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An augmented risk-based paradigm for structural health monitoring

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Abstract

The notion of risk is comprised of *two* components: the likelihood of an adverse event occurring, and the severity of the consequences. Probabilistic risk assessment (PRA) is an established methodology for quantifying risks, used by engineers in a range of industries to inform decisions regarding the design and operation of safety-critical or high-value structures and systems.

In comparison, a salient motivation for implementing a structural health monitoring (SHM) system is to facilitate the decision-making process throughout the lifetime of a structure. Oftentimes, there is uncertainty when assessing the damage state of a structure. As such, a statistical pattern recognition (SPR) approach to structural health monitoring is employed in which data acquired from a structure of interest are processed to yield features indicative of the damage state. The current paper details how decision-making under uncertainty can be aided by augmenting the established structural health monitoring paradigm to incorporate risk, utilising a framework based on probabilistic graphical models.

The modelling of failure events as fault trees is a core process in conducting a PRA and, by modelling key failure modes of interest for a given structure in this way, provides a convenient and rigorous basis for formulating risk-based SHM problems. As statements in Boolean logic, fault trees are limited to representing binary damage states. Fortunately, it is possible to map fault trees into Bayesian networks which are capable of representing multi-state variables whilst also affording other benefits.

Risk is incorporated into the framework by introducing utility nodes into the probabilistic graphical model, thereby attributing costs to failure events. Decision nodes are also included, enabling the evaluation of potential courses of action such that a strategy that maximises utility may be determined.

Keywords: Structural health monitoring; probabilistic risk assessment; probabilistic graphical models

1 Introduction

Probabilistic Risk Assessment (PRA) is a method for identifying potential adverse events and quantifying the associated likelihood of occurrence and the severity of the consequences. For decades, PRA has been employed in a range of industries, including nuclear and aerospace, to inform decision-making regarding the design and operation of safety critical systems and structures, such as nuclear power plants [1] and reusable space vehicles, like the space shuttle [2].

In comparison, *Structural Health Monitoring* (SHM) is a field of engineering concerned with the detection of damage within aerospace, civil or mechanical infrastructure via continual acquisition and online processing of data [3]. A primary incentive for the development and implementation of SHM systems is to gain the ability to make informed decisions regarding the operation and management of structures so as to improve safety and/or reduce costs. A decision-making agent in the context of SHM is required to specify action policies whilst being robust to uncertainties that arise due to having imperfect information regarding the damage state of a structure. Additionally, structures can be complex and a decision maker may have to take into account multiple failure modes and/or manage a population of structures simultaneously. The problem of decision-making for SHM is highly involved and therefore demands a rigorous and structured approach.

Thus far, the majority of research in the field of SHM has been focussed on the identification, localisation and classification of damage. There have been fewer attempts to address decision-making processes and to incorporate risk into SHM problems. Flynn and Todd successfully applied a Bayes risk approach to the decision problem of sensor placement for an SHM system on square, gusset and T-shaped plates in [4]. The approach considered the risk of false positives and false negatives of damage identification in discrete regions of the plates. An approach proposed in [5], facilitates cost-efficient reliability-based maintenance. As it is a reliability-based approach rather than a risk-based approach, the costs of failure events and maintenance are not explicitly modelled. Hence, whilst the maintenance strategies developed may be cost-efficient for given safety parameters, they are not necessarily cost-optimal. There has been some research into the risk-based operation and maintenance of structures and components. Nielsen details a risk-based approach that is utilised for the operation and maintenance of off-shore wind turbines in [6], using probabilistic graphical modelling. Similarly, Hovgaard and Brincker provide a case study demonstrating a risk-based approach to the monitoring and maintenance of a finite element model of a wind turbine tower experiencing circumferential cracking in [7]. In [8] a continuous-state partially observable Markov decision process (POMDP) was demonstrated on artificial data for maintenance planning on a deteriorating bridge.

The current paper aims to address the lack of a generalised framework for conducting risk-based monitoring of structures at the full-system scale by augmenting the current SHM paradigm with practices employed in probabilistic risk assessment and thereby facilitating the decision-making processes that motivate the implementation of SHM systems. An overview is given of the current paradigms for conducting PRA and SHM as outlined in the literature. Also provided is background theory regarding the key technologies required for mapping PRA onto SHM; namely, probabilistic graphical models in the form of Bayesian networks and influence diagrams. A notation is established before the augmented risk-based paradigm for SHM is detailed. Finally, a discussion around the framework is made and further challenges in the SHM decision-process are identified.

