ECONOMIC RESEARCH
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WORKING PAPER SERIES

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Working Paper Number	2005-003A
Citable Link	https://doi.org/10.20955/wp.2005.003
Suggested Citation	Guidolin, M., Timmermann, A.; An Econometric Model of Nonlinear Dynamics in the Joint Distribution of Stock and Bond Returns, Federal Reserve Bank of St. Louis Working Paper 2005-003. URL https://doi.org/10.20955/wp.2005.003

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An Econometric Model of Nonlinear Dynamics in the Joint Distribution of Stock and Bond Returns*

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July 2004

Abstract

This paper considers a variety of econometric models for the joint distribution of US stock and bond returns in the presence of regime switching dynamics. While simple two- or three-state models capture the univariate dynamics in bond and stock returns, a more complicated four state model with regimes characterized as crash, slow growth, bull and recovery states is required to capture their joint distribution. The transition probability matrix of this model has a very particular form. Exits from the crash state are almost always to the recovery state and occur with close to 50 percent chance suggesting a bounce-back effect from the crash to the recovery state.

1 Introduction

This paper studies a variety of econometric models for the joint distribution of US stock and bond returns. We show that although there are well-defined regimes in the marginal

*We thank two anonymous referees and the editor, Dick van Dijk, for many helpful suggestions. We are also grateful to seminar participants at CERP University of Turin, University of Houston, University of Rochester, Federal Reserve Bank of St. Louis and at the Tinbergen Centenary conference for comments on an earlier version of the paper.

distributions of both stock and bond returns, there is very little coherence between these regimes. This complicates models for the joint dynamics of stock and bond returns and suggests that a richer model with several states is required. We study in detail a richly specified model with four regimes broadly corresponding to 'crash', 'slow growth', 'bull' and 'recovery' states.

Unfortunately the vast majority of work on regime switching considers univariate models. Examples include studies of economic variables such as exchange rates (Engel and Hamilton (1990)), output growth (Hamilton (1989)), interest rates (Gray (1996), Ang and Bekaert (2002b)), commodity indices (Fong and See (2001)), and stock returns (Rydén, Teräsvirta, and Asbrink (1998), Turner, Startz and Nelson (1989), and Whitelaw (2001)).

Exceptions to the focus on univariate models include Ang and Bekaert (2002a) and Perez-Quiros and Timmermann (2000) who consider bivariate regime switching models fitted to stock market portfolios tracking either country indices or portfolios based on market capitalization. Hamilton and Lin (1996) also consider a bivariate model for stock returns and growth in industrial production. There appears to be no clear guidelines for how to generalize univariate nonlinear models to the general multivariate case, however. Simple generalizations easily yield overwhelmingly large models. To see this, suppose that stock returns are divided into two states based on periods of high and low volatility, while bond returns are divided into recession, low growth and high growth states. Also suppose that the pair of state variables are only weakly correlated. In this case a six-state model $-$ comprising low and high volatility recessions, low and high volatility states with low growth and low and high volatility states with high growth $-$ is required to capture the joint distribution of stock and bond returns. In general such models are not feasible to estimate or will be poorly identified since most states are likely only to be visited very few times during the sample.[1]

The plan of the paper is as follows. Section 2 studies regimes in the individual asset returns. Section 3 considers their joint distribution and discusses at some length a four-state specification. Section 4 extends out setup to include additional predictor variables such as

[1]For a further discussion of multivariate regime switching models see Franses and van Dijk (2000), pp 132-134.

the dividend yield. Section 5 concludes.

2 Stock and bond returns under regime switching: Univariate models

In this section we consider the dynamics in the univariate or separate distributions of stock and bond returns. An understanding of the univariate dynamics of the returns for the individual asset classes is an important starting point for an analysis of their joint distribution. We study three major US asset classes, namely stocks, bonds and T-bills although we simplify the analysis to just stocks and bonds by analyzing their excess returns over and above the T-bill rate. We further divide the stock portfolio into large and small stocks in light of the empirical evidence suggesting that these stocks have very different risk and return characteristics across different regimes, c.f. Perez-Quiros and Timmermann (2000).

2.1 Data

All data is obtained from the Center for Research in Security Prices. Our analysis uses monthly returns on all common stocks listed on the NYSE. The first and second size-sorted CRSP decile portfolios are used to form a portfolio of small firm stocks, while deciles 9 and 10 are used to form a portfolio of large firm stocks. We also consider the return on a portfolio of 10-year T-bonds. Returns are calculated applying the standard continuous compounding formula, $y_{t+1} = \ln S_{t+1} - \ln S_t$, where S_t is the asset price, inclusive of any cash distributions (dividends or coupons) between time t and $t+1$. To obtain excess returns, we subtract the 30-day T-bill rate from these returns. Dividend yields are also used in the analysis and are computed as dividends on a value-weighted portfolio of stocks over the previous twelve month period divided by the current stock price. Our sample is January 1954 - December 1999, a total of 552 observations.

2.2 Regimes in the individual series

Before proceeding to the joint model for stock and bond returns we consider the presence of regimes in the individual asset return series. The objective is to assess the degree of coherence across the state variables characterizing the regimes (if any) in the returns on small and large firms and on long-term bonds. A high degree of coherence would naturally suggest a substantial reduction in the overall number of regimes, k, required in a joint model for stock and bond returns. Each of the univariate return series (indexed by $i = 1,..,n$, where n is the number of assets), y_{it}, is modeled as a simple Markov switching process whose parameters are driven by an asset-specific state variable, S_{it}, taking values $s_{it} = 1,..,k_i$, where k_i is the number of states for the ith series:

$$y_{it} = \mu_{is_{it}} + \sum_{j=1}^{p} a_{ij,s_{it}} y_{it-j} + \sigma_{is_{it}} u_{it}, \quad i = 1,...,n, \quad u_{it} \sim IIN(0,1), \tag{1}$$

where state transitions are governed by a constant transition probability matrix

$$P(S_{it} = s_{it} | S_{it-1} = s_{it-1}) = p_{s_{it}s_{it-1}}, \qquad s_{it}, s_{it-1} = 1,...,k_i. \tag{2}$$

Thus each regime is assumed to be the realization of a first-order, homogeneous, irreducible and ergodic Markov chain. For each series, y_i, the number of states, k_i, is a key parameter in the proposed model. If $k_i = 1$, we are back to the standard linear model used in much of the literature. As k_i rises, it becomes increasingly easy to fit complicated dynamics and deviations from the normal distribution in asset returns. However, this comes at the cost of having to estimate more parameters which can lead to deteriorating out-of-sample forecasting performance.

Economic theory offers little guidance to the most plausible non-linear model capable of adequately fitting the data. If recurrent shifts only affect the diversifiable component of portfolio returns (idiosyncratic risk), regime switching in well-diversified portfolios such as those we study here should only show up in the form of regime-dependent heteroskedasticity giving rise to a model of the type

$$y_{it} = \mu_i + \sigma_{s_{it}} u_{it}. \tag{3}$$

On the other hand, when shifts occur in the systematic risk component, then most economic models would suggest regime dependence both in the risk premium (μ) and in the variance:

$$y_{it} = \mu_{is_t} + \sum_{j=1}^{p} a_{ij,s_{it}} y_{it-j} + \sigma_{s_{it}} u_{it}, \tag{4}$$

The presence of autoregressive lags may proxy for omitted state variables tracking time-varying risk premia. This ambiguity about the correct theoretical model suggests we should consider a wide range of models.

To determine k_i, we undertake an extensive specification search, considering values of $k_i = 1, 2, 3$ and different values of the autoregressive order, p. We consider up to three states because of the existing evidence in the literature of either two (Schwert (1989) and Turner, Startz, and Nelson (1989)) or three (Kim, Nelson and Startz (1998)) regimes in the mean and volatility of U.S. asset returns (see also Rydén et al. (1998)). It is of course important to determine whether multiple states are needed in the first place, i.e. whether $k_i > 1$. Testing a model with k_i states against a model with $k_i - 1$ states is complicated because some of the parameters of the model with k_i states are unidentified under the null of $k_i - 1$ states and test statistics follow non-standard distributions.[2] To check if the linear model ($k_i = 1$) is misspecified, we computed the test proposed by Davies (1977) which accounts for the unidentified nuisance parameter problem. To determine the number of states, we adopted the Hannan-Quinn information criterion for model selection (c.f. Rydén et al. (1998)). This trades off the improved fit resulting from adding more parameters as k_i grows against the decreasing parsimony.

Table 1 reports the parameter estimates of two- and three-state models fitted to the returns on our three portfolios along with linearity tests and values of the Hannan-Quinn information criterion. The left panels (A and C) set $p = 0$ (no autoregressive terms), while the right panels (B and D) assume that $p = 1$. For all three assets the single-state model is strongly rejected in favour of a multistate model.[3] The Hannan-Quinn criterion points to

[2] See, e.g., Davies (1977), Garcia (1998) and Hansen (1992).

[3] In addition to the Hannan-Quinn information criterion we also considered the Akaike and Schwarz information criteria. Two of three information criteria applied to the univariate series suggested a two-state model for stock returns while all criteria selected a three-state model for bond returns.

a two-state specification for both stock market portfolios and a three-state specification for bonds. Furthermore, there is evidence of first-order autoregressive terms in the small stock and bond return series.

Each of the two regimes identified in the two stock return series has a clear economic interpretation. The first regime captures a bear state with high volatility and low expected returns: large stocks are characterized by negative mean excess returns and an annual volatility of 22.2%, small stocks by relatively low mean excess returns of 5.4% per annum and volatility of 29.5%. Conversely, the second - more persistent - regime is associated with high mean returns (large stocks earn an annualized premium of 11.7%, small stocks a premium of 13%) and low volatility. The estimates of the transition probability matrices for small and large stocks are also quite similar although small stocks tend to stay longer in bear states. The states identified in the bond returns have a similar interpretation: Regime 1 captures economic recessions during which interest rates tend to fall or stay roughly constant so that long-term bonds earn low but positive average excess returns (1.8% per annum), while their volatility is above-average (8.5%). Regime 2 captures economic booms with rising interest rates and negative excess returns on bonds.

To further assist with the economic interpretation of these states, Figure 1 shows smoothed state probability plots for the two-state models fitted to the individual return series. Although the matching between the high volatility states identified for the two stock portfolios is by no means perfect, there are clearly strong similarities between the two and many well-known historical episodes trigger similar regime switches in both portfolios, e.g., the Vietnam War in the 1960s, the oil shocks of the 1970s, the volatility surge of 1987-1988, the early 1990s recession, and the Asian flu of 1998. As a result, the correlation between the smoothed probability of state 1 across the two stock return series is 0.52.

In contrast, there is not much similarity between the regimes identified in the stock and bond return series. Indeed the correlations between the smoothed state probabilities inferred from bond returns and the probabilities implied by both small and large stock returns are close to zero (0.15-0.16). This impression is further enhanced by the scatter plots of smoothed state probabilities shown in Figure 2, indicating no strong correlations between the states identified in stock and bond returns. Furthermore, many episodes associated with regime switches in the stock market portfolios (e.g., the early 1980s recession and the 1987 crash) are not reflected in similar switches in bond returns.

