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An asymptotic closure theory for irradiance in the sea and its inversion to obtain the inherent optical properties

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Abstract

An expression is derived for the rate at which the diffuse attenuation coefficient for vector irradiance approaches its asymptotic value in a homogeneous medium. The asymptotic approach rate is shown to be a function of boundary conditions at the surface and the asymptotic diffuse attenuation coefficient which is an inherent optical property. The asymptotic approach rate is then used to derive the vertical structure of the vector and scalar irradiances, the vector and scalar diffuse attenuation coefficients, the average cosine of the light field, and the remotely sensed reflectance at the surface, based only on the surface values of the vector and scalar irradiances and the vector and scalar diffuse attenuation coefficients. This theory is inverted and combined with previously derived radiative transfer relations to show that in principle the vertical structure of the absorption, scattering, attenuation, and backscattering coefficients can be derived from the vertical structure of the scalar and vector irradiances and the nadir radiance. An example for the western North Atlantic Ocean is provided.

The vertical structure of the light in the sea is important to many disciplines. Sunlight is the energy source for the biological food chain, and the amount and spectrum of solar energy available at a given depth must be known if accurate productivity calculations are to be made. The oceanic biota in return are a major factor in determining the distribution of light through their absorption and scattering characteristics. Solar energy that is absorbed by the ocean plays a role in physical oceanography, in particular in the structure of the mixed layer. Optical remote sensing allows us to study the large-scale structure of biological and optical parameters by measuring radiance emanating from the ocean. Remote sensing is essential in investigating the global effects of stratospheric ozone depletion and the greenhouse effect.

The behavior of the light field in the sea is described by the equation of radiative transfer, which relates the light field and its derivative to the inherent optical properties via the beam attenuation coefficient and the volume scattering function. An analytical solution to this equation for a homogeneous medium was first given by Chandrasekhar (1950). Many other solutions have been given (*see* Prieur and Morel 1973; Zaneveld 1974; Jerlov 1976). Preisendorfer (1976) has provided an extensive treatise on the equation of radiative transfer and its applications to optical oceanography. Implementation of the analytical solution to the full equation of radiative transfer is cumbersome because it requires use of the full radiance distribution as well as the volume scattering function.

Purely numerical solutions based on Monte Carlo routines were developed early in the last decade (Plass and Kattawar 1972; Gordon et al. 1975) and have developed into powerful tools for the study of the forward problem, i.e. the derivation of the structure of the underwater light field as a function of the inherent optical properties. Another genre of numerical model, based on the deterministic solution of differential equations rather than on a probabilistic Monte Carlo solution, is now available (Mobley 1989). Either type of model can generate the equivalent of an experimental data set.

Approximate solutions based on the two-

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Significant symbols

a	Absorption coefficient, m^{-1}
b	Scattering coefficient, m^{-1}
b_b	Backscattering coefficient, m^{-1}
b_f	Forward scattering coefficient, m^{-1}
b_p	Particulate scattering coefficient, m^{-1}
β	Volume scattering function, m^{-1} sr^{-1}
c	Beam attenuation coefficient, m^{-1}
c_w	Beam attenuation coefficient of water, m^{-1}
E	Vector irradiance, W m^{-2}
E_0	Scalar irradiance, W m^{-2}
E_{0d}	Downwelling scalar irradiance, W m^{-2}
f_b	Shape factor for scattering
f_L	Shape factor for radiance
K	Diffuse attenuation coefficient, m^{-1}
K_a	Vertical structure coefficient for absorption, m^{-1}
K_E	Attenuation coefficient for vector irradiance, m^{-1}
K_K	Vertical structure coefficient for K_E, m^{-1}
$K_{\bar{\mu}}$	Vertical structure coefficient for average cosine, m^{-1}
K_0	Attenuation coefficient for scalar irradiance, m^{-1}
K_{0d}	Attenuation coefficient for downwelling scalar irradiance, m^{-1}
K_∞	Asymptotic diffuse attenuation coefficient, m^{-1}
L	Radiance, W m^{-2} sr^{-1}
λ	Wavelength, nm
$\bar{\mu}$	Average cosine
$\bar{\mu}_\infty$	Asymptotic average cosine
P	Asymptotic approach rate, m^{-1}
RSR	Remotely sensed reflectance, sr^{-1}
z	Depth, m

stream model of Schuster (1905) have been described in great detail by Preisendorfer (1976). The two-stream models are quite restrictive in their assumptions if realistic solutions for the oceans are to be obtained (Aas 1987; Stavn and Weidemann 1989).

