NASA Contractor Report 178266

ICASE REPORT NO. 87-17

ICASE

AN ASYMPTOTIC ANALYSIS OF SUPERSONIC REACTING MIXING LAYERS

T. L. Jackson

M. Y. Hussaini

Contract No. NAS1-18107

March 1987

INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING
NASA Langley Research Center, Hampton, Virginia 23665

Operated by the Universities Space Research Association

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

AN ASYMPTOTIC ANALYSIS OF SUPERSONIC REACTING MIXING LAYERS

T. L. Jackson

Department of Mathematical Sciences
Old Dominion University
Norfolk, Virginia 23508
and
Institute For Computer Applications In Science And Engineering
NASA Langley Research Center
Hampton, Virginia 23665

M. Y. Hussaini

Institute For Computer Applications In Science And Engineering
NASA Langley Research Center
Hampton, Virginia 23665

Abstract. The purpose of this paper is to present an asymptotic analysis of the laminar mixing of and the simultaneous chemical reaction between parallel supersonic streams of two reacting species. The study is based on a one-step irreversible Arrhenius reaction and on large activation energy asymptotics. Essentially it extends the work of Linan and Crespo to include the effect of free shear and Mach number on the ignition regime, the deflagration regime and the diffusion flame regime. It is found that the effective parameter is the product of the characteristic Mach number and a shear parameter.

This work was supported under the National Aeronautics and Space Administration under NASA Contract NAS1-18107 while the authors were in residence at the Institute for Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, VA 23665.

1. Introduction. The study of supersonic combustion flames has become extremely important in view of the scramjet engine for the propulsion of hypersonic aircraft and cruise missile. The aerodynamic processes governing such reacting flows are exceedingly complex involving strong interaction between chemical and fluid dynamical effects. The simplest relevant problem of a combustion flame may be formulated by considering two uniform parallel streams of hydrogen and air, both supersonic and semi-infinite in extent, which begin to mix at some origin (Fig.1). The physical processes underlying this problem are 1) the evolution of the mixing flow-field, 2) the evolution of the temperature field through conduction and convection, and through the heat generated by viscous dissipation and chemical reaction, and 3) molecular diffusion of each gaseous component into the other together with the annihilation of the combustible gas and the consequent production of combustion products. Any realistic modelling of the complex kinetics will necessitate a full scale numerical solution (e.g., Ferri (1973), Drummond, Rogers and Hussaini (1986)). Then questions arise as to the reaction rates and their dependence on temperature and the relative importance of various reactions which entail thus rather large uncertainties.

In a mathematical treatment of combustion, the kinetic model has to be necessarily simple. The one-step irreversible Arrhenius model has been extensively used with significant success in the study of combustion (Buckmaster and Ludford (1982)). It was employed by Marble and Adamson (1954) who considered the laminar mixing of two parallel streams, one of a cold combustible and the other of a hot inert gas. They discussed questions of ignition, location of the flame relative to the initial contact point, and of its detailed development in the isovelocity case where both the streams have the same uniform velocity. The method of solution involved expansions in terms of the streamwise coordinate and the classical von Karman integral technique for solving the resulting set of ordinary differential equations. A similar problem of the continuous evolution from nearly frozen flow to near equilibrium flow in unsteady diffusion flames was analyzed by Linan and Crespo (1976). Their analysis, which employs a two-reactant model, elucidated the existence of the ignition, deflagration and diffusion flame·regimes. Although their study was confined to the isovelocity case and one-dimensional situation, it captured surprisingly well the rather complex picture.

The present study extends their analysis to the more general problem of the chemical reaction between two parallel streams with different velocities. It is assumed that the characteristic chemical time and the fluid dynamical time are of the same order. The kinetic model consists of a one-step two-species irreversible Arrhenius process with high activation energy. This model appears to cover the essential physics of the problem. In the last decade or so, the asymptotic studies of combustion based on this model have significantly enhanced our understanding of ignition, of flame stability and of diffusion flame structure in low subsonic flows. There is as yet no reason to believe that this model will not play an equally significant role in enhancing our understanding of supersonic combustion. In any event, this idealization makes the problem amenable to asymptotic

analysis and thus provides a semi-analytical solution. The results can then certainly verify and in turn be verified by full numerical simulations. The asymptotic results of the present study bring out the effect of free shear and Mach number. They indicate that the important parameter is the product of Mach number and shear. Further, they predict the existence of the ignition, deflagration and diffusion-flame regimes even in the supersonic case. The numerical solutions cited above for analogous problems appear to miss the ignition and the deflagration regimes. Perhaps, careful fine-tuning of the numerics will uncover these regimes; this will be the subject of future effort.

