An Economic Dispatch Algorithm as Combinatorial Optimization Problems

Kyung-Il Min, Su-Won Lee, and Young-Hyun Moon*

Abstract: This paper presents a novel approach to economic dispatch (ED) with nonconvex fuel cost function as combinatorial optimization problems (COP) while most of the conventional researches have been developed as function optimization problems (FOP). One nonconvex fuel cost function can be divided into several convex fuel cost functions, and each convex function can be regarded as a generation type (G-type). In that case, ED with nonconvex fuel cost function can be considered as COP finding the best case among all feasible combinations of G-types. In this paper, a genetic algorithm is applied to solve the COP, and the λ-P table method is used to calculate ED for the fitness function of GA. The λ-P table method is reviewed briefly and the GA procedure for COP is explained in detail. This paper deals with three kinds of ED problems, namely ED considering valve-point effects (EDVP), ED with multiple fuel units (EDMF), and ED with prohibited operating zones (EDPOZ). The proposed method is tested for all three ED problems, and the test results show an improvement in solution cost compared to the results obtained from conventional algorithms.

Keywords: Combinatorial optimization problems (COP), economic dispatch (ED), function optimization problems (FOP), genetic algorithm (GA), multiple fuel units, prohibited operating zones, valve point effects, λ-P table method.

1. INTRODUCTION

Economic dispatch (ED) is defined as finding the optimal distribution of system load to generators to minimize total generation cost. Generally, ED problems with quadratic fuel cost function can be solved by the Lagrangian multiplier method [1]. In real power systems, however, nonconvexity such as prohibited operating zones, valve-point effect, and multifuel options should be considered in ED. In recent decades, a considerable number of studies have been conducted on ED with nonconvex fuel cost functions. Most of these researches are based on heuristic optimization techniques such as genetic algorithm (GA) [2-4], simulated annealing (SA) [5], Hopfield neural network (HNN) [6], tabu search (TS) [7], evolutionary programming (EP) [8-10], and particle swarm optimization (PSO) [11-14].

Optimization problems seem to divide naturally into two categories: those with continuous variables, and those with discrete variables - that is, function optimization problems (FOP) and combinatorial optimization problems (COP), respectively [15]. All heuristic approaches published up to the present have regarded ED with nonconvex fuel cost functions as FOP. Assuming that a generator has several smooth fuel cost functions dividing the nonconvex fuel cost function, the problem can be considered as a COP, which finds the best case among all feasible combinations. To take the case of GA, FOP encodes a chromosome as all output powers of generators, but COP as generation type (G-type) which is a convex fuel cost function. To calculate fitness function, COP can use conventional ED methods guaranteeing local optimum with respect to a combination of G-types.

COP can be solved by mixed integer programming (MIP) [16] or Lagrangian relaxation (LR) [16,17] as well as various heuristic approaches. However, this paper adopts GA, renowned as a highly efficient heuristic approach for COP.

The proposed algorithm utilizes the λ-P table method [18] to calculate the fitness function of GA. The λ-P table method uses cost tables by sampling the incremental cost function, which can be applied to ED with non-quadratic fuel cost functions. If practical fuel cost curves are directly sampled instead of converted to polynomial functions, approximated error can be remarkably reduced. Moreover, the λ-P table method is useful for repeating the ED process since it is fast

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and easy to treat constraints such as generating capacity limits.

This paper deals with three kinds of ED with nonconvex fuel cost functions:

- ED considering valve-point effects (EDVP)
- ED with multiple fuel units (EDMF)
- ED with prohibited operating zones (EDPOZ)

The proposed algorithm is applied to test systems for three ED problems and compared to other heuristic approaches.

2. FORMULATION OF ED PROBLEMS

2.1. Formulation of the ED Problem

The ED can be formulated as an optimization as follows:

$$Min \sum_{i=1}^{n_g} F_i(P_i), \tag{1}$$

$$s.t. \sum_{i=1}^{n_g} P_i = P_D, \tag{2}$$

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad \text{for } i = 1,\ldots,n_g, \tag{3}$$

where

F_i	fuel cost function of generator i
P_i	power output of generator i
P_D	total system demand
$P_i^{\min}$	minimum output of generator i
$P_i^{\max}$	maximum output of generator i
n_g	number of generators.

