An Asset Allocation Puzzle: Comment

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The purpose of this note is to look at the rationale behind popular advice on portfolio allocation among cash, bonds, and stocks. We argue that the typical investment advice is not inconsistent with the behavior of risk-averse expected-utility maximizers. We propose an additional solution to the asset allocation puzzle posed by Niko Canner et al. (1997), who argue that popular advice contradicts financial theory because it is inconsistent with the capital asset pricing model (CAPM) mutual-fund separation theorem. The CAPM asserts that investors should hold the same selection of risky assets, while popular advice is that investors should hold a proportion of bonds to stocks that increases with risk aversion.

Using mean-variance (MV) analysis and the CAPM, Canner et al. show that recommended portfolios are far from optimal and that losses from the apparent failure of optimization are not substantial. However, they failed to explain the popular advice within an economic model.

We offer a rational model based on stochastic dominance to demonstrate that all popular financial advice portfolios belong to the efficient set for all risk-averse investors. Using the historical annual real returns on bonds and stocks in Canner et al., we cannot ascertain that investment advisors indeed offer bad advice. Rather, we maintain that acting as agents for numerous clients, advisors recommend portfolios that are not inefficient for all risk-averse investors.

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I. Overview

Portfolio managers and financial advisors are in the business of offering investment alternatives to clients and the public in general. In the case of general public printed advice, the advisors mostly do not know the specific risk aversion of those who would use the advice, and thus propose portfolios that fit a wide range of investors. As advice is collective, it is impossible to tailor it to a specific client's needs.

To investigate whether a piece of advice is reasonable, we suggest a test. We ask whether the proposed portfolio is not inefficient in the sense that one cannot find an alternative portfolio that is preferred by all risk-averse clients. Identification of an alternative portfolio that is preferred by all risk-averse investors—and that advisors fail to find—would indicate that advisors are not acting in clients' best interests (assuming that clients and advisors agree about asset returns distributions). If investors disagree on which portfolio is preferred over the recommended one, then it is unjustified to ask the advisor to come up with a preferred portfolio when a preferred portfolio cannot be found.

To test the efficiency of the proposed portfolios, we use second-degree stochastic dominance (SSD) which states the conditions that would allow all risk-averse expected-utility maximizers to prefer one portfolio over another. SSD is originally calculated by comparing the areas under the cumulative distributions of portfolio returns [Josef Hadar and William R. Russell (1969), Giora Hanoch and Haim Levy (1969), and Michael Rothschild and Joseph E. Stiglitz (1970)].

Anthony F. Shorrocks (1983) later developed SSD conditions in terms of generalized (or absolute) Lorenz curves (hereafter referred to as the *Lorenz*), which are the cumulative expected returns on the portfolio. In essence, for all risk-averse investors to prefer one portfolio of assets over another, its *Lorenz* must lie above the *Lorenz* of the alternative. To evaluate the popular advice on a particular investment allocation, we ought to compare its *Lorenz* with the *Lorenz* of all alternative portfolios, given the historical distribution of returns. This approach, however, does not provide practical results as it involves an infinite number of pairwise comparisons of portfolios. Furthermore, constructing dominating portfolios according to SSD is bound to fail as one can always find a combination yielding higher expected returns.

Rather than build an optimal portfolio, we would like to determine whether a given portfolio belongs to the SSD efficient set so that it is impossible to find an alternative portfolio that is pairwise preferred by all risk-averse investors. That is, instead of finding the entire SSD efficient set, we restrict ourselves to the simpler problem of whether a given portfolio belongs to the efficient set.

II. Methodology

We use marginal conditional stochastic dominance (MCSD) as developed by Shalit and Yitzhaki (1994). It allows us to address a somewhat easier question. Assume that a client chooses to embrace a popular advice portfolio. Given that choice, we ask if one can find an alternative portfolio by marginally changing the allocation so that the proposed portfolio will be inferior in the eyes of every risk-averse client. If this is possible, the proposed popular advice would be inefficient. If, on the other hand, it cannot be shown that every risk-averse client would prefer an alternative portfolio, it is unjustified to claim that financial advisors, not knowing the client's exact utility function, could suggest a superior portfolio by marginally changing asset proportions.

