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AN APPROXIMATION OF A LOSS SYSTEM WITH TWO HETEROGENEOUS TYPES OF CALLS

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Abstract This paper presents an efficient and fairly accurate approximation method for a loss system having two types of calls and three types of server groups. In this model, two types of calls arrive at the individual server group for each call type. If an arriving call finds all the servers of a corresponding group busy, the call overflows to a common server group. If the overflowed call again finds all the common servers busy, the call is lost. This model is constructed around a telephone based ticket reservation system, as well as around integrated services digital networks which will be introduced in the near future. By approximating the overflow process using an interrupted Poisson process (IPP), the main problem is reduced to solving a queueing model with a common server group having two independent IPP inputs with different mean service times. The proposed method makes it possible to compute loss probabilities in a far shorter time as compared with the exact method employing the lumping method. The accuracy of the results is shown to be sufficiently good for practical use carrying out numerical experiments.

1. Introduction

This paper presents an efficient and fairly accurate approximation method for a loss system accommodating two types of customers or calls. The system was first analyzed in detail by Sato and Mori [9]. Their model was constructed around a telephone based ticket reservation system for an airlines company. In the system, two types of reservation calls are received and answered by reservation clerks. A call is either for domestic travel or international travel. The clerks are divided into three groups: for domestic service, international service, and capable of handling both services. The call first arrives at the location of the corresponding reservation clerks. If all the clerks are busy, the call overflows to the location of well-trained clerks who can handle both types of calls. If all the well-trained clerks are also busy, the call is lost. The mean service times for calls may differ between both call types and clerk types.

In their investigation, in order to get the exact numerical values of the steady-state probabilities, they used the lumping method, which was originally proposed and applied to a GI/G/C queueing system [11]. The lumping method, as reported by Sato and Mori [9], makes it possible to calculate steady-state probabilities within a reasonable computation time for a rather small system, but a longer computation time is required if the number of clerks or servers becomes larger.

To overcome this drawback, IPPs (Interrupted Poisson Processes) devised by Kuczura [4] are introduced to approximate overflow processes from loss systems. The overflow from the clerks for domestic service is well approximated with another IPP. Thus, the problem is reduced to solving a queueing model containing only common well-trained clerks having two IPP inputs and different mean service times.

Several studies have recently been made on queueing models having multiple IPP input streams in connection with the traffic design of telephone networks and packet switched networks [5, 6, 7, 8, 10, 12]. All of these studies, however, concentrate on cases where the mean service times of all calls are the same, and little attention has been paid to the cases where the IPPs or overflow inputs have different mean service times. If calls types are different, their mean service times would be different under actual conditions.

The present model, centered around a loss system having two IPP inputs with different mean service times, includes the case where one of the inputs is Poissonian. Even this simplified model does not appear to have been studied yet.

The present model appears to be useful for the traffic design of an ISDN (Integrated Services Digital Network) scheduled to be introduced in the near future. In the initial stage of ISDN development, it is difficult to accurately forecast demand for individual new communication services. To alleviate this problem, resources such as transmission lines or memory storage, should be shared between different call types for economy, and it is felt that the present model will provide a good evaluation of loss probability in the system.

In the next section, an exact model and approximation model are described. In Section 3, steady-state equations are obtained for the approximation model and a numerical method is proposed for solving the equations. In Section 4, the numerical results for the loss probabilities are shown and compared with the results for the exact model to confirm the accuracy of the approximation method.

2. Model Description

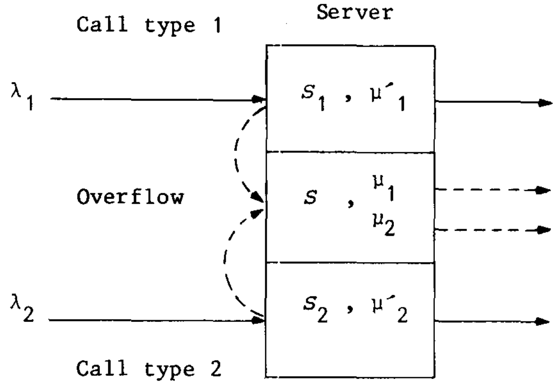

Fig. 1 Original exact model.

