AN EDDY TRANSFER COEFFICIENT MODEL FOR TURBULENT FREE JETS WITH VARIABLE DENSITY

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A generalized eddy transfer coefficient model has been experimentally obtained for axisymmetric, nonisothermal variable-density jets, the initial density ratio of which ranges from 0.66 to 5.1. The local variation in the dynamic eddy transfer coefficients for momentum and heat has been measured in the developed region: both transfer coefficients on the jet axis are proportional to the square root of the initial density ratio, but are almost independent of the axial distance from the nozzle exit; the transfer coefficients for momentum and a chemical species remain constant in the inner region whereas the transfer coefficient for heat decreases slightly with increasing radial distance; all the transfer coefficients decrease rapidly in the outer region owing to the intermittent entrainment of the quiescent ambient fluid. The present model can perdict local values of the transfer coefficients over the range of initial density ratio from 0.66 to 5.1 if the initial jet condition alone is specified at a nozzle exit.

Introduction

If a free jet of gas has a difference in temperature between the jet and the ambient receiving medium, the turbulent transport phenomena greatly deviate from that for a constant-density jet, even when the effect of natural convection is sufficiently small. For a heated jet, owing to the local variation in fluid density, the potential core becomes shorter and the rates of radial spreading and axial decay faster than for incompressible jets.

The target of the present work is to establish a generalized model of turbulent mixing in nonisothermal variable-density flows. Various eddy diffusivity models are available for the analytical description of the free mixing of jet flows.

Prandtl's incompressible turbulent diffusion model[15] gives an eddy diffusivity of the form

$$\varepsilon_M = K_p b |U_m - U_\infty| \tag{1}$$

where K_p is the mixing rate parameter and b the width of the mixing region. For axisymmetric isothermal or incompressible jets exhausting into a stationary atmosphere (i. e. $U_\infty=0$), b increases linearly with the axial distance Z while U_m decreases at a rate of $1/Z$. Hence, ε_M remains constant over the whole fully-developed region. Most of the models proposed for variable-density jet flows are directly or indirectly based upon Prandtl's model. Warren[20] and Donaldson and Gray[3] applied a compressibility correction to the mixing rate parameter of Prandtl's model for high-velocity compressible jet flows. Donaldson and Gray found the correction parameter to be a function of local Mach number at the half-velocity radius. Ferri *et al.*[5] proposed another form of the eddy viscosity modeled along the lines of Prandtl's constant-density model:

$$\rho\varepsilon_M = K_F r_{1/2M} |\rho_\infty U_\infty - \rho_m U_m| \tag{2}$$

They recommended a value of 0.025 for their mixing rate parameter K_F. For a compressible jet exhausting into a quiescent receiving medium of unequal density, the dynamic eddy transfer coefficient $\rho\varepsilon_M$ was expressed in terms of the initial jet condition as

$$\overline{\rho\varepsilon_M} = 0.0125\bar{\rho}_\infty^{1/2} \tag{3}$$

Here, $\bar{\rho}_\infty$ is the jet-to-ambient fluid density ratio called the "initial density ratio". This equation indicates that the dimensionless eddy transfer coefficient is proportional to the square root of the initial density ratio. Still the applicability of this model remains to be ascertained over a wide range of the initial density ratio by taking into account the local variation in the fluid density and velocity of compressible jets. Kleinstein[11] tried to solve the mixing problem in turbulent axisymmetric compressible jets by linearization of the conservation equations in the plane of the von Mises vari-