The problem of inverting the vertical structure of the radiance field to obtain the inherent optical properties is also of considerable importance. It is not yet possible to measure the spectral inherent optical properties in the ocean routinely and accurately, although significant improvement in this regard is expected soon (Zaneveld et al. 1988). Even so, the determination of inherent optical properties by direct instrumentation uses only small measurement volumes while those with radiance and irradiance use very large volumes. Comparison of the two will give considerable insight into the contribution to the radiance distribution by large particles.

A theoretical solution of the inverse problem—the derivation of the inherent optical properties from the radiance and its derivative with depth—was first given by Zaneveld (1974) based on the work of Zaneveld and Pak (1972). Due to the unavailability of suitable radiance and scattering function data, the inversion has not yet been tested.

Much of the interest in two-stream solutions derives from their potential use to invert the vertical structures of the up- and downwelling irradiances to obtain the vertical structure of the inherent optical properties, especially the backscattering coefficient (Preisendorfer and Mobley 1984; Aas 1987; Stavn and Weidemann 1989). As mentioned above, these solutions require very restrictive assumptions in order to obtain general solutions. As shown here, a more general inverse solution for the vertical structure of the backscattering coefficient can be obtained from the inversion of an analytic solution for the remotely sensed reflectance (upwelling nadir radiance divided by the downwelling scalar irradiance) derived by Zaneveld (1982).

There continues to be a need to provide realistic models of the vertical structure of irradiance and upwelling radiance as a function of the inherent optical properties. Models that allow inversion of irradiance and radiance to obtain the inherent optical properties are particularly useful. In this paper I develop such a model based on the observation that the attenuation coefficients for irradiance eventually become constant. This is the so-called asymptotic regime in which the shape of the radiance distribution is constant and the magnitude of the radiances and irradiances all decrease at the same exponential rate.

A semiempirical proof of the existence of the asymptotic regime was given by Preisendorfer (1959). The final theoretical proof was provided by Højerslev and Zaneveld (1977). A review of other work regarding the asymptotic light field is given by Zaneveld (1974). Particularly useful is the analysis of Prieur and Morel (1971) which gives the relationship between the asymptotic attenuation coefficient, K_∞, and the inherent optical properties. K_∞ is an inherent optical property as it does not depend on the initial light field at the surface. The theory dictates that the shape of the light field eventually becomes a function of the inherent optical properties only. We thus see a gradual transformation of a surface-dependent light field (an apparent optical property) to one that is an inherent optical property. This phenomenon also shows that the diffuse attenuation coefficient K is not constant in a homogeneous medium. The rate at which K is transformed from its surface value to its asymptotic value has not been derived previously. This is an important rate because the asymptotic K as well as the shape of the asymptotic light field are inherent optical properties.

In this paper advantage is taken of the knowledge that the transformation to the asymptotic state exists. The rate at which this transformation occurs is then calculated and its dependence on the surface lighting conditions and the inherent optical properties is derived. When the rate of approach to the asymptotic state is known, the vertical structure of the irradiance and upwelling nadir radiance can be calculated, as well as the remotely sensed reflectance just beneath the surface. Once the light field is asymptotic its properties are inherent and, as will be shown, can be inverted to obtain the inherent optical properties, including the absorption, scattering, attenuation, and backscattering coefficients.

Relationships between the diffuse attenuation coefficients and the vertical structure coefficients for absorption and the average cosine

The equation of radiative transfer for a medium without internal sources, or any cross-wavelength effects, and for which the horizontal gradients are negligible compared to the vertical ones is given by:

$$\cos(\theta)\frac{dL(\theta,\phi,z)}{dz} = -cL(\theta,\phi,z) + \int_0^{2\pi}\int_0^{\pi} \beta(\theta,\theta',\phi,\phi')L(\theta',\phi',z)\sin(\theta')\,d\theta'\,d\phi' \quad (1)$$

where z is taken to be positive downward (units given in list of symbols). Integration over 4π sr gives

$$-\frac{dE(z)}{dz} = a(z)E_0(z). \quad (2)$$

This is Gershun's (1939) equation, which allows us to calculate $a(z)$ when we know the vertical structure of $E(z)$ and $E_0(z)$. The vector irradiance is defined as

$$E(z) = \int_0^{2\pi}\int_0^{\pi} L(\theta, \phi, z)\cos(\theta)\sin(\theta)\,d\theta\,d\phi, \quad (3)$$

and the scalar irradiance is defined as

$$E_0(z) = \int_0^{2\pi}\int_0^{\pi} L(\theta, \phi, z)\sin(\theta)\,d\theta\,d\phi. \quad (4)$$