Figure 1 illustrates the original exact model. Two independent Poisson streams, call types 1 and 2, arrive at a servicing facility at arrival rates of λ_1 and λ_2, respectively. The total number of servers in the system is S_T. These servers are composed of three groups: sole use servers S_1 and S_2 corresponding to types 1 and 2, respectively, and common servers S for calls of both types which overflow from S_1 or S_2. We use the same symbols S_k $(k=1,2)$ and S for indicating both the names of the groups and the number of servers in each group. A call of type k arrives first at S_k $(k=1,2)$. If it finds that all of the S_k servers are occupied, it overflows to S. Further, if it finds all of the S servers occupied, it is cleared from the system. Service times are assumed to be independently and exponentially distributed; however, their means may differ between both different call types and different server groups. Namely, the mean service time of a call of type k $(k=1,2)$ is $1/\mu'_k$ at an S_k server and $1/\mu_k$ at an S server.

The above model is easily formulated as a Markov process, and the steady-state probabilities can be obtained by solving a system of linear equations. However, the number of states is equal to $N=(S_1+1)(S_2+1)(S+1)(S+2)/2$. This becomes very large if the number of servers is large. Sato and Mori devised an efficient numerical method by applying the lumping method to solve the state equations [9]. Even in their approach, however, much time is required for computing the steady-state probabilities.

Recently, IPPs are being increasingly used to approximate overflow processes from loss systems. An IPP is a process which is alternately turned on for an exponentially distributed time and then turned off for another exponentially and independently distributed time. Therefore, an IPP has three parameters. The present paper also introduces IPPs for approximating the overflow processes from S_1 and S_2. This approximation makes it possible to treat the servicing process at S independently from servicing processes at S_1 and S_2. Let IPP_k denote the IPP for call type k (k=1,2). The servicing process at S_k is stochastically equivalent to that in an $M/M/S_k/S_k$ loss model, and the overflow probability is easily obtained using Erlang's loss formula. Thus, the servicing process at S is approximated by a model having two IPP inputs with different mean service times. This is shown in Fig. 2. When

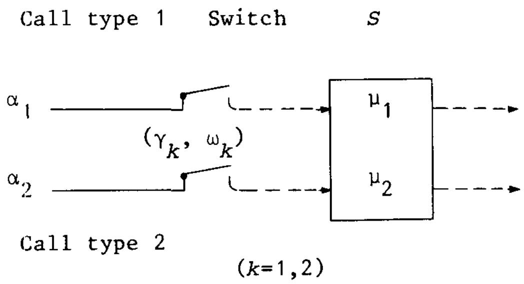

Fig. 2 Approximate model.

$\mu_1 = \mu_2$, a loss system with two IPP inputs can be solved more easily and efficiently than the method described in the present paper (See [10]). We denote the IPP_k parameters by $(\alpha_k, \gamma_k, \omega_k)$, ($k$=1,2), where α_k is the mean Poisson arrival rate at the on/off switch which generates IPP_k, and γ_k and ω_k are the reciprocals of the means of the on- and off- time periods of the switches, which are distributed independently and exponentially. A three-moment matching method is used to determine the IPP parameters, $(\alpha_k, \gamma_k, \omega_k)$ [3]. The three parameters included in an IPP are determined as follows. We consider fictitious infinite servers, to which an exact overflow process or an IPP is fed. The first three moments of number of calls in the fictitious servers are calculated for the two input processes, and the matching of these three moments produces the three IPP parameters. The number of states in the approximate model for S is $M=2(S+1)(S+2)$. The loss probability in the original model is approximated by the loss probability in the approximate model for S.

3. State Equations and Numerical Solution Methods

The state of the system can be expressed by the number of calls of each type in S and by the on-off state of each switch for IPP. The following notations are introduced to describe the system state of the model shown in Fig. 2.

i : number of type 1 calls,

j : number of type 2 calls,

where $i \geqq 0$, $j \geqq 0$ and $i+j \leqq S$.

m : the state of the switch for IPP_1,

n : the state of the switch for IPP_2.