Of course, this analysis may not fully reveal possible similarities between the nonlinear components in stock and bond returns since we identified three states in bond returns. We therefore next consider three-state models for stock and bond returns. Panels C and D in Table 1 report parameter estimates for these models while Figure 3 plots the smoothed state probabilities for the univariate three-state models fitted to the two stock return series and bond returns. Interpretation of the three states in stock returns is difficult. As we move from regime 1 to 3 the risk premium on large stocks changes from -20.3% to 44.5% per annum and the volatility declines from 25% to 6.3%. For small stocks there is no great difference in the volatility estimates for states 1 and 3 while their mean returns (-29.3% and 104% per annum, respectively) are very different.

In contrast, the three-state model marks a clear improvement over the two-state model fitted to bond returns. In this case the three states are easier to interpret. Regime 1 has relatively high volatility (11.8%) and high mean excess returns (3.6%), and therefore represents periods of declining short-term interest rates and strong growth following a recession. Regime 2 corresponds to periods of rising short-term interest rates (leading to negative mean excess returns on long-term bonds) and downward sloping, stable yield curves. The third state is the most frequently visited regime in our sample, characterized by moderately positive mean excess returns (0.7%) and moderate volatility (6.2% per annum). The steady growth of the 1990s with stable interest rates and monetary policy falls almost entirely in this regime. This classification of the sample period into regimes is more sensible than that provided by the two-state model for bond returns.[4]

[4]There is in fact an interesting association between some of the regime shifts appearing in Figure 3 for bonds and changes in monetary policy. For instance, out of roughly 15 major switches, as many as four can be linked to the classical Romer and Romer (1989) (contractionary) monetary policy shock dates, in the sense that these switches occur within six months of Romer and Romer's dates. In particular, the 1968:12 and 1979:10 episodes are associated with almost contemporaneous shifts to regime 2, consistent with tight monetary conditions and increasing interest rates; similarly, the 1955:09 and 1974:04 dates precede switches

The lack of coherence between regimes in stock and bond returns encountered in the two-state models is even clearer in the three-state models. The correlation between the smoothed state probabilities for stock and bond returns shown in Figure 3 is systematically negative or close to zero, irrespective of how the states are ordered.

Interestingly, all of the results on the presence of regimes in stock and bond returns, their interpretation, and the coherence between regimes in stock and bond returns are insensitive to the inclusion of autoregressive terms. For instance, the coefficients of correlations across stock and bond portfolios are similar to those reported above when $p = 1$ and the state probabilities resulting from this model are practically indistinguishable from those in Figure 2.

In conclusion, while there is a strong correlation between the process driving regimes in large and small firms' stock returns, bond returns appear to be governed by a very different process. This is already suggested by the fact that a two-state model is selected for stock returns while a three-state model is chosen for bond returns and is further stressed by the difference in the state transition probability estimates of the two-state models.[5] The fact that a three-state specification fits excess bond returns much better than a simpler, two-regime model and that these states are weakly correlated with those identified in the stock portfolios indicates that multiple regimes are needed to capture the joint distribution of stock and bond returns.

3 A joint model for stock and bond returns

Earlier studies of regime switching in stock and bond returns focused on separately modeling stock returns or the evolution in interest rates, but do not consider their joint distribution. When considering the joint stochastic process of returns on stocks and bonds, we have to

to state 3, in which bond returns are moderate. As explained by Romer and Romer (1989), their dates are supposed to detect only pure, contractionary monetary shocks. This explains why we find more shifts than their dates. We thank an anonymous referee for leading us to explore these issues.

[5] While bond returns imply that the average duration of a 'bear market' is almost 13 months, the stock returns suggest an estimate between four (large stocks) and nine (small stocks) months.

carefully determine the number of states in their joint distribution and need to pay attention to differences in their individual state characteristics.

To capture the possibility of regimes in the joint distribution of asset returns, consider an $n \times 1$ vector of returns in excess of the T-bill rate, $\mathbf{y}_t = (y_{1t}, y_{2t}, ..., y_{nt})'$. Suppose that the mean, covariance and possibly also serial correlation in returns are driven by a common state variable, S_t, that takes integer values between 1 and k:

$$\mathbf{y}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^{p} \mathbf{A}_{j,s_t} \mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t. \tag{5}$$

Here $\boldsymbol{\mu}_{s_t} = (\mu_{1st}, ..., \mu_{ns_t})'$ is an $n \times 1$ vector of mean returns in state s_t, $\mathbf{A}_{j,s_t}$ is the $n \times n$ matrix of autoregressive coefficients associated with lag $j \geq 1$ in state s_t, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{nt})' \sim N(\mathbf{0}, \Omega_{s_t})$ follows a multivariate normal distribution with zero mean and state-dependent covariance matrix, Ω_{s_t}, given by

$$E\left[\left(\mathbf{y}_t - \boldsymbol{\mu}_{s_t} - \sum_{j=1}^{p} \mathbf{A}_{j,s_t} \mathbf{y}_{t-j}\right)\left(\mathbf{y}_t - \boldsymbol{\mu}_{s_t} - \sum_{j=1}^{p} \mathbf{A}_{j,s_t} \mathbf{y}_{t-j}\right)' |s_t\right] = \Omega_{s_t} \tag{6}$$

Regime switches in the state variable, S_t, are assumed to be governed by the transition probability matrix, P, with elements

$$\Pr(S_t = s_t|, S_{t-1} = s_{t-1}) = p_{s_t s_{t-1}}, \quad s_t, s_{t-1} = 1, .., k. \tag{7}$$

Each regime is thus the realization of a first-order Markov chain with constant transition probabilities.

While simple, this model allows asset returns to have different means, variances and correlations in different states. This means that the risk-return trade-off can vary over states in a way that can have strong implications for investors' asset allocation. For example, knowing that the current state is a persistent bull market will make most risky assets more attractive than in a bear state. Likewise, if stock market volatility is higher in recessions than in expansions, equity investments are less attractive in recessions unless their mean return rises commensurably.

Estimation of the parameters of the joint model is relatively straightforward and proceeds by optimizing the likelihood function associated with (5) - (7). Since the underlying state variable, S_t, is unobserved we treat it as a latent variable and use the EM algorithm to update our parameter estimates, c.f. Hamilton (1989).

3.1 Determination of the number of states

Before turning to the selection of the number of states for the joint model, we first consider the implications of the analysis of the univariate series in Section 2. Suppose that each of the n univariate return series is governed by a Markov switching process of the form (1) - (2). Also assume that the innovation terms are simultaneously correlated,

$$E\left[\left(y_{it}-\sum_{j=1}^{p}a_{ij,s_{it}}y_{it-j}-\mu_{is_{it}}\right)\left(y_{mt}-\sum_{j=1}^{p}a_{mj,s_{mt}}y_{mt-j}-\mu_{ms_{mt}}\right)|s_{it},s_{mt}\right]=\sigma_{ims_{it}s_{mt}}, \tag{8}$$

although for all $i \neq m$ and all $q \neq 0$, $E[(y_{it-q}-\sum_{j=1}^{p}a_{ij,s_{it}}y_{it-q-j}-\mu_{is_{it-q}})\,(y_{mt}-\sum_{j=1}^{p}a_{mj,s_{mt}}r_{mt-j}-\mu_{ms_{mt}})]=0$ (no serial correlation or cross-correlation).

Under no further restrictions on the relationship between the individual state variables $\{s_{1t},...,s_{nt}\}$ the states (S_t) for the joint process $\{y_{1t},....,y_{nt}\}$ can be obtained from the product of the individual states:

$$S=\prod_{i=1}^{n}S_i=S_1\times S_2\times...\times S_n. \tag{9}$$

This gives a total of $k=\prod_{i=1}^{n}k_i$ possible states and $k(k-1)$ state transition probabilities. Under independence between the individual states, the transition probability matrix defined on the joint outcome space is simply the Kronecker product of the individual transition matrices and the number of transition probability parameters to be estimated reduces to $\sum_{i=1}^{n}k_i(k_i-1)$ which can be considerably smaller than $k(k-1)$ when n is large. For example, in the bivariate case ($n=2$) we have

$$\Pr(s_{1t}=a,s_{2t}=a^*|s_{1t-1}=b,s_{2t-1}=b^*)=P_{ab}[1]\otimes P_{a^*b^*}[2]. \tag{10}$$

Obviously, the original n-variable Markov switching process with $\prod_{i=1}^{n}k_i$ states is perfectly equivalent to a modified univariate Markov switching process characterized by $k=\prod_{i=1}^{n}k_i$

different regimes and a single $(\prod_{i=1}^{n} k_i)\times(\prod_{i=1}^{n} k_i)$ −dimensional transition probability matrix

$$P = P_1 \otimes P_2 \otimes ... \otimes P_n. \tag{11}$$

In practical multivariate problems of even moderate size this representation is not, of course, feasible to use. For example, in the case with three variables each of whose marginal distribution has three states ($n = 3$, $k_i = 3$) the total number of states would be 27, involving the estimation of 702 parameters in the transition probability matrix alone. This suggests the need for carefully considering ways for the econometric modeler to reduce the set of states required to capture the essential dynamics of the joint distribution.

To determine the number of states for the joint model, k, we undertake an extensive specification search, considering values of $k = 1, 2, 3, 4, 5$ and different values of the autoregressive order, p. Results from the specification analysis are presented in Table 2. In all cases linearity is very strongly rejected no matter how many states and lags are present in the regime switching model. The Hannan-Quinn information criterion supports four states. There is only weak evidence of an autoregressive component in asset returns. We therefore settle on a four-state regime switching model without autoregressive terms.[6]

3.2 Interpretation of the States

Having determined the number of states we next focus on their economic interpretation. Table 3 reports the parameters of the four-state regime switching model while Figure 4 plots the associated smoothed state probabilities. For reference we also show the estimates of a single-state model with no autoregressive terms.

It is relatively straightforward to interpret the four regimes. Regime 1 is a 'crash' state characterized by large, negative mean excess returns and high volatility. It includes the two oil price shocks in the 1970s, the October 1987 crash, the early 1990s, and the 'Asian

[6]The number of parameters involved in our model depends on the number of assets, n, the number of states, k, and the number of autoregressive lags and is equal to $(nk + pn^2k + k\frac{n(n+1)}{2} + k(k-1))$. For the preferred model $n = 3$, $k = 4$, $p = 0$, so we have 48 parameters and 1,656 data points for a saturation ratio (the number of data points per parameter) of 35.

flu'. Regime 2 is a low growth regime characterized by low volatility and small positive mean excess returns on all assets. Regime 3 is a sustained bull state in which stock prices — especially those of the small stocks— grow rapidly on average. Interest rates frequently increase in this state and excess returns on long-term bonds are negative on average. The drawback to the high mean excess returns on small stocks is their rather high volatility, while large stocks and bonds have less volatile returns. Notice the big difference between mean returns on small and large stocks in regimes 2 and 3. In state 2 the mean return of large stocks exceeds that of small stocks by about 7% per annum, while this is reversed in state 3. Regime 4 is a "bounce-back" regime with strong market rallies and high volatility for small stocks and bonds.[7] Mean excess returns, at annualized rates of 27%, 55%, and 12%, are very large in this state as is their volatility.