2. Governing Equations. Consider two reactants initially separated by a finite length splitter plate coming in at different velocities and temperatures (see Fig. 1). The nondimensional shear-layer equations with zero pressure gradient which govern this mixing process are given by (Williams (1985))

$$(\rho u)_{x'} + (\rho v)_{y'} = 0, \qquad 1 = \rho T, \tag{2.1a}$$

$$\rho(uu_{x'} + vu_{y'}) = (\mu u_{y'})_{y'}, \tag{2.1b}$$

$$\rho(uT_{x'} + vT_{y'}) = (\mu T_{y'})_{y'} + M^2(\gamma-1)\mu u_{y'}^2 + \alpha\Omega, \tag{2.1c}$$

$$\rho(uY_{i_{x'}} + vY_{i_{y'}}) = (\mu Y_{i_{y'}})_{y'} - \alpha_i\Omega, \quad i = 1,2, \tag{2.1d}$$

$$\Omega = D\rho^{a+b+1}Y_1^aY_2^be^{-\theta/T}, \tag{2.1e}$$

where ρ is the density, u the velocity in the x'-direction, v the velocity in the y'-direction and T the temperature. Y_1 and Y_2 are the mass fractions above and below the plate, respectively, with reaction assumed to be irreversible and of Arrhenius-type. The viscosity μ is assumed to be a function of temperature. The other nondimensional parameters appearing above are:

$\alpha = QY_{10}/C_PT_{10}\nu_1W_1$ — Heat release number,

$\theta = E/RT_{10}$ — Activation energy,

$D = \dfrac{l_o}{u_{10}} \dfrac{B\rho_{10}^{a+b}Y_{10}^{a+b-1}\nu_1W_1}{W_1^aW_2^b}$ — Damkohler number,

$\alpha_i = \nu_iW_i/\nu_1W_1$ — Parameter involving stoichiometry,

$M = u_{10}/a_{10}$ — Mach number,

where a and b are the reaction orders of Y_1 and Y_2 , respectively; ν_j the stoichiometric coefficient for species j; W_j the molecular weight of species j; E the dimensional activation energy; R the universal gas constant; B the preexponential constant in rate expression; a_{10} the speed of sound referred to T_{10} ; Q the chemical heat release; γ the specific-heats ratio; and finally C_p the specific heat at constant pressure. The equations were nondimensionalized by selecting the freestream values T_{10}, ρ_{10}, u_{10}, Y_{10} above the plate for the temperature, density, velocities and mass fractions, respectively. Lengths are referred to l_o , some characteristic length scale of the flow. We have assumed unit Prandtl and Lewis numbers in writing down these equations. The assumption of unit Lewis number allows us to consider linear combinations of (2.1c) and (2.1d) to eliminate the source term, which then admits similarity-type solutions (see remarks after (2.10)).

The boundary conditions consistent with (2.1) are

$$T = u = Y_1 = 1\ ,\ Y_2 = 0 \qquad x' = 0,\ y' > 0, \text{ and } at\ x' > 0,\ y' \to \infty,$$

$$T = \beta_T\ ,\ u = \beta_U\ ,\ Y_1 = 0\ ,\ Y_2 = Y_{2f} \qquad x' = 0,\ y' < 0, \text{ and } at\ x' > 0,\ y' \to -\infty. \tag{2.2a}$$

For definiteness, we take the maximum initial temperature to occur above the plate, and so we require $\beta_T \leq 1$. We make no such restriction on where the maximum initial velocity can occur, we only require both streams to be supersonic; i.e., $M - 1 > 0$ and $\beta_U M - 1 > 0$. In addition, matching the pressure across the mixing layer results in the compatibility equation

$$\frac{1}{\sqrt{M^2 - 1}} \nu(x',\infty) = \frac{-1}{\sqrt{M^2 \beta_U^2 \beta_T^{-1} - 1}} \frac{\beta_U}{\beta_T} \nu(x',-\infty), \tag{2.2b}$$

which reduces to that given by Ting (1959) for $\beta_T = 1$.

