

An approximate solution method based on tabu search for k -minimum spanning tree problems

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Abstract—This paper considers k -minimum spanning tree problems. An existing solution algorithm based on tabu search, which was proposed by Katagiri et al., includes an iterative solving procedure of minimum spanning tree (MST) problems for subgraphs to obtain a local optimal solution of k -minimum spanning tree problems. This article provides a new tabu-search-based approximate solution method that does not iteratively solve minimum spanning tree problems. Results of numerical experiments show that the proposed method provides a good performance in terms of accuracy over those of existing methods for relatively high cardinality k .

I. INTRODUCTION

A k -minimum spanning tree (k -MST) problem is a combinatorial optimization problem to find a subtree with exactly k edges, i.e., k -subtree, such that the sum of the weights is minimal. The k -MST problem was firstly introduced by Hamacher et al. [6] in 1993, and it can be applied to many real-world problems in wide variety of decision making, e.g. in telecommunications [13], facility layout [10], open pit mining [1], oil-field leasing [6], matrix decomposition [16], [17] and quorum-cast routing [18].

Since the k -MST problem is NP-hard [11], [12], it is difficult to solve large-scale problems within a practically acceptable time. Therefore, it is very important to construct solving methods which quickly obtain a near optimal solution.

As for existing approximate solution methods for k -MST problems, Urosevic et al. [3] provided approximate solution methods based on Variable Neighborhood Search (VNS). Blum et al. [2] also proposed several metaheuristic approaches. Recently, Katagiri et al. [5] developed a solution method which uses a combination of tabu search and an iterative solving procedure for minimum spanning tree (MST) problems. They showed that their method provides a better performance than existing methods for dense graphs with high cardinality k through some numerical experiments.

In this paper, we propose a new tabu-search-based approximate solution method with an efficient local search algorithm. Our local search algorithm obtains local optimal solutions of k -MST not solving MST problems iteratively. In order to demonstrate efficiency of the proposed solution method, we compare the performances of the proposed method with those of existing methods by Blum et al. and Katagiri et al.

This paper is organized as follows: Section 2 provides problem formulation. In Section 3, we introduce a summary of tabu search.

II. PROBLEM FORMULATION

Given that a graph $G = (V, E)$ where V is a set of vertices and E is a set of edges, k -subtree T_k is defined as

$$T_k \in G, \quad k \leq |V| - 1.$$

Then a k -minimum spanning tree problem is formulated as

$$\begin{aligned} & \text{minimize} && \sum_{e \in E(T_k)} w(e) \\ & \text{subject to} && T_k \in \mathcal{T}_k, \end{aligned}$$

where \mathcal{T}_k is the set of all k -subtree T_k in G , $E(T_k)$ denotes the edges of T_k and $w(e)$ is a weight attached to an edge e . The above problem is to seek a k -subtree with minimum sum of weights. If the problem size is small, the problem can be easily solved by finding the k -subtree with the minimum sum of weights after enumerating all possible k -subtrees in a given graph.

Even if the size of problem is not so large, it can be solved by some exact solution algorithm. As for exact solution algorithms for k -MST problems, a branch and bound method [18] and a branch and cut algorithm [7] have been developed and implemented.

However, it has been shown that the k -MST problem is NP-hard even if the edge weight is in $\{1, 2, 3\}$ for all edges, or if a graph is fully connected. The problem is also NP-hard for planar graphs and for points in the plane [12]. Therefore, it is impossible to solve large-scale problems within a practically acceptable time even if the problems is solved by efficient exact solution methods.

Therefore, it is important to construct not only exact solution methods but also efficient approximate solution methods. Metaheuristic approaches such as genetic algorithms are useful for getting an approximate optimal solution. Blum et al. [2] proposed three metaheuristic approaches to k -minimum spanning tree problems, namely, evolutionary computation, tabu search and ant colony optimization. They compared their performances through benchmark instances [8] and showed that the performance of their metaheuristics depends not only on the instances but also on the cardinality k . For example, an ant colony optimization approach is the best for relatively small k s, whereas a tabu search approach has an advantage for large k s in terms of accuracy.

Recently, Katagiri et al. [5] proposed a tabu-search-based approximate solution method which includes a procedure of

iteratively solving minimum spanning tree (MST) problems. They showed that their algorithm has a better performance in terms of accuracy in comparison with those of existing methods for dense graph with large ks .