Correlations between returns also vary substantially across regimes. The correlation between large and small firms' returns varies from a high of 0.82 in the crash state to a low of 0.50 in the recovery state. The correlation between large cap and bond returns even changes signs across different regimes and varies from 0.37 in the recovery state to -0.40 in the crash state. Finally, the correlation between small stock and bond returns goes from -0.26 in the crash state to 0.12 in the slow growth state.

Mean returns and volatilities are greater in absolute terms in the crash and recovery regimes, so it is perhaps unsurprising that persistence also varies considerably across states. The crash state has low persistence and on average only two months are spent in this regime. Interestingly, the transition probability matrix has a very particular form. Exits from the crash state are almost always to the recovery state and occur with close to 50 percent chance suggesting that, during volatile markets, months with large, negative mean returns cluster with months that have high positive returns. The slow growth state is far more persistent

[7] The volatility estimate may seem low for the large stocks. However, it should be recalled that, for each state, the volatility estimate is measured around the mean return for that state. Estimates of the conditional volatility starting from state four also depend on the probability of shifting to another state, multiplied by the squared value of the difference between that state's mean and the mean return in state four, summed across states 1-3.

with an average duration of seven months. The bull state is the most persistent state with a 'stayer' probability of 0.88. On average the market spends eight successive months in this state. Finally, the recovery state is again not very persistent and the market is expected to stay just over three months in this state. The steady state probabilities, reflecting the average time spent in the various regimes are 9% (state 1), 40% (state 2), 28% (state 3) and 23% (state 4). Hence, although the crash state is clearly not visited as often as the other states, it is by no means an 'outlier' state that only picks up extremely rare events.

It is interesting to relate these states to the underlying business cycle. Correlations between smoothed state probabilities and NBER recession dates are 0.32 (state 1), -0.13 (state 2), -0.21 (state 3), and 0.18 (state 4). Notice that since the state probabilities sum to one, by construction if some correlations are positive, others must be negative. This suggests that indeed, the high-volatility states - states 1 and 4 - occur around official recession periods.[8]

3.3 Mean and Variance Restrictions

The preferred four-state regime switching model is characterized by a large number of parameters so it is therefore legitimate to ask whether a more parsimonious specification can be constructed by imposing further restrictions on the parameter space, as in, e.g., Ang and Bekaert (2002a, pp. 1148-1150).

Although the results reported in Table 3 confirm that most of the mean excess returns parameters in μ_{s_t} are significantly different from zero and differ from each other, it is commonly found that mean asset returns are difficult to estimate precisely, suggesting that the fit of our model would not be greatly reduced by restricting the intercept vector μ to be identical across regimes:

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \quad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \Omega_{s_t}), \tag{12}$$

[8] It could be argued that the state probabilities backed out from movements in financial asset returns should lead economic recession months. Indeed, the correlation between the state-1 probability lagged 6 months and the NBER recession indicator rises to 0.40.

Table 4 reports the parameter estimates from this restricted model. The imposed restrictions lead to important changes in the transition dynamics. Regime 1 in the restricted model has no persistence and is best characterized as a purely transient state that leads to regime 4 ($\hat{P}[1,4] = 0.99$). Furthermore, regime 1 itself is likely to be accessed mostly from regime 4 ($\hat{P}[4,1] = 0.24$), so the resulting model implies a sequence of relatively calm periods (regimes 2 and 3) briefly interspersed by a period with highly volatile markets (regimes 1 and 4). In view of the similarity between $\hat{\Omega}_1$ and $\hat{\Omega}_4$ in this model, effectively the constrained model is an overparameterized version of a much simpler three-state model with regime-independent $\boldsymbol{\mu}$. The parametric restrictions implied by the null hypothesis that mean returns do not vary across states are strongly rejected using a likelihood-ratio test,

$$LR = 2(3462.91 - 3447.88) = 30.06.$$

This yields a p-value of 0.0004.

Another restriction naturally suggested by the results in Tables 3 and 4 is that the covariance matrices are identical in the highly volatile crash and recovery regimes. To investigate this possibility, we estimated the four-state model (5) subject to the restriction $\hat{\Omega}_1 = \hat{\Omega}_4$. Results are provided in Table 5. The resulting estimates of the high-variance covariance matrix are, as expected, an average of the unrestricted estimates of $\hat{\Omega}_1$ and $\hat{\Omega}_4$. The six parameter restrictions implied by the null hypothesis that $\hat{\Omega}_1 = \hat{\Omega}_4$ were strongly rejected by means of a likelihood-ratio test,

$$LR = 2(3462.91 - 3449.09) = 27.64,$$

which implies a p-value of 0.0001. Clearly the data supports correlations and volatilities that are different even in the two regimes with the highest volatility.

4 Additional Predictor variables

Equation (5) can easily be extended to incorporate an $m \times 1$ vector of additional predictor variables, $\mathbf{x}_{t-1}$. Define the $(m+n) \times 1$ vector $\mathbf{z}_t = (\mathbf{y}_t' \; \mathbf{x}_t')'$. Then (5) is readily extended to

$$\mathbf{z}_t = \begin{pmatrix} \boldsymbol{\mu}_{s_t} \\ \boldsymbol{\mu}_{\mathbf{x}s_t} \end{pmatrix} + \sum_{j=1}^{p} \mathbf{A}^*_{j,s_t} \mathbf{z}_{t-j} + \begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\varepsilon}_{\mathbf{x}t} \end{pmatrix}, \quad (s_t = 1, ..., k) \tag{13}$$

where $\boldsymbol{\mu}_{\mathbf{x}s_t} = (\mu_{x1s_t}, ..., \mu_{xms_t})'$ is the intercept vector for $\mathbf{x}_t$ in state s_t, $\{\mathbf{A}^*_{j,s_t}\}_{j=1}^{p}$ are now $(n+m)\times(n+m)$ matrices of autoregressive coefficients in state s_t, and $(\boldsymbol{\varepsilon}_t' \; \boldsymbol{\varepsilon}_{xt}')' \sim MN(\mathbf{0}, \Omega^*_{s_t})$, where $\Omega^*_{s_t}$ is an $(n+m \times n+m)$ covariance matrix.

In this extended model predictability of returns occurs through two channels. Most obviously, if the autoregressive terms or lagged predictor variables are significant, the conditional mean of stock and bond returns are predictable. Even in the absence of time-varying predictor variables or autoregressive terms, predictability arises in general as long as there are two states, s_t and s_t' for which $\mu_{s_t} \neq \mu_{s_t'}$. Variation in the state probabilities over time will then lead to time-variation in expected returns. Variations in the covariance matrix across states, will lead to further predictability in higher order moments such as volatility, correlations and skews.

This setup is directly relevant to the large literature in finance that has reported evidence of predictability in stock and bond returns. While many predictor variables have been proposed, one of the key instruments is the dividend yield; see, e.g., Campbell and Shiller (1988) and Fama and French (1988, 1993).

Notice that when $k = 1$, equation (13) simplifies to a standard vector autoregression. Our model thus nests as a special case the standard linear (single-state) model used in much of the asset allocation literature; see e.g. Barberis (2000).

4.1 Empirical Results

Again we conducted a battery of tests to determine the best model specification. To select the lag order for the extended model we first estimate a range of VAR(p) models, where

p is gradually augmented and information criteria used to evaluate the effect of including additional lags.[9] All information criteria as well as a sequential likelihood ratio test pointed towards a VAR(1) model. This is unsurprising given the strong persistence of the dividend yield.

Turning next to the search across different numbers of states, k, table 6 suggests that, although the model has now been extended by an autoregressive term, a four-state model continues to provide the best trade-off between fit and parsimony.[10] Table 7 shows the parameter estimates for the preferred model specification. Results for a comparable single state VAR(1) model are shown to provide a benchmark for the richer four-state model.

In the linear model the dividend yield predicts returns on small stocks but does not appear to be significant in the equations for returns on large stocks and long-term bonds.[11] As expected, the dividend yield is highly persistent and the estimated correlation matrix shows a strong positive correlation between the returns of small and large stocks while stock returns are strongly negatively correlated with simultaneous shocks to the dividend yield.

Estimates of the autoregressive matrices, $\hat{\mathbf{A}}_j$, suggest that the effect of changes in the dividend yield on asset returns continues to be strong in the multi-state model. Inclusion of the dividend yield therefore does not weaken the evidence of multiple states, nor does the presence of such states in a framework that allows for heteroskedasticity remove the predictive power of the dividend yield over asset returns.[12]

As in the pure return regime-switching model, the transition probability matrix continues to have a very special structure. Exits from states 1 and 2 are almost always to the bull-burst

[9] As suggested by Krolzig (1997, p. 128) the autoregressive order p in a regime switching model can conveniently be pre-selected as the maximal lag order p obtained in the single state VAR.

[10] There is clear evidence of separate regimes in the univariate dividend yield series. Independently of the specific form of the estimated regime switching model, the null of linearity was rejected using Davies' (1977) upper bounds for the p-values of likelihood ratio tests in the presence of nuisance parameters.

[11] A one standard deviation increase in the dividend yield incrases the annualized mean excess return on small stocks by 1.2%. The corresponding figures for large stocks and bonds are 0.23% and 0.25%, respectively.

[12] After controlling for regime switching in a univariate model for the returns of a value-weighted portfolio of stocks, Schaller and van Norden (1997) find that the dividend yield remains significant in a regime switching model with homoskedastic shocks but is insignificant once the volatility is allowed to be state-dependent.

state 4, while exits from states 3 and 4 are predominantly to the crash state 1.

To assist with the economic interpretation of the four regimes, Figure 5 plots the smoothed state probabilities. Regime 1 continues to pick up market crashes, characterized by negative, double-digit (on an annualized basis) mean excess returns (-38% and -49% for large and small firms and -10% for bonds).[13] The dividend yield is relatively high in this state (4%) and volatility is also above average. The probability of regime 1 is highest around the oil price shocks of the 1970s, the recession of the early 1980s, the October 1987 stock market crash, the Kuwait Invasion in 1990 and the Asian flu. It matches the beginning of major U.S. business cycle contractions and also picks up many well-known episodes with low returns and high volatility. In steady state this regime occurs 15% of the time although it has an average duration of only two months. The autoregressive coefficients indicate substantial predictability of small and large firms' returns in this state. Lagged bond returns and dividend yields have the strongest predictive power and small stocks' returns are also strongly serially correlated. The dividend yield is highly persistent but unpredictable from past asset returns in this state.

Regime 2 is a slow growth state characterized by single-digit mean excess stock returns (9.9% and 8.8% for large and small firms, respectively) and moderate volatility. Long periods of time was spent in this state during the stagnating markets of the mid-1970s and the first half of the 1990s. This state is highly persistent, lasting on average almost 16 months and occurring close to one-third of the time. There is less predictability of returns in this regime although the dividend yield still affects stock returns, again with the expected positive sign.