For simplicity, the system loss is omitted here with the assumption of P_D accounting for the system loss. The fuel cost function may have a high degree of nonlinearity. However, the cost function is usually approximated as a second order polynomial for practical field applications as in [1].

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2, \tag{4}$$

where a_i, b_i, and c_i are the cost coefficients of the generator i.

2.2. EDVP

The fuel cost function considering valve-point effects is given as [2]

$$\begin{aligned} F_i(P_i) &= a_i + b_i P_i + c_i P_i^2 \\ &\quad + \left| e_i \sin\left(f_i\left(P_{i\min} - P_i\right)\right)\right|, \end{aligned} \tag{5}$$

where a_i, b_j, and c_i are the cost coefficients of the i-th generator, and e_j, and f_i are the cost coefficients of the i-th generator with valve-point effects.

2.3. EDMF

The ED problem with multiple fuel units can be formulated by using piecewise quadratic functions [19]. In this case, the fuel cost has the following form.

$$F_i(P_i) = \begin{cases} a_{i1} + b_{i1}P_i + c_{i1}P_i^2 & \text{if } P_{i1}^{\min} \le P_i \le P_{i1}^{\max} \\ a_{i2} + b_{i2}P_i + c_{i2}P_i^2 & \text{if } P_{i2}^{\min} \le P_i \le P_{i2}^{\max} \\ \vdots & \vdots \\ a_{in} + b_{in}P_i + c_{in}P_i^2 & \text{if } P_{in}^{\min} \le P_i \le P_{in}^{\max}, \end{cases} \tag{6}$$

where a_{ij}, b_{ij}, and c_{ij} are the cost coefficients of fuel j for unit i and $P_{ij}^{\max}$ is equal to $P_{i,j+1}^{\min}$.

2.4. EDPOZ

The fuel cost function of the generator with POZ is represented as follows [20,21].

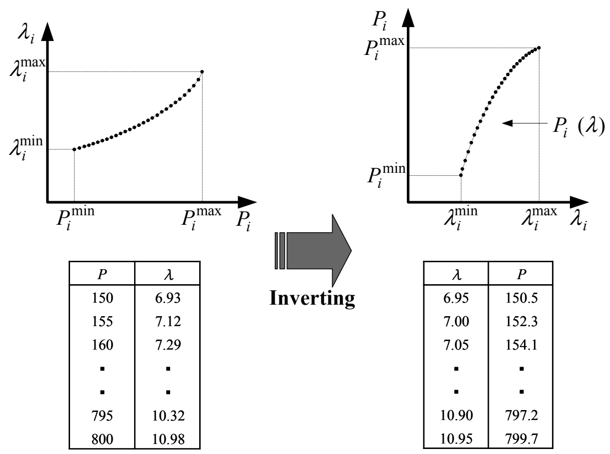
$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i, \begin{cases} P_i^{\min} \le P_i \le P_{i,1}^{l} \\ P_{i,j-1}^{u} \le P_i \le P_{i,j}^{l} \\ P_{i,n_{Pi}}^{u} \le P_i \le P_i^{\max}, \end{cases} \tag{7}$$

where $P_{i,j}{}^{l}$ and $P_{i,j}{}^{u}$ are the lower and upper bounds of the j-th POZ of unit i, and n_{Pi} is number of POZs in unit i.

3. OVERVIEW OF ED ALGORITHM BY Λ-P TABLE METHOD

The λ-P table method [18] is based on the duality theory and its fundamental principle is found in [22], [23]. The main feature of this method is to use the inverse of the incremental fuel cost tables sampled in regular intervals, as illustrated in Fig. 1. The inverse tables can be easily obtained by linear interpolation.