III. SUMMARY OF TABU SEARCH AND VARIABLE NEIGHBORHOOD SEARCH

A. *tabu search*

Tabu search [4] is one of metaheuristics and is the extension of local search. Let \mathbf{x}^c be a current solution. Local search generally improves the current solution because it moves from the current solution \mathbf{x}^c to a solution $\mathbf{x}' \in N(\mathbf{x}^c)$ which is better than the current solution, where $N(\cdot)$ is a given neighborhood structure. For simplicity, suppose that \mathbf{x}^c is a local minimum solution and that the next solution \mathbf{x}' is selected as the best solution among $N(\mathbf{x}^c)$. If the local search is applied for \mathbf{x}' , then \mathbf{x}' is moved back to \mathbf{x}^c because \mathbf{x}^c is the best solution among a neighborhood $N(\mathbf{x}')$. In this way, cycling among solutions often occurs around local minima. In order to avoid such cycling, TS algorithms use a short-term memory. The short-term memory is implemented as a set of tabu lists that store solution attributes. Attributes usually refer to components of solutions, moves, or differences between two solutions. Tabu lists prevent the algorithm from returning to recently visited solutions.

Aspiration criteria permit a part of moves in the tabu list to cancel any tabu status. The typical aspiration criterion is to accept a tabu move if it leads to a new solution better than the current best solution.

The outline of TS is as follows:

- Step 1** Generate an initial solution \mathbf{x} and initialize a tabu list TL .
- Step 2** Find the best solution $\mathbf{x}' \in N(\mathbf{x})$ such that $\mathbf{x}' \notin TL$, and set $\mathbf{x} := \mathbf{x}'$.
- Step 3** Stop if a termination condition is satisfied. If not, then update TL and return to Step 2.

In Step 2, a tabu list memorizes solution attributes. A tabu tenure, i.e., the length of the tabu list determines the behavior of the algorithm. A larger tabu tenure forces the search process to explore larger regions, because it forbids revisiting a higher number of solutions.

In step 3, it is checked whether the algorithm satisfies a termination condition. The termination condition is usually related to the iteration number of the algorithm and/or the iteration number of not updating the current best solution.

B. *variable neighborhood search*

Variable neighborhood search proposed by Mladenovic and Hansen [15] is summarized as follows:

Variable Neighborhood Search (VNS)

Initialization. Select the set of neighborhood structures $\mathcal{N}_p, p = 1, \dots, p_{max}$, that will be used in the search; find an initial solution T ; choose a stopping condition;

Repeat the following sequence until the stopping condition is met:

- 1) Set $p \leftarrow 1$;
- 2) Until $p = p_{max}$, repeat the following steps:
 - a) (Shaking) Generate a tree T' at random from the p^{th} neighborhood of $T (T' \in \mathcal{N}_p(T))$;
 - b) (Local search) Apply some local search method with T' as initial solution; denote with T'' the so obtained local optimum;
 - c) (Move or not) If this local optimum is better than the incumbent, move there ($T \leftarrow T''$), and continue the search with $\mathcal{N}_1 (p \leftarrow 1)$; otherwise, set $p \leftarrow p + 1$;

IV. PROPOSED ALGORITHM

A. *Initial solution*

At first, an edge $e = \{v, v'\}$ is chosen uniformly at random. With this edge, a 1-subtree T_1 with is generated. Then $k - 1$ edges are added to the subtree so as to construct a k -subtree. Finally, a solution algorithm for MST problems is applied for the subgraph of which vertices are selected as the k -subtree. In this way, an initial solution of k -minimum spanning tree is obtained.

B. *Neighborhood*

Let us introduce a distance $\eta(T_1, T_2)$ between any two solutions (trees with cardinality k) T_1 and T_2 as a cardinality of difference between their edge sets, i.e.,

$$\eta(T_1, T_2) = |V_{T_1} \setminus V_{T_2}| + |V_{T_2} \setminus V_{T_1}|$$

Note that the distance functions above may be viewed as Hamming distances if each solution is represented by 0–1 vectors having 1 if an edge belongs to the solution T and 0 otherwise. The neighborhood $\mathcal{N}_p(T_1)$ consists of all solutions (subtrees) with distance p from $T_1 : T_2 \in \mathcal{N}_p(T_1) \Leftrightarrow \eta(T_1, T_2) = p$. It is clear that this function is metric.