Equations (2.1) can be transformed into incompressible equations by the use of the Howarth-Dorodnitzyn transformation (Stewartson (1964))

$$x = \frac{x'}{l_c}, \quad z = \frac{1}{\sqrt{l_c}} \int_0^{y'} \rho dy', \quad w = \sqrt{l_c}\,(\rho v + u \int_0^{y'} \rho_{x'}\, dy'), \tag{2.3}$$

where l_c is a scaling factor based on a characteristic chemical length, to be chosen later. The transformed equations in the (x,z) plane are

$$u_x + w_z = 0, \tag{2.4a}$$

$$u u_x + w u_z = C u_{zz}, \tag{2.4b}$$

$$uT_x + wT_z = CT_{zz} + C(\gamma-1)M^2u_z^2 + l_c\frac{\alpha}{\rho}\Omega, \tag{2.4c}$$

$$uY_{i_x} + wY_{i_z} = CY_{i_{zz}} - l_c\frac{\alpha_i}{\rho}\Omega, \quad i = 1,2, \tag{2.4d}$$

where the constant C appearing above is given by Chapman's (1950) linear viscosity law

$$\mu = CT. \tag{2.5}$$

These equations may also be written in terms of the variables x and η , where

$$\eta - \eta_o = \frac{z}{2\sqrt{xC}} \tag{2.6}$$

is the similarity variable for the chemically frozen heat conduction problem, and η_o corresponds to a shift in the origin determined uniquely by (2.2b). (We note here that if the initial velocities were subsonic, then the compatability condition (2.2b) would be trivially satisfied, and thus η_o would remain indeterminate (Klemp and Acrivos (1972)). By introducing the relationships

$$u = f'(\eta) \ , \ w = \frac{\sqrt{C}}{\sqrt{x}}(\eta f' - f), \tag{2.7}$$

we see that the continuity equation (2.4a) is automatically satisfied, while the η -momentum equation (2.4b) reduces to the well known Blasius equation for free shear

$$f''' + 2ff'' = 0, \quad f'(\infty) = 1, \ f'(-\infty) = \beta_U, \ f(0) = 0, \tag{2.8}$$

which represents the mean flow profile of the wake velocity. Numerical solutions are given, for example, by Lock (1951). For our purposes, we shall consider the velocity field completely known.

Once f is known, the temperature and mass fractions can be determined from the equations

$$4xf'T_x - 2fT_\eta - T_{\eta\eta} - (\gamma-1)M^2(f'')^2 = \frac{4xl_c\alpha\Omega}{\rho} \ , \tag{2.9}$$

$$4xf'Y_{i_x} - 2fY_{i_\eta} - Y_{i_{\eta\eta}} = -\frac{4xl_c\alpha_i\Omega}{\rho} \ , \ i = 1,2. \tag{2.10}$$

These do not have similarity solutions due to the presence of the nonlinear source term. However, the combinations $T+\alpha Y_1$ and $T+\frac{\alpha}{\alpha_2}Y_2$ do satisfy the heat equation without source terms, and are given by the form

$$4xf'\phi_x - 2f\phi_\eta - \phi_{\eta\eta} - (\gamma-1)M^2(f'')^2 = 0.$$

Thus, a similarity solution exists (Stewartson (1964)) and is given by

$$\phi = c_1 + g(\eta) + \frac{\gamma-1}{2}M^2(f' - f^2),$$

where g satisfies the differential equation

$$g'' + 2fg' = 0,$$

with solution

$$g(\eta) = c_2 \int_0^\eta exp(-\int_0^x 2f(s)ds)\, dx = c_2 I(\eta);$$

c_1 and c_2 being arbritary constants. When the boundary conditions (2.2a) are taken into account, we find that

$$T + \alpha Y_1 = 1 + \alpha - (1 - \beta_T + \alpha)\xi + \frac{\gamma-1}{2}M^2(1-\beta_U)^2\xi(1-\xi), \tag{2.11}$$

$$T + \frac{\alpha}{\alpha_2}Y_2 = 1 - (1 - \beta_T - \frac{\alpha}{\alpha_2}Y_{2f})\xi + \frac{\gamma-1}{2}M^2(1-\beta_U)^2\xi(1-\xi), \tag{2.12}$$

where

$$\xi = \frac{I(\infty) - I(\eta)}{I(\infty) - I(-\infty)} = \frac{1 - f'}{1 - \beta_U}. \tag{2.13}$$

Note that as $\eta \to \infty$, $\xi \to 0$, and as $\eta \to -\infty$, $\xi \to 1$.