2 Current structural health monitoring paradigm

Structural health monitoring is a field of engineering that is concerned with determining the health state of a structure over time via a damage identification strategy. Due to the uncertainties that arise when dealing with problems in engineering, statistical pattern recognition (SPR) is a natural approach to SHM that has been the focus of much research over the past three decades. The established SPR paradigm for an SHM system is composed of four procedures [3]:

1. Operational Evaluation.
2. Data acquisition.
3. Feature selection.
4. Statistical modelling for feature discrimination.

2.1 Operational evaluation

The operational evaluation process seeks to answer questions regarding the implementation of an SHM system, specifically:

- What is the justification (safety and/or economic) for implementing an SHM system?
- How is damage defined for the system and what are the critical damage states?
- What are the environmental and operational conditions that the monitoring system is required to perform under?
- How does the operational environment limit data acquisition?

A large amount of information is required for the operational evaluation process, and significant effort may be necessary to obtain a sufficient amount. Examples of required information include: the financial cost and reliability of the proposed SHM system; possible damage states of the structure and thresholds at which they can be considered to have occurred; and the temperature variation experienced by the structure.

2.2 Data acquisition

The data acquisition process is informed by the operational evaluation. The process aims to finalise the types, number and locations of sensors to be used in the SHM system. The data acquisition, storage and transmittal hardware must also be selected. The process is constrained by both economic restrictions and the limitations enforced by the expected environmental conditions; for this reason, the data acquisition process is context dependent and relies heavily on the information gathered during the operational evaluation stage.

2.3 Feature selection

Once the data have been acquired, a set of features must be constructed that indicate whether or not there is damage present in the structure. This procedure often involves processing the data acquired from the structure; common practices include domain transformation, dimensionality reduction, and normalisation.

2.4 Statistical modelling for feature discrimination

Statistical models must be developed to exploit the discrepancy between features that indicate differing damage states. The degree of knowledge regarding the damage state obtained from an SHM system is highly dependent on the statistical model employed and can be evaluated in terms of Rytter's Hierarchy [9]:

1. Is there damage in the system?
2. Where is the damage located?
3. What type of damage is present?
4. How severe is the damage?
5. How much useful life remains?

Whilst Rytter's hierarchy, in itself, does not lead to decisions being made, as SHM systems progress up the hierarchy, the information they yield becomes increasing useful to agents tasked with deciding upon a course of action for a structure. Within the field of decision theory, cost and utility are metrics used ubiquitously for the comparison of courses of action and their consequences. By combining the cost/utility of a given consequence with the respective likelihood, one can arrive at the notion of *risk*.

3 Probabilistic risk assessment paradigm

Probabilistic risk assessment (PRA) is a method that is widely used for evaluating risks and making decisions associated with the design and management of safety-critical systems and high-value assets. In the context of PRA, risk is characterised by the likelihood of an adverse event occurring and the severity of the consequences of the event. The likelihood of occurrence for uncertain adverse events is quantified through probabilistic event sequence and system modelling. The consequences and expected costs/gains are compared and evaluated by finding an appropriate utility metric - obvious examples include financial cost and loss of human life; however, in many applications these are overly simplistic [10]. Probabilistic risk assessment is applied in a range of industries including nuclear, aerospace and chemical process. Whilst the exact methodology used for conducting PRA differs between industries, they generally adhere to the key steps as outlined by the US Nuclear Regulatory Commission (USNRC) and the International Atomic Energy Agency (IAEA) [1]:

1. Initial information collection.
2. Event-tree development.
3. System modelling.
4. Reliability modelling.

5. Failure sequence quantification.
6. Consequence analysis.

Further detail is provided for each step in the following subsections.

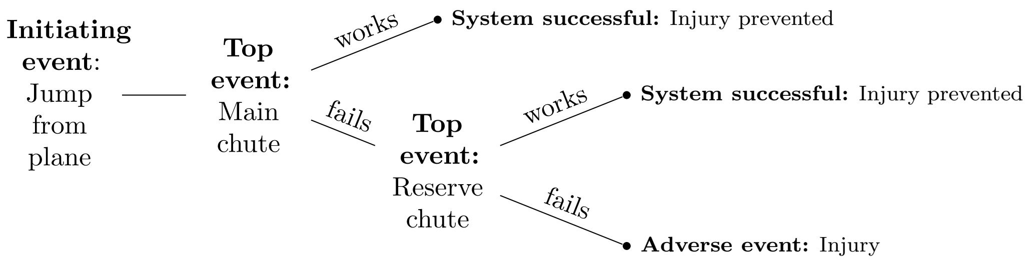

Figure 1: An example event tree for a parachute system to prevent fall injuries [11].