The inherent optical properties are defined as follows: a is the absorption coefficient; b is the volume scattering coefficient, where

$$b = 2\pi\int_0^{\pi} \beta(\gamma)\sin(\gamma)\,d\gamma; \quad (5a)$$

c is the beam attenuation coefficient, and

$$c = b + a. \quad (5b)$$

We define the diffuse attenuation coefficients for vector and scalar irradiance by

$$K_E(z) = -\frac{d}{dz}\ln E(z) = -\frac{1}{E(z)}\frac{dE(z)}{dz}, \quad (6a)$$

so that

$$E(z) = E(0)\exp\left[-\int_0^{z} K_E(z)\,dz\right], \quad (6b)$$

and

$$K_0(z) = -\frac{d}{dz}\ln E_0(z) = \frac{-1}{E_0(z)}\frac{dE_0(z)}{dz}, \quad (7a)$$

so that

$$E_0(z) = E_0(0)\exp\left[-\int_0^z K_0(z)\,dz\right]. \quad (7b)$$

Substitution of Eq. 6a in 2 gives

$$K_E(z)E(z) = a(z)E_0(z). \quad (8)$$

Differentiation of Eq. 8 then yields

$$E(z)\frac{d}{dz}K_E(z) + K_E(z)\frac{dE(z)}{dz} = a(z)\frac{dE_0(z)}{dz} + E_0(z)\frac{da}{dz}. \quad (9)$$

We now divide the left-hand side by $K_E(z)E(z)$ and the right-hand side by $a(z)E_0(z)$. These factors are equal as shown by Eq. 8. We then get

$$\frac{1}{K_E(z)}\frac{dK_E(z)}{dz} + \frac{1}{E(z)}\frac{dE(z)}{dz} = \frac{1}{E_0(z)}\frac{dE_0(z)}{dz} + \frac{1}{a(z)}\frac{da(z)}{dz}. \quad (10)$$

We now define vertical structure coefficients similarly to the diffuse attenuation coefficients. Thus the vertical structure coefficient for the diffuse attenuation coefficient of vector irradiance is defined by

$$K_K(z) = -\frac{1}{K_E(z)}\frac{dK_E(z)}{dz}, \quad (11a)$$

so that

$$K_E(z) = K_E(0)\exp\left[-\int_0^z K_K(z)\,dz\right]. \quad (11b)$$

The vertical structure coefficient for the absorption coefficient is defined by

$$K_a(z) = -\frac{1}{a(z)}\frac{da(z)}{dz}, \quad (12a)$$

so that

$$a(z) = a(0)\exp\left[-\int_0^z K_a(z)\,dz\right]. \quad (12b)$$

Substitution of Eq. 6, 7, 11, and 12 into 10 gives

$$K_K(z) + K_E(z) = K_0(z) + K_a(z). \quad (13)$$

The average cosine used by Jerlov (1976) and others is given by

$$\bar{\mu}(z) = \frac{E(z)}{E_0(z)} = \frac{a(z)}{K_E(z)}. \quad (14)$$

We then define the vertical structure coefficient for the average cosine by

$$K_{\bar{\mu}}(z) = -\frac{1}{\bar{\mu}(z)}\frac{d\bar{\mu}(z)}{dz}. \quad (15)$$

Differentiation of Eq. 14 and a similar manipulation as that used to obtain Eq. 13 then yields

$$K_{\bar{\mu}}(z) = K_E(z) - K_0(z). \quad (16)$$

A combination of Eq. 13 and 16 gives the following important result:

$$K_{\bar{\mu}}(z) = K_a(z) - K_k(z) = K_E(z) - K_0(z). \quad (17)$$

These new relations constitute the differential form of Gershun's equation and are valid at any depth in any horizontally homogeneous but vertically inhomogeneous medium. The relationship for a homogeneous ocean, $-K_K(z) = K_E(z) - K_0(z)$, was reported earlier by Højerslev and Zaneveld (1977).

An interesting result from Eq. 17 is that, in principle, we should be able to obtain the vertical structure coefficient of the absorption coefficient from the vertical structure of $K_E(z)$, $K_0(z)$ and $K_K(z)$, i.e.

$$K_a(z) = K_E(z) - K_0(z) + K_K(z). \quad (18)$$

This is different from Eq. 8 and 2 in that no intercalibration of the vector and scalar irradiances is necessary. The term E/E_0 does not occur. Applying Eq. 12b and setting $a(0) = K_E(0)\bar{\mu}(0)$ allows us to calculate the vertical structure of $a(z)$. Only $\bar{\mu}(0)$ needs to be estimated.