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ables. He investigated the dynamic eddy transfer coefficient suggested by Ferri *et al.* and took the numerical constant of Eq. (3) as 0.00915 by comparison with Corrsin and Uberoi's heated air jet data[2]. Kleinstein's constant is, however, too small to extrapolate his model to incompressible flows. Tomich and Weger[19] and Witze[21] tried a modification of Kleinstein's model for high subsonic Mach number heated jets exhausting into quiescent air. The Schetz unified model[17] of the eddy transfer coefficient was based upon the idea of compressible displacement thickness, but it was developed for coaxial flows only. Zelazny[22] modified the Schetz unified model for the treatment of a jet flowing into a quiescent atmosphere. Still, his model must be used sufficiently downstream of the initial region. The Ting-Libby model[18] tried to analyze mainly two-dimensional and coaxial variable-density flows by the constant-density flow equations through the use of suitable transformations. Cohen and Guile[1] investigated the dependence of Prandtl's mixing rate parameter upon the density and velocity of the ambient stream by taking into account the effect of preturbulence. The use of their model for a heated jet flowing into a stationary ambient is not adequate. Sforza and Mons[18] extended the Reichardt inductive theory[16] of free turbulence for a variable-density free jet. The transfer coefficient of their conservation equations, which is similar to the eddy transfer coefficient, should also be evaluated by experiment. A more fundamental model is the turbulent kinetic-energy model suggested by Nevzgljadov[14]. According to a simpler model of the turbulent kinetic energy described by Dryden[4], local turbulent shear stress is linearly related to the local kinetic energy of turbulence at the same position. With this hypothetical relationship, an equation for the conservation of turbulent kinetic energy may be used as an additional equation to determine the turbulent shear field. The model was also used by Harsha and Lee[6] for a turbulent free-mixing analysis. In their approach, the definition of the turbulent kinetic energy level was necessary at the initial station. In addition, a difficulty remains in the use of the hypothetical linear relationship between the turbulent shear stress and kinetic energy of turbulence: the situation in which the turbulent shear stress approaches zero while the local kinetic energy of turbulence remains above zero generally exists in the vicinity of zero velocity-gradient of the mean flow. At the present stage, from the practical viewpoint, the kinetic energy model is not yet convenient for compressible heated jets.

All the eddy transfer coefficient models were based upon the assumption of constant transfer coefficients in the radial direction. There exists little work on the eddy transfer coefficient for heat transfer. The previous paper by Kataoka and Takami[8] was on the extension of the model of Ferri *et al.* However, the local eddy transfer coefficients for momentum, heat, and a chemical species were directly measured in a high-temperature turbulent free jet produced by the combustion of methane gas. Still more experimental work is necessary to establish a generalized model.

The objective of the present paper is to obtain a generalized eddy transfer coefficient model applicable over a wide range of initial density ratio, on the lines of the previous paper[8]. Experiments were performed by measuring the axial and radial variations of velocity and temperature in variable-density free jets.

1. Experimental Apparatus and Procedure

The jet mixing experiments were performed by using the following three kinds of free jets exhausting into a quiescent atmosphere: (i) non-isothermal jets of burned gas produced by the combustion of methane, (ii) isothermal jets of air-CO_2 mixture, and (iii) isothermal jets of air. The experimental apparatus is the same as used in the previous paper[9]. In essence, it consists of a vertically mounted nozzle assembly and a measuring system with a traversing device for probes as shown schematically in **Fig. 1**.

The nozzle assembly of stainless steel, shown in **Fig. 2**, consists of a 14 mm ID venturi throat, a 60 mm ID combustion (or calming) chamber with a straight section 135 mm long, and a 10 mm ID circular convergent nozzle with a contraction ratio of 1/36. The inner surface of the convergent nozzle was approximated by a cosine curve so as to make the exit velocity profile uniform. The combustion chamber and the convergent nozzle section were joined by smooth curved surfaces to provide low initial-turbulence jets. The nozzle outlet was arranged flush with, and central to, the horizontal surface of an insulating firebrick, not only to make the exit temperature profile uniform but also to prevent entrainment of ambient air from behind the nozzle.

As shown in the previous paper[8], it was confirmed that the distributions of velocity and temperature were uniform over about 90% of the 10 mm nozzle diameter. For the high-temperature jet experiment, the molar flow rates of methane oxygen, and nitrogen were kept at $CH_4:O_2:N_2=1:3.3:5.2$ to make the nitrogen/oxygen ratio of the burned gas equal to that of the ambient air. That is, excess oxidizer was supplied for complete combustion. It was confirmed by a composition analysis of the gas sample taken at the nozzle exit that no burning mixture issued from the nozzle. The average molecular weight of the burned gas was nearly equal to that of air. For simplicity, the physical properties of air were used as those of the burned gas with negligibly small error. The hot jet of