Here, $m,n=0$ if the switch is in the off state, and $m,n=1$ if the switch is in the on state.

Thus, the system state is represented by the quadruple (i,j,m,n), and the system behaves as a Markov process. Since the Markov process is irreducible and aperiodic, a steady state always exists. The system is reduced to a birth-and-death process because all time intervals, including interarrival times to each IPP switch, on- and off- time periods for the switches and service times for each call type are exponentially distributed. From the theory of the birth-and-death process, steady-state equations are easily obtained as follows: the rate of flow of probability out of state (i,j,m,n) is balanced by the rate of flow of probability into the state (i,j,m,n) from the neighboring states.

Letting p_{ijmn} denote the steady-state probabilities, the following set of state equations are obtained, where $\delta_{ii}=1$ and $\delta_{ij}=0$, if $i \neq j$, and $\bar{\delta}_{ij}=1-\delta_{ij}$.

$$(i\mu_1+j\mu_2+\omega_1+\omega_2)p_{ij00}$$

$$= \bar{\delta}_{i+j,S}\{(i+1)\mu_1 p_{i+1,j,0,0} + (j+1)\mu_2 p_{i,j+1,0,0}\} + \gamma_1 p_{ij10} + \gamma_2 p_{ij01}, \tag{3.1}$$

$$(i\mu_1+j\mu_2+\bar{\delta}_{i+j,S}\alpha_2+\omega_1+\gamma_2)p_{ij01}$$

$$= \bar{\delta}_{i+j,S}\{(i+1)\mu_1 p_{i+1,j,0,1} + (j+1)\mu_2 p_{i,j+1,0,1}\} + \bar{\delta}_{j0}\alpha_2 p_{i,j-1,0,1} \tag{3.2}$$

$$+ \omega_2 p_{ij00} + \gamma_1 p_{ij11},$$

$$(i\mu_1+j\mu_2+\bar{\delta}_{i+j,S}\alpha_1+\gamma_1+\omega_2)p_{ij10}$$

$$= \bar{\delta}_{i+j,S}\{(i+1)\mu_1 p_{i+1,j,1,0} + (j+1)\mu_2 p_{i,j+1,1,0}\} + \bar{\delta}_{i0}\alpha_1 p_{i-1,j,1,0} \tag{3.3}$$

$$+ \omega_1 p_{ij00} + \gamma_2 p_{ij11},$$

$$\{i\mu_1 + j\mu_2 + \bar{\delta}_{i+j,S}(\alpha_1+\alpha_2) + \gamma_1 + \gamma_2\}p_{ij11}$$

(3.4)
$$= \bar{\delta}_{i+j,S}\{(i+1)\mu_1 p_{i+1,j,1,1} + (j+1)\mu_2 p_{i,j+1,1,1}\} + \bar{\delta}_{i0}\alpha_1 p_{i-1,j,1,1} + \bar{\delta}_{j0}\alpha_2 p_{i,j-1,1,1} + \omega_1 p_{ij01} + \omega_2 p_{ij10} .$$

If calls of one of the call types, say of type 1, arrive at S by a Poisson process instead of via IPP_1, the switch for IPP_1 may always be thought of as on. In this case, p_{ij0n}'s are made equal to 0 and Eqs. (3.1) and (3.2) are omitted.