This method is developed on the basis that each



P	λ
150	6.93
155	7.12
160	7.29
.	.
.	.
795	10.32
800	10.98

λ	P
6.95	150.5
7.00	152.3
7.05	154.1
.	.
.	.
10.90	797.2
10.95	799.7

Fig. 1. Sampling and inverting process of the incremental cost function using duality theory.

output power of the generators can be determined by the incremental cost λ. Once the incremental cost λ is determined, then the total generating power, P_{Gttl}, can be directly calculated and can be denoted as a function of λ by:

$$P_{Gttl}(\lambda) = \sum_{i=1}^{n_g} P_{Gi}(\lambda). \tag{8}$$

Here, it is noted that $P_{Gttl}(\lambda)$ is nondecreasing. Given the total demand of the system, the optimal incremental cost λ^* can be obtained by solving the following.

$$P_{Gttl}(\lambda) = \sum_{i=1}^{n_g} P_{Gi}(\lambda) = P_D \tag{9}$$

The nondecreasing property of P_{Gttl} allows utilization of the bisection or linear interpolation methods in order to obtain the optimal incremental cost λ^*. It should be noted that the Kuhn-Tucker conditions need not be checked, since $P_{Gi}(\lambda)$ provides all the information of the limitation of the generation outputs and the must-run conditions. Fig. 2 shows an illustrative example with a 3-generator system. Gen. 1 and Gen. 3 are operated in must-run condition where each generator must produce its minimum output, while Gen. 2 is stopped because its economical efficiency is below a certain marginal cost.

The λ-P table method is composed of the following 4 steps:

Step 1: Establish the P-λ tables by sampling the incremental fuel cost function, and construct the λ-P tables by interpolating the P-λ tables for all of the generators.

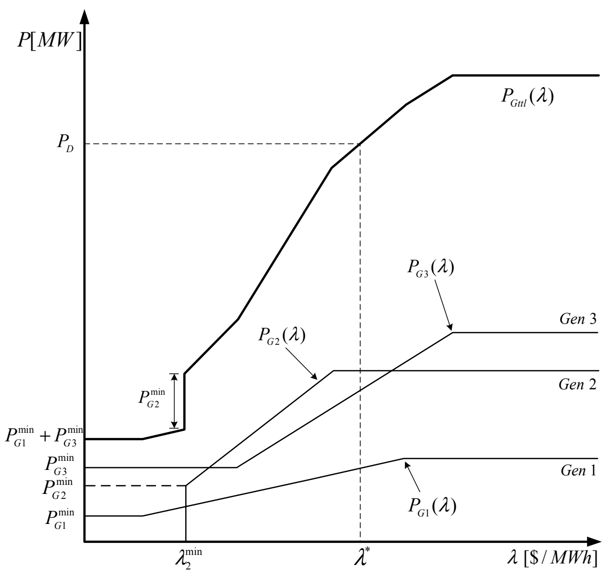


Fig. 2. The summation of three generators' output power.

Step 2: Construct the total generation table $P_{Gttl}(\lambda)$ by summing up the λ-P tables for all the generators.

Step 3: Calculate the optimal λ^* by solving (9) and by using the bisection method and/or linear interpolation.

Step 4: Calculate the optimal dispatch for each generator with $P_{Gi}(\lambda^*)$.

In case of considering the effect of line losses, the above method can be employed in the same manner by applying penalty factors to the λ-P table. The detailed explanation will not be treated here.

4. GA FOR COMBINATORIAL OPTIMIZATION PROBLEMS

GA is renowned as an efficient method to resolve COP, and a variety of GA strategies have been conducted. However, there is a typical GA structure commonly revealed [24,25]. The execution of GA iteration is basically a two stage process. It starts with the current population. Selection is applied to create an intermediate population (mate pool). Then, crossover and mutation are applied to the intermediate population to create the next generation of potential solutions. The coding scheme and the fitness function are the most important aspects of any GA, which are problem dependent. In this section, we will examine the proposed genetic algorithms for combinatorial optimization problems (GA-COP).

4.1 Generation type and encoding

A G-type means the ordered number to each convex curve when a nonconvex fuel cost function is divided into several monotonous convex curves. That is, G-type is valve number in EDVP, fuel option number in EDMF, and operating zone number in EDPOZ, as illustrated in Fig. 3. In case of EDVP, minimum and maximum limit power is added at every convex curve.