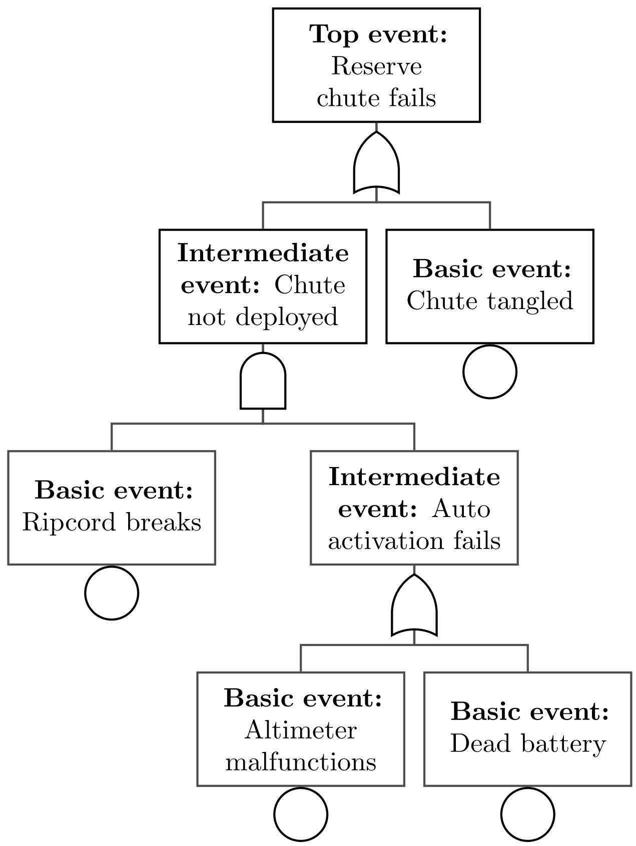

Figure 2: An example fault tree for a reserve parachute system [11].

3.1 Initial information collection

Information regarding the design and operation of the structure in question is collated. Details such as component specifications, loading and environmental conditions are considered. The information gathered at this stage is used to inform the subsequent steps. Given the large quantity of information required for conducting PRA, an important factor to be considered at this stage is the method by which the necessary information is represented, stored and managed. A common practice is to utilise a database [10].

3.2 Event-tree development

Event trees outline potential accident sequences - combinations of initiating events and the subsequent system failures or successes that may result in an adverse consequence. The sequences of system failures and successes are known as *top events*. The system failures identified in the event-tree development stage are subsequently modelled as fault trees. An example event tree for a system designed to prevent injury following a jump from a plane is shown in Figure 1.

3.3 Fault tree development

Fault trees are used in PRA to facilitate the quantification of system failure probabilities. The development of fault trees involves expressing the causal relationships between component failures and subsystem failures using Boolean logic gates. The level of detail captured in the fault tree (the level of components which are incorporated) is determined by the component level for which meaningful reliability data can be obtained. An example fault tree for the deployment of a reserve parachute is shown in Figure 2.

3.4 Reliability modelling

Information regarding the reliability of system components and the frequency of initiating events is necessary to quantify the probabilities of top events. The is typically gleaned from data and applying appropriate reliability models.

3.5 Failure sequence quantification

By assigned the components in the fault trees with failure rates, the probability of top events may be computed. Propagating the probabilities of the initiating events and top events through the event tree allows the probability of each possible outcome in the event tree to be calculated.

3.6 Consequence analysis

With the probability of an adverse event quantified, a cost/utility metric must be chosen so that the risk associated with the failure sequence may be assessed. The risk assessment may then be used to inform design decisions, such as increasing safety by introducing additional redundancies in the system, or optimising cost by removing components that do not cause the risk to fall below an acceptable threshold. The risk assessment may also be used to inform risk-based inspection for a system that is in operation.

4 Probabilistic graphical models

Probabilistic graphical models (PGMs) are a powerful framework for reasoning and decision-making under uncertainty - a core problem in SHM. Probabilistic graphical models are representations of joint probability distributions, in which nodes denote a set of random variables and edges connecting nodes imply dependency between variables. The probabilistic graphical model representation provides benefits over a flat (non-graphical) representation [12]:

- They provide a compact and intuitive representation of complex probability distributions which makes them easier to understand, communicate and learn.
- They facilitate efficient computation by exploiting local independence structures.

A PGM over a set of N variables X may be specified by a set of M local functions $f(Y_i)$, where Y_i is some subset of X, and a graph G comprised of nodes/vertices V and edges E. The joint probability distribution represented by the graph is obtained by:

$$P(X_1, X_2, \ldots, X_N) = K \prod_{i=1}^{M} f(Y_i) \tag{1}$$

where K is a normalisation factor ensuring the probabilities sum to unity.