The forward solution in a homogeneous medium using asymptotic closure

We wish to calculate the vertical structure of the apparent optical properties when the inherent optical properties are known. Højerslev and Zaneveld (1977) have proven that the diffuse attenuation coefficient $K_E(z)$ asymptotically reaches a constant value K_∞ at great depth. Such asymptotic behavior can be modeled by

$$K_E(z) = [K_E(0) - K_\infty]\exp(-Pz) + K_\infty. \quad (19)$$

This structure for $K_E(z)$ is supported by observations (Preisendorfer 1959). That is, the diffuse attenuation coefficient for vector irradiance changes rapidly at first and then asymptotically approaches K_∞ at great depth.

The rate at which $K_E(z)$ approaches the asymptotic value is governed by P. In a homogeneous medium the asymptotic rate parameter, P, is assumed to be constant. This allows the solution of the simplified radiative transfer equations presented here. The term "asymptotic closure" has been adopted to describe the present theory. We now need to derive an expression for the asymptotic approach rate based on the boundary conditions at the surface of the ocean.

Differentiation of Eq. 19 gives

$$\frac{dK_E(z)}{dz} = -P[K_E(0) - K_\infty]\exp(-Pz). \quad (20)$$

The vertical structure coefficient for the diffuse attenuation coefficient is then given by

$$K_k(z) = -\frac{1}{K_E(z)}\frac{dK_E(z)}{dz} = \frac{P[K_E(0) - K_\infty]\exp(-Pz)}{[K_E(0) - K_\infty]\exp(-Pz) + K_\infty} \quad (21a)$$

or

$$K_K(z) = \frac{P[K_E(z) - K_\infty]}{K_E(z)}. \quad (21b)$$

Solving for P yields

$$P = \frac{K_K(z)K_E(z)}{K_E(z) - K_\infty} \quad (22)$$

for homogeneous water. Alternately, by substitution of Eq. 18, we get

$$P = \frac{K_0(z) - K_E(z)}{1 - \dfrac{K_\infty}{K_E(z)}}. \quad (23)$$

This equation is correct at any depth in a homogeneous ocean. Setting $z = 0$ allows us to relate P to the boundary conditions at the surface. The rate at which the vector irradiance approaches the asymptotic value thus depends on the difference of the diffuse attenuation coefficients for scalar and vector irradiance at the surface, $K_0(0) - K_E(0)$, and the ratio of the asymptotic diffuse attenuation coefficient and the diffuse attenuation coefficient for vector irradiance at the surface, $K_\infty/K_E(0)$. Use of the boundary conditions allows us to calculate P, and substitution into Eq. 19 gives the vertical structure of $K_E(z)$ given K_∞ (the relationship between K_∞ and the inherent optical properties is discussed later). With the vertical structure of the vector irradiance attenuation coefficient in hand, we must now derive the vertical structure of the irradiances and the remotely sensed reflectance.

Using the definition of Eq. 6b, we can integrate the expression for $K_E(z)$ in Eq. 19 and obtain

$$E(z) = E(0)\exp x \quad (29)$$

where

$$x = -\{K_\infty z - \frac{1}{P}[\exp(-Pz)] \cdot [K_E(0) - K_\infty]\}. \quad (24)$$

We then calculate the scalar irradiance from

$$E_0(z) = \frac{K_E(z)E(z)}{a} = \frac{1}{a}\{[K_E(0) - K_\infty] \cdot \exp(-Pz) + K_\infty\}E(0)\exp x. \quad (25)$$

The average cosine is given by

$$\bar{\mu}(z) = \frac{a}{K_E(z)} = \frac{a}{[K_E(0) - K_\infty]\exp(-Pz) + K_\infty}. \quad (26a)$$

Rewriting Eq. 26a gives

$$\frac{1}{\bar{\mu}(z)} = \left[\frac{1}{\bar{\mu}(0)} - \frac{1}{\bar{\mu}_\infty}\right] \cdot \exp(-Pz) + \frac{1}{\bar{\mu}_\infty}. \quad (26b)$$

We thus see that the asymptotic approach rate describes the rate at which the inverse of the average cosine, also known as the distribution function, approaches its asymptotic value K_∞/a.

Zaneveld (1982) has derived a theoretical relationship of the vertical structure of the inherent optical properties and the remotely sensed reflectance (RSR) just beneath the surface of the ocean. He has shown that

$$\frac{L(\pi, 0)}{E_{0d}(0)} = \text{RSR}(0) = \int_0^\infty \frac{f_b(z')b_b(z')}{2\pi} \cdot \exp\left[-\int_0^{z'} c(z'') - f_L(z'')b_f(z'') + K_{0d}(z'')\,dz''\right] dz' \quad (27)$$

where $L(\pi, 0)$ is the nadir radiance at the surface, $E_{0d}(z)$ the downwelling scalar irradiance and $K_{0d}(z)$ the associated attenuation coefficient, f_b and f_L are shape factors for the scattering function and the radiance distribution, b_f is the forward scattering coefficient, and b_b the backscattering coefficient. Equation 27 is exact in any horizontally homogeneous, vertically stratified ocean. It was shown that the shape functions are close to unity for all oceanic conditions. Setting f_L and f_b equal to unity, and setting $c - b_f = a + b_b$, it can then be shown that in a homogeneous ocean the upwelling nadir radiance at the surface, $L(\pi, 0)$, is given by

$$L(\pi, 0) = \int_0^\infty \frac{b_b}{2\pi} E_0(z)\exp[-(a + b_b)z]\,dz. \quad (28)$$