The above four equations (3.1)-(3.4) may be rewritten into the following single equation.

$$\{i\mu_1 + j\mu_2 + \delta_{m0}\omega_1 + \delta_{m1}(\bar{\delta}_{i+j,S}\alpha_1+\gamma_1) + \delta_{n0}\omega_2 + \delta_{n1}(\bar{\delta}_{i+j,S}\alpha_2+\gamma_2)\}p_{ijmn}$$

(3.5)
$$= \bar{\delta}_{i+j,S}\{(i+1)\mu_1 p_{i+1,j,m,n} + (j+1)\mu_2 p_{i,j+1,m,n}\} + \bar{\delta}_{i\,0}\delta_{m1}\alpha_1 p_{i-1,j,m,n} + \bar{\delta}_{j0}\delta_{n1}\alpha_2 p_{i,j-1,m,n} + \delta_{m0}\gamma_1 p_{ij1n} + \delta_{m1}\omega_1 p_{ij0n} + \delta_{n0}\gamma_2 p_{ijm1} + \delta_{n1}\omega_2 p_{ijm0}$$

The steady-state probabilities p_{ijmn} are determined by Eq. (3.5) together with the normalizing condition that their sum in the defined state space is equal to 1. Let P_{mn} be the steady-state probability that the joint state of switches for IPP_k's is (m,n). Namely,

(3.6)
$$P_{mn} = \sum_{i=0}^{S}\sum_{j=0}^{S-i} p_{ijmn} \quad ; \quad m=0,1, \quad n=0,1.$$

Probabilities P_{mn} can also be represented by using IPP_k parameters γ_k and ω_k.

(3.7)
$$P_{00} = \gamma_1\gamma_2/\nu, \quad P_{01} = \gamma_1\omega_2/\nu,$$
$$P_{10} = \omega_1\gamma_2/\nu, \quad P_{11} = \omega_1\omega_2/\nu,$$
$$\text{where} \quad \nu = (\gamma_1+\omega_1)(\gamma_2+\omega_2).$$

Equation (3.7) is another representation of the normalizing condition on p_{ijmn}'s. In the case where a type 1 call arrives at S by a Poisson process, Eq. (3.7) is reduced to

(3.8)
$$P_{10} = \gamma_2/(\gamma_2+\omega_2), \quad P_{11} = \omega_2/(\gamma_2+\omega_2).$$

The system of equations (3.5) and (3.7) appears very difficult to solve analytically and neat solutions for state probabilities are not easily obtained. Therefore, a numerical calculation method must be used. Since the system of state equations is linear, any of the conventional numerical methods for linear equations is applicable. The SOR (Successive Over-Relaxation) method [1] was applied to our calculations.

Let $p_{ijmn}^{(k)}$ be the state probabilities at the k-th iteration and $p_{ijmn}^{(0)}$ be their initial values. Comparatively simple initial values are assumed here:

$$p_{ijmn}^{(0)} = P_{mn}/((S+1)(S+2)/2) \tag{3.9}$$

for all possible set of values of i, j, m and n. Namely we start with a uniform distribution for individual pair (m,n).

An iteration scheme is shown below, in which r is the over-relaxation factor. The Gauss-Seidel method is the special case of $r=1$. Whether an optimal r value can be obtained depends on both the model structure and load factor. Thus, it is difficult to select an optimal r value analytically. However, it is observed that over-relaxation with $r \fallingdotseq 1.3$ is often faster than simple Gauss-Seidel iteration [1]. At each step in the following equations, k-th iterative values should be normalized by using Eq. (3.7).

$$\begin{aligned} p_{ij00}^{(k)} = {} & [r[\bar{\delta}_{i+j,S}\{(i+1)\mu_1 p_{i+1,j,0,0}^{(k-1)} + (j+1)\mu_2 p_{i,j+1,0,0}^{(k-1)}\} \\ & + \gamma_1 p_{ij10}^{(k-1)} + \gamma_2 p_{ij01}^{(k-1)}] + (1-r)p_{ij00}^{(k-1)}] \,/\, (i\mu_1+j\mu_2+\omega_1+\omega_2), \end{aligned} \tag{3.10}$$