There are two classes of problem associated with PGMs: *inference* and *learning*. Inference is concerned with obtaining the marginal or conditional probabilities of a subset of variables Z given any other subset Y. i.e. $P(Z|Y)$. Learning is concerned with obtaining the graph structure and parameters given a complete, or incomplete, set of observed data values for X. i.e. $G, f(Y_i)|X$. The remainder of the current paper will be primarily concerned with inference problems and their application in a risk-informed SHM framework.

4.1 Bayesian networks

Bayesian networks (BNs) are a form of PGM. Specifically, they are directed acyclic graphs (DAGs) in which nodes represent random variables and edges connecting nodes represent conditional dependencies between variables. For discrete random variables, the local functions that describe the conditional probability distributions (CPDs) between variables are conditional probability tables (CPTs), and in the case of continuous random variables are conditional probability density functions (CPDFs).

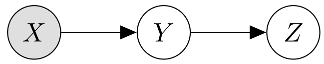

Figure 3: An example Bayesian network.

Figure 3 shows a simple Bayesian network where X is a parent of Y and an ancestor of Z; Z is said to be the child of Y and a descendant of X. Node X is independent of of other nodes and so is specified by the unconditional distribution $P(X)$. Observed variables are shaded grey.

Given observations on a subset of nodes in a BN, inference algorithms can be applied to obtain posterior distributions over the unobserved random variables. In some cases, analytical solutions of posterior distributions may be found by using exact inference methods. To solve the inference problem using direct computation from the joint probability distribution, the computational complexity increases exponentially with the size of the graph and quickly becomes intractable. Fortunately, algorithms have been developed that allow efficient computation [13].

4.2 Influence diagrams

For the purposes of decision-making, discrete Bayesian networks can be augmented with nodes representing decision variables and utility functions - these augmented networks are known as *influence diagrams*. Decision nodes are denoted by squares and utility nodes are denoted by rhombi, as shown in Figure 4. Edges connecting random variables to utility nodes denote that the utility function is dependent on the state of that variable. Similarly, edges connecting decision nodes to utility nodes denote that the utility function has a dependence on the action decided upon. Edges connecting random variables or decision nodes to other decision nodes denote order, that is to say the random variable or decision is observed prior to the decision being made; such edges are referred to as *informational links* as they do not imply a functional dependence but rather that the information regarding the state of the variable is required for the decision to be made.

The influence diagram shown in Figure 4 can be interpreted as a binary decision process regarding whether to go out for a walk or stay in and watch TV under uncertainty in the future weather condition W_c given an observed weather forecast W_f. The nodes W_f and W_c can be considered as binary random variables representing the weather forecast and actual weather condition, respectively, with possible states $domain(W_f) = domain(W_c) = \{bad, good\}$ and the weather forecast is dependent on the weather condition. The possible actions can be summarised as $domain(D) = \{TV, walk\}$. The utility U achieved is then dependent on both the weather condition experienced and the decided action.

In general, a policy δ is a mapping from all possible observations to possible actions. The problem of inference in influence diagrams is to determine an optimal strategy $\mathbf{\Delta}^* = \{\delta_1^*, \ldots, \delta_n^*\}$ given a set of observations on random variables where δ_i^* is the policy for the i^{th} decision to be made in a strategy $\mathbf{\Delta}^*$ that yields the *maximum expected utility* (MEU). The expected utility is a function of probability and utility; and by this definition is equivalent to risk.

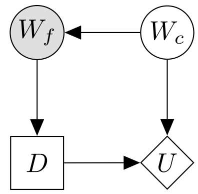

Figure 4: An example influence diagram representing the decision of whether to go outside or stay in under uncertainty in the future weather condition given an observed forecast.

5 Definitions

To establish a framework for mapping PRA onto SHM, some fundamental concepts will first be defined. In addition, a notation will be established for describing structures that can be expressed as hierarchical graphs.

One must start with a physical structure or system of interest $\boldsymbol{S}$. It is assumed that $\boldsymbol{S}$ may be defined in terms of constituent units; components c, joints j and substructures $\boldsymbol{s}$. Components and joints are considered irreducible base units of $\boldsymbol{S}$ whereas substructures are compound units and may be comprised of joints, components and other substructures.