To complete the argument, we rely on Yitzhaki and Joram Mayshar (1997) who show that if a portfolio is not SSD-dominated by a local alternative portfolio, then it is also not dominated by any (global) alternative portfolio. MCSD states the probabilistic conditions under which all risk-averse investors, given a portfolio of assets, prefer to increase the proportion of one risky asset at the expense of another. MCSD conditions are formulated in terms of absolute concentration curves (ACCs), which are defined as the cumulative expected returns on an asset conditional on the return on the entire portfolio. Given a specific portfolio, all risk-averse investors will prefer to increase the proportion of the asset whose ACC lies entirely above the ACC of another.

Using this criterion for a given portfolio, pairwise dominating and dominated securities assets can be determined, and investors can improve expected utility by marginally increasing the dominating assets at the expense of the dominated ones. According to MCSD, the lack of any pair of such assets is a necessary condition for efficiency of the portfolio. Our idea is to apply MCSD to each popular advice portfolio and check whether one can find dominating and dominated sets of assets given the portfolio. Should each portfolio pass our MCSD-based test, all of the portfolios cannot be considered inefficient. Any portfolio failing our test is inefficient.

III. Findings and Explanations

To compare our results with those of Canner et al. (1997), we use their data on asset allocation recommended by financial advisors, as presented in Table 1. We also adopt their time span and use historical annual real returns on stocks, bonds, and bills from 1926 to 1992 (Ibbotson Associates, 1999).

We assume that financial advisors consider only risk-averse expected-utility maximizers, i.e., investors whose marginal utility is positive and declining with wealth. Since investors have different unknown marginal utilities, advisors are unable to tailor their recommendations to specific clients' needs.

Our purpose is to evaluate advice as follows. One can state that a given recommendation is not efficient if *all* investors agree on changing the proportions of the assets in that portfolio. If, on the other hand, one concludes that investors have conflicting opinions on which proportions to increase and which to decrease, one cannot refute the advice and declare it inefficient.

For a given recommended allocation, we use the asset annual real returns for all realized states of nature to compute portfolio returns. We then rank the states of nature according to portfolio returns. Since utility is defined over wealth, the *ranking* of states of nature with

TABLE 1—ASSET ALLOCATIONS RECOMMENDED BY FINANCIAL ADVISORS

Advisor and investor type	Percent of portfolio: Cash	Bonds	Stocks	Annual real mean return (percent)	Standard deviation (percent)
Fidelity Investments					
Conservative	50	30	20	2.76	6.82
Moderate	20	40	40	4.59	10.34
Aggressive	5	30	65	6.54	14.53
Merrill Lynch					
Conservative	20	35	45	4.94	10.99
Moderate	5	40	55	5.85	13.00
Aggressive	5	20	75	7.23	16.18
Jane Bryant Quinn					
Conservative	50	30	20	2.76	6.82
Moderate	10	40	50	5.43	12.09
Aggressive	0	0	100	9.04	20.76
The New York Times					
Conservative	20	40	40	4.59	10.34
Moderate	10	30	60	6.12	13.57
Aggressive	0	20	80	7.65	17.18

Source: Canner et al. (1997).

respect to portfolio returns yields the same results as if the ranking were by declining marginal utility for each investor. All investors concur with this ranking because it is based only on portfolio returns that are assumed to be their only wealth. Investors may not exhibit the same marginal utility from portfolio returns, but they all agree upon the *ranking* of the marginal utilities of these returns. Hence, the ranking of the states of nature with respect to portfolio returns is the only information relevant to provide a ranking with respect to marginal utility.