The relations (2.11) and (2.12) gives the mass fractions in terms of T and η , enabling us to reduce our problem to that of finding T from (2.9). If we choose for l_c a nondimensional characteristic chemical length based on the maximum initial temperature T_{10} and initial velocity u_{10} ,

$$l_c = (\frac{l_o}{u_{10}})^{-1} \frac{C_p T_{10}}{Q} \frac{RT_{10}}{E} \frac{1}{B} (\frac{W_1}{\rho_{10}Y_{10}})^a (\frac{W_2}{\rho_{10}Y_{20}})^b e^{E/RT_{10}}, \tag{2.14}$$

we obtain

$$4xf'T_x - 2fT_\eta - T_{\eta\eta} - (\gamma-1)M^2(f'')^2 = \frac{4x}{\theta}Y_{2f}^{-b}T^{-(a+b)}Y_1^aY_2^b\exp\{\theta(\frac{T-1}{T})\}, \tag{2.15}$$

subject to the boundary conditions

$$T \to 1 \quad as\ \eta \to \infty,\ x > 0 \text{ and } x = 0,\ \eta > 0, \tag{2.16a}$$

$$T \to \beta_T \ \ as\ \eta \to -\infty,\ x > 0 \text{ and } x = 0,\ \eta < 0. \tag{2.16b}$$

Eqn. (2.14) implies, in effect, that the characteristic chemical time and characteristic flow time are of the same order.

For the special case $\beta_U = 1$ (no shear), (2.8) gives $f = \eta$ corresponding to u=1, v=0 everywhere. Then from (2.13),

$$\xi = \xi_o = \frac{1}{2}(1-erf\eta) \tag{2.17}$$

and we see that (2.11)-(2.17) is equivalent to Linan and Crespo's (1976) equations (13)-(17); with x in our paper playing the role of the time-like variable t in theirs. Therefore, the analysis of Linan and Crespo (1976), hereafter referred to as I, can be considered a limiting case of the above system. The goal of this work is to assess the effect of shear on mixing and ignition, as well as the coupling effect of differing initial temperatures.

3. Ignition Stage. In the absence of chemistry, equation (2.15) reduces to

$$4xf'T_x - T_{\eta\eta} - 2fT_\eta - (\gamma-1)M^2(f'')^2 = 0 \tag{3.1}$$

which posesses the (inert) similarity solution

$$T_I = 1 - (1 - \beta_T)\xi + \frac{\gamma-1}{2}M^2(1-\beta_U)^2\xi(1-\xi), \tag{3.2}$$

which is also a solution of (2.15) for small x. We note here that for $\beta_U = 1$, the Mach number M plays no role. Substituting (3.2) into (2.11) and (2.12) yields for the (inert) mass fractions

$$Y_{1I} = 1 - \xi\ , \quad Y_{2I} = Y_{2f}\xi\ . \tag{3.3}$$

Since ξ increases as β_U increases (see (2.13)), we see from (3.3) that Y_{1I} decreases while Y_{2I} increases. Thus, the effect of shear is to enhance the initial mixing process that would otherwise be present only by interdiffusion of the reactants.

As x increases, more of the combustible mixes until, at some finite distance downstream of the plate, a thermal explosion occurs characterized by significant departure from the inert. The exponent of (2.15) suggests that, in the limit $\theta \to \infty$ being considered here, this will occur in regions where $T - 1 = O(\theta^{-1})$. Therefore, T = 1 is the switch-on temperature for the chemical reaction.

To describe the ignition stage, we follow I and set

$$\phi = \theta(T - T_I). \tag{3.4}$$

The basic equation (2.15) becomes

$$4xf'\phi_x - 2f\phi_\eta - \phi_{\eta\eta} = 4xY_{2f}^{b}(T_I + \theta^{-1}\phi)^{-(a+b)}Y_1^aY_2^b\exp\{\frac{\phi + \theta(T_I-1)}{T_I + \theta^{-1}\phi}\}, \tag{3.5}$$

where

$$Y_1 = 1 - \xi - \theta^{-1}\frac{\phi}{\alpha}, \quad Y_2 = Y_{2f}\xi - \theta^{-1}\frac{\alpha_2}{\alpha}\phi \tag{3.6}$$

from (2.11) and (2.12), respectively.

The term $\theta(T_I - 1)$ in the exponent of (3.5) can be either positive or negative, depending on the amount of initial shear. For $T_I - 1 = O(1)$, the rate term is either exponentially small or exponentially large, depending on its sign. The latter case shows that ignition takes place immediately at the trailing edge of the plate. A realistic model for this case should include the boundary layer effects at the trailing edge of the plate, which the present model neglects. Modifications of this model to allow for very short ignition distances will be the subject of a future paper. The former case shows that chemical activity is quenched. Thus, we confine our attention to regions where $T_I - 1 = O(\theta^{-1})$, consistent with (3.4) with T - 1 = $O(\theta^{-1})$.