The remotely sensed reflectance can be obtained by substituting Eq. 7b into 28:

$$\text{RSR}(0) = \frac{L(\pi, 0)}{E_0(0)} = \int_0^\infty \frac{b_b}{2\pi} \cdot \exp\left[-\int_0^z K_0(z') + a + b_b\,dz'\right] dz. \quad (29)$$

RSR(0) can then be obtained by numerical integration over depth, using the vertical structure of $K_0(z)$ derived in Eq. 24. Setting $E_{0d}(z) = E_0(z)$ entails only a small error as the reflectance is very small. An analytical solution also exists but is somewhat cumbersome. It is derived next. Substitution of Eq. 25 into 28 gives

$$L(\pi, 0) = \int_0^\infty b_b \frac{E(0)}{2\pi a} \{[K_E(0) - K_\infty]\exp(-Pz) + K_\infty\}\exp y\,dz \quad (30)$$

where

$$y = -\{(K_\infty + a + b_b)z - \left[\frac{1}{P}\exp(-Pz) - 1\right] \cdot [K_E(0) - K_\infty]\}.$$

Let $M = K_E(0) - K_\infty$ and $N = K_\infty + a + b_b$, so that

$$L(\pi, 0) = \frac{b_b}{2\pi a} E(0) \int_0^\infty M\exp(-M) \times \left\{\exp\left[-(N + P)z + \frac{M}{P}\exp(-Pz)\right]\right\} + K_\infty \exp(-M) \times \left\{\exp\left[-(Nz) + \frac{M}{P}\exp(-Pz)\right]\right\} dz. \quad (31)$$

Expansion of the terms in braces into power series and integration gives the desired result:

$$\frac{L(\pi, 0)}{E(0)} = \frac{b_b}{2\pi a}\left[\frac{M\exp(-M)}{P}\sum_{n=0}^{\infty}\frac{\left(\frac{M}{P}\right)^n}{n!\left(\frac{N+P}{P} + n\right)} + K_\infty \frac{\exp(-M)}{P}\sum_{n=0}^{\infty}\frac{\left(\frac{M}{P}\right)^n}{n!\left(\frac{N}{P} + n\right)}\right]. \quad (32)$$

In order to express the remotely sensed reflectance as the ratio of nadir radiance and scalar irradiance, the result must be multiplied by the average cosine:

$$\mathrm{RSR}(0) = \frac{L(\pi, 0)}{E_0(0)} = \frac{L(\pi, 0)}{E(0)} \bar{\mu}(0). \quad (33)$$

We have thus derived analytical expressions for RSR(0), $E(z)$, $E_0(z)$, $K(z)$, $K_E(z)$, and $\bar{\mu}(z)$ based only on boundary conditions and the inherent optical properties of the medium. These expressions form the theoretical basis for the transformation of the apparent optical properties at the surface, RSR(0), $E(0)$, $E_0(0)$, $K(0)$, $K_E(0)$, and $\bar{\mu}(0)$ to the inherent optical properties RSR_∞, K_∞, and $\bar{\mu}_\infty$ at asymptotic depths.

The inverse problem

Considerable effort has been expended recently (Preisendorfer and Mobley 1984; Aas 1987; Stavn and Weidemann 1989) in trying to invert two-flow models of irradiance to obtain the vertical structure of the backscattering coefficient. A problem with two-flow models is that they require severe restrictions in order to solve them. Preisendorfer and Mobley assumed that the backscattering coefficients for the upwelling and downwelling streams are the same. Stavn and Weidemann showed this to be incorrect for many more turbid oceanic cases. They in turn assumed that the ratio of the backscattering coefficients for the upwelling and downwelling streams is known in order to provide a solution. This assumption implies considerable knowledge of the nature of the water being studied.

A far less restrictive solution to the inverse problem can be based on the work of Zaneveld (1982) and the asymptotic closure model presented here. In the inversion model presented here, it is assumed that the vertical structure of the vector irradiance, $E(z)$, the scalar irradiance, $E_0(z)$, and the upwelling nadir radiance, $L(\pi, z)$, are known. If $E_0(z)$ and $E(z)$ are known, the vertical structure of $K_0(z)$ and $K_E(z)$ can then be calculated by means of Eq. 6a and 7a.