We start with a given financial recommendation such as the conservative portfolio proposed by Fidelity Investments (Table 1). For this portfolio allocation, we compute the annual real returns and rank them from the lowest return to the highest as shown in Figure 1. On the horizontal axis of the figure, the states of nature are ranked from the ones generating the lowest portfolio returns to the ones generating the highest returns. The lowest portfolio returns yield the highest marginal utility, and the highest portfolio returns the lowest marginal utility; thus, states of nature are ranked according to decreasing marginal utility.

Cumulative portfolio returns up to a specific state of nature are on the vertical axis. The *Lorenz* associates cumulative portfolio returns with marginal utility. In Figure 1, the mean annual real return on the portfolio (2.76 percent) is the last point (F) on the *Lorenz,* where all states of nature are accounted for. The *Lorenz* intersects the horizontal axis at point H from which one concludes that in 68 of the states of nature with the highest marginal utility (or lowest returns) the cumulative mean return on the conservative portfolio by Fidelity Investments is zero.

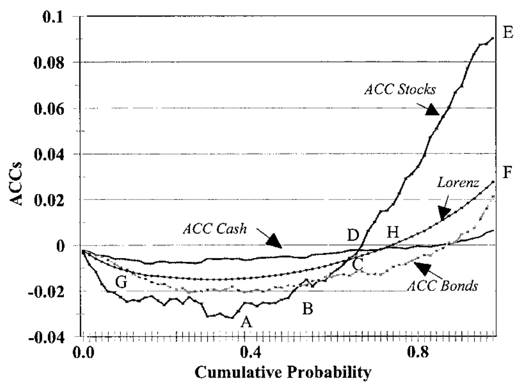

FIGURE 1. ACCs FOR CASH, BONDS, AND STOCKS COMPUTED FOR A CONSERVATIVE ASSET ALLOCATION RECOMMENDATION BY FIDELITY INVESTMENTS

Following Shorrocks (1983) and Shalit and Yitzhaki (1994), a portfolio is preferred over another by all risk-averse investors if its *Lorenz* is not below the *Lorenz* of the alternative portfolio.[1] In order to show that a recommended asset allocation is not dominated by an alternative portfolio, it is sufficient to show that there is no other portfolio with a higher *Lorenz* everywhere. The search for such a portfolio is confined to the vicinity of the proposed allocation because substantial changes in asset proportions might alter the ranking of states of nature, and thus the original portfolio *Lorenz* might cease to represent the ranking according to marginal utility.

To see whether in the vicinity of the recommended portfolio there is an allocation with a higher *Lorenz* everywhere, we first evaluate the impact of a small shift in the proportion of one asset on the portfolio *Lorenz.* This is expressed by the ACC that is the derivative of the *Lorenz* with respect to the proportion of an asset.

For the state of nature with the highest marginal utility, the effect of marginally changing an asset share will be proportional to the return to that asset in that state of nature. For the next state of nature (with a lower marginal utility), the effect will be the cumulative rates of return on that asset in those two states of nature. We continue until we reach the state of nature with the lowest marginal utility.

The curve we obtain is the ACC that expresses the cumulative annual real returns accruing to the portfolio by specific assets for all states of nature ranked according to decreasing marginal utility. Such ACCs appear in Figure 1 for stocks, bonds, and cash. For example, if one marginally increases the proportion of stocks in the portfolio, the effect will be shown along the ACC of stocks. For the entire data set, the total effect is the mean annual real return of 9 percent (point E in Figure 1). In 65 percent of the states of nature (to the left of point C), the ACC of stocks is below the *Lorenz*, implying that increasing the proportion of stocks worsens the return on the portfolio in those states of nature with the highest marginal utility. In the remaining 35 percent (to the right of point C), increasing the proportion of stocks raises portfolio returns.

Point D represents the intersection of the ACC of stocks with the horizontal axis at zero return. For the 60 percent of states of nature with the highest marginal utility, the mean cumulative annual real return on stocks is zero. Furthermore, in one-third of the states of nature, i.e., point A, the mean annual real return on stocks is -3 percent so that a more risk-averse investor will not want to increase the proportion of stocks in the portfolio, while a risk-neutral investor might.