Regime 3 is a bull state in which the annualized mean excess return on large and small stocks is 11% and 14%, respectively. This state includes the long expansions of the 1950s and 1960s, the high growth periods of 1971-1973, the protracted boom of the 1980s as well as

[13]The mean excess return in each regime (k) is estimated as the weighted sample average of mean excess returns:

$$\left\{\sum_{t=1954:02}^{1999:12} \widehat{\pi}_{k,t}\right\}^{-1} \left\{\sum_{t=1954:02}^{1999:12} \widehat{\pi}_{k,t} E_{t-1}[\mathbf{y}_t | s_{t-1} = k]\right\}$$

where $E_{t-1}[\mathbf{y}_t | s_{t-1} = k] = \hat{\boldsymbol{\mu}}_{s_{t-1}=k} + \hat{A}_{s_{t-1}=k}\mathbf{y}_{t-1}$.

some periods in the early 1990s. It is often accompanied by interest rate cuts and therefore by positive mean excess returns on long-term bonds. At 2.8%, the mean dividend yield, on the other hand, is low. Return volatilities reach intermediate levels. This regime is also highly persistent and occurs one third of the time, lasting on average almost 15 months. Return predictability is weak in this state although the dividend yield remains positively correlated with stock returns.

Finally, regime 4 is again a bull-burst regime with strong stock market rallies accompanied by substantial volatility. Annualized mean excess returns on large and small stocks are 57% and 95%, respectively, while long-term bonds have mean excess returns of 17%. This state thus picks up either the initial and more impetuous stages of business cycle upturns or market 'rebounds' following crashes. Many peaks of U.S. expansions and market booms such as 1985-1986, or the 'new economy' of 1997-1999 occurred during this state which does not last long with an average duration of only 2 months. Nevertheless, at 18%, its steady-state probability is quite high. As in the first state, there is some predictability and the dividend yield forecasts returns on small caps and long-term bonds in the fourth state.

4.2 Relation to Fama-French Factors

Fama and French (1993) proposed a number of factors to explain the cross-sectional variation in stock and bond returns. For stock returns they considered the market portfolio, a portfolio capturing book-to-market effects (HML) and a portfolio capturing size (SMB). For bond returns they considered a default premium and a term premium factor.

Although the analysis of Fama and French (1993) was primarily concerned with explaining patterns in the cross-section of returns on stock and bond returns by means of factors measured during the same period, while our analysis is concerned with predictive patterns in returns, it is interesting to relate expected returns implied by our four-state model to the five Fama-French factors. To do so, we estimate univariate predictive regressions of the expected stock and bond returns implied by the regime switching model ($\hat{y}_{it}$) on the lagged values of the Fama-French factors:

$$\hat{y}_{it} = \beta_{0i} + \beta_{1i} HML_{t-1} + \beta_{2i} SMB_{t-1} + \beta_{3i} r_{t-1}^{MKT} + \beta_{4i} DEF_{t-1} + \beta_{5i} TERM_{t-1} + \varepsilon_{it}, \quad (14)$$

where HML is the return on the Fama-French 'High-minus-Low Book-to-Market' stock portfolio, SMB is the return on the Fama-French 'Small-minus-Big Size' stock portfolio, r^{MKT} is the excess return on the market (the value-weighted CRSP portfolio), DEF is the default premium (difference between the yield on Moody's BAA and AAA corporate bonds), and $TERM$ is the term premium (difference between the return on long-term government bond yields and 30-day T-bill rates). Results are shown in Figure 6. The correlation coefficients between expected returns calculated from (14) vs. the ones implies by the four-state regime model estimated in Section 3 are 0.053 for bonds, 0.273 for small caps and 0.331 for large caps. Hence, there is a positive but weak relationship between the lagged Fama-French factors and expected returns under the regime switching model.

5 Conclusion

The joint process of stock and bond returns follows a rich and complex dynamic pattern. We found evidence that standard linear models do not capture essential features of this distribution and that four regimes are required to capture the time-variation in the mean, variance and correlation between large and small firms' stock returns and long-term bond returns. Two regimes capture periods with high volatility and low persistence and two regimes are intermediate states with higher persistence. Furthermore, transitions between these regimes take a very special form with exits from the highly volatile bear state mostly being to the volatile recovery state with high expected returns, suggesting the presence of bounce-back effects after a period with large negative returns. These conclusions do not change when we add the dividend yield as a predictor in our model.

There are several extensions of this work that would be interesting to consider. First, while we used diagnostic tests and information criteria to choose the number of regimes in the univariate and multivariate models, another possibility is to select the preferred model on the basis of its forecasting performance in an out-of-sample experiment. It is a common finding in economics that nonlinear models provide good in-sample fits, but perform worse out-of-sample. One could select the architecture of the regime switching model - primarily the number of states and the number of autoregressive terms - on the basis of its out-of-sample forecasting performance.

A second extension of our results is to consider their asset allocation implications. This is done in Guidolin and Timmermann (2003). It turns out that the regime switching model not only affects the optimal level of asset holdings across a range of preference specifications, but also affects how the optimal asset allocation relates to the investor's time horizon, bear states giving rise to upward sloping demand for stocks while bull states give rise to a downward sloping demand for stocks as a function of the investment horizon.

References

[1] Ang A., and G., Bekaert, 2002a, "International Asset Allocation with Regime Shifts", *Review of Financial Studies*, 15, 1137-1187.

[2] Ang, A., and G., Bekaert, 2002b, "Regime Switches in Interest Rates", *Journal of Business and Economic Statistics*, 20, 163-182.

[3] Barberis, N., 2000, "Investing for the Long Run When Returns Are Predictable", *Journal of Finance*, 55, 225-264.

[4] Campbell, J., and R. Shiller, 1988, "The Dividend Price Ratio and Expectations of Future Dividends and Discount Factors", *Review of Financial Studies,* 1, 195-228.

[5] Davies, R., 1977, "Hypothesis Testing When a Nuisance Parameter Is Present Only Under the Alternative", *Biometrika*, 64, 247-254.

[6] Engel, C., and J., Hamilton, 1990, "Long Swings in the Dollar: Are They in the Data and Do Markets Know It?", *American Economic Review*, 80, 689-713.

[7] Fama, E., and K., French, 1988, "Dividend Yields and Expected Stock Returns", *Journal of Financial Economics*, 22, 3-25.

[8] Fama, E., and K., French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds", *Journal of Financial Economics*, 33, 3-56.

[9] Fong, W.-M., and K.H., See, 2001, "Modelling the Conditional Volatility of Commodity Index Futures as a Regime Switching Process", *Journal of Applied Econometrics,* 16, 133-163.

[10] Franses, P.H. and D. van Dijk, 2000, Non-linear Time Series Models in Empirical Finance. Cambridge University Press.

[11] Garcia, R., 1998, "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models", *International Economic Review,* 39, 763-788.

[12] Gray, S., 1996, "Modeling the Conditional Distribution of Interest Rates as Regime-Switching Process", *Journal of Financial Economics*, 42, 27-62.

[13] Guidolin, M., and A., Timmermann, 2003, "Strategic Asset Allocation under Regime Switching", mimeo, University of Virginia and UCSD.

[14] Hamilton, J., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle", *Econometrica*, 57, 357-384.

[15] Hamilton, J., and G., Lin, 1996, "Stock Market Volatility and the Business Cycle", *Journal of Applied Econometrics*, 11, 573-593.

[16] Hansen, B., 1992, "The Likelihood Ratio Test Under Non-Standard Conditions: Testing the Markov Switching Model of GNP", *Journal of Applied Econometrics*, 7, S61-S82.

[17] Kim, C.-J., C., Nelson, and R., Startz, 1998, "Testing for Mean Reversion in Heteroskedastic Data Based on Gibbs-Sampling-Augmented Randomization", *Journal of Empirical Finance*, 5, 131-154.

[18] Krolzig, H.-M., 1997, *Markov-Switching Vector Autoregressions*, Berlin, Springer-Verlag.

[19] Perez-Quiros, G. and A., Timmermann, 2000, "Firm Size and Cyclical Variations in Stock Returns", *Journal of Finance*, 55, 1229-1262.

[20] Romer, C. and D., Romer, 1989, "Does Monetary Policy Matter? A New Test in the Spirit of Friedman and Schwartz", NBER working paper No. 2966.

[21] Rydén, T., T., Teräsvirta, and S., Asbrink, 1998, "Stylized Facts of Daily Return Series and the Hidden Markov Model", *Journal of Applied Econometrics*, 13, 217-244.

[22] Schaller, H., and S., van Norden, 1997, "Regime Switching in Stock Market Returns", *Applied Financial Economics*, 7, 177-191.

[23] Schwert, G., 1989, "Why Does Stock Market Volatility Change over Time?", *Journal of Finance*, 44, 1115-1153.

[24] Turner, C., R., Startz, and C., Nelson, 1989, "A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market", *Journal of Financial Economics*, 25, 3-22.

[25] Whitelaw, R., 2001, "Stock Market Risk and Return: An Equilibrium Approach", *Review of Financial Studies*, 13, 521-548.

Table 1

Univariate Regime Switching Models for Stock and Bond Returns

This table reports estimation results for the model

$$y_{it} = \mu_{is_t} + \sum_{i=1}^{p} a_{i,s_t} y_{it-i} + \sigma_{is_t} u_{it} ,$$

where s_t is governed by an unobservable, discrete, first-order Markov chain that can assume *k* values (states). u_{ti} is IIN(0,1). *i* =1, 2, 3 indexes excess returns on portfolios of large and small stocks and 10-year T-bonds. Data are monthly and obtained from the CRSP tapes. The sample period is 1954:01 – 1999:12. For likelihood ratio tests we report in square brackets the *p*-value based on the $\chi^2(r)$ distribution (*r* is the number of restrictions) and in curly brackets the *p*-value based on Davies' (1977) upper bound.

