{connectivity} = f \left(\sum_{j \in J} R_{1j} Z_{1j}, \sum_{j \in J} R_{2j} Z_{2j}, \dots, \sum_{j \in J} R_{ij} Z_{ij}, \dots, \sum_{j \in J} R_{nj} Z_{nj} \right), \quad (19)$$

where: $R_{net}^{connectivity}$ is a mixed integer nonlinear function of network performance reliability obtained from relation 17 replacing R_i with $\sum_{j \in J} R_{ij} Z_{ij}$, n is the number of arcs in the network and Z_{ij} is a binary state variable in case if investment level $j \in J$ is assigned to arc $i \in I$, then its value equals 1, otherwise is 0.

Constraints of the model are:

$$\sum_{j \in J} \sum_{i \in I} E_{ij} \cdot Z_{ij} \leq E_{\max}; \quad (20)$$

$$\sum_{j \in J} Z_{ij} = 1, i = 1, 2, \dots, n; \tag{21}$$

$$Z_{ij} \in \{0, 1\}, \forall i \in I, \forall j \in J, \tag{22}$$

where: E_{ij} is maximum budget or the allowable total investment. Constraint 20 indicates that the sum of investments must not be greater than maximum budget. Constraint 21 shows that each arc can achieve only one investment level and constraint 22 guarantees that decision variable Z_{ij} is binary.

It is to be noted that the reliability of arc performance alter along with arc capacities because the reliability of arc performance depends on arc volumes (e. g. equation 18) and arc volumes are due to the conservation of user equilibrium condition provided by equations 13, 9, 10 and 11. Thus, a bi-level approach must be considered to keep optimality. We utilized a standard Genetic Algorithm (GA) to solve the proposed bi-level resource allocation model the detail algorithm of which will be presented in another article.

3.7. Considerations on Network Size

When a large-scale network is under consideration, not only the exact calculation of connectivity reliability but also its lower an upper approximation will be computationally intractable. To concern this problem, some approximation methods may be applied, like those presented by Shier and Liu (1992); Li and Silvester (1984); Rosenthal (1977). For even more approximations, these methods can be used along with different types of k -shortest path algorithms. Interested readers may find k -shortest path algorithms in Takaoka (1998).

4. Numerical Results

As explained in the previous sections, this paper concerns with two aspects of network reliability evaluation – performance reliability analysis and resource allocating with respect to reliability optimization. The subjects of those parts dealt with reliability analysis are similar in some existing works, and therefore for the sake of comparing the numerical results obtained from the proposed and other methods, we have examined a simple test network employed repeatedly in network reliability analysis, particularly in the case of capacity reliability studies (e. g. Chen *et al.* 2002b). As shown in Fig. 1, the network consists of five nodes, seven arcs and two OD pairs. The base demand for OD pairs (1,4) and (1,5) are 20 and 25, respectively. The Bureau of Public Road (BPR) arc travel time function is used:

$$t_i = t_i^f \left(1 + 0.25 \left(\frac{v_i}{C_i} \right)^4 \right), \tag{23}$$

where: v_i , t_i^f and C_i are flow, free-flow travel time and random capacity on arc i , respectively.

Table 1 gives the results of user equilibrium traffic assignment and general information about the network arcs.

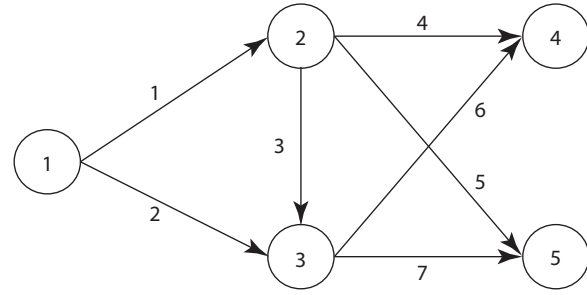


Fig. 1. An example of the network

Table 1. General properties of the arcs and assigned flow volumes

Arc No.	Free-flow travel time	Statistical properties of arc capacities				Flow volume
		Mean	Standard deviation	Lower bound	Upper bound	
1	4.0	18.75	3.61	12.5	25.0	24.95
2	5.2	18.75	3.61	12.5	25.0	20.05
3	1.0	11.25	2.17	7.5	15.0	5.86
4	5.0	11.25	2.17	7.5	15.0	7.84
5	5.0	11.25	2.17	7.5	15.0	11.25
6	4.0	11.25	2.17	7.5	15.0	12.16
7	4.0	11.25	2.17	7.5	15.0	13.75

4.1. Reliability Analysis

Because Monte Carlo simulation is a powerful technique to analyze probabilistic networks, especially when enough samples are drawn, it may result in an acceptable solution. For comparisons, therefore, we will consider Monte Carlo simulation method.

Reliability assessment procedure presented in this paper has been applied to the test network with the assumption of $(v/C)_i = 1$.

Fig. 2 compares the connectivity reliability of OD pair (1,4) obtained from the proposed method and Monte Carlo simulation (with 5000 samples) for different demand levels.

As mentioned in sub-section 3.3, the LOS (in the form of v/C ratio) can be considered to calculate network reliability in the proposed method.