The budget constraint does not allow us to increase the proportion of one asset without changing that of other assets. The complete effect of a change in the shares of several assets on the portfolio returns will be reflected by the sum of changes in shares multiplied by the ACCs. This gives us the entire effect on the *Lorenz.*

In the Appendix we show mathematically that the *Lorenz* is the sum of the ACCs, weighted by the share of each asset. In Figure 1, we can see that the ACC of stocks is lower than the ACC of cash until we reach point D. The effect of an increase in the proportion of stocks with an equal reduction in the proportion of cash can be seen by drawing a curve that equals the ACC of stocks minus the ACC of cash.

To see if an alternative portfolio is preferred by all risk-averse clients, we search for a weighted sum of concentration curves that will be above zero for every state of nature. If such a mix of assets can be found, this means we have found a portfolio with a higher *Lorenz.*

All three ACCs in Figure 1 intersect with each other. This means that we cannot increase the proportion of one asset and equally reduce the proportion of another in order to find a stochastically dominating portfolio. One might argue that, although we fail to find a dominating portfolio by altering the proportions of two assets, perhaps we can find a combination of changes in the proportions of three assets so that the resulting portfolios will be MCSD-superior to the Fidelity conservative recommendation. To verify this possibility, we adjust the methodology developed in Mayshar and Yitzhaki

[1] This can be seen by looking at the specific utility function with the marginal utility equating 1 up to P percent of the worst states of nature and 0 afterward. With this utility function, type P investor chooses the portfolio with the highest *Lorenz* at P because it yields the highest return and hence maximizes expected utility. If Lorenz curves intersect, different investors will prefer different portfolios, depending on their specific P. In that case, investors are not unanimous on the preferred allocation.

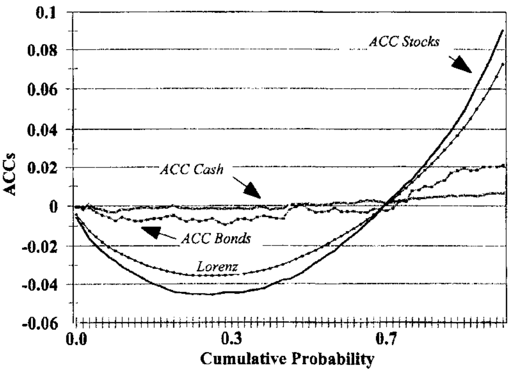

FIGURE 2. ACCS FOR CASH, BONDS, AND STOCKS COMPUTED FOR AN AGGRESSIVE ASSET ALLOCATION RECOMMENDATION BY MERRILL LYNCH

(1995), who use a numerical optimization algorithm to find a vector of small changes in asset proportions according to which the resulting portfolio dominates (SSD) the reference portfolio. The Appendix presents the algorithm used to find a dominating portfolio by changing the proportions of the three assets constituting the portfolio. We fail to find such an alternative portfolio that *all* risk-averse investors would agree dominates, and therefore we cannot declare that the Fidelity conservative portfolio is inefficient.

Figure 2 presents the analysis of the aggressive portfolio recommended by Merrill Lynch. We again reach the same conclusions. As all ACCs intersect, one cannot find a pair of dominating and dominated assets; i.e., no two assets exist such that *all* investors agree as to which proportion to increase and which to reduce. Following MCSD, the recommended portfolio cannot be improved by marginally changing the proportions of assets.

When we perform the analysis for all the portfolios recommended by the financial advisors in Table 1, we could not find alternative portfolios that are MCSD-superior to financial advisor recommendations.

IV. Conclusion

We conclude that certain portfolios proposed by financial advisors cannot be SSD-dominated by local alternative portfolios. Yitzhaki and Mayshar (1997) have shown that under relatively mild continuity conditions of the utility function, if a given portfolio is not dominated by a local (marginal) alternative portfolio, then it is also not dominated by any other portfolio. We tested all the portfolios in Table 1, and could not find dominating portfolios. Therefore, there is no way to conclude that the recommended portfolios are inefficient because they cannot be dominated by alternative portfolios.