The absorption coefficient is obtained from Gershun's equation (Eq. 2):

$$a(z) = K_E(z) \frac{E(z)}{E_0(z)}. \quad (34)$$

This procedure has been used frequently in the past. A review is given by Jerlov (1976).

Similar to the irradiances, the vertical structure of the attenuation coefficient for nadir radiance can then be calculated from

$$K(\pi, z) = -\frac{1}{L(\pi, z)} \frac{dL(\pi, z)}{dz}. \quad (35)$$

We can then use Zaneveld's (1982) relation

$$\mathrm{RSR}(z) = \frac{L(\pi, z)}{E_{0d}(z)} = \frac{b_b(z)/2\pi}{K(\pi, z) + a(z) + b_b(z)} \quad (36)$$

to calculate $b_b(z)$:

$$b_b(z) = \frac{\mathrm{RSR}(z)[K(\pi, z) + a(z)]}{\frac{1}{2\pi} - \mathrm{RSR}(z)}. \quad (37)$$

If necessary, the remotely sensed reflectance

$$\mathrm{RSR}(z) = \frac{L(\pi, z)}{E_{0d}(z)} \quad (38)$$

can be approximated to within a few percent by

$$\mathrm{RSR}(z) = \frac{L(\pi, z)}{E_0(z)} \quad (39)$$

as the reflectivity for scalar irradiance is small.

It should be noted that Eq. 34 and 37 are not dependent on vertical homogeneity of the water column and can be applied whatever the vertical structure of the inherent optical properties.

The asymptotic closure theory presented here allows us, in principle, to calculate the scattering and beam attenuation coefficients in addition to the absorption and backscattering coefficients. In a homogeneous layer between $z = z_0$ and $z = z_2$, Eq. 19 can be applied in the form

$$K_E(z) = [K_E(z_0) - K_\infty]\exp(-Pz) + K_\infty. \quad (40)$$

The parameters P and K_∞ apply only to the layer (z_0, z_2). In other homogeneous layers different parameters P and K_∞ apply. We now measure $K_E(z_0)$, $K_E(z_2)$, and $K_E(z_1)$ where z_1 is halfway between z_0 and z_2 and then solve for K_∞:

$$K_\infty = \frac{K_E(z_0)K_E(z_2) - K_E(z_1)^2}{K_E(z_0) + K_E(z_2) - 2K_E(z_1)}. \quad (41)$$

We can thus, in theory, obtain K_∞ in a homogeneous layer even though $K_E(z)$ in that layer remains far from asymptotic.

Prieur and Morel (1971) and Timofeeva (1971) have derived relationships between K_∞ and b/c. If we use the observation that in the ocean particulate scattering is much larger than molecular scattering, the results of Prieur and Morel's theoretical calculations as well as Timofeeva's experimental results can be fit by the expression

$$\frac{K_\infty}{c} = 1 - 0.52\frac{b}{c} - 0.44\frac{b^2}{c^2}. \quad (42)$$

The average cosine for the asymptotic radiance distribution can be obtained by use of Eq. 14 and 42:

$$\bar{\mu}_\infty = \frac{a}{K_\infty} = \frac{1 - (b/c)}{1 - 0.52(b/c) - 0.44(b^2/c^2)}. \quad (43)$$

$\bar{\mu}_\infty$ is thus a function of b/c only.

If a has been calculated from Gershun's (1939) equation, we can then solve for b and c if $\bar{\mu}_\infty$ is known.

$$b = a\left[\frac{b/c}{1 - (b/c)}\right]. \quad (44)$$

In principle we have thus shown that the asymptotic closure theory allows us to calculate b and c in addition to a from Gershun's (1939) equation and b_b from Zaneveld's (1982) equation. The equations above can be used in a homogeneous layer even though K_E has not yet reached its asymptotic limit. It should be noted that there are practical limitations to this approach. The accurate measurement of apparent optical properties is a serious problem due to their very nature. They are readily influenced by a myriad of effects such as ship's shadow, varying cloud conditions, waves, orientation of the instrument platform, etc. Separation of the medium into homogeneous layers may also be difficult, although the vertical structure of $a(z)$ and $b_b(z)$ could be derived first and used as guidance. More research is needed to extend the asymptotic closure theory to inhomogeneous cases.