C. *Shake*

We use the procedure of Shake proposed by Mladenovic and Urosevic [14].

The distance function η is used in our shaking step. In order to obtain $T' \in \mathcal{N}_p(T)$, the following procedure is done with p cycles.

- Step 1** Choose at random a set of a deleted vertex $v_{del} \in V(T)$ and an added vertex $v_{add} \notin V(T)$.
- Step 2** Apply the **Transition Algorithm** (see Section V). If the transition is infeasible, then return to Step 1. Otherwise, terminate.

D. *Local search*

We use tabu search as a local search for an initial solution which is obtained by Shaking step. The flowchart of tabu search is as follows (see Fig.1):

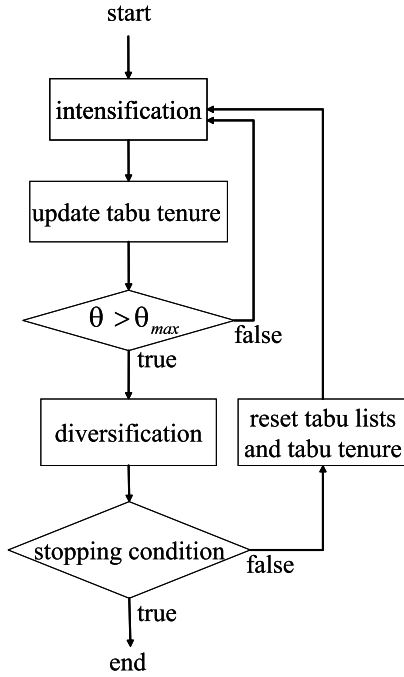


Fig. 1. Flowchart of local search

1) *Intensification*: The procedure of *Intensification* repeats transitions based on hill climbing with tabu lists and aspiration criterion:

a) *Transition strategy*: There are two major transition strategies; one is *best improvement strategy* and the other is *first improvement strategy*. The best improvement strategy is a search strategy which selects the best solution of all solutions in neighborhood as the next solution. On the other hand, the first improvement strategy selects the firstly found solution of which objective function value is better than that of the current solution. In this paper, we use the first improvement strategy because it takes much computational time for search a neighborhood if we use the best improvement strategy. These strategies are also applied when **Transition Algorithm** is used in *Intensification* procedure.

b) *Tabu list*: Tabu lists store the induces of the edges that were recently added or removed. As described before, every move consists of two steps; the first step is to remove one edge $e \in T^c$ from the current spanning tree T^c , and the second step is to add an edge in $N_p(T^c - e) \setminus \{e\}$ to $T^c - e$. The status of the forbidden moves are explained as: If a vertex v_j is in the tabu list, then our algorithm forbids the addition or deletion of the vertex v_j . In this paper, we use two tabu lists *InList* and *OutList*, which keep the induces of removed edges and to sore that of added edges, respectively. Tabu tenure, denoted by θ , is a period for which it forbids vertices in the tabu lists from deleting or adding.

c) *Aspiration criterion*: When an attribute is declared tabu, all solutions possessing this attribute are implicitly declared tabu. However, some of these solutions may have never been considered by the search. To remedy this, an aspiration

criterion is defined to override the tabu status of a solution. One common aspiration criterion is to allow tabu solutions yielding better solution values than that of the best known solution.

In our TS implementation, we apply an extension of the aspiration level concept by associating an attribute to each vertex of the graph. The tabu status of an attribute can be revoked if it leads to a solution with smaller cost than that of the best solution identified having that attribute. The aspiration level γ_v of an attribute is initially set equal to the cost of the initial solution T^{int} if vertex v belongs to this solution, and to ∞ otherwise. At every iteration, the aspiration level of each attribute $v \in V(T)$ of the current solution is updated to $\min\{\gamma_v, f(T)\}$, where $f(T)$ stands for the cost value of solution T .

2) *Update tabu tenure*: Let nic_{max} and θ_{inc} be given parameters. If the current best solution is not updated nic_{max} times, then we regards this situation as cycling and increase tabu tenure using (1).

$$\theta \leftarrow \theta + \theta_{inc} \quad (1)$$

3) *Diversification strategy*: A diversification procedure, using the residence frequency memory function, will lead to the exploration of region of the solution space not previously visited. The residence frequency memory records the number of times a specific element has been part of the solution.

Frequency-based memory is one of the long-term memories and consists of gathering pertinent information about the search process so far. In our algorithm, we use residence frequency memory, which keeps each track of the number of iterations where vertices have been explored.