An intuitive explanation for our results can be found by observing the differences between the annual real mean returns for cash, bonds, and stocks which are 0.64 percent, 2.12 percent, and 9.04 percent, respectively. These large mean return differences imply that these asset classes are not very good substitutes for one another. For SSD of a recommended portfolio, the alternative one must have at least the same return. Hence, to keep the portfolio mean return unchanged, one needs a combination of 82¢ in cash and 18¢ in stocks to offset a reduction in $1.00 in bonds.

But stocks are riskier than bonds and cash, and, as can be seen from the ACCs of Merrill Lynch's aggressive allocation in Figure 2, this alternative combination does not suffice to raise the combined ACCs of cash and stocks above the ACC of bonds. As can be seen from Figure 2, at low rates of return of the portfolio, the ACC of cash is not high enough above the ACC of bonds to allow for an increase in the proportion of stocks without increasing the riskiness of the portfolio. A similar story can be told with respect to the other recommended portfolios. The riskiness of stocks, as expressed by its ACC, is high and cannot be compensated for by increasing cash.

In summary, according to historical annual real returns from 1926 until 1992, we can state that all the popular advice portfolios recommended in the early 1990's were not inefficient in the sense that it is impossible to find better portfolios for *all* risk-averse investors.

APPENDIX: LORENZ DOMINANCE AND MCSD

MCSD provides the conditions for dominance in the case of two assets, given a portfolio. We here extend the analysis to allow for changes in many securities of a portfolio in order to find a dominating allocation. For the portfolio proposed by a financial advisor, we seek to find an alternative combination of stocks, bonds, and cash that would have been preferred by all risk-averse investors. Hence, we extend MCSD to Lorenz dominance.

Let p be the return on portfolio $\{\boldsymbol{\alpha}\}$ defined as $p = \sum \alpha_i r_i$, where r_i is the return on asset i and the portfolio is defined by $\sum \alpha_i = 1$. Let $f_{\boldsymbol{\alpha}}$ be the probability distribution of the portfolio. Following Shalit and Yitzhaki (1994, p. 673), the ACC of asset i with respect to portfolio $\{\boldsymbol{\alpha}\}$ is defined by the cumulative conditional expected return on asset i, $\mu_i(t)$, given portfolio return t as a function of the portfolio cumulative distribution $F_{\boldsymbol{\alpha}}(p)$:

$$ACC_i[F_{\boldsymbol{\alpha}}(p)] = \int_{-\infty}^{p} \mu_i(t) f_{\boldsymbol{\alpha}}(t)\, dt \qquad \text{for } \infty \geq p \geq -\infty.$$

Similarly, the *Lorenz* for portfolio $\{\boldsymbol{\alpha}\}$ is defined as:

$$L[F_{\boldsymbol{\alpha}}(p)] = \int_{-\infty}^{p} t f_{\boldsymbol{\alpha}}(t)\, dt \quad \text{for } \infty \geq p \geq -\infty.$$

The *Lorenz* of a portfolio can then be written as the weighted sum of the assets' ACCs in the portfolio:

$$L[F_{\boldsymbol{\alpha}}(p)] = \sum \alpha_i ACC_i[F_{\boldsymbol{\alpha}}(p)].$$

According to second-degree stochastic dominance (SSD), portfolio $\{\boldsymbol{\alpha}_1\}$ is preferred to portfolio $\{\boldsymbol{\alpha}_0\}$ by all risk-averse investors if:

$$L[F_{\boldsymbol{\alpha}_1}(p)] \geq L[F_{\boldsymbol{\alpha}_0}(p)] \quad \text{for all } p.$$