Parameter	Large caps	Small caps	Bonds	Large caps	Small caps	Bonds
	Panel A – Two-State AR(0) Models			**Panel B – Two-State AR(1) Models**		
μ_1	-0.0083	0.0045	0.0015	-0.0239	0.0042	0.0012
μ_2	0.0097	0.0109	-0.0012	0.0154	0.0070	-0.0007
a_1	NA	NA	NA	0.4400	0.1555	0.0645
a_2	NA	NA	NA	-0.1639	0.2553	0.2989
σ_1	0.0641	0.0852	0.0246	0.0444	0.0873	0.0247
σ_2	0.0335	0.0360	0.0070	0.0347	0.0366	0.0071
P_{11}	0.7298	0.8910	0.9721	0.3819	0.8768	0.9757
P_{22}	0.9424	0.9218	0.9196	0.8521	0.9285	0.9315
Log-likelihood	996.3292	804.2038	1394.8273	993.5284	816.2982	1399.0809
Linear Log-lik.	976.9035	756.5298	1334.0423	975.1871	765.8890	1333.1040
LR test of linearity	38.8514 [0.000] {0.000}	95.3481 [0.000] {0.000}	121.5699 [0.000] {0.000}	36.6825 [0.000] {0.000}	100.8184 [0.000] {0.000}	131.9537 [0.000] {0.000}
Hannan-Quinn	-3.5698	-2.8737	-5.0137	-3.5528	-2.9095	-5.0248
	Panel C – Three-State AR(0) Models			**Panel D – Three-State AR(1) Models**		
μ_1	-0.0169	-0.0245	0.0029	-0.0289	-0.0155	0.0000
μ_2	0.0061	0.0121	-0.0014	0.0057	0.0070	-0.0003
μ_3	0.0371	0.0867	0.0006	0.0306	0.1106	0.0026
a_1	NA	NA	NA	0.3804	0.1215	0.0948
a_2	NA	NA	NA	-0.0290	0.2612	0.5497
a_3	NA	NA	NA	-0.2615	-0.3356	0.0486
σ_1	0.0722	0.0744	0.0337	0.0452	0.0753	0.0170
σ_2	0.0354	0.0365	0.0056	0.0300	0.0359	0.0029
σ_3	0.0181	0.0762	0.0181	0.0371	0.0726	0.0334
p_{11}	0.7356	0.8578	0.9799	0.4578	0.8776	0.9809
p_{22}	0.9663	0.9232	0.9206	0.9562	0.9347	0.8932
p_{33}	0.6716	0.4533	0.9726	0.7155	0.3433	0.9800
p_{12}	0.0017	0.0011	0.0069	0.0079	0.0014	0.0118
p_{21}	0.0313	0.0645	0.0001	0.0418	0.0592	0.1067
P_{31}	0.0052	0.0029	0.0077	0.2129	0.0082	0.0116
Log-likelihood	1004.7285	814.9706	1420.7636	1005.6759	826.5749	1429.0516
Linear Log-lik.	976.9035	756.5298	1334.0423	975.1871	765.8890	1334.1040
LR test of linearity	55.6501 [0.000] {0.000}	116.8817 [0.000] {0.000}	173.4425 [0.000] {0.000}	60.9775 [0.000] {0.000}	121.3719 [0.000] {0.000}	191.8951 [0.000] {0.000}
Hannan-Quinn	-3.5602	-2.8727	-5.0676	-3.5501	-2.9000	-5.0868

Table 2

Model Selection for Stock and Bond Returns (joint model)

This table reports values of the log-likelihood function, linearity tests and information criterion values for the multivariate Markov switching conditionally heteroskedastic VAR model:

$$y_t = \mu_{s_t} + \sum_{j=1}^{p} A_{js_t} \mathbf{r}_{t-j} + \varepsilon_t$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, A_{js_t} is the matrix of autoregressive coefficients at lag j = 1 in state s_t and $\boldsymbol{\varepsilon}_t=[\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}]' \sim N(\mathbf{0},\Omega_{s_t})$. S_t is governed by a first-order Markov chain that can assume *k* values. *p* autoregressive terms are considered. The three monthly return series comprise a portfolio of large stocks (ninth and tenth CRSP size decile portfolios), a portfolio of small stocks (first and second CRSP deciles), and 10-year T-bonds. Returns are measured in excess of the 30-day T-bill rate. The data was obtained from the CRSP tapes. The sample period is 1954:01 – 1999:12. MMSIA is short for Multivariate Markov Switching with regime-dependent Intercept and Autoregressive terms, while MMSIAH introduces regime-dependent heteroskedasticity.

Model (k,p)	Number of parameters	Log-likelihood	LR test for linearity	Hannan-Quinn
		Base model: MSIA(1,0)		
MMSIA(1,0)	9	3290.82	NA	-11.8632
MMSIA(1,1)	18	3314.34	NA	-11.9099
MMSIA(1,2)	27	3314.72	NA	-11.8618
		Base model: MSIA(2,0)		
MMSIA(2,0)	14	3316.24	50.8244 (0.000)	-11.8552
MMSIAH(2,0)	20	3392.79	203.9312 (0.000)	-12.1592
MMSIAH(2,1)	38	3436.99	245.2865 (0.000)	-12.2213
MMSIAH(2,2)	56	3438.51	253.5739 (0.000)	-12.1285
		Base model: MSIA(3,0)		
MMSIA(3,0)	21	3340.86	100.0658 (0.000)	-11.9643
MMSIAH(3,0)	33	3418.03	254.4206 (0.000)	-12.1639
MMSIAH(3,1)	60	3468.10	307.5043 (0.000)	-12.1871
MMSIAH(3,2)	87	3480.08	336.7194 (0.000)	-12.0721
		Base model: MSIA(4,0)		
MMSIA(4,0)	30	3380.29	178.9327 (0.000)	-12.0471
MMSIAH(4,0)	48	3462.91	344.1803 (0.000)	**-12.2263**
MMSIAH(4,1)	84	3517.36	406.0404 (0.000)	-12.2054
MMSIAH(4,2)	120	3554.56	485.6775 (0.000)	-12.1218
MMSIAH(4,3)	156	3589.30	550.8718 (0.000)	-12.0291

Table 2 (continued)

Model Selection for Stock and Bond Returns (joint model)

Model (k,p)	Number of parameters	Log-likelihood	LR test for linearity	Hannan-Quinn
Base model: MSIA(5,0)				
MMSIA(5,0)	41	3406.45	231.2536 (0.000)	-12.0685
MMSIAH(5,0)	65	3485.78	389.9136 (0.000)	-12.1957
MMSIAH(5,1)	110	3546.33	463.9703 (0.000)	-12.1367
MMSIAH(5,2)	155	3599.75	576.0651 (0.000)	-12.0517

Table 3

Estimates of Regime Switching Model for Stock and Bond Returns

This table reports parameter estimates for the multivariate regime switching model

$$y_t = \mu_{s_t} + \varepsilon_t,$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t and $\boldsymbol{\varepsilon}_t=[\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}]' \sim N(\mathbf{0},\Omega_{s_t})$. S_t is governed by a first-order Markov chain that can assume four values. The three monthly return series comprise a portfolio of large stocks (ninth and tenth CRSP size decile portfolios), a portfolio of small stocks (first and second CRSP deciles), and 10-year T-bonds. Returns are measured in excess of the 30-day T-bill rate. The data was obtained from the CRSP tapes. The sample is 1954:01 – 1999:12. The first panel refers to the single-state benchmark case (k = 1). Values on the diagonals of the correlation matrices are annualized volatilities. Asterisks attached to correlation coefficients refer to covariance estimates. For mean coefficients and transition probabilities, standard errors are reported in parentheses.

	Panel A – Single State Model		
	Large caps	Small caps	Long-term bonds
1. Mean excess return	0.0066 (0.0018)	0.0082 (0.0026)	0.0008 (0.0009)
2. Correlations/Volatilities			
Large caps	0.1428***		
Small caps	0.7215**	0.1481***	
Long-term bonds	0.2516	0.1196	0.0748***

	Panel B – Four State Model			
	Large caps	Small caps	Long-term bonds	
1. Mean excess return				
Regime 1 (crash)	-0.0510 (0.0146)	-0.0810 (0.0219)	-0.0131 (0.0047)	
Regime 2 (slow growth)	0.0069 (0.0027)	0.0008 (0.0033)	0.0009 (0.0016)	
Regime 3 (bull)	0.0116 (0.0032)	0.0167 (0.0048)	-0.0023 (0.0007)	
Regime 4 (recovery)	0.0226 (0.0055)	0.0458 (0.0098)	0.0098 (0.0033)	
2. Correlations/Volatilities				
Regime 1 (crash):				
Large caps	0.1625***			
Small caps	0.8233***	0.2479***		
Long-term bonds	-0.4060*	-0.2590	0.0902***	
Regime 2 (slow growth):				
Large caps	0.1118***			
Small caps	0.7655***	0.1099***		
Long-term bonds	0.2043***	0.1223	0.0688***	
Regime 3 (bull):				
Large caps	0.1133***			
Small caps	0.6707***	0.1730***		
Long-term bonds	0.1521	-0.0976	0.0261***	
Regime 4 (recovery):				
Large caps	0.1479***			
Small caps	0.5013***	0.2429***		
Long-term bonds	0.3692***	-0.0011	0.1000***	
3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1 (crash)	0.4940 (0.1078)	0.0001 (0.0001)	0.02409 (0.0417)	0.4818
Regime 2 (slow growth)	0.0483 (0.0233)	0.8529 (0.0403)	0.0307 (0.0110)	0.0682
Regime 3 (bull)	0.0439 (0.0252)	0.0701 (0.0296)	0.8822 (0.0403)	0.0038
Regime 4 (recovery)	0.0616 (0.0501)	0.1722 (0.0718)	0.0827 (0.0498)	0.6836

* significant at the 10% level, ** significant at the 5% level, *** significant at the 1%. level

Table 4

Estimates of Multivariate Regime Switching Model for Stock and Bond Returns Under Mean Restrictions

This table reports parameter estimates for the multivariate regime switching model

$$y_t = \mu + \varepsilon_t,$$

where $\boldsymbol{\varepsilon}_t=[\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}]' \sim N(\mathbf{0},\Omega_{s_t})$ is the vector of unpredictable return innovations. The model is estimated under the restriction that the vector of mean excess returns ($\boldsymbol{\mu}$) is regime-independent. The unobserved state variable, S_t, is governed by a first-order Markov chain that can assume four values. The three monthly return series comprise a portfolio of large stocks (ninth and tenth CRSP size decile portfolios), a portfolio of small stocks (first and second CRSP deciles), and 10-year T-bonds. Returns are measured in excess of the 30-day T-bill rate. The data was obtained from the CRSP tapes. The sample is 1954:01 – 1999:12. The first panel refers to the single-state benchmark (k = 1). Values reported on the diagonals of the correlation matrices are annualized volatilities. Asterisks attached to correlation coefficients refer to covariance estimates. For mean coefficients and transition probabilities, standard errors are reported in parentheses.

Panel A – Single State Model

	Large caps	Small caps	Long-term bonds
1. Mean excess return	0.0066 (0.0018)	0.0082 (0.0026)	0.0008 (0.0009)
2. Correlations/Volatilities			
Large caps	0.1428***		
Small caps	0.7215**	0.1481***	
Long-term bonds	0.2516	0.1196	0.0748***

Panel B – Four State Model under Mean Restrictions

	Large caps	Small caps	Long-term bonds
1. Mean excess return	0.0066 (0.0017)	0.0082 (0.0021)	0.0008 (0.0007)
2. Correlations/Volatilities			
Regime 1 (crash)			
Large caps	0.1897***		
Small caps	0.9683***	0.3032***	
Long-term bonds	-0.5424**	-0.4251**	0.0625***
Regime 2 (slow growth):			
Large caps	0.1069***		
Small caps	0.7087***	0.1074***	
Long-term bonds	0.0789	0.0688	0.0617***
Regime 3 (bull):			
Large caps	0.1163***		
Small caps	0.7324***	0.1637***	
Long-term bonds	0.2473**	0.0196	0.0064***
Regime 4 (recovery):			
Large caps	0.1661***		
Small caps	0.6520***	0.2682***	
Long-term bonds	0.4821***	0.2502	0.1012***

3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1 (crash)	0.0001 (0.0469)	0.0000 (0.0230)	0.0000 (0.0123)	0.9999
Regime 2 (slow growth)	0.0385 (0.0197)	0.9234 (0.0285)	0.0000 (0.0181)	0.0381
Regime 3 (bull)	0.0007 (0.0322)	0.0436 (0.0265)	0.9182 (0.0332)	0.0375
Regime 4 (recovery)	0.2350 (0.1041)	0.0434 (0.0267)	0.0413 (0.0193)	0.6803

* significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level.