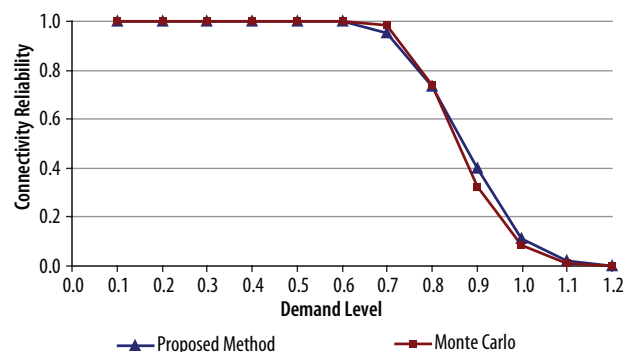


Fig. 2. Connectivity reliability of OD pair (1,4) at various levels of demand

Fig. 3 shows how the reliability of network connectivity reacts to the different amounts of v/C . When $v/C < 0.75$ (the network is expected to be connected with a higher LOS), the network is nearly unreliable. On the other hand, for $v/C > 0.9$, lower service quality is expected, and accordingly, the network is almost reliable. Differently from the above amounts of v/C , when v/C is between 0.75 and 0.9, the rate of change in the reliability of network connectivity is considerable, thus it would be important to know the LOS or its related v/C the network is expected to operate at and, therefore, how much the decision maker must spend to ascertain the desired performance of the network.

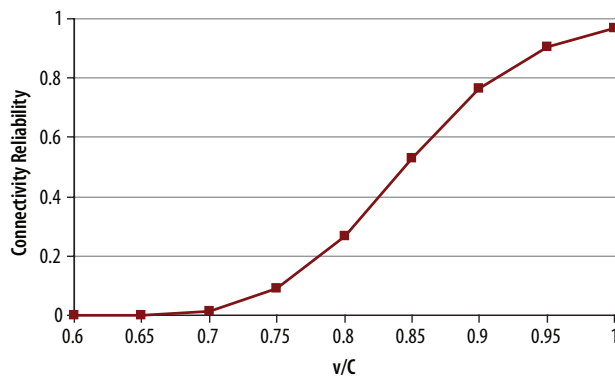


Fig. 3. The reliability of network connectivity for different levels of v/C

4.2. Resource Allocation

Before computing the reliability of arc performance for different investment levels, we need to determine investment functions. It is assumed that all arcs can be modified by investment quantities 1, 2.5 and 5 each of which can alter only the mean of capacity PDFs and all standard deviations would be constants. For different investment levels, the mean capacities of the arcs and related calculated reliability for different amounts of v/C are displayed in Table 2.

Fig. 5 reflects the result summary of implementing the proposed reliability optimization model to the test network for various amounts of budget from 0 to 35. The figure also exhibits the sensitiveness of the resource allocation model for different v/C s. As shown in Fig. 5, when v/C decreases from 1 to 0.7, the reliability of network connectivity declines rapidly and for amounts lower than 0.7, the reaction of connectivity reliability is not so considerable.

In Table 3, the arcs that ought to be invested for some possible budget levels and their related investment levels are presented. Table 3 shows that various budget levels would result in different arc investment combinations. For example, with the budget level of 1 unit ($E_{max} = 1$), the proposed optimization model necessitates arc 1 to be invested at investment level 1, whereas, for budget level 2, we have to invest 1 unit in both arcs 2 and 6.

Fig. 4 reflects a summary of results implementing the proposed reliability optimization model to the test network for various amounts of budget from 0 to 35. The figure also exhibits the sensitiveness of the resource allocation model for different v/C s. As shown in the figure, when v/C decreases from 1 to 0.7, the reliability of network connectivity declines rapidly and for amounts lower than 0.7, the reaction of connectivity reliability is not so considerable.

In order to display how much capability the proposed reliability optimization method has, we have applied uniform investment allocation in all arcs for three possible levels of budget: 7 (1 investment unit for each arc), 17.5 (2.5 investment units for each arc), and 35 (5 investment units for each arc). Fig. 5 compares the proposed method and uniform allocation which clearly indicates that the capability of the proposed method will extremely appear when we are confronted with investment limitation. Namely, the difference between the results obtained from different methods for budget level 7 is too much higher than those obtained from 17.5 and 35.

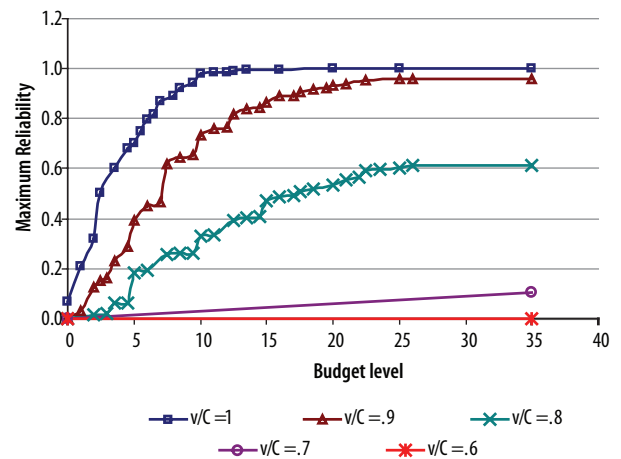


Fig. 4. Maximum reliability of network connectivity for different levels of budget