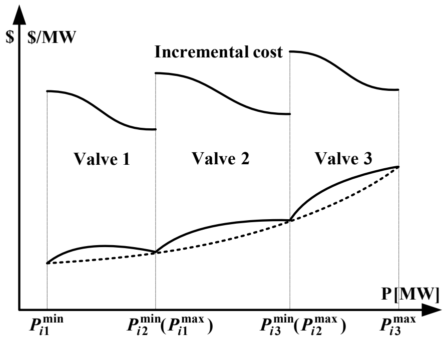
In the majority of GA applications, the chromosomes use a binary alphabet and their length is constant during the whole generation process. In FOP, a chromosome is encoded as a generation vector including all generators as illustrated in Fig. 4(a) [2]. Fig. 4(b) represents integer implementation of COP, in which each gene corresponds to G-types of generators.

The length of COP encoding is much shorter than the FOP. The short encoding length may cause a rise in the probability of premature convergence. To improve this problem, mutation rate or population size should be raised.

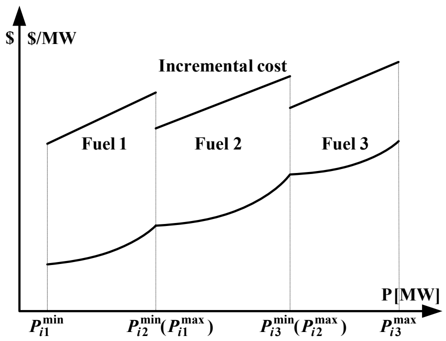
4.2 Fitness function

Fitness function of the proposed GA-COP is represented as

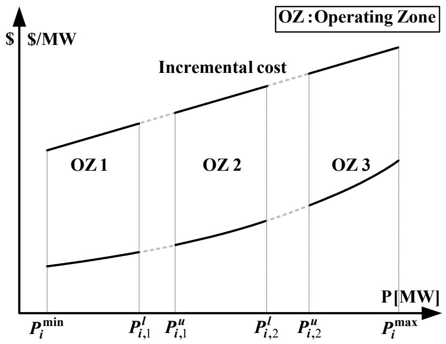
$$f_i = (C_w - C_i) + (C_w - C_b)/(p_s - 1), \ \ p_s > 1, \tag{10}$$



(a) EDVP.

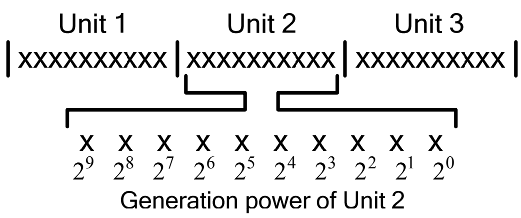


(b) EDMF.

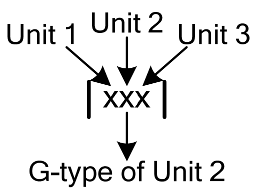


(c) EDPOZ.

Fig. 3. Fuel cost functions and incremental fuel cost functions of EDVP, EDMF, and EDPOZ.



(a) FOP – Binary implementation.



(b) COP – Integer implementation.

Fig. 4. Encoding illustrations in a 3-generator system.

```
produce n-initial chromosomes;
calculate ED and fitness function;
repeat {
        for i = 1 to k {
            select two chromosomes, p1, p2;
            offspring_i = crossover(p1, p2);
            offspring_i = mutation(offspring_i);
        }
        replace offspring_1, …, offspring_k with
            k-chromosomes in the population;
        calculate ED and fitness function;
} until (termination-condition);
return the best chromosome;
```

Fig. 5. Fundamental structure of GA-COP.

where C_w, C_b, and C_i are the worst cost in the solution set, the best cost in the solution set, and the cost of the i-th solution, respectively. Selection pressure, p_s is the degree to which superior chromosomes are favored. The higher the selection pressure, the more superior chromosomes are favored.