Table 5

Estimates of the Multivariate Regime Switching Model for Stock and Bond Returns Under Covariance Restrictions

This table reports parameter estimates for the multivariate regime switching model:

$$y_t = \mu_{s_t} + \varepsilon_t$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t and $\boldsymbol{\varepsilon}_t=[\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}]' \sim N(\mathbf{0},\Omega_{s_t})$ is the vector of unpredictable return innovations. The model is estimated under the restriction that $\Omega_1 = \Omega_4$. S_t is governed by a first-order Markov chain that can assume four values. The three monthly return series comprise a portfolio of large stocks (ninth and tenth CRSP size decile portfolios), a portfolio of small stocks (first and second CRSP deciles), and 10-year bonds. Returns are measured in excess of the 30-day T-bill rate. The data was obtained from the CRSP tapes. The sample is 1954:01 – 1999:12. The first panel refers to the single-state benchmark (k = 1). Values reported on the diagonals of the correlation matrices are annualized volatilities. Asterisks attached to correlation coefficients refer to covariance estimates. For mean coefficients and transition probabilities, standard errors are reported in parentheses.

	Panel A – Single State Model			
	Large caps	Small caps	Long-term bonds	
1. Mean excess return	0.0066 (0.0018)	0.0082 (0.0026)	0.0008 (0.0009)	
2. Correlations/Volatilities				
Large caps	0.1428***			
Small caps	0.7215**	0.1481***		
Long-term bonds	0.2516	0.1196	0.0748***	
	Panel B – Four State Model with $\Omega_1 = \Omega_4$			
	Large caps	Small caps	Long-term bonds	
1. Mean excess return				
Regime 1 (crash)	-0.0574 (0.0143)	-0.0922 (0.0208)	-0.0100 (0.0055)	
Regime 2 (slow growth)	0.0061 (0.0028)	0.0001 (0.0034)	0.0001 (0.0017)	
Regime 3 (bull)	0.0103 (0.0033)	0.0133 (0.0059)	-0.0015 (0.0008)	
Regime 4 (recovery)	0.0210 (0.0055)	0.0424 (0.0099)	0.0066 (0.0033)	
2. Correlations/Volatilities				
Regimes 1-4 (high volatility)				
Large caps	0.1514***			
Small caps	0.5885***	0.2380***		
Long-term bonds	0.1786*	-0.0246	0.0967***	
Regime 2 (slow growth):				
Large caps	0.1103***			
Small caps	0.7687***	0.1118***		
Long-term bonds	0.1496***	0.1541	0.0687***	
Regime 3 (bull):				
Large caps	0.1096***			
Small caps	0.6929***	0.1676***		
Long-term bonds	0.0291	-0.0071	0.0228***	
3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1 (crash)	0.4543 (0.1169)	0.0000 (0.0572)	0.0102 (0.0298)	0.5355
Regime 2 (slow growth)	0.0487 (0.0244)	0.8538 (0.0379)	0.0000 (0.0491)	0.0975
Regime 3 (bull)	0.0491 (0.0293)	0.0657 (0.0338)	0.8817 (0.0389)	0.0035
Regime 4 (recovery)	0.0346 (0.0350)	0.1521 (0.0819)	0.0963 (0.0894)	0.7170

* significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level.

Table 6

Selection of Regime Switching Model for Stock and Bond Returns, Dividend Yield

This table reports estimation results for the extended regime switching model

$$y_t = \mu_{s_t} + \sum_{j=1}^{p} A_{js_t} y_{t-j} + \varepsilon_t,$$

where $\mathbf{y}_t$ is a (n+m x 1) random vector collecting excess asset returns in the first n positions followed by m predictor variables, $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, A_{js_t} is the matrix of autoregressive coefficients associated with lag j = 1 in state s_t and $\boldsymbol{\varepsilon}_t=[\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}\ \varepsilon_{4t}]' \sim N(\mathbf{0}, \Omega_{s_t})$. The unobserved state variable, S_t, is governed by a first-order Markov chain that can assume k distinct values. The three monthly return series comprise a portfolio of large stocks (ninth and tenth CRSP size decile portfolios), a portfolio of small stocks (first and second CRSP deciles), and 10-year T-bonds. Returns are measured in excess of the 30-day T-bill rate. The predictor is the dividend yield. The data was obtained from the CRSP tapes. The sample period is 1954:01 – 1999:12. MMSIA is short for Multivariate Markov Switching with regime-dependent Intercept and Autoregressive terms, while MMSIAH introduces regime-dependent heteroskedasticity. Models of the class MMSIAH(1, p) correspond to Gaussian VAR models of order p.

Model (k,p)	Number of parameters	Log-likelihood	LR test for linearity	Hannan-Quinn
Base model: MSIA(1,0)				
MMSIA(1,0)	14	5131.15	NA (NA)	-18.4976
MMSIA(1,1)	30	6673.70	NA (NA)	-24.0233
MMSIA(1,2)	46	6674.33	NA (NA)	-23.9549
Base model: MSIA(3,0)				
MMSIA(3,0)	28	5549.51	836.7187 (0.000)	-19.9200
MMSIAH(3,0)	48	5594.02	925.7533 (0.000)	-19.9477
MMSIAH(3,1)	96	6960.39	573.3915 (0.000)	-24.6226
MMSIAH(3,2)	144	6978.27	611.8806 (0.000)	-24.4109
Base model: MSIA(4,0)				
MMSIA(4,0)	38	5503.87	745.4456 (0.000)	-19.6879
MMSIAH(4,0)	68	5611.97	961.6513 (0.000)	-19.8792
MMSIAH(4,1)	132	7029.66	711.9237 (0.000)	**-24.6333**
MMSIAH(4,2)	196	7083.13	821.5979 (0.000)	-24.4439
MMSIAH(4,3)	260	7155.59	958.2742 (0.000)	-24.3232

Table 7

Estimates of Regime Switching Model for Stock and Bond Returns and the Dividend Yield

This table reports parameter estimates for the multivariate regime switching model

$$z_t = \mu^*_{s_t} + A^*_{s_t} z_{t-1} + \varepsilon^*_t,$$

where $\mathbf{z}_t$ is a 4×1 vector collecting excess asset returns in the first three positions plus an additional prediction variable (the dividend yield), $\mu^*_{s_t}$ is the intercept vector in state s_t, $A^*_{s_t}$ is the matrix of autoregressive coefficients associated with lag 1 in state s_t and $\boldsymbol{\varepsilon}^*_t=[\varepsilon_t\ \varepsilon_{xt}]' \sim N(\mathbf{0}, \Omega^*_{s_t})$. The unobservable state s_t is governed by a first-order Markov chain that can assume four distinct values. The three monthly return series comprise a portfolio of large stocks (ninth and tenth CRSP size decile portfolios), a portfolio of small stocks (first and second CRSP deciles), and 10-year bonds all in excess of the return on 30-day T-bills. The predictor is the dividend yield. The first panel refers to the single-state benchmark (k = 1). Asterisks attached to correlation coefficients refer to covariance estimates. For mean coefficients and transition probabilities, standard errors are reported in parentheses.

Panel A – VAR(1) (single state) Model				
	Large caps	Small caps	Long-term bonds	Dividend Yield
1. Intercept term	0.0021 (0.0070)	-0.0160 (0.0102)	-0.0032 (0.0036)	0.0004(0.0003)
2. VAR(1) Matrix				
Large caps	-0.0466 (0.0635)	0.0370 (0.0925)	0.2299(0.0330)	0.1261(0.0024)
Small caps	0.1236 (0.0412)	0.1244 (0.0600)	0.2624 (0.0214)	0.6641 (0.0016)
Long-term bonds	-0.0442 (0.0839)	-0.0261 (0.1223)	0.1070 (0.0436)	0.1322 (0.0032)
Dividend Yield	-0.0005 (0.2028)	-0.0005 (0.2953)	-0.0098 (0.1054)	0.9856 (0.0077)
3. Correlations/Volatilities				
Large caps	0.1417***			
Small caps	0.7285***	0.2063***		
Long-term bonds	0.2466*	0.1353	0.0736***	
Dividend Yield	-0.9243***	-0.7695***	-0.2413	0.0056***

Panel B – Four State Model				
	Large caps	Small caps	Long-term bonds	Dividend Yield
1. Intercept term				
Regime 1 (crash)	-0.0848 (0.1065)	-0.1152 (0.1528)	-0.0150 (0.0396)	0.0014 (0.0514)
Regime 2 (slow growth)	-0.0232 (0.0338)	-0.0188 (0.0516)	-0.0016 (0.0115)	0.0011 (0.0010)
Regime 3 (bull)	0.0122 (0.0539)	-0.0323 (0.0471)	0.0048 (0.0278)	0.0002 (0.0021)
Regime 4 (recovery)	0.0370 (0.0490)	0.0179 (0.0940)	-0.0038 (0.0324)	0.0007 (0.0019)
2. VAR(1) Matrix				
Regime 1 (crash):				
Large caps	-0.0494 (0.5360)	0.2391 (0.3875)	0.3092 (0.7164)	1.2089 (2.8282)
Small caps	-0.0357 (0.9401)	0.2424 (0.6332)	0.7277 (1.0894)	1.5972 (4.0047)
Long-term bonds	0.0136 (0.4381)	-0.0059 (0.2641)	-0.0215 (0.4246)	0.1838 (1.0283)
Dividend Yield	0.0002 (0.0262)	-0.0076 (0.0192)	-0.0170 (0.0301)	1.0074 (0.1302)
Regime 2 (slow growth):				
Large caps	-0.0563 (0.3064)	-0.0311 (0.1609)	0.0526 (0.4049)	1.1417 (1.1539)
Small caps	-0.0029 (0.5142)	0.2710 (0.2795)	-0.0077 (0.7227)	0.8963 (1.7180)
Long-term bonds	-0.0430 (0.1539)	-0.0056 (0.0896)	0.4234 (0.1888)	0.0813 (0.3948)
Dividend Yield	0.0010 (0.0096)	0.0007 (0.0051)	-0.0013 (0.0132)	0.9552 (0.0340)
Regime 3 (bull):				
Large caps	-0.0535 (0.3682)	-0.0789 (0.3452)	-0.0800 (0.4560)	-0.0810 (1.4631)
Small caps	0.0200 (0.3399)	0.1878 (0.3256)	-0.1707 (0.4503)	1.0675 (1.2817)
Long-term bonds	-0.0272 (0.1568)	-0.0550 (0.1518)	-0.0057 (0.1809)	-0.1571 (0.7925)
Dividend Yield	-0.0022 (0.0124)	0.0032 (0.0113)	0.0055 (0.0162)	0.9924 (0.0566)