We first consider the important case of ignition for nearly equal initial temperatures and velocities. To this end, we set

$$\beta_T = 1 - \theta^{-1}\tilde{\beta}_T \ , \quad \beta_U = 1 - \theta^{-1/2}\tilde{\beta}_U, \tag{3.7}$$

and seek an expansion for $\theta \to \infty$ about the inert, with $\tilde{\beta}_T$, $\tilde{\beta}_U$ and M fixed and O(1) constants. In particular, we see that

$$\xi = \xi_o + O(\theta^{-1/2}), \quad f = \eta - \theta^{-1/2}\tilde{\beta}_U\xi_o + O(\theta^{-1}), \tag{3.8}$$

from (2.13) and (2.8), respectively, with ξ_o defined in (2.17). The inert solution can now be expanded as

$$T_I = 1 + \theta^{-1}\{-\tilde{\beta}_T\xi_o + \frac{\gamma-1}{2}M^2\tilde{\beta}_U^2\xi_o(1-\xi_o)\} + O(\theta^{-2}), \tag{3.9}$$

$$Y_{1I} = 1 - \xi_o + O(\theta^{-1/2}), \quad Y_{2I} = Y_{2f}\xi_o + O(\theta^{-1/2}). \tag{3.10}$$

Substitution of (3.8)-(3.10) into (3.5) yields the following equation for the disturbance temperature:

$$4x\phi_x - 2\eta\phi_\eta - \phi_{\eta\eta} = 4x\xi_o^b(1-\xi_o)^a\exp\{\phi - \tilde{\beta}_T\xi_o + \frac{\gamma-1}{2}M^2\tilde{\beta}_U^2\xi_o(1-\xi_o)\} \tag{3.11}$$

subject to the boundary conditions

$$\phi = 0 \quad at \quad x = 0 \quad \text{and} \; at \; \eta \to \pm\infty. \tag{3.12}$$

The expansions are based on the assumption that $\theta^{-1} \gg Re^{-1/2}$, where Re is the Reynolds number based on the free-stream values of the flow. (Typical values for θ are on the order of twenty, and for Re, the order of several millions). Except for the last term in the exponent, eqn. (3.11) is identical to (33I). Thus, the important new parameter governing ignition is the product of Mach number M and shear parameter $\tilde{\beta}_U$, which measures the deviation from the isovelocity case.

The solution to this problem was computed numerically. The equation was discretized in η, and the resulting system of ordinary differential equations was solved via the Gear package. Computations were done for various values of the parameters $\tilde{\beta}_T$ and $M\tilde{\beta}_U$ and for reaction orders a = b = 1. Integration was terminated when ϕ became infinite, thus identifying the ignition point (x_*,η_*). Results of these computations are displayed in the table, which is discussed below.

The first entry of the table ($M\tilde{\beta}_U = 0$) corresponds to that of I. Clearly, as $\tilde{\beta}_T$ increases, the igntion distance x_* increases, while the ignition location (x_*,η_*) moves into the region of higher initial temperature. Thus, the effect of cooling the gas below the plate is to retard ignition.

$M\tilde{\beta}_U = 0$

$\tilde{\beta}_T$	x_*	η_*
0	5.839	.0
2	13.203	.3
5	27.595	.62
10	54.850	.84

$\tilde{\beta}_T = 0$

$M\tilde{\beta}_U$	x_*	η_*
0	5.839	0
2	4.899	0
5	1.912	0
10	0.057	0

$\tilde{\beta}_T = 2$

$M\tilde{\beta}_U$	x_*	η_*
0	13.203	.30
2	11.270	.28
5	4.666	.18
10	0.149	.08

$\tilde{\beta}_T = 10$

$M\tilde{\beta}_U$	x_*	η_*
0	54.85	.84
2	51.01	.82
5	32.86	.70
10	2.92	.36

Table. Ignition points for various values of $\tilde{\beta}_T$ and $M\tilde{\beta}_U$, and for a = b = 1.

The other entries corresponds to fixed values of $\tilde{\beta}_T$. We note that these results do not depend on the sign of $\tilde{\beta}_U$; i.e., does not depend on where the maximum initial velocities occur. As $|\tilde{\beta}_U|$ increases, with M fixed and O(1), the ignition distance moves closer to the trailing edge of the splitter plate, while the ignition location (x_*,η_*) approaches the center line $\eta = 0$, where the maximum amount of shearing occurs. Thus, the effect of increase in $|\tilde{\beta}_U|$ is to enhance mixing, and hence to enhance ignition. On the other hand, as M becomes large (i.e., hypersonic speeds), for fixed $|\tilde{\beta}_U|$, ignition is found to occur rapidly even with a small amount of mixing! This is understandable since increasing the Mach number increases the inert temperature, which in turn feeds in more energy into the system causing it to ignite sooner. Consistent with the results of I, if we fix $M|\tilde{\beta}_U|$ and vary $\tilde{\beta}_T$, we again see that ignition is retarded, moving away from the edge of the plate.