4.3 Procedures of GA-COP

4.3.1 Pre-processing

To apply the λ-P table method to ED for a G-type arbitrary combination, it is necessary to construct the λ-P table prior to the ED. Incremental fuel cost functions of all generators are sampled with respect to P at regular intervals, and P-λ tables are constructed. And then, λ-P tables, equivalent to the inverse function of incremental fuel cost, are obtained by linearly interpolating the P-λ tables with respect to λ at regular intervals.

4.3.2 GA processing

The procedure of GA-COP is similar to general GA procedures. Operators of GA-COP are selected as generally used ones - roulette wheel selection, k-point crossover, and uniform mutation. According to the length of the chromosomes, k is adjusted to 2~3. The chromosomes are randomly initialized to 0 or 1 for every gene satisfying that the total load is within the generation capacity of the chromosome. Fitness function is calculated by applying the cost of ED to (10). The fundamental structure of GA-COP is illustrated in Fig. 5.

5. CASE STUDIES

The proposed GA-COP was directly coded using real values and was implemented on a personal computer (Pentium D CPU 3.00 GHz) in Microsoft Visual C++ 6.0. To construct the P-λ table and λ-P table, the sampling interval is set to 0.001.

5.1 EDVP

GA-COP is applied to test the ED with a 40-

generator system [9]. The total system demand is set to 10500 MW. In this test, 3-point crossover is adopted, and GA parameters are given as follows.

- p_s	5.0
- crossover rate	0.3
- mutation rate	0.1
- population	800

The test results of the proposed GA-COP are compared with CEP [9], FEP [9], MFEP [9], IFEP [9], MPSO [11], PSO_SQP [12], and NPSO-LRS [13].

Mean cost, maximum cost, and minimum cost of the proposed method and other heuristic methods for 100 trials are summarized in Table 1. Minimum cost of the proposed method is \$121,525.23, which is the best solution in comparison with other methods.

Table 2 represents relative frequency of convergence after 100 trials. All trials of the proposed GA-

Table 1. Comparison of the test results after 100 trials.

Method	Mean cost (\$)	Maximum cost (\$)	Minimum cost (\$)
CEP	124793.48	126902.89	123488.29
FEP	124119.37	127245.59	122679.71
MFEP	123489.74	124356.47	122647.57
IFEP	123382.00	125740.63	122624.35
MPSO	-	-	122252.27
PSO_SQP	-	-	122094.67
NPSO-LRS	122209.32	122981.59	121664.43
GA-COP	121714.52	122243.37	121525.23

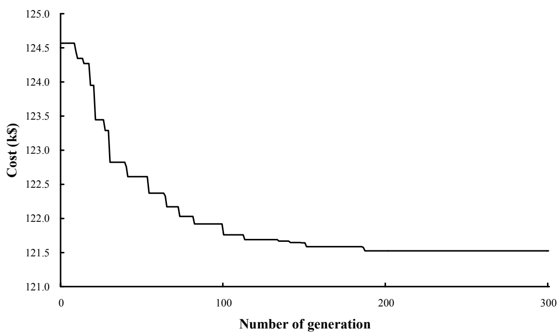


Fig. 6. Convergence characteristics of GA-COP.

Table 3. Generation, cost, valve number, and number of valves of each generator in a 40-generator system.

Gen. No.	Generation	Cost	Valve no. (G-type)	Number of valves
1	110.7548	925.1027	2	3
2	113.955	977.418	3	3
3	97.3549	1190.431	1	2
4	179.6881	2143.455	2	3
5	96.955	852.5211	2	2
6	139.955	1596.326	2	2
7	259.5547	2612.818	2	3
8	284.5547	2779.774	2	3
9	284.5547	2798.165	2	3
10	204.7548	3618.179	1	3
11	168.7548	2959.178	1	4
12	168.7548	2977.171	1	4
13	214.7148	3791.899	1	5
14	394.2344	6414.668	2	5
15	304.4746	5171.065	2	5
16	304.4746	5171.065	3	5
17	489.2344	5296.687	3	4
18	489.2344	5288.742	3	4
19	511.2344	5540.899	3	4
20	511.2344	5540.879	3	4
21	523.2344	5071.324	3	4
22	523.2344	5071.324	3	4
23	523.2259	5057.27	3	4
24	523.2259	5057.27	3	4
25	523.2344	5275.111	3	4
26	523.2344	5275.111	3	4
27	10	1140.524	1	4
28	10	1140.524	1	4
29	10	1140.524	1	4
30	96.955	852.5211	2	2
31	189.955	1643.814	3	3
32	189.955	1643.814	3	3
33	189.955	1643.814	3	3
34	164.7548	1585.518	1	2
35	164.7548	1539.859	1	2
36	164.7548	1539.859	1	2
37	109.955	1219.903	3	3
38	109.955	1219.903	3	3
39	109.955	1219.903	3	3
40	511.2344	5540.899	3	4
Total	10500	121525.2		