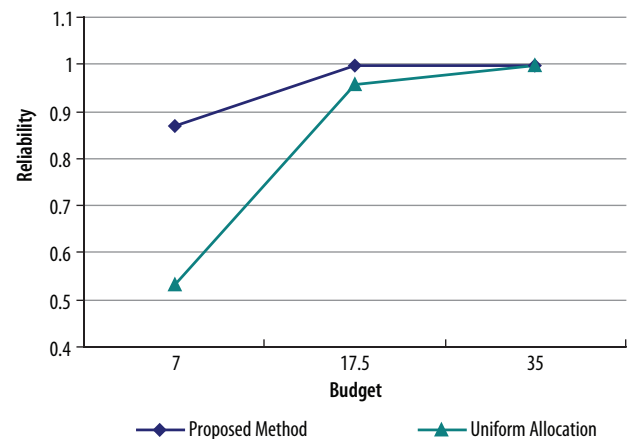


Fig. 5. Comparison between the reliability of network connectivity obtained from the proposed method and uniform allocation

Table 2. Effort levels and related changes in performance reliability and the mean of each arc

		Arc No.							
Effort Level, E_{ij}	Property	1	2	3	4	5	6	7	
Base (0)	Mean Capacity	18.75	18.75	11.25	11.25	11.25	11.25	11.25	
	Reliability	$v/c = .6$	0	0	0.7761	0.1739	0	0	0
		$v/c = .7$	0	0	0.945	0.51	0	0	0
		$v/c = .8$	0	0	1	0.7707	0.0606	0	0
		$v/c = .9$	0	0.1337	1	0.9137	0.2623	0.1165	0
		$v/c = 1$	0.0014	0.3466	1	0.9825	0.5	0.3226	0.0902
1	Mean Capacity	22.5	22.5	13.5	13.5	13.5	13.5	13.5	
	Reliability	$v/c = .6$	0	0	0.9992	0.5864	0	0	0
		$v/c = .7$	0	0.003	1	0.888	0.083	0	0
		$v/c = .8$	0	0.2151	1	0.9977	0.3884	0.1907	0.0029
		$v/c = .9$	0.0352	0.5268	1	1	0.6938	0.4978	0.1794
		$v/c = 1$	0.2258	0.7742	1	1	0.8822	0.7528	0.4499
2.5	Mean Capacity	26.25	26.25	15.75	15.75	15.75	15.75	15.75	
	Reliability	$v/c = .6$	0	0	1	0.9278	0.0452	0	0
		$v/c = .7$	0	0.2313	1	1	0.4357	0.2025	0
		$v/c = .8$	0.048	0.6406	1	1	0.8074	0.6092	0.2313
		$v/c = .9$	0.3273	0.8976	1	1	0.9726	0.8809	0.594
		$v/c = 1$	0.6534	0.9986	1	1	1	0.9923	0.8511
5	Mean Capacity	28.00	28.00	17.00	17.00	17.00	17.00	17.00	
	Reliability	$v/c = .6$	0	0.0273	1	1	0.1834	0.0263	0
		$v/c = .7$	0	0.4229	1	1	0.6808	0.4258	0.076
		$v/c = .8$	0.1603	0.8187	1	1	0.9499	0.8238	0.4624
		$v/c = .9$	0.5335	0.9839	1	1	1	0.9869	0.8125
		$v/c = 1$	0.8283	1	1	1	1	1	0.9726

Table 3. The arcs that should be invested in various budgets to achieve optimum resource allocation in the network, $v/C=1$

		Budget Level, E_{max}									
E_{ij}	1	2	3.5	4.5	5	5.5	6.5	8.5	9.5	16	35
1	1	2,6	5	2,7		2,5,7	2,5,6,7	6	5,6	5	
2.5			1	1	1,7	1	1	1,2,7	1,2,7	2,6	
5										1,7	All Arcs

5. Concluding Remarks

We have presented a reliability optimization method concerned with resource allocation modelling so as it would be possible to identify which arcs with how much effort must be modified to meet the required levels of connectivity reliability. This study has two main properties. The first one is dealt with a new approach to performance reliability computed based on the reliability of arc performance and taking into account the required LOS as a function of v/C ratio and arc capacity PDFs. The second property is the ability to adopt the above discussed ap-

proach to the evaluation of performance reliability in a way it can be embedded in an optimization model.

However, the most important result of this research is that it establishes a tool enabling approximate improvement in transport networks under uncertainty. In the proposed reliability optimization model, the arcs can be modified with different levels of investment which consequently would make different combinations of arc capacities in a manner that selecting one of these combinations would result in approximate improvement in the network. Although this paper presents a method comprising

possible influencing factors, reliability optimization in transportation networks is still at its preliminary stages. However, future research may take advantage of extending the proposed method considering the interdependency of arc capacity distributions, the role of multiple objective decision making methods in the optimization model and time scheduling to minimize traffic disturbance during the execution of improvement actions on the arcs.

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