As $\tilde{\beta}_T$ and $|\tilde{\beta}_U|$ increase, with M remaining O(1), the expansions (3.7) break down. For the special case of no shear or zero Mach number, the analysis proceeds as in I. For O(1) shear, the location of the flame critically depends on the amount of initial tempertaure differences. If the two streams are nearly equal in temperature, then ignition ocurrs at the trailing edge of the plate, and a more delicate analysis is needed, as mentioned above. If, on the other hand, the two streams are sufficiently different in temperature, then ignition could occur away from the plate, and the analysis again proceeds as in I. We postpone the analysis for O(1) velocity and temperature differences as a topic of future research.

After ignition has occured, a pair of well-defined deflagration waves emerges according to classical thermal explosion theory. Since the effect of the Mach number and shear is to only quantitatively alter the direction and speed of the waves, we do not present any results here. We only note that they must exist, penetrating the mixing layer until all of the deficient reactant is consumed (Fig. 1). Just downstream of the deflagration wave a diffusion flame exist where the flow is in chemical equilibrium, and is described in the following section.

4. Diffusion Flame. As in I, a thin diffusion flame exists behind the premixed flame and is characterized by near-equilibrium conditions; $Y_1 = 0$ on one side of the flame with $Y_2 = 0$ on the other. From the relations (2.11) and (2.12), we see that the flow can be described by

$$Y_1 = 1 - (1 + \frac{Y_{2f}}{\alpha_2})\xi\ ,\ \ Y_2 = 0, \tag{4.1a}$$

$$T = 1 - (1 - \beta_T - \frac{\alpha}{\alpha_2}Y_{2f})\xi + \frac{\gamma-1}{2}M^2(1-\beta_U)^2\xi(1-\xi) \tag{4.1b}$$

for $\eta > \eta_e$, and

$$Y_1 = 0, \quad Y_2 = -\alpha_2\{1 - (1 + \frac{Y_{2f}}{\alpha_2})\xi\}, \tag{4.2a}$$

$$T = 1 + \alpha - (1 - \beta_T + \alpha)\xi + \frac{\gamma-1}{2}M^2(1-\beta_U)^2\xi(1-\xi), \tag{4.2b}$$

for $\eta < \eta_e$, where η_e gives the location of the flame sheet where both reactants vanish, and T takes the adiabatic flame value T_e, given by

$$T_e = 1 - (1 - \beta_T - \frac{\alpha}{\alpha_2}Y_{2f})\xi_e + \frac{\gamma-1}{2}M^2(1-\beta_U)^2\xi_e(1-\xi_e) \tag{4.3}$$

and

$$\xi_e = \frac{1 - f'_e}{1 - \beta_U} = (1 + \frac{Y_{2f}}{\alpha_2})^{-1} \tag{4.4}$$

defines the flame location. We remark here that the location of the diffusion flame depends not only on the initial concentration, but also on the product of Mach number and shear, a feature absent from I. Fig. 2 shows η_e for various values of $M(1-\beta_U)$. In particular, as β_U decreases (or, equivalently, as M decreases), η_e moves into the region of lower initial velocity, for fixed values of $(1 + Y_{2f}/\alpha_2)^{-1}$.

To study the structure of the diffusion flame, which appears as a discontinuity on the η, or ξ scale, we introduce the variables

$$\xi = \xi_e + \theta^{-1}BZ, \quad T = T_e - \theta^{-1}T_e^2T_1, \tag{4.5}$$

and expand for $\theta \to \infty$. Here, B is a scaling factor defined by

$$B = \frac{2T_e}{A}\,(1 + \frac{Y_{2f}}{\alpha_2})^{-1}.$$

Following the notation of I, we introduce the relations

$$Y_1 = T_e\theta^{-1}(\Gamma - Z)/A, \quad Y_2 = \alpha_2T_e\theta^{-1}(\Gamma + Z)/A, \tag{4.6}$$

with A defined by (83I). By substituting (4.5)-(4.6) into (2.15), we obtain equation (84) of I (where a (-) minus sign is missing from the exponent), with Γ given by (81I), except now the constant g of (85I) is replaced by

$$g = (1 + \frac{Y_{2f}}{\alpha_2})^{-1}\,\{1 + (\frac{2\alpha_2}{\alpha Y_{2f}}(1-\beta_T)-1)\frac{Y_{2f}}{\alpha_2} - \frac{\gamma-1}{\alpha}M^2(1-\beta_U)^2(1-2\xi_e)\}. \tag{4.7}$$

Hence, the analysis of I for the structure of the diffusion flame also holds when shear is taken into account, with an appropiate interpretation of g.