The diversification procedure begins at the situation that some spanning tree is formed. Let $d(T_k) = \sum_{v \in V(T_k)} Freq(v)$ denote a criterion for diversification. In a manner similar to intensification strategy as described above, **Transition Algorithm** is repeated with k cycles, where $Freq(v)$ is the frequency of vertex v to be searched.

4) *Reset tabu*: *InList* and *OutList* are set empty. A parameter θ is reset the default value.

5) *Stopping condition*): If the iteration is beyond a given value, then terminate.

V. NEW TRANSITION ALGORITHM

The most important feature of the proposed algorithm is that it does not apply a minimum spanning tree algorithm iteratively for a subgraph with exactly $k + 1$ vertices unlike the solution method by Katagiri et al. [5]. Since minimum spanning tree algorithms find an optimal spanning tree for a fixed subgraph, the obtained solution is considered as a local minimum of k -minimum spanning tree problem. In this sense, MST algorithms is worth using for local search. However, there are many cases where it dose not need to use MST algorithms in order to find a local optimal solution. Therefore,

in this paper, we consider a new transition algorithm which move the current solution to a local minimum solution, not using MST algorithms. As described in the previous section, this algorithm is applied for vertex transition in Shaking and any local search of Intensification and Diversification procedures.

Our transition algorithm which obtains a local optimal solution of k -minimum spanning tree, not using the algorithm for solving MST problems, consists of two stages. Since the algorithm is a little complex, we explain the outline of the algorithm using a simple example (see Fig.2). Let $v_{del} \in V(T_k)$ and $v_{add} \notin V(T_k)$ be the vertices selected as the deleted and added vertices, respectively.

Transition algorithm for the first stage

Step 1 Let $S_t, t = 1, 2, 3, \dots$ be a set of sub-trees which is constructed by deleting v_{del} . Merge each S_t into a vertex, called *super-vertex* and let S_0 be v_{add} . Then construct $G'(V', E')$ according to following equation (see Fig.3):

$$V' \leftarrow \{S_t | t = 0, 1, 2, \dots\}$$

$$E' \leftarrow \{(S_i, S_j) | (S_i, S_j) \in E, i \neq j\}$$

Step 2 Obtain a minimum spanning tree problem using some algorithm such as Prim method or Kruskal method (see Fig. 4).

Step 3 Go to the second stage and apply the transition algorithm for the second stage.

Transition algorithm for the second stage

Let the dotted lines denote the edges between super-vertices (see Fig.5).

Step 1 Find edge e_{max} whose weight is the maximum from among all the edges included in super-vertices. Find edge e_{min} whose weight is the minimum from among all the dotted edges.

Step 2 If $w(e_{max}) > w(e_{min})$, then go to Step 3. Otherwise, go to Step 4.

Step 3 Delete the edge e_{max} and add the edge e_{min} . Go to Step 5.

Step 4 Attach label "explored" to a set of dotted lines that connects subgraphs which are derived by deleting the edge e_{max} .

Step 5 If all the dotted line is labeled "explored", then terminate. Otherwise, return to Step 1.

The above algorithm is applied for all vertex transition procedures in the subroutine of shaking and local search.

VI. NUMERICAL EXPERIMENTS

In order to compare the performances of our method with those of representative existing solution algorithms, we solve some benchmark instances which includes the instances provided by Blum [8] and our new instances. Tables I and II show the results for instances by Blum [8] and our new instances, respectively.

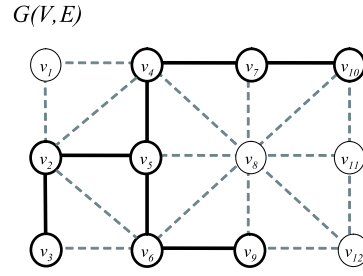


Fig. 2. Example of $|V| = 12, |E| = 23, k = 7$ (Bold lines are edges which form the current solution)