$$\begin{aligned} p_{ij01}^{(k)} = {} & [r[\bar{\delta}_{i+j,S}\{(i+1)\mu_1 p_{i+1,j,0,1}^{(k-1)} + (j+1)\mu_2 p_{i,j+1,0,1}^{(k-1)}\} \\ & + \bar{\delta}_{j0}\alpha_2 p_{i,j-1,0,1}^{(k)} + \omega_2 p_{ij00}^{(k-1)} + \gamma_1 p_{ij11}^{(k-1)}] + (1-r)p_{ij01}^{(k-1)}] \\ & /\, (i\mu_1 + j\mu_2 + \bar{\delta}_{i+j,S}\,\alpha_2 + \omega_1 + \gamma_2) , \end{aligned} \tag{3.11}$$

$$\begin{aligned} p_{ij10}^{(k)} = {} & [r[\bar{\delta}_{i+j,S}\{(i+1)\mu_1 p_{i+1,j,1,0}^{(k-1)} + (j+1)\mu_2 p_{i,j+1,1,0}^{(k-1)}\} \\ & + \bar{\delta}_{i0}\alpha_1 p_{i-1,j,1,0}^{(k)} + \omega_1 p_{ij00}^{(k)} + \gamma_2 p_{ij11}^{(k-1)}] + (1-r)p_{ij10}^{(k-1)}] \\ & /\, (i\mu_1 + j\mu_2 + \bar{\delta}_{i+j,S}\alpha_1 + \gamma_1 + \omega_2), \end{aligned} \tag{3.12}$$

$$p^{(k)}_{ij11} = [r[\bar{\delta}_{i+j,S}\{(i+1)\mu_1 p^{(k-1)}_{i+1,j,1,1} + (j+1)\mu_2 p^{(k-1)}_{i,j+1,1,1}\}$$

$$(3.13) \qquad + \bar{\delta}_{i0}\alpha_1 p^{(k)}_{i-1,j,1,1} + \bar{\delta}_{j0}\alpha_2 p^{(k)}_{i,j-1,1,1} + \omega_1 p^{(k)}_{ij01} + \omega_2 p^{(k)}_{ij10}$$

$$+ (1-r)p^{(k-1)}_{ij11}] \;/\; \{i\mu_1 + j\mu_2 + \bar{\delta}_{i+j,S}(\alpha_1+\alpha_2) + \gamma_1 + \gamma_2\}.$$

If type 1 calls arrive at S by a Poisson process, omit Eqs. (3.10) and (3.11) and put $p^{(k)}_{ij0n}=0$.

There exist several possible convergence criteria for stopping the iteration. For example, for a sufficiently small $\varepsilon>0$,

(a) $|p^{(k)}_{ijmn} - p^{(k-1)}_{ijmn}| < \varepsilon$ for any (i,j,m,n),

(b) $|p^{(k)}_{ijmn} - p^{(k-1)}_{ijmn}| \;/\; p^{(k-1)}_{ijmn} < \varepsilon$ for any (i,j,m,n),

(c) $\sum\sum |p^{(k)}_{ijmn} - p^{(k-1)}_{ijmn}| < \varepsilon$ where summation covers all possible states (i,j,m,n).

In the present paper, criterion (a) is adopted.

4. Performance Measures and Numerical Results

The loss probabilities for the original model shown in Fig. 1 can be obtained by using the steady-state probabilities obtained in Section 3 and Erlang's loss formula.

Let B_1 and B_2 be the loss probabilities for call types 1 and 2, respectively, in the approximation model for the S servers in Fig. 2. Then, B_k is equal to the conditional probability that the S servers are all busy under the condition that the switch for IPP$_k$ is in the on state (k=1,2).
Hence

$$(4.1) \qquad B_1 = \sum_{i=0}^{S} (p_{i,S-i,1,0} + p_{i,S-i,1,1})/R_1,$$

$$(4.2) \qquad B_2 = \sum_{i=0}^{S} (p_{i,S-i,0,1} + p_{i,S-i,1,1})/R_2,$$

where R_i is the probability that the switch for IPP$_k$ is in the on state; namely, (4.3) $R_1 = P_{10} + P_{11} = \omega_1/(\gamma_1+\omega_1)$,

(4.4) $R_2 = P_{00} + P_{01} = \omega_2/(\gamma_2+\omega_2)$.