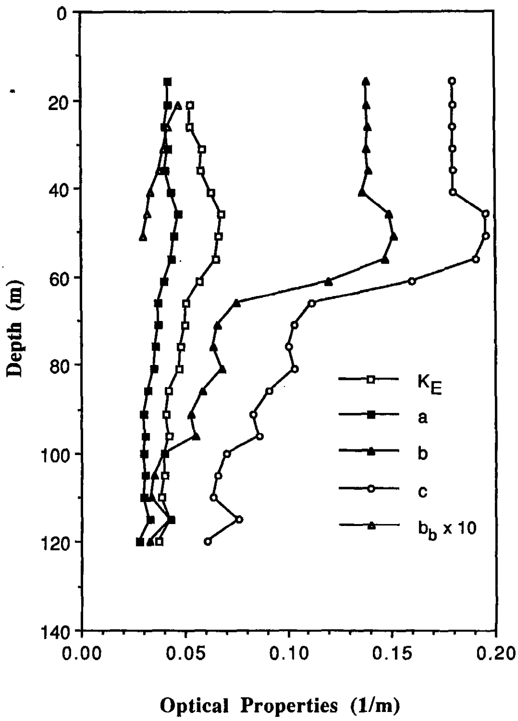

Fig. 1. Inherent and apparent optical properties derived from the vertical structure of vector and scalar irradiance and nadir radiance.

Sample analysis

There are few data sets which lend themselves readily to the analysis proposed above. What is needed is a numerically generated set of $E_0(z)$, $E(z)$, and $L(\pi, z)$ along with known values of a, b, b_b, etc. so that the inversion algorithms can really be tested. What follows here is an analysis of Biowatt 85 station 19-56. This data set has problems in that $E_0(z)$ was measured on a different instrument platform than $E(z)$ and $L(\pi, z)$. Furthermore, the depth gauges of the two instruments did not correspond. This was

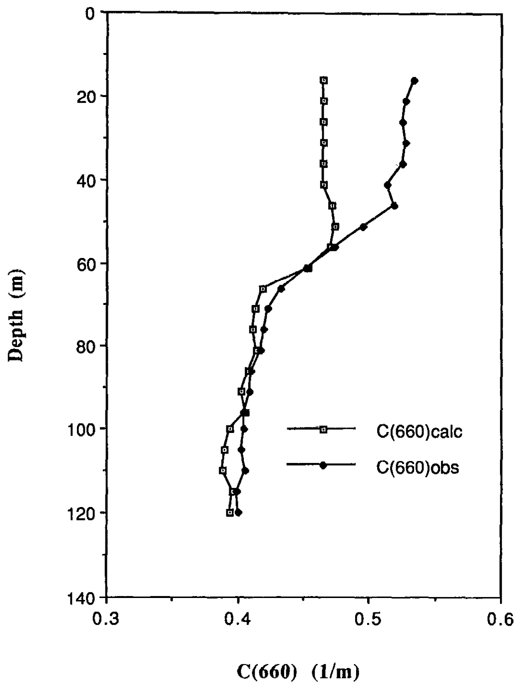

Fig. 2. A comparison of observed and calculated beam attenuation coefficients.

corrected by comparing beam attenuation data as taken with Sea Tech transmissometers. During occupation of this station clear skies prevailed, however ship drift and orientation may have biased the data. Nevertheless it seems to be the best set available. The analysis was carried out for $\lambda = 488$ nm as $L_u(z)$ penetrated the deepest at that wavelength. Figure 1 shows the vertical structure of $K_E(z)$, $a(z)$ as calculated from Eq. 34, and $b_b(z)$ calculated from Eq. 37. These calculations are relatively straightforward.

The absorption coefficient at 488 nm shows a nearly constant value to ~36 m depth, after which it climbs to a maximum at 46 m and then decreases to 81 m after which it is constant. The backscattering coefficient decreases steadily down to 50 m, after which the instrument is not sufficiently sensitive to detect any further changes.

Next, we wish to calculate b and c from the vertical structure of K_0 and K_E. The data are far too noisy for direct application of Eq. 41. However a slightly less accurate, but less sensitive approach can be used.

Equations 19 and 23 can be combined; we set

$$r = \frac{K_0(z_2) - K_E(z_2)}{K_0(z_1) - K_E(z_1)} \approx \frac{K_E(z_2) - K_\infty}{K_E(z_1) - K_\infty} \qquad (45)$$

where it is assumed that

$$K_\infty \gg [K_E(z) - K_\infty]\exp(-Pz).$$

K_∞ is then calculated from

$$K_\infty = \frac{K_E(z_2) - rK_E(z_1)}{1 - r}. \qquad (46)$$

This equation was applied to the depth interval 16–36 m. K_∞ was found to be 0.061 m^{-1}. We then calculate $\bar{\mu}_\infty = a/K_\infty = 0.68$. b/c can then be calculated from Eq. 43. In this case $b/c = 0.77$. Use of Eq. 23 allows us to calculate $P = 0.076$. With this value for P Eq. 19 can be used to show that $K_E(z)$ is within 5% of its asymptotic value for $z = 40$ m. Therefore below 40 m we will assume that $K_E = K_\infty$. K_E continues to change with depth, however, as $a(z)$ changes. As long as the absorption coefficient changes relatively slowly $K_E(z)$ will remain asymptotic. Therefore as long as changes in $a(z)$ are $\lesssim \exp(-Pz)$, we can set $K_E(z) = K_\infty(z)$. We then calculate b/c from $\bar{\mu}_\infty = a/K_\infty$ and Eq. 43.