Table 2. Relative frequency of convergence in the ranges of cost for EDVP.

Method	Range of Cost [k\$]									
	127.0 ~ 126.5	126.5 ~ 126.0	126.0 ~ 125.5	125.5 ~ 125.0	125.0 ~ 124.5	124.5 ~ 124.0	124.0 ~ 123.5	123.5 ~ 123.0	123.0 ~ 122.5	122.5 ~ 120.0
CEP	10	4	-	16	22	42	4	2	-	-
FEP	6	-	4	2	10	20	26	24	6	-
MFEP	-	-	-	-	-	14	26	50	10	-
IFEP	-	-	2	-	4	4	18	50	22	-
MPSO	-	-	-	-	-	-	-	-	53	47
NPSO-LRS	-	-	-	-	-	-	-	-	21	79
GA-COP	-	-	-	-	-	-	-	-	-	100

COP are included from \$120,000 to \$122,500. From Tables 1 and 2, the results show that the proposed method has the most predominant convergence characteristics in comparison with other methods.

Fig. 6 illustrates convergence characteristics of the GA-COP. The horizontal axis is the generation number and the vertical axis is the corresponding cost. The cost is decreased drastically up to around the 100th iteration and converged at round 200th iteration as seen in Fig. 6. The generation outputs, the costs, and the valve number (G-type) of the best solution are provided in Table 3.

5.2 EDMF

The proposed GA-COP is applied to the ED problems with a 10-generator system [19]. During the tests, the total system demand is varied from 2400 MW to 2700 MW with 100 MW increments. In this test, 2-point crossover is adopted, and GA parameter is set as follows.

- p_s 2.0
- crossover rate 0.2
- mutation rate 0.1
- population 100

The solution is compared with results of various heuristic approaches including HM [19], IEP [8], IGA_MU [4], AHNN [6], MPSO [11], AIS [26], and A-Life [27]. In the IGA_MU and A-Life, the results are compared with the case whose total demand is 2700 MW and 2400 MW respectively, since the cited papers provide only these cases of the solutions. The results of the proposed algorithms and the various heuristic approaches mentioned above are summarized in Tables 4-7.

As shown in Tables 4-7, the GA-COP provides the best solution except the case of 2600 MW of HM and from 2400 MW to 2600 MW of AHNN.

Table 8 provides frequency of convergence after 100 trials. All solutions are converged to the cases

Table 4. Comparison of conventional methods and GA-COP for EDMF (Demand = 2400 MW).

S	U	HM		AHNN		IEP		MPSO		AIS		A-Life		GA-COP	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	1	193.2	1	189.1	1	190.9	1	189.7	1	189.68	1	189.74	1	189.74
	2	1	204.1	1	202.0	1	202.3	1	202.3	1	202.40	1	202.34	1	202.34
	3	1	259.1	1	254.0	1	253.9	1	253.9	1	253.81	3	253.90	1	253.90
	4	3	234.3	3	233.0	3	233.9	3	233.0	3	233.02	3	233.05	3	233.05
2	5	1	249.0	1	241.7	1	243.8	1	241.8	1	241.94	1	241.83	1	241.83
	6	1	195.5	1	233.0	3	235.0	3	233.0	3	233.06	3	233.05	3	233.05
	7	1	260.1	1	254.1	1	253.2	1	253.3	1	253.37	1	253.27	1	253.27
3	8	3	234.3	3	232.9	3	232.8	3	233.0	3	232.85	3	233.05	3	233.05
	9	1	325.3	1	320.0	1	317.2	1	320.4	1	320.45	1	320.38	1	320.38
	10	1	246.3	1	240.3	1	237.0	1	239.4	1	239.40	1	239.40	1	239.40
TP		2401.2		2400		2400		2400		2400		2400		2400	
TC		488.5		481.7		481.779		481.723		481.723		481.72		481.723	