The overflow probability B'_k, k=1,2, from the S_k server is given by Erlang's loss formula

(4.5) $B'_k = B(S_k, a_k)$,

where $a_k = \lambda_k/\mu'_k$ and

(4.6) $B(C,a) = (a^C/C!)/(\sum_{i=0}^{C} a^i/i!)$.

Hence the loss probability P_k for call type k in the original model is given by

(4.7) $P_k = B'_k B_k$, k=1,2.

A single loss probability is sometimes required as a characteristic quantity of the model. In that case, the average of P_1 and P_2 is taken. There are two approaches to taking this average [2].

One way is to take the average using the arrival rates of individual call types as weights. Thus, the first average loss probability P_t is given by

(4.8) $P_t = (\lambda_1 P_1 + \lambda_2 P_2)/(\lambda_1 + \lambda_2)$.

This loss probability represents the percentage of lost calls.

The second way is to take the average using the offered traffic intensities of individual call types as weights. However, in the present model, the traffic intensity for each call type is not always clearly defined. This is because mean service times are different for the S_k server (k=1,2) and S server. Here, let traffic intensities be determined by the S_k server (k=1,2) which serves only type k calls. Thus, the second average loss probability, denoted as P_e, is given by

(4.9) $P_e = (a_1 P_1 + a_2 P_2)/(a_1 + a_2)$.

This probability represents the percentage of lost traffic intensity.

The values P_t and P_e will differ, except when $\mu'_1 = \mu'_2$. Which performance measure a system designer should use as a global criterion depends on his area of interest.

Numerical results are shown in Tables 1 and 2. In both tables, the approximate values are obtained by the present method using r=1.3 and $\varepsilon=10^{-5}$.

Table 1 Loss probabilities when $\mu_1=\mu_2$

$s_1+s_2+s=10$, $\lambda_1=3.0$, $\lambda_2=2.0$,

$\mu_1'=1.0$, $\mu_2'=1.0$, $\mu_1=0.9$, $\mu_2=0.9$

s_1	s_2	s	P_1 Approx.	P_1 Exact	P_2 Approx.	P_2 Exact	P_t Approx.	P_t Exact
9	1	0	.00270	.00270	.66667	.66667	.26829	.26829
8	1	1	.00557	.00557	.41741	.41740	.17030	.17030
7	1	2	.00975	.00976	.23755	.23754	.10087	.10087
6	1	3	.01449	.01453	.12726	.12722	.05959	.05961
5	1	4	.01878	.01887	.07100	.07094	.03967	.03970
4	1	5	.02227	.02238	.04622	.04615	.03185	.03188
3	1	6	.02523	.02532	.03565	.03560	.02940	.02943
2	1	7	.02804	.02809	.03070	.03068	.02910	.02913
1	1	8	.03089	.03089	.02811	.02811	.02978	.02977
0	1	9	.03382	.03382	.02667	.02667	.03096	.03096
8	2	0	.00813	.00813	.40000	.40000	.16488	.16488
7	2	1	.01325	.01324	.22382	.22383	.09748	.09748
6	2	2	.01877	.01878	.11725	.11724	.05816	.05816
5	2	3	.02359	.02364	.06376	.06372	.03966	.03968
4	2	4	.02742	.02750	.04056	.04055	.03267	.03272
3	2	5	.03068	.03076	.03078	.03078	.03072	.03077
2	2	6	.03381	.03385	.02622	.02625	.03078	.03081
1	2	7	.03702	.03701	.02382	.02387	.03174	.03175
0	2	8	.04035	.04033	.02247	.02252	.03320	.03321
7	3	0	.02186	.02186	.21053	.21053	.09733	.09733
6	3	1	.02872	.02870	.10694	.10695	.06000	.06000
5	3	2	.03444	.03446	.05596	.05596	.04305	.04306
4	3	3	.03893	.03898	.03430	.03432	.03708	.03712
3	3	4	.04280	.04285	.02535	.02539	.03582	.03587
2	3	5	.04660	.04662	.02123	.02129	.03645	.03649
1	3	6	.05056	.05054	.01906	.01913	.03796	.03798
0	3	7	.05473	.05470	.01782	.01789	.03996	.03998
6	4	0	.05216	.05216	.09524	.09524	.06939	.06939
5	4	1	.05925	.05925	.04689	.04688	.05430	.05430
4	4	2	.06483	.06484	.02703	.02703	.04971	.04972
3	4	3	.06980	.06982	.01911	.01913	.04952	.04954
2	4	4	.07484	.07485	.01557	.01562	.05113	.05116
1	4	5	.08024	.08022	.01373	.01379	.05364	.05365
0	4	6	.08601	.08598	.01267	.01273	.05667	.05668
5	5	0	.11005	.11005	.03670	.03670	.08071	.08071
4	5	1	.11696	.11697	.01915	.01914	.07783	.07784
3	5	2	.12352	.12352	.01259	.01260	.07915	.07915
2	5	3	.13048	.13048	.00984	.00986	.08222	.08223
1	5	4	.13810	.13808	.00846	.00849	.08624	.08624
0	5	5	.14636	.14635	.00766	.00770	.09088	.09089