Once $a(z)$ and $b(z)/c(z)$ have been calculated, $b(z)$ and $c(z)$ can be obtained from Eq. 44 and 5b. We can then also calculate $b_b(z)/b(z)$. Figure 1 shows the calculated a, b, c, and b_b at 488 nm. It is of interest to compare the calculated beam attenuation coefficient at 488 nm to the one measured with the Sea Tech transmissometer at 660 nm. We do this by assuming the particulate scattering coefficient, $b_p(\lambda)$, to vary approximately as λ^{-1}. We then obtain $c(660) \approx b_p(488) \times 488/660 + c_w(660)$, where $c_w(660)$ is the beam attenuation coefficient for pure water. Figure 2 shows the comparison between $c(660)$ calculated from the apparent optical properties and the measured $c(660)$.

Discussion and conclusions

The introduction of the vertical structure coefficients for the average cosine, the absorption coefficient, and the diffuse attenuation coefficient allows us to study the difference between the scalar and vector diffuse attenuation coefficients, which is of interest because this difference is more readily measured than the ratio of the vector and scalar irradiances since no intercalibration of the sensors is needed. The vertical structure coefficients play an important role in derivation of the asymptotic approach rate P. P must always be positive as can be seen from the following argument. If $K_E(0)$ is $<K_\infty$ in a homogeneous ocean, the denominator in Eq. 23 is negative. $K_E(z)$ must then increase and therefore $K_K(z)$ must be negative. P thus is positive. Similarly when $K_E(0)$ is $>K_\infty$, $K_K(z)$ must be positive and P again is positive.

In the theory presented here it is assumed that the boundary conditions $K_0(0)$ and $K_E(0)$ are known. For modeling purposes it will be necessary to derive the boundary conditions from first principles, i.e. express them in terms of the inherent optical properties and incoming radiance field. For inversion this is not necessary. The theory presented here needs to be extended to the case of a vertically inhomogeneous ocean. The emphasis in this paper has been on the inversion. I have shown that judicious use of the asymptotic closure theory allows us to calculate the vertical structure of the inherent optical properties. This is certainly reasonable once $K_E(z)$ is nearly asymptotic. The mixed layer being nearly homogeneous in structure allowed us to calculate the inherent optical properties there also. The theory is at present not well suited to the normally extremely noisy apparent optical properties data. To obtain a reasonably accurate value for the asymptotic approach rate and K_∞ requires that the homogeneous layer be quite thick, which may not always be the case.

The measurement of inherent optical properties is not subject to the various environmental perturbations that naturally affect the apparent properties, and so they are potentially far less noisy. They also can be measured at any time of the day or night. As the capability of measuring the inherent optical properties routinely and accurately develops, it is thus useful to develop theories that readily allow the subsequent calculation of the apparent optical properties.

I have demonstrated that in addition to $a(z)$, $b_b(z)$ can routinely be determined provided that the upwelling nadir radiance is measured. It would be useful to increase the sensitivity of radiometers so that the backscattering coefficient can be deduced to greater depths. Values for b_b/b determined here varied from 3.4 to 2%. The lower backscattering ratios were found in a region of increased a, b, and fluorescence, indicative of simultaneous increases in phytoplankton and pigment concentrations. This corresponds well with Morel and Bricaud's (1981) observation that backscattering in pure phytoplankton cultures is small.

The comparison between the calculated attenuation coefficient at 660 nm and the measured one is excellent considering that two different instrument platforms were used in deriving the former, as well as a λ^{-1} scattering dependence. This type of analysis constitutes a form of optical closure in that the absorption and scattering coefficients were calculated from the vector and scalar irradiances and the resultant beam attenuation coefficient was compared with an entirely independent measurement. The procedure shows that the inversion may be useful to compare large-volume inherent properties derived from apparent optical properties with small-volume inherent optical properties. In that case measurements must be made at the same wavelengths, and the scalar and vector irradiances must be measured from the same platform and be properly intercalibrated.

In conclusion it has been shown that the asymptotic closure theory for irradiance is useful for understanding the vertical structure of irradiance in the sea as well as inversion of that structure to obtain the inherent optical properties.

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