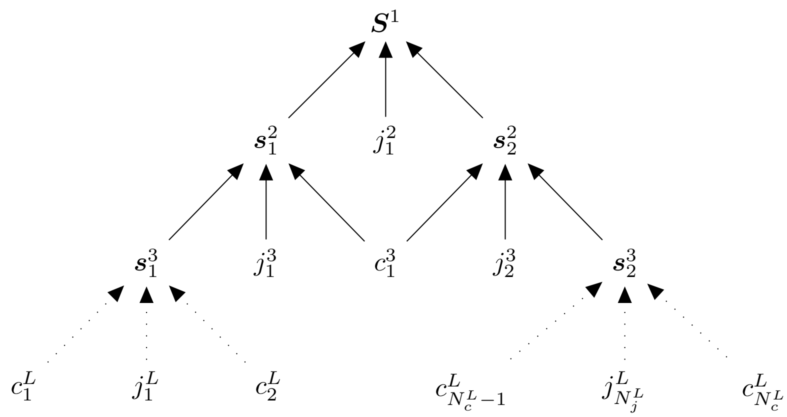

Figure 5: A hierarchical graphical representation of a generic structure $\boldsymbol{S}$. Superscripts denote the level in the hierarchy and subscript indexes each type of constituent unit in a given level. Dotted edges imply an arbitrary structuring between levels.

Figure 5 depicts a graphical representation of a hierarchical structure that may be considered without loss of generality. Nodes represent the global structure and its constituent units and edges represent the dependence of a (sub)structure on its constituent units. At the top, or level 1, of the hierarchy is the global structure with the hierarchy level denoted in the superscript. It can be seen that the global structure $\boldsymbol{S}^1$ is comprised of two substructures $\boldsymbol{s}_1^2$, $\boldsymbol{s}_2^2$ and a joint j_1^2, i.e. $\boldsymbol{S}^1 = \left\{\boldsymbol{s}_1^2, j_1^2, \boldsymbol{s}_2^2\right\}$. These units form the second level of the hierarchy. $\boldsymbol{s}_1^2$ and $\boldsymbol{s}_2^2$ may in turn be expanded to yield $\boldsymbol{S}^1 = \left\{\{\boldsymbol{s}_1^3, j_1^3, c_1^3\}, j_1^2, \{c_1^3, j_2^3, \boldsymbol{s}_2^3\}\right\}$. Progressing down the hierarchy levels, one can continue to expand the substructures into constituent units until the L^{th} level of the hierarchy which is comprised solely of base units. By taking the expansion of $\boldsymbol{S}^1$ into its constituent base units and discarding the repeated units arising from substructures that share components, one obtains a list of the base units that form a given structure $\boldsymbol{S}$. Within each level of the hierarchy, units are numbered via a subscript from 1 to N_u^i where N_u^i is the number of a constituent unit type u in the i^{th} level of the hierarchy. The notation u_n^i where i is an integer from 1 to L and n is an integer from 1 to N_u^i provides a unique identifier for each unit within a structure.

It is assumed that there exists a set of features $\boldsymbol{\nu}$, observable from $\boldsymbol{S}$, that are produced according to a generative latent state model with latent state $\boldsymbol{H}^*(t)$, where $\boldsymbol{H}^*(t)$ is the true health state of $\boldsymbol{S}$ and may be expressed in terms of the true health states of the constituent components and joints $h^*c_n^i(t)$ and $h^*j_n^i(t)$, respectively, i.e. $\boldsymbol{H}^*(t) = \left\{h^*c_1^2(t), \dots, h^*c_{N_c^L}^L(t), h^*j_1^2(t), \dots, h^*j_{N_j^L}^L(t)\right\}$.

The structure $\boldsymbol{S}$ also has a predicted time-dependent health state vector $\boldsymbol{H}(t) = \big\{hc_1^2(t), \dots, hc_{N_c^L}^L(t), hj_1^2(t), \dots,$

$h^{j^L_{N^L_j}}(t)\big\}$. Health-state vectors can be constructed from any subset of components and joints.

For the structure/system $\boldsymbol{S}$, there must exist a set of failure modes of interest $\boldsymbol{F} = \big\{F_1, \ldots, F_{N_F}\big\}$ whereby $\boldsymbol{S}$ ceases to be fit for purpose. It is assumed that a given failure mode is dependent on the health states of a subset of components, joints and substructures for which a health-state vector can be constructed. In addition, each failure state has an associated utility U_{F_n}.

Finally, for the structure $\boldsymbol{S}$, there also exists a set of decisions $\boldsymbol{d} = \big\{d_1, \ldots, d_{N_d}\big\}$ which affect $\boldsymbol{H}^*$, each having an associated utility U_{d_i}. In addition, there will exist some set of environmental conditions $\boldsymbol{e} = \big\{e_1(t), \ldots, e_{n_e}(t)\big\}$ that will alter the distribution of $\boldsymbol{\nu}$.