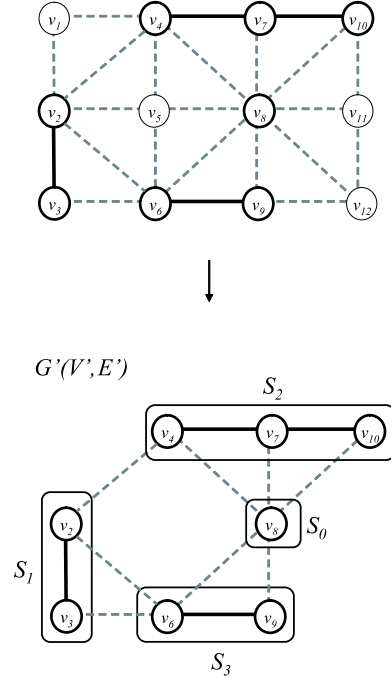


Fig. 3. Example of generating G' from G ($v_{del} = v_5$ and $v_{add} = v_8$)

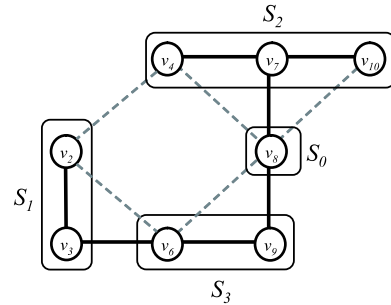


Fig. 4. Graph of super-vertices and its minimum spanning tree

We use C as the programming language and compiled all software with C-Compiler: Microsoft Visual C++ 7.1. All the metaheuristic approaches were tested on a PC with Celeron 3.06GHz CPU and Ram 1GB under Microsoft Windows XP. In the tables shown, TSI, TSK and TSB represent tabu search approaches by this paper, Katagiri et al. and Blum et

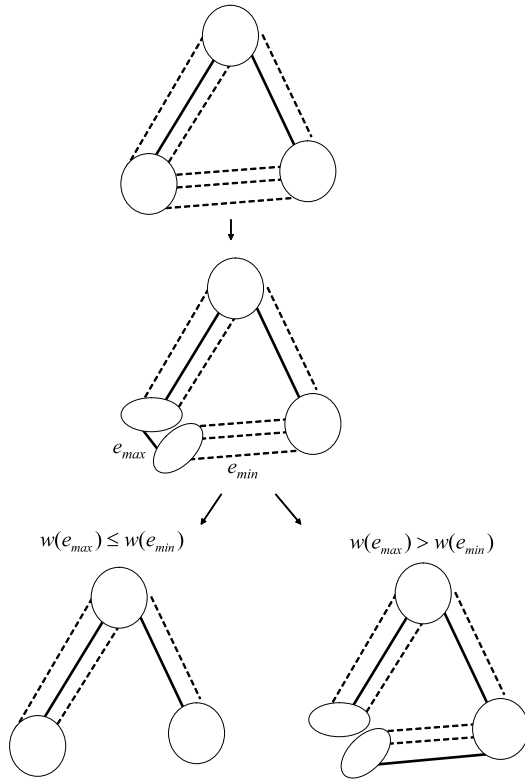


Fig. 5. Transition algorithm in the second stage

al., respectively. We executed each method in 30 runs and computed the *best*, *mean* and *worst* objective function values for each method. represents a mean of computational time.

Table I shows that the performance of the proposed method is clearly better than that of the existing method by Katagiri et al. Also, our algorithm provides better performance than the method by Blum et al., for high cardinality k , whereas the performance of the method by Blum et al. is the best for low cardinality k .

Table II shows the results for new instances which are more dense than the existing benchmark instances. It is observed from Table II that our algorithm provides as good a performance as the method by Katagiri et al.. Although the best values are often obtained by the method by Blum et al., the performances of our algorithm are fairly better than the method by Blum et al. in respect to the mean and worse objective function values. Therefore, we conclude that our algorithm provides a better robustness performance than the method by Blum et al.

VII. CONCLUSION

In this paper, we have proposed a new solution method based on tabu search for k -minimum spanning tree problems and compared the performance of the proposed method with those of existing methods though numerical experiments with several benchmark instances. It has been shown that the proposed method has an advantage of robustness over the existing methods. However, numerical experiments executed