Table 5. Comparison of conventional methods and GA-COP for EDMF (Demand = 2500 MW).

S	U	HM		AHNN		IEP		MPSO		AIS		GA-COP	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	2	206.6	2	206.0	2	203.1	2	206.5	1	205.88	2	206.52
	2	1	206.5	1	206.3	1	207.2	1	206.5	1	206.33	1	206.46
	3	1	265.9	1	265.7	1	266.9	1	265.7	3	266.48	1	265.74
	4	3	236.0	3	235.7	3	234.6	3	236.0	3	235.79	3	235.95
2	5	1	258.2	1	257.9	1	259.9	1	258.0	1	256.87	1	258.02
	6	3	236.0	3	235.9	3	236.8	3	236.0	3	236.65	3	235.95
	7	1	269.0	1	269.6	1	270.8	1	268.9	1	269.20	1	268.86
3	8	3	236.0	3	235.9	3	234.4	3	235.9	3	235.51	3	235.95
	9	1	331.6	1	331.4	1	331.4	1	331.5	1	332.23	1	331.49
	10	1	255.2	1	255.4	1	254.9	1	255.1	1	255.02	1	255.06
TP		2501.1		2500		2500		2500		2500		2500	
TC		526.7		526.23		526.304		526.239		526.24		526.239	

Table 6. Comparison of conventional methods and GA-COP for EDMF (Demand = 2600 MW).

S	U	HM		AHNN		IEP		MPSO		AIS		GA-COP	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	2	216.4	2	215.8	2	213.0	2	216.5	2	216.01	2	216.54
	2	1	210.9	1	210.7	1	211.3	1	210.9	1	210.77	1	210.91
	3	1	278.5	1	279.1	1	283.1	1	278.5	3	278.73	1	278.54
	4	3	239.1	3	239.1	3	239.2	3	239.1	3	239.47	3	239.1
2	5	1	275.4	1	276.3	1	279.3	1	275.5	1	275.25	1	275.52
	6	3	239.1	3	239.1	3	239.5	3	239.1	3	238.55	3	239.10
	7	1	285.6	1	286.0	1	283.1	1	285.7	1	286.55	1	285.72
3	8	3	239.1	3	239.1	3	239.2	3	239.1	3	239.27	3	239.10
	9	1	343.3	1	342.8	1	340.5	1	343.5	1	343.07	1	343.49
	10	1	271.9	1	271.9	1	271.9	1	272.0	1	272.32	1	271.99
TP		2600		2600		2600		2600		2600		2600	
TC		574.03		574.37		574.473		574.381		574.381		574.381	

Table 7. Comparison of conventional methods and GA-COP for EDMF (Demand = 2700 MW).

S	U	HM		AHNN		IEP		MPSO		IGA_MU		AIS		GA-COP	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	2	218.4	2	225.7	2	219.5	2	218.3	2	218.12	2	218.38	2	218.25
	2	1	211.8	1	215.2	1	211.4	1	211.7	1	211.68	1	211.66	1	211.66
	3	1	281.0	1	291.8	1	279.7	1	280.7	1	280.86	3	280.54	1	280.72
	4	3	239.7	3	242.3	3	240.3	3	239.6	3	239.65	3	239.69	3	239.63
2	5	1	279.0	1	293.7	1	276.5	1	278.5	1	278.63	1	278.30	1	278.50
	6	3	239.7	3	242.3	3	239.9	3	239.6	3	239.61	3	239.65	3	239.63
	7	1	289.0	1	302.8	1	289.0	1	288.6	1	288.57	1	288.57	1	288.58
3	8	3	239.7	3	242.3	3	241.3	3	239.6	3	239.71	3	239.84	3	239.63
	9	3	429.2	1	355.1	3	425.1	3	428.5	3	428.45	3	428.42	3	428.52
	10	1	275.2	1	288.8	1	277.2	1	274.9	1	274.7	1	274.95	1	274.87
TP		2702.2		2700		2700		2700		2700		2700		2700	
TC		625.18		626.24		623.851		623.809		623.8093		623.809		623.8092	