Conclusions. The laminar mixing of and the simultaneous chemical reaction between parallel supersonic streams of two reacting species has been discussed in the framework of high activation energy asymptotics. Attention is confined to the special but important case of low shear parameter, which is a measure of departure from the isovelocity case, and small temperature differences between the streams. It is found that the regimes of ignition, deflagration and diffusion flame uncovered by Linan and Crespo (1976) are found to exist in the supersonic case also. In particular, it is found that two parameters governs the ignition regime: a parameter giving the temperature difference across the plate (Linan and Crespo), and a second parameter which is the product of the characteristic Mach number and the shear parameter. The location of the ignition point for a given temperature difference is a function of the latter parameter. It is implied that the ignition point moves upstream as the value of this parameter increases.

This work suggests resolution requirement for the numerical simulation of the problem to capture the ignition and deflagration regimes. The next improvement of this model, to be more realistic, should include the effect of the finite boundary layer thickness at the trailing edge.

Acknowledgments. The authors would like to thank A. Kapila for helpful comments in connection with this work.

REFERENCES

Buckmaster, J. D. and Ludford, G. S. S. (1982). Theory of Laminar Flames, Cambridge University Press.

Chapman, D. R. (1950). "Laminar mixing of a compressible fluid", NACA Rep. 958.

Drummond, J. P., Rogers, R. Clayton and Hussaini, M. Y. (1986). "A detailed numerical model of a supersonic reacting mixing layer", AIAA Paper No. 86-1427.

Ferri, A. (1973). "Mixing-controlled supersonic combustion", Annual Review of Fluid Mechanics, Vol. 5, pp. 301-338.

Klemp, J. B. and Acrivos, A. (1972). "A note on the laminar mixing of two uniform parallel semi-infinite streams", J. Fluid Mech., 55, pp. 25-30.

Linan, A. and Crespo, A. (1976). "An asymptotic analysis of unsteady diffusion flames for large activation energies", Comb. Sci. Tech., 14, pp. 95-117.

Lock, R. C. (1951). "The velocity distribution in the laminar boundary layer between parallel streams", Quart. J. Mech. and Appl. Math., 4, pp. 42-63.

Marble, F. E. and Adamson, T. C. (1954). "Ignition and combustion in a laminar mixing zone", Jet Propulsion, 24, pp. 85-94.

Stewartson, K. (1964). The Theory of Laminar Boundary Layers in Compressible Fluids, Oxford University Press, Great Britain.

Ting, L. (1959). "On the mixing of two parallel streams", J. Math. Phys., 28, pp. 153-165.

Williams, F. A. (1985). Combustion Theory, Benjamin Cummings Publishing Company.

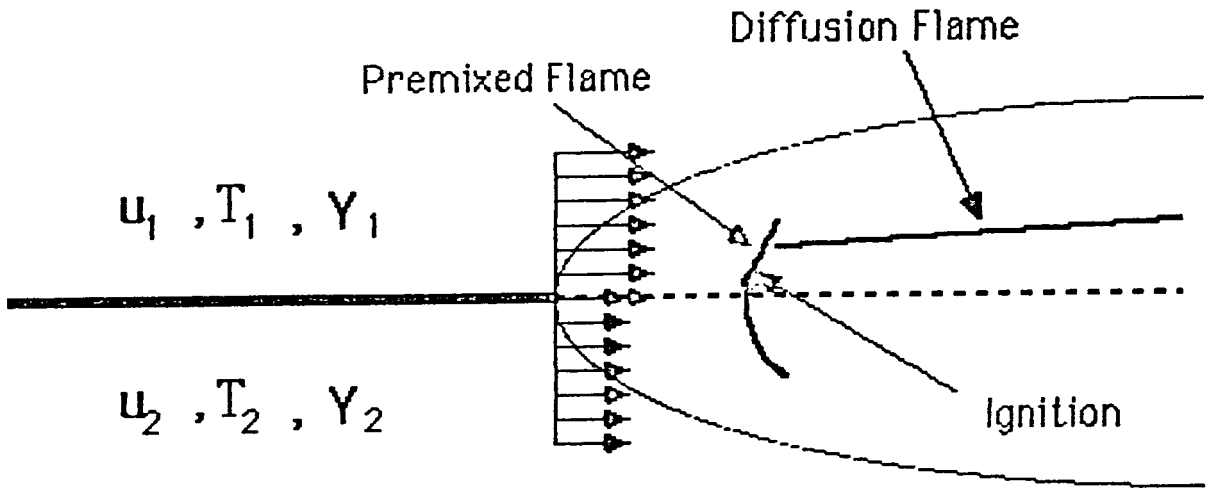

Figure 1. Schematic showing the reacting mixing layer, with $T_1 > T_2$.