6 Mapping PRA onto SHM

Upon examination, it becomes apparent that there are both differences and similarities between the paradigms for SHM and PRA that can be examined to determine which aspects of PRA will be useful for SHM. Whilst it is clear that both SHM and PRA are utilised for the purpose of making decisions in the face of uncertainty, PRA is conducted offline for a system experiencing a set of anticipated initiating events. In contrast, the decision processes for which SHM is implemented are online and require continual predictions of the damage state of the structure. It is for this reason that the event-tree development stages and failure sequence quantification stages in PRA are less applicable to SHM.

Both paradigms begin with collating information regarding the structure and defining the context in which decisions are to be made. In fact, the first three stages of the PRA paradigm involve expressing the structure and context in a logical way which facilitates the quantification of risk and the decision-making process. It is in this formal expression of the structure that the decision-making process in the SHM paradigm stands to benefit. An overview of the risk-based SHM paradigm is as follows:

1. Operational evaluation.
2. Failure-mode modelling.
3. Decision modelling.
4. Data acquisition.
5. Feature selection.
6. Statistical modelling for feature discrimination.

6.1 Operational evaluation

With the aim of justifying the use and defining the context of a risk-based SHM system, the operational evaluation stage seeks to answer many of the same questions as in the standard paradigm. However, some questions require an approach that facilitates the failure-mode modelling and decision-modelling stages.

Foremost, information regarding the components $\boldsymbol{c}$, joints $\boldsymbol{j}$, substructures $\boldsymbol{s}$, and the dependencies between them is required.

When identifying the critical damage states of the structure $\boldsymbol{S}$, one should aim to identify the failure modes of interest $\boldsymbol{F}$. Critical components, joints and substructures/subsystems that contribute to $\boldsymbol{F}$ should also be identified at this stage. The predicted damage states of these components h should be defined. The damage states of the critical substructures/subsystems $\boldsymbol{H}$ should be defined as a vector in terms of h.

For each failure mode in $\boldsymbol{F}$, potential decisions $\boldsymbol{d}$ should be identified and the ways in which the actions influence the structure or likelihood of failure modes occurring should be determined. Utility values $\boldsymbol{U}_F$ and $\boldsymbol{U}_d$ for all $\boldsymbol{F}$ and all $\boldsymbol{d}$, respectively, should be quantified. The selection of utility values will determine the behaviour of the decision-making agent, and is analogous to setting a decision threshold in a standard SHM paradigm.

Environmental influences $\boldsymbol{e}$ should also be identified. It should also be decided whether the SHM system is to evaluate the health of the structure at static, independent instances in time, or predict future health states, thereby requiring a model forecasting the degradation of the structure.

For large, complex structures it may be beneficial to borrow the data management techniques used in PRA, such as databases, to organise the information obtained during the Operational Evaluation stage. This will allow for a rigorous and structured approach to the information collection and allow for the identification of aspects of the SHM system that require further specification or more information. Having a formal information structure will also expedite the subsequent failure-mode modelling step which requires detailed knowledge of the physical structure.

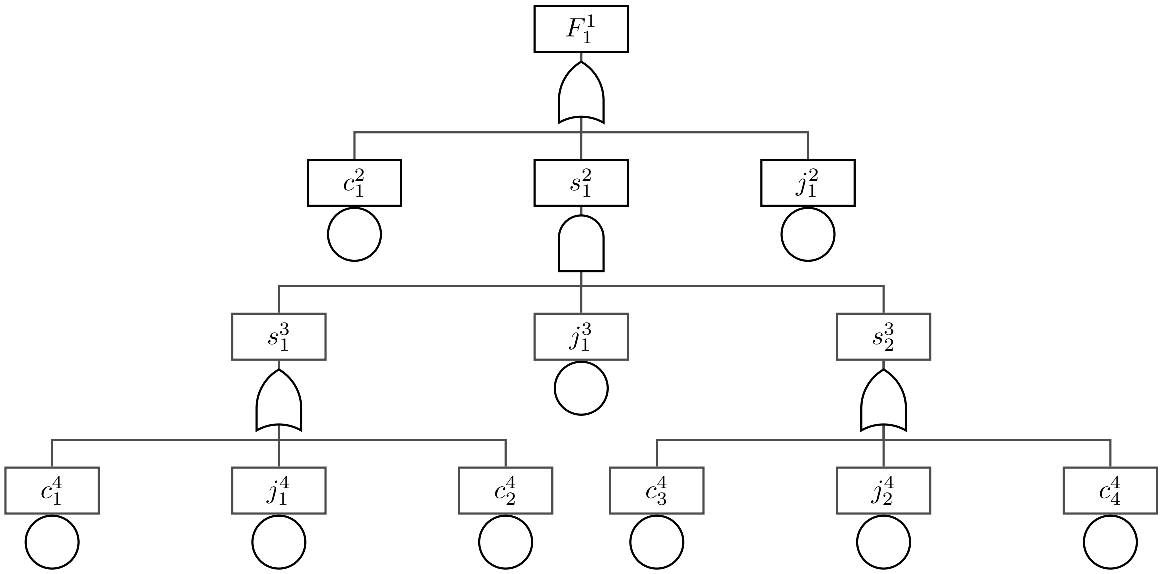

Figure 6: A fault tree of a failure mode F_1 where the superscript denotes the hierarchy level and the subscript is an identifier.