Table 8. Frequency of convergence after 100 trials for EDMF.

Demand [MW]	Cost [$]	Hit	Fuel combination (G-type)
2400	481.7226	91	1 1 1 3 1 3 1 3 1 1
	481.8281	7	2 1 1 3 1 3 1 3 1 1
	486.3992	2	2 1 1 3 1 3 1 3 3 1
2500	526.2388	86	2 1 1 3 1 3 1 3 1 1
	526.4551	11	1 1 1 3 1 3 1 3 1 1
	528.8229	3	2 1 1 3 1 3 1 3 3 1
2600	574.3808	100	2 1 1 3 1 3 1 3 1 1
2700	623.8092	100	2 1 1 3 1 3 1 3 3 1

given in Table 8. In the cases of 2400 MW and 2500 MW, the best solutions are hit 91 and 86 times respectively, and the remaining solutions are also considerably close to the best solutions. In cases of 2600 MW and 2700 MW, all 100 trials hit the best solutions.

5.3 EDPOZ

The FM algorithm for EDPOZ is applied to a 15-generator system [20]. Load demand is set up by 2650 MW. 2-point crossover is adopted, and GA parameters are given as follows.

- p_s 2.0
- crossover rate 0.2
- mutation rate 0.1
- population 100

The prohibited operating zones are described in Table 9. As seen in Table 10, the results of the proposed algorithm are compared with the λ-δ iterative method [20], FCEP [28], and MIQP [29]. All three methods obtain the same generations and costs except the cost of the λ-δ iterative method.

The best solution is 32544.97 $/h, and the

Table 9. Prohibited operating zones of 15-generator system.

Unit	Zone 1 [MW]	Zone 2 [MW]	Zone 3 [MW]
2	[185, 225]	[305, 335]	[420, 450]
5	[180,200]	[260, 335]	[390, 420]
6	[230, 255]	[365, 395]	[430, 455]
12	[30, 55]	[65, 75]	-

Table 10. Comparison of generations and costs for EDPOZ.

Unit No.	$\lambda - \delta$ Iterative method [MW]	FCEP [MW]	MIQP [MW]	GA-COP [MW]
1	450	450	450	450
2	450	450	450	450
3	130	130	130	130
4	130	130	130	130
5	335	335	335	335
6	455	455	455	455
7	465	465	465	465
8	60	60	60	60
9	25	25	25	25
10	20	20	20	20
11	20	20	20	20
12	55	55	55	55
13	25	25	25	25
14	15	15	15	15
15	15	15	15	15
Cost [$/h]	32549.8	32544.97	32544.97	32544.97

corresponding combination of operating zones (G-type) is [1 4 1 1 3 4 1 1 1 1 1 2 1 1 1]. The proposed algorithm to EDPOZ is tested 100 times, and all the trials hit the best solution.

6. CONCLUSIONS

This paper presents a novel approach to ED with nonconvex fuel cost functions as a COP instead of as a FOP, as adopted by other heuristic approaches. To solve COP and calculate fitness functions, GA and the λ-P table method are used respectively. The proposed algorithm is applied to EDVP, EDMF, and EDPOZ, and simulated to compare the test results with various heuristic approaches. The proposed algorithms have provided superior solutions to other heuristic approaches in most of the cases. The results show that the proposed algorithm is efficient for solving ED with nonconvex fuel cost functions. This paper is the first step in the study of ED with nonconvex fuel cost function as COP. A further direction for this study will be to apply other heuristic approaches as COP.

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