(The exact values are cited from Table 1 in reference [9].)

Table 2 Loss probabilities when $\mu_1 \neq \mu_2$

$S_1+S_2+S=30$, $S_2=10$, $\lambda_1=5$, $\lambda_2=3$,
$\mu_1'=0.3125$, $\mu_1=0.9\mu_1'$, $\mu_2=0.9\mu_2'$

μ_2'	S_2	S	P_1 Approx.	P_1 Exact	P_2 Approx.	P_2 Exact	P_t Approx.	P_t Exact	P_e Approx.	P_e Exact
	20	0	.06441	.06441	.21458	.21458	.12073	.12073	.12217	.12217
	19	1	.07332	.07333	.17631	.17627	.11194	.11193	.11293	.11292
.3	18	2	.08155	.08159	.14604	.14599	.10574	.10574	.10636	.10636
	17	3	.08895	.08903	.12284	.12280	.10166	.10169	.10199	.10202
	16	4	.09551	.09562	.10545	.10546	.09924	.09931	.09933	.09940
	20	0	.06441	.06441	.09954	.09954	.07759	.07759	.07562	.07562
	19	1	.06913	.06914	.07453	.07449	.07116	.07115	.07086	.07085
.4	18	2	.07314	.07316	.05753	.05747	.06729	.06728	.06816	.06815
	17	3	.07659	.07662	.04624	.04618	.06521	.06521	.06690	.06691
	16	4	.07965	.07969	.03874	.03872	.06431	.06433	.06659	.06661

(The exact values are provided by the authors of reference [9].)

The exact values represent the results obtained by the method reported on Sato and Mori [9]. In the case of $S=0$, the loss probabilities are obtained by using only Erlang's loss formula. Therefore, this approximation method gives exact results. Since IPP accurately represents the overflow process from a single server loss system, for the cases of $S_1 \leqq 1$ and $S_2 \leqq 1$, this approximation method also gives exact results. In the other cases in Tables 1 and 2, the approximate values agree fairly well with the exact values. The differences between the two sets of values are smaller than 10^{-4} for almost all cases.

In the lumping method proposed by Sato and Mori [9], the total states are partitioned into $(S_1+1)(S_2+1)$ groups each having $(S+1)(S+2)/2$ states and several methods of speeding up the algorithm were devised. However, it was reported that much computation time was required to obtain steady-state probabilities.

On the other hand, the present approximation method reduces the number of states for the main routine to $2(S+1)(S+2)$ and the computational burden time is little affected by the S_1 and S_2 values. Therefore, the present algorithm can save much computation time as compared with the exact method. For example, for the case of ($S_T=30$; $S_1=17$, $S_2=10$, $S=3$) which is included in Table 2, numerical experiments show that the present method is several thousand times faster than the exact method proposed by Sato and Mori.*

5. Conclusion

An efficient and fairly accurate approximation method has been proposed for a loss model. The model involves three groups of servers and two types of calls arriving at individual servers. Two individual overflow streams from the individual servers are routed to a common server group. Replacing each overflow process with an interrupted Poisson process makes it possible to compute loss probabilities for both call types within a far shorter time period than required with the exact method employing lumping. The results of the proposed method are shown to be sufficiently good for practical application.