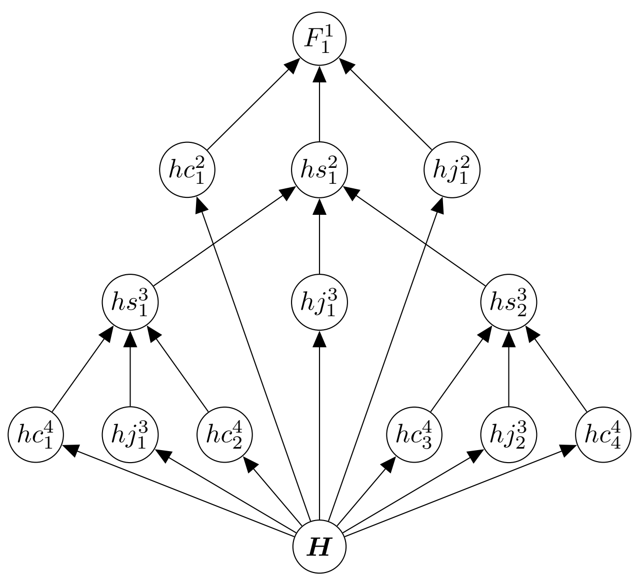

Figure 7: A Bayesian network of failure mode in F_1.

6.2 Failure-mode modelling

For each of the failure modes of interest in $\boldsymbol{F}$, one should proceed to construct a fault tree, such as that shown in Figure 6, based upon the health states of the relevant components, joints and substructures/subsystems. It should be noted here that, in many cases, the exact nature of the failure modes will be unknown and so a best estimate based on engineering judgement may be used.

Fault trees offer a rigorous and consistent structure for expressing the failure modes; however, as statements in Boolean logic they are limited in their flexibility. In the context of SHM, it is desirable to represent the components in a fault tree as having multiple damage states, and it is for this reason that one should map the constructed fault trees into Bayesian networks. Bobbio *et al* outline a convenient mapping from fault trees into Bayesian networks in [14], whilst also highlighting the additional flexibility that is granted by doing so. Additionally, Bayesian networks are used to represent structural failures in [15].

In the example shown in Figure 7, the component health states, substructure health states and failure event are represented as random variables where the substructure health states are conditioned on the component health states and the failure events is conditioned on the substructure health state. The random variables are defined using a conditional probability distribution (CPD) which may be discrete or continuous.

A node representing the health-state vector of the critical components and joints $\boldsymbol{H}$ should be included in the fault tree, as this latent state will be predicted during the statistical modelling process. To define the vector $\boldsymbol{H}$ within the Bayesian network, the conditional dependence between the nodes representing the local health states of the components and joints and $\boldsymbol{H}$ are expressed as a binary logic table.

One function of the failure-mode Bayesian network is to allow the flow of information from the statistical model to the decision, whilst parsing the information in a way that facilitates the defining of the failure events $\boldsymbol{F}$. The network also allows the computation of marginal distributions for the probability of failure in each component, joint, or substructure allowing for damage localisation.

6.3 Decision modelling

Modelling the decision process involves augmenting the Bayesian network developed in the failure modelling stage with nodes for each decision in $\boldsymbol{d}$ and for utilities $\boldsymbol{U}_F$ and $\boldsymbol{U}_d$ to produce an influence diagram. Decision nodes in which the actions alter the probability of a state or event should modify the CPDs accordingly. Utility nodes are constrained to be leaf nodes and should be dependent on the appropriate failure events or decisions.

For static problems, it may be convenient to model the decision process in a separate influence diagram which receives information regarding failure probabilities from the fault tree. This issue is because it is implicit that the decision is made after observations are made; if one attempts to solve a network in which a decision is made that yields a state that is inconsistent with the observed state, a conflict arises.

6.4 Data acquisition

The data acquisition process should not differ from that in the standard SHM paradigm. Here, there is a subtlety that the data acquisition system should be designed so as to optimise the decision-making rather than damage identification.

6.5 Feature selection

The feature selection process should not differ from that in the standard SHM paradigm. Again, there is the subtlety that the features should be selected so as to optimise the decision-making.