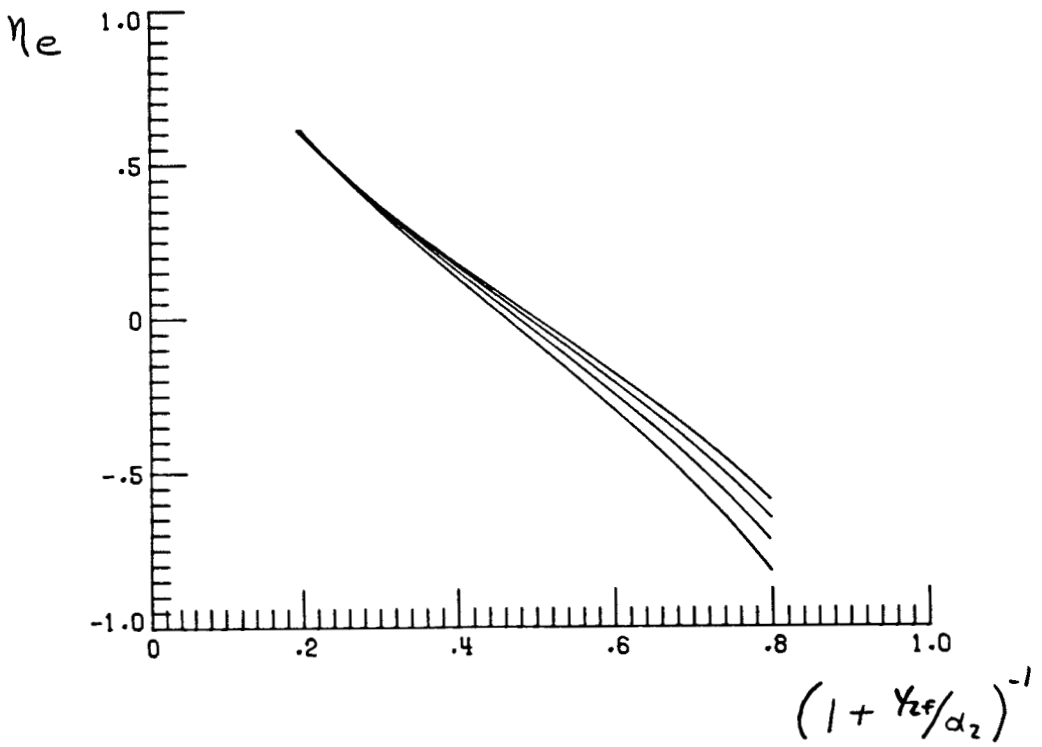

Figure 2. Diffusion flame location η_e as a function of initial concentrations.

Standard Bibliographic Page

1. Report No. NASA CR-178266 ICASE Report No. 87-17	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle AN ASYMPTOTIC ANALYSIS OF SUPERSONIC REACTING MIXING LAYERS		5. Report Date March 1987 6. Performing Organization Code
7. Author(s) T. L. Jackson, M. Y. Hussaini		8. Performing Organization Report No. 87-17
9. Performing Organization Name and Address Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23665-5225		10. Work Unit No. 11. Contract or Grant No. NAS1-18107
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Contractor Report 14. Sponsoring Agency Code 505-90-21-01

15. Supplementary Notes

Langley Technical Monitor:
J. C. South

Submitted to Combustion Science and Technology

Final Report

16. Abstract

The purpose of this paper is to present an asymptotic analysis of the laminar mixing of the simultaneous chemical reaction between parallel supersonic streams of two reacting species. The study is based on a one-step irreversible Arrhenius reaction and on large activation energy asymptotics. Essentially it extends the work of Linan and Crespo to include the effect of free shear and Mach number on the ignition regime, the deflagration regime and the diffusion flame regime. It is found that the effective parameter is the product of the characteristic Mach number and a shear parameter.

17. Key Words (Suggested by Authors(s))	18. Distribution Statement
chemical mixing, supersonic flow, asymptotics	59 - Mathematical and Computer Sciences (General) Unclassified - unlimited

19. Security Classif.(of this report)	20. Security Classif.(of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	16	A02

For sale by the National Technical Information Service, Springfield, Virginia 22161