TABLE I

graph	k		Objective value		
			TSI	TSK	TSB
$ N = 1000$ $ E = 1250$ $d = 2.5$ $\sigma(d) = 1.57$	200	best	4131	4098	3609
		mean	4147.6	4373.2	3685.9
		worst	4174	4587	3771
	400	best	9624	9936	8976
		mean	9841.0	10088.0	9091.0
		worst	9918	10202	9301
	600	best	16299	17243	16282
		mean	16319.0	17320.1	16323.7
		worst	16349	17330	16454
	800	best	26429	27170	26552
		mean	26429	27172.9	26687.3
		worst	26429	27173	26755
	900	best	32981	32981	33147
		mean	32984.5	33284.0	33187.6
		worst	32985	33459	33233
$ N = 400$ $ E = 800$ $d = 4.00$ $\sigma(d) = 0.00$	80	best	1627	1478	1466
		mean	1627	1562.4	1477.5
		worst	1627	1627	1500
	160	best	3330	3361	3217
		mean	3346.1	34522.5	3240.0
		worst	3369	3449	3259
	240	best	5264	5270	5215
		mean	5281.6	5432.5	5224.6
		worst	5325	5531	5234
	320	best	7682	7684	7682
		mean	7687.8	7697.9	7682.9
		worst	7689	7719	7684
	360	best	9249	9256	9250
		mean	9249	9257.8	9257
		worst	9249	9259	9260
$ N = 1000$ $ E = 5000$ $d = 10.0$ $\sigma(d) = 3.22$	200	best	1100	1130	1047
		mean	1141.1	1166.5	1063.7
		worst	1175	1225	1078
	400	best	2577	2682	2499
		mean	2602.6	2698.9	2535.9
		worst	2639	2725	2604
	600	best	4570	4681	4516
		mean	4590.4	4705.6	4548.6
		worst	4608	4718	4603
	800	best	7324	7405	7281
		mean	7325.8	7418.9	7324.7
		worst	7359	7434	7405
	900	best	9248	9375	9291
		mean	9248	9375.0	9323.9
		worst	9248	9376	9372
$ N = 450$ $ E = 8168$ $d = 36.30$ $\sigma(d) = 16.83$	90	best	139	138	135
		mean	141.3	145.1	136.7
		worst	145	155	140
	180	best	346	349	337
		mean	353.8	352.2	346.5
		worst	357	356	434
	270	best	631	643	630
		mean	632.1	649.2	653.4
		worst	637	654	728
	360	best	1060	1062	1060
		mean	1060.1	1062	1098.7
		worst	1064	1070	1158
	405	best	1388	1389	1391
		mean	1388.0	1389.4	1410.8
		worst	1389	1390	1467

are not enough to conclude such advantage is still valid for other types of benchmark instances. In the near future, we will provide additional benchmark instances such as random graphs, geometric graphs or small-world graphs, and execute more numerical experiments to clarify the advantage of our method.

TABLE II

graph	k		Objective value		
			TSI	TSK	TSB
$ N = 300$ $ E = 20000$ $d = 133.3$ $\sigma(d) = 36.57$	60	best	554	546	554
		mean	579.9	572.0	628.8
		worst	603	606	2180
	120	best	1236	1267	1229
		mean	1294.2	1304.4	1613.4
		worst	1346	1384	3189
	180	best	2179	2241	2169
		mean	2184.5	2256.5	2838.9
		worst	2208	2264	4517
	240	best	3564	3564	3566
		mean	3571.7	3568.8	4313.7
		worst	3572	3581	5906
270	best	4690	4690	4690	
	mean	4690	4690	5326.3	
	worst	4690	4690	6635	
$ N = 300$ $ E = 30000$ $d = 200.00$ $\sigma(d) = 38.99$	60	best	355	354	357
		mean	361.6	359.8	503
		worst	367	364	2552
	120	best	891	897	877
		mean	898.5	912.4	1228.2
		worst	920	922	3047
	180	best	1737	1661	1653
		mean	1738.2	1746.6	2194.9
		worst	1764	1784	3947
	240	best	2760	2737	2740
		mean	2760.3	2753.0	3290.3
		worst	2765	2765	5019
270	best	3491	3491	3491	
	mean	3491	3491	4322.2	
	worst	3491	3491	5730	
$ N = 300$ $ E = 40000$ $d = 266.67$ $\sigma(d) = 24.61$	60	best	237	224	224
		mean	243.3	238.1	405.4
		worst	255	257	2430
	120	best	566	547	554
		mean	572.3	567.4	1075.3
		worst	605	589	2787
	180	best	1016	1031	986
		mean	1034.9	1053.5	1653.1
		worst	1055	1066	3271
	240	best	1671	1656	1647
		mean	1678.2	1659.3	2493.3
		worst	1696	1676	3939
270	best	2107	2107	2107	
	mean	2107	2108.5	2845.5	
	worst	2107	2109	4440	

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