6.6 Statistical modelling

The purpose of the statistical model is to predict the critical health states $\boldsymbol{H}$ given the selected feature set $\boldsymbol{\nu}$. As aforementioned, it is assumed that $\boldsymbol{x}$ is produced through a generative latent state model, with latent-state $\boldsymbol{H}$. Probabilistic classifiers that output a probability distribution over all possible states of $\boldsymbol{H}$, such as relevance vector machines (RVMs) and Gaussian mixture models (GMMs), are compatible. Probabilistic classifiers are instrumental in building robustness to the uncertainty surrounding the true health state of $\boldsymbol{S}$ into the decision process. Ideally, the chosen statistical model will be capable of consistently identifying the actual health state under all identified operating and environmental conditions $\boldsymbol{e}$, or at least appropriately reflect the uncertainty caused by varying conditions in the prediction.

Finally, if a model describing the degradation of $\boldsymbol{S}$ (i.e. a transition model for $\boldsymbol{H}$) is required for forecasting failure events in the time-dependent case, the CPDs defining $P(\boldsymbol{H}_t|\boldsymbol{H}_{t-1}, \boldsymbol{d})$ should be specified accordingly.

7 Discussion

The framework described provides an approach to conducting risk-based SHM that incorporates useful stages of the PRA procedure into the SHM paradigm. Decision-making is facilitated through the inclusion of risk, thereby allowing for the comparison of actions and the selection of one that maximises expected utility.

By considering the nature of the decision problems associated with PRA and SHM, inapplicable aspects of the PRA procedure were identified. Specifically, due to fact that the decision process for SHM are required to be online and continual, the event-tree development, reliability modelling and failure sequence quantification stages were deemed unnecessary.

On the other hand, PRA provided a numbers of methods that facilitate the implementation of SHM systems capable of informed decision-making. Firstly, formalising the operational evaluation procedure by organising the information specifying the structure and monitoring system in a database will assist with ensuring all the necessary information required for subsequent stages is acquired and it will provide a structured method for the retrieval of applicable information at each stage. The fault tree development process of PRA provides the key novelty of this risk-based approach to SHM. Targeting selected failure modes of interest for a structure and modelling them as fault trees allows the scope of the decision-maker to be limited thereby making the problem more approachable. Mapping the fault trees into Bayesian networks enables the framework to retain information regarding the uncertainties in the health states thereby allowing robustness in the decision-making. Finally, the consequence analysis process of PRA motivates the explicit incorporation of risk into the SHM framework. Utilities are attributed to the selected failure events and possible courses of action enabling the computation of expected utilities within an influence diagram and the comparison of strategies so that a utility-optimal decision can be made.

Whilst the framework presented addresses some of the problems surrounding the SHM decision process, there remain a number of challenges. One challenge, that has been widely acknowledged in the SHM community, is that data from the damage states of interest for a structure are seldom available prior to the implementation of an SHM system. This poses an issue in the development of the probabilistic classifiers on which the decision process is highly dependent and a choice must be made regarding the approach to the statistical modelling. One option is to take a model-driven approach [16] that utilises outputs from physics-based models of the structure in its damage states of interest to learn a classifier in a supervised manner pre-implementation of the SHM system. Subsequently, the classifier can be continuously updated and validated with data obtained during the monitoring campaign. Alternatively, a semi-supervised approach can be taken in which a clustering algorithm is applied to the data acquired throughout the monitoring campaign. Clusters are attributed damage state labels through the incorporation of labelled data into the clustering algorithm [17]; damage state labels for data points may be obtained through inspection of the structure [18].

In addition to being dependent on the statistical classifier used, the optimality of decisions in temporal processes is highly contingent on the appropriateness of the transition model used; if the degradation of the structure is not accurately modelled, erroneous actions may be taken. Facing a similar issue to the statistical modelling process, oftentimes, data describing the transitions between the health states of interest are not held *a priori*. Again, one is faced with the choice of taking a model-driven approach involving the simulation of the degradation, or a data-driven approach that utilises data obtained during the monitoring campaign.

A necessary step in the risk-based framework is to assign utilities/costs to failure events and actions. Currently, within the literature there is no formal approach to how these values should be elicited, nor is there a consensus on how the risk preferences of an SHM decision-maker should be specified; should an agent be risk averse, risk neutral, or risk seeking? The issue at hand is one of both a technical and ethical nature, and whilst it will not be discussed in further detail in the current paper, it is highlighted to stimulate the conversations required for progress in the area of risk-informed decision-making for SHM.

In summary, a risk-based framework for structural health monitoring was presented. Borrowing practices frequently used in probabilistic risk assessment, such as the use of fault trees to model system failures, the framework facilitates robust decision-making under uncertainty.

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