Table 7 (continued)

Estimates of Regime Switching Model for Stock and Bond Returns and the Dividend Yield

	Panel B (cont'd) – MMSIAH(4,1) Model			
	Large caps	Small caps	Long-term bonds	Dividend Yield
2. VAR(1) Matrix (cont'd)				
Regime 4 (recovery):				
Large caps	-0.1994 (0.4243)	-0.0419 (0.2394)	0.2603 (0.4992)	-0.0123 (1.3605)
Small caps	0.3832 (0.7902)	-0.1739 (0.4847)	0.0481 (1.0007)	1.1191 (2.6891)
Long-term bonds	-0.1465 (0.3439)	-0.0113 (0.1973)	0.0606 (0.3846)	0.4777 (0.8776)
Dividend Yield	0.0047 (0.0154)	0.0024 (0.0086)	-0.0105 (0.0180)	0.9428 (0.0504)
3. Correlations/Volatilities				
Regime 1 (crash):				
Large caps	0.1206*			
Small caps	0.7530	0.2044*		
Long-term bonds	-0.2128	-0.1487	0.0906*	
Dividend Yield	-0.9289	-0.7885	0.1688	0.0056
Regime 2 (slow growth):				
Large caps	0.0896***			
Small caps	0.7496***	0.1513***		
Long-term bonds	0.2344	0.0006	0.0431***	
Dividend Yield	-0.9322***	-0.7939***	-0.1808	0.0027***
Regime 3 (bull):				
Large caps	0.1224***			
Small caps	0.7524***	0.1239***		
Long-term bonds	0.1083**	0.1450	0.0577***	
Dividend Yield	-0.9099***	-0.7261***	-0.1174	0.0043***
Regime 4 (recovery):				
Large caps	0.1191*			
Small caps	0.3668	0.2189***		
Long-term bonds	0.2600	-0.1320	0.0949**	
Dividend Yield	-0.9312*	-0.5573	-0.1909	0.0041*
3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1 (crash)	0.4606 (0.1868)	0.0623 (0.1117)	4.51e-19 (0.0733)	0.4771
Regime 2 (slow growth)	2.29e-05 (0.0541)	0.9151 (0.0670)	9.07e-15 (0.0440)	0.0848
Regime 3 (bull)	0.0598 (0.0727)	5.71e-22 (0.0106)	0.9329 (0.0696)	0.0074
Regime 4 (recovery)	0.3223 (0.1939)	0.0809 (0.0935)	0.1160 (0.1063)	0.4808

* significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level.

Figure 1

Smoothed State Probabilities from Two-State Models for Stock and Bond Returns

The graphs plot the smoothed probability of regime 1 estimated from the Markov switching model

$$y_{it} = \mu_{is_{it}} + c_{is_{it}} u_{it},$$

where s_{it} is governed by an unobservable first-order Markov chain that can assume two distinct values (states). u_{it} is IIN(0,1). i =1, 2, 3 are indexes for returns on large stocks, small stocks and 10-year T-bonds portfolio. The data are monthly and obtained from the CRSP tapes. Excess returns are calculated as the difference between portfolio returns and the 30-day T-bill rate. The sample period is 1954:01 – 1999:12.

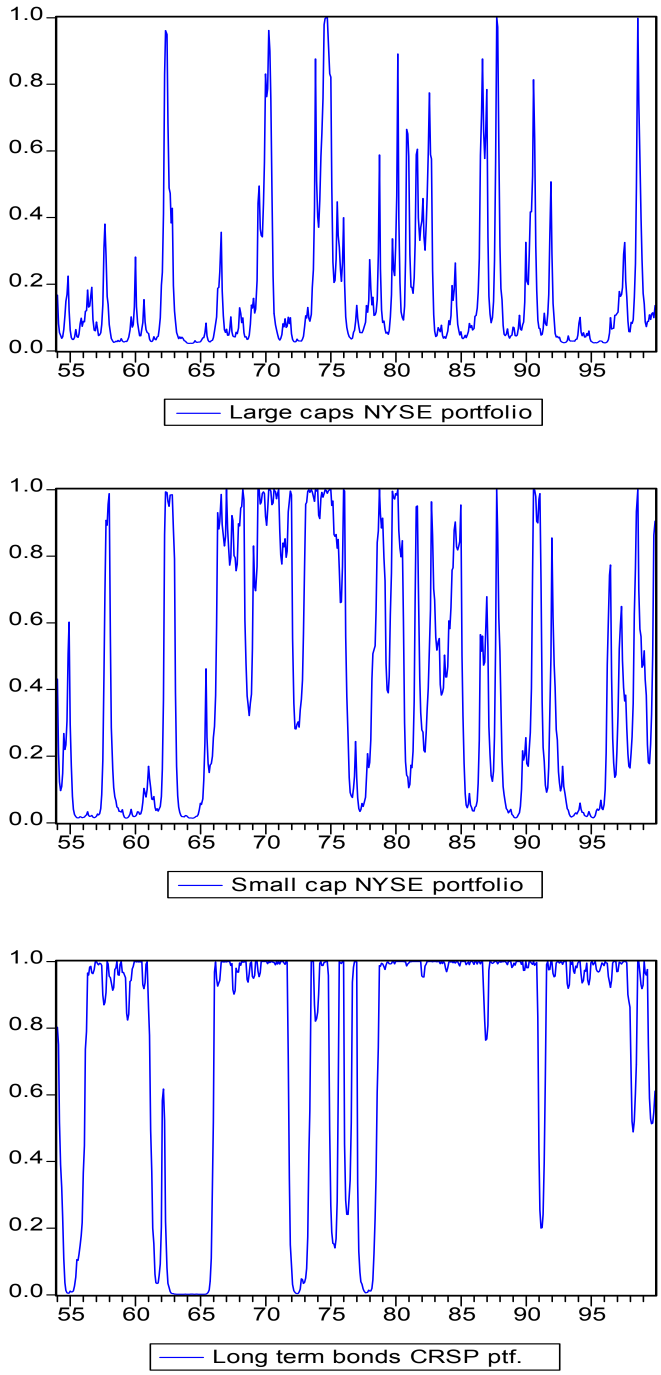

Figure 2

Smoothed State Probabilities from Two-State Models for Stock and Bond Returns

The graphs plot pair-wise scatter diagrams of smoothed probabilities of state 1 estimated from the Markov switching model

$$y_{it} = \mu_{is_{it}} + \sigma_{is_{it}} u_{it},$$

where s_{it} is governed by an unobservable, discrete, first-order Markov chain that can assume two distinct values (states). u_{it} is IIN(0,1). i =1, 2, 3 indexes excess returns on a large cap portfolio (ninth and tenth size deciles), a small cap portfolio (first and second deciles), and a 10-year T-bond portfolio. The data are monthly and obtained from the CRSP tapes. Excess returns are calculated as the difference between portfolio returns and the 30-day T-bill rate. The sample period is 1954:01 – 1999:12.

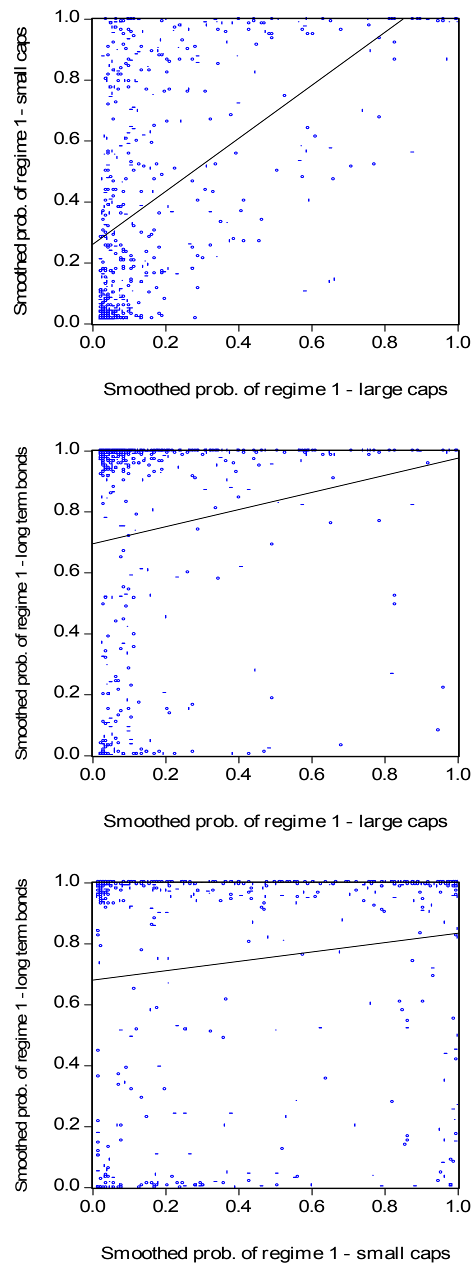

Figure 3

Smoothed State Probabilities from Three-State Models for Stock and Bond Returns

The graphs plot the smoothed probability of regimes 1-3 from the Markov switching model

$$y_{it} = \mu_{is_t} + \sigma_{is_t} u_{it},$$

where s_t is governed by an unobservable, first-order Markov chain that can assume three distinct values (states). u_{it} is IIN(0,1). j =1, 2, 3 are indexes for returns on large stocks, small stocks and 10-year T-bonds. The data are monthly and obtained from the CRSP tapes. Excess returns are calculated as the difference between portfolio returns and the 30-day T-bill rate. The sample period is 1954:01 – 1999:12.