The main focus of the present method is on the analysis of a loss model having two IPP inputs with different mean service times. The analysis also covers the case where one input stream is Poissonian. The results obtained can be readily incorporated into the following models having two heterogeneous IPP inputs.

(i) Trunk reservation systems and buffer reservation systems in communications networks.

(ii) Queueing models having loss discipline and delay discipline with a finite waiting room, which may be useful in a hybrid transmission system, e.g. integrated voice and data system.

Moreover, explicit and neat approximation formulas are desired for broader use in the practical traffic design of an ISDN. This is one area where further studies are required.

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* Sato and Mori [9] treat $(S_1+1)(S_2+1)(S+1)(S+2)/2$ dimensional state space, while we treat only $2(S+1)(S+2)$ dimensional state space. It is well known that it takes a computation time proportional to N^3 to numerically solve an N dimensional linear equation when using the Gauss elimination method. Thus, if the elimination method is used, the ratio, g, of the computation time for the exact model by Sato and Mori to that for the present approximate model is found to be $g=((S_1+1)(S_2+1)/4)^3$. When $S_1=17$ and $S_2=10$, g is about 10^5. This value is about twice as large as that for the numerical experiment. It appears that this difference is caused by the use of the iterative method to solve the linear equations in both papers.

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References

[1] Cooper, R. B.: *Introduction to Queueing Theory*. 2nd ed. Edward Arnold Ltd., London, 1972.

[2] Fischer, M. J.: Priority Loss Systems - Unequal Holding Times. *AIIE Transactions*, 12 (1980),47-53.

[3] Heffes, H.: Analysis of First-come First-served Queuing Systems with Peaked Inputs. *Bell System Technical Journal*, Vol.52, No.7 (1973), 1215-1228.

[4] Kuczura, A.: The Interrupted Poisson Process as an Overflow Process. *Bell System Technical Journal*, Vol.52, No.3 (1973), 437-448.

[5] Machihara, F.: On the Overflow Processes from the $PH_1+PH_2/M/S/K$ Queue. *Proceedings of the 10th International Teletraffic Congress*, 1983, Session 4.1, Paper 5.

[6] Matsumoto, J. and Watanabe, Y.: Analysis of Individual Traffic Characteristics for Queueing Systems with Multiple Poisson and Overflow Inputs. *Proceedings of the 10th International Teletraffic Congress*, 1983, Session 5.3, Paper 1.

[7] Reneby, L.: On Individual and Overall Losses in Overflow Systems. *Proceedings of the 10th International Teletraffic Congress*, 1983, Session 5.3, Paper 5.

[8] Sanders, B. and de Jonge, R.: An Optimization Tool for Overflow Systems with respect to Individual Losses. *Proceedings of the 1st Telecommunications Networks Planning Symposium*, 1980, 94-100.

[9] Sato, T. and Mori, M.: An Application of the Lumping Method to a Loss System with Two Types of Customers. *Journal of the Operations Research Society of Japan*, Vol.26, No.1 (1983), 51-59.

[10] Songhurst, D. J.: Protection against Traffic Overload in Hierarchical Networks employing Alternative Routing. *Proceedings of the 1st Telecommunications Networks Planning Symposium*, 1980, 214-220.

[11] Takahashi, Y. and Takami, Y.: A Numerical Method for the Steady-state Probabilities of a GI/G/C Queueing System in a General Class. *Journal of Operations Research Society of Japan*, Vol.19, No.2 (1976), 147-157.

[12] Wallström, B. and Reneby, L.: On Individual Losses in Overflow Systems. *Proceedings of the 9th International Teletraffic Congress*, 1979, WALLSTRÖM/RENEBY-1-8.

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