MCSD, however, considers marginal changes in asset proportions conditional on holding a portfolio. Hence one needs to look at the change in the portfolio *Lorenz* resulting from the changes in the marginal asset proportions such that the total of the proportion changes remains invariant: $\sum d\alpha_i = 0$. Given a portfolio $\{\boldsymbol{\alpha}_0\}$ an alternative portfolio $\{\boldsymbol{\alpha}_1\} = \{\boldsymbol{\alpha}_0 + \mathbf{d}\boldsymbol{\alpha}\}$ is preferred by all risk-averse investors if:

$$\sum_i \frac{\partial L[F_{\boldsymbol{\alpha}}(p)]}{\partial \alpha_i} d\alpha_i \geq 0 \quad \text{or}$$

$$\sum_i ACC_i[F_{\boldsymbol{\alpha}}(p)] d\alpha_i \geq 0 \quad \text{for all } 0 \leq F_{\boldsymbol{\alpha}} \leq 1.$$

Hence, to decide whether a given portfolio is efficient for all risk-averse investors, one needs to establish that there is no set of changes $\{\mathbf{d}\boldsymbol{\alpha}\}$ that satisfies $\sum_i ACC_i[F_{\boldsymbol{\alpha}}(p)] d\alpha_i \geq 0$ subject to $\sum d\alpha_i = 0$, for all $0 \leq F_{\boldsymbol{\alpha}} \leq 1$.

This is a simpler problem than the one Mayshar and Yitzhaki (1995) use to find a tax reform plan that improves upon all concave social welfare functions. We use their algorithm as follows: As we have three assets (stocks, bonds, and cash) and one constraint, two parameters remain to be chosen. Of these two, one is chosen as the numeraire, which can be either positive or negative. For three assets and a sample of n observations, the problem is reduced to minimize the target function:

$$\min_{\mathbf{d}\boldsymbol{\alpha}} \sum_{t=1}^{n} \left\{ \max\left[-\sum_{i=1}^{3} ACC_i(F_{\boldsymbol{\alpha}})\, d\alpha_i,\, 0 \right] \right\}^2$$

subject to $d\alpha_1 = -1$ or 1 and $\sum_i d\alpha_i = 0$.

If the minimum value of the target function reaches 0, a dominating portfolio is found. If on the other hand, the minimum value is positive, a dominating portfolio does not exist.

This problem is optimized using a numerical algorithm as in Mayshar and Yitzhaki (1995). The resulting calculations show that all 12 popular advice portfolios are marginally efficient in the sense that one cannot find, at the margin, a dominated portfolio.

REFERENCES

Canner, Niko; Mankiw, N. Gregory and Weil, David N. "An Asset Allocation Puzzle." *American Economic Review*, March 1997, *87*(1), pp. 181–91.

Hadar, Josef and Russell, William R. "Rules for Ordering Uncertain Prospects." *American Economic Review*, March 1969, *59*(1), pp. 25–34.

Hanoch, Giora and Levy, Haim. "The Efficiency Analysis of Choices Involving Risk." *Review of Economic Studies*, July 1969, *36*(3), pp. 335–46.

Ibbotson Associates. *Stocks, bonds, bills, and inflation: 1999 yearbook*. Chicago: Ibbotson Associates, 1999.

Mayshar, Joram and Yitzhaki, Shlomo. "Dalton-Improving Indirect Tax Reform." *American Economic Review*, September 1995, *85*(4), pp. 793–807.

Rothschild, Michael and Stiglitz, Joseph E. "Increasing Risk I: A Definition." *Journal of Economic Theory*, March 1970, *2*(1), pp. 66–84.

Shalit, Haim and Yitzhaki, Shlomo. "Marginal Conditional Stochastic Dominance." *Management Science*, May 1994, *40*(5), pp. 670–84.

Shorrocks, Anthony F. "Ranking Income Distributions." *Economica*, February 1983, *50*(1), pp. 3–17.

Yitzhaki, Shlomo and Mayshar, Joram. "Characterizing Efficient Portfolios." Mimeo, Hebrew University of Jerusalem, 1997.