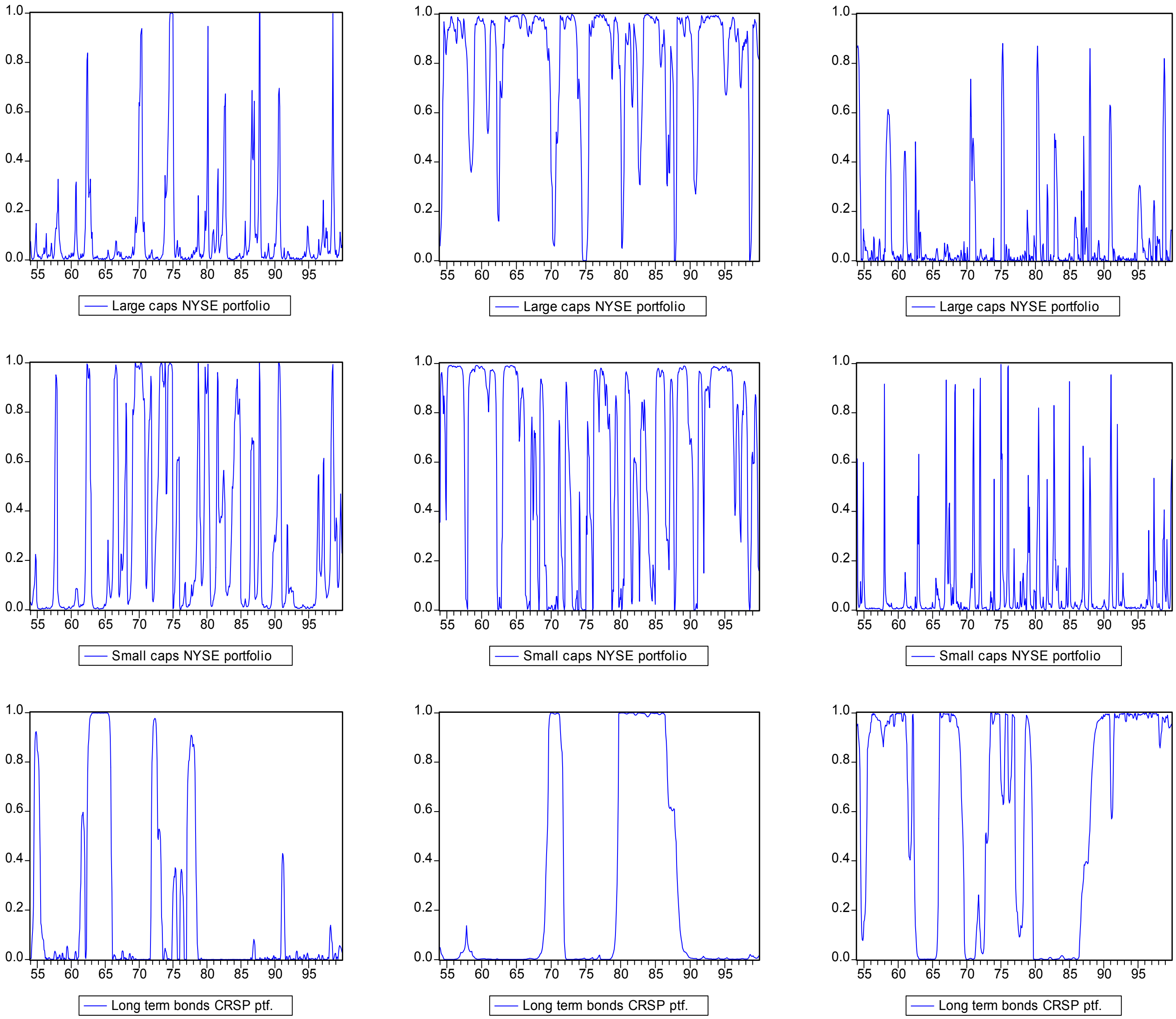

Figure 4

Smoothed State Probabilities: Four-State Model for Stock and Bond Returns

The graphs plot the smoothed probabilities of regimes 1-4 for the multivariate Markov Switching model comprising returns on large and small firms and 10-year T-bonds all in excess of the return on 30-day T-bills.

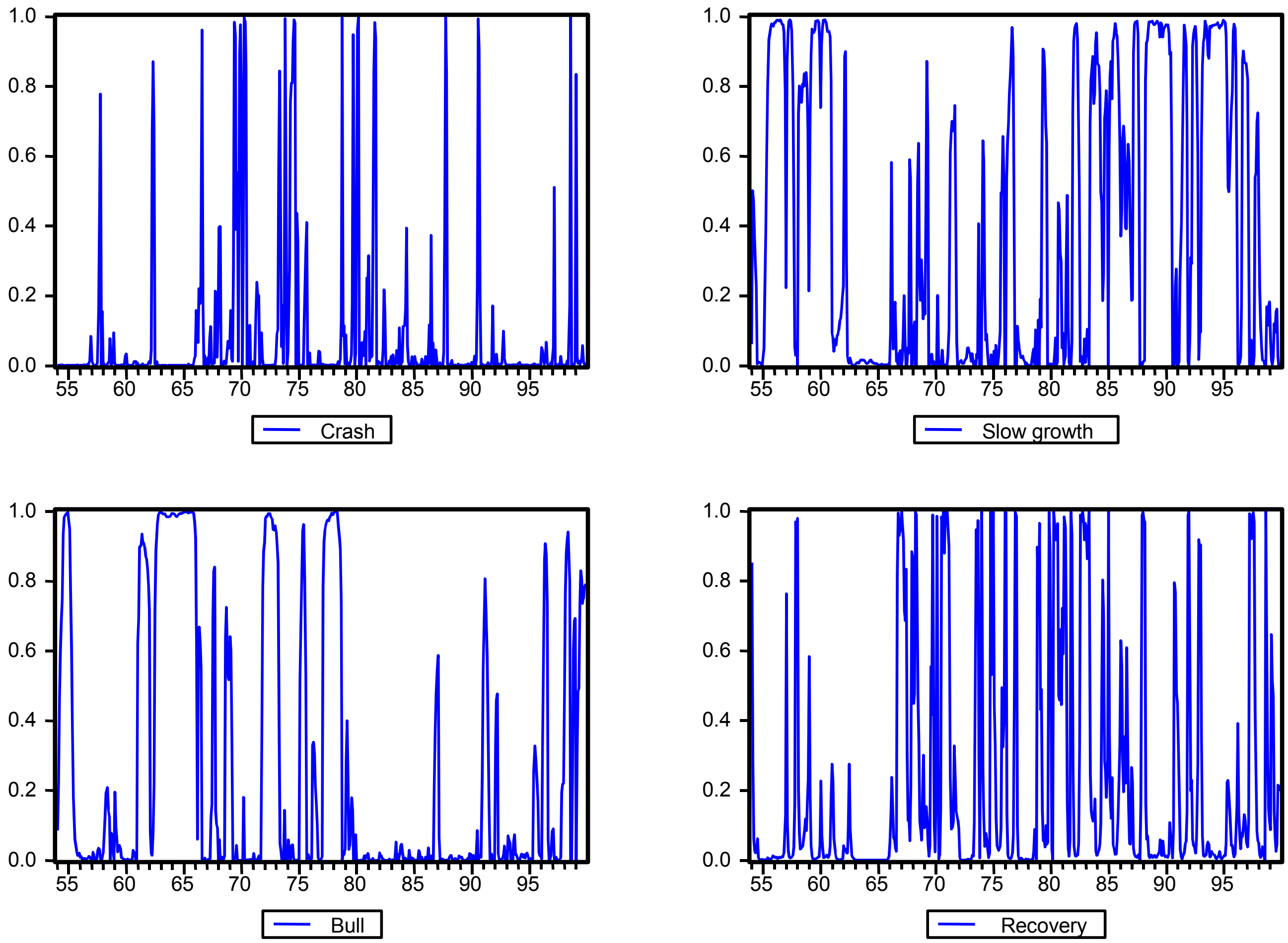

Figure 5

Smoothed State Probabilities: Four-State Model for Stock and Bond Returns and the Dividend Yield

The graphs plot the smoothed probabilities of regimes 1-4 for the multivariate Markov Switching model comprising returns on large and small firms and 10-year bonds all in excess of the return on 30-day T-bills and extended by the dividend yield.

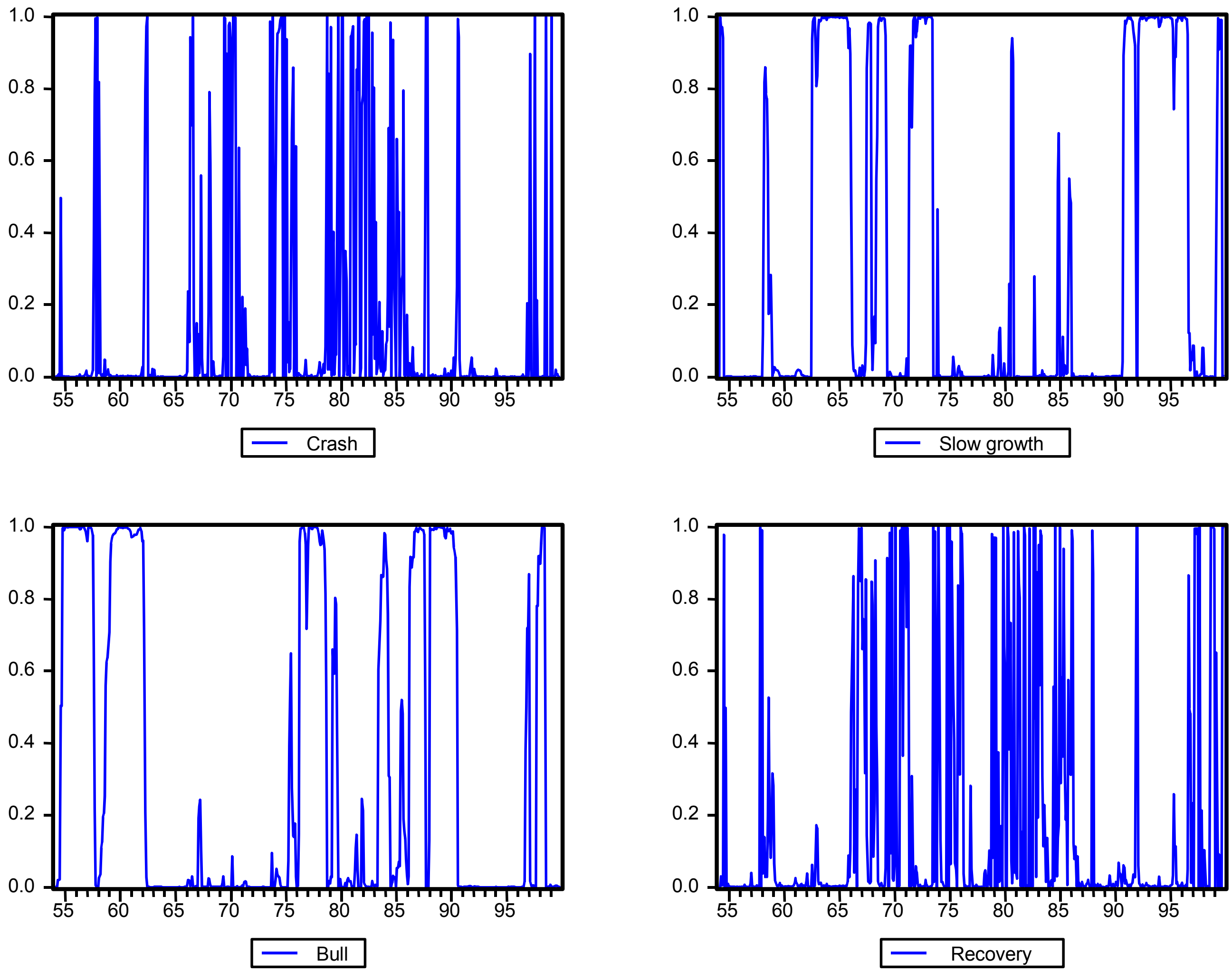

Figure 6

Expected Returns from Four-state Markov Switching Model and from 5-Factor Fama-French Linear Model