Table 1 Experimental conditions

Nonisothermal jet of burned gas into air		
	U_o	7.4 –22.7 m/s
	T_o	823 –1531 K
	$\bar{\rho}_\infty$	2.73–4.98
	Re_o	857 –1810
Isothermal jet of air-CO_2 mixture into air		
	U_o	5.76–7.55 m/s
	X_o	0.48–1.0
	$\bar{\rho}_\infty$	0.66–0.80
	Re_o	7100–7600
Isothermal air jet into air		
	U_o	7.4 –12.6 m/s
	Re_o	4900–8300

burned gas was exhausted vertically upward into quiescent ambient air at room temperature. By using the same estimation as in the previous paper[8], the effect of free convection due to the great temperature differences between the jet and the ambient was considered to be negligibly small, except in the outer intermittent region. The effect of radiative transfer was also neglected because the centerline temperature and the mole fractions of CO_2 and H_2O under the condition of the highest nozzle-exit temperature were lower than 687 K and 0.025, respectively, in the developed region ($Z/D \geq 8$).

For the isothermal jet experiment, carbon dioxide gas was mixed far upstream of the venturi throat with the air supplied by a compressor. The gas mixture was exhausted at the same temperature as the ambient air.

Local gas temperatures were measured directly by a 0.1 mm diameter Pt/PtRh thermocouple. With the aid of the measured temperatures, local velocities were calculated from the dynamic pressures measured by a miniature Pitot tube of 1.4 mm OD quartz tube, which also served as a sampling tube for gas composition analysis. Local gas compositions were determined in terms of the carbon dioxide/air mole fraction measured by means of a gas chromatograph. Measurements of the radial and axial property variations were made every 1 mm in the radial direction and every 10 mm in the axial direction, mainly in the developed region. These probes could be placed at any position by means of the traversing device with accurate lead screws that allowed their locations to be determined with an accuracy of 0.1 mm. The experimental conditions are given in **Table 1**.

2. Dynamic Eddy Transfer Coefficient Model

Suppose that two eddies are exchanged over the so-called mixing length (l_M, l_H, l_D) in a shear flow field which has large thermal and density gradients in the transverse direction, as shown in **Fig. 3**. It is assumed that these eddies do not lose their original quantities, that is, (U_1, T_1, X_1, ρ_1) and (U_2, T_2, X_2, ρ_2) while they

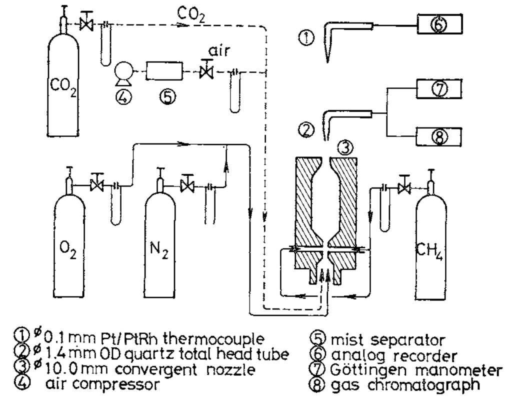

Fig. 1 General arrangement of experimental apparatus

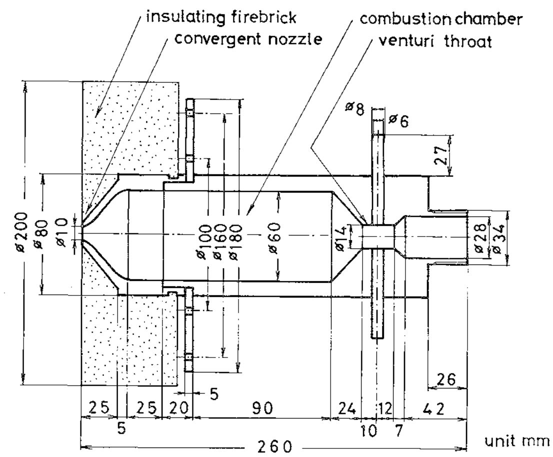

Fig. 2 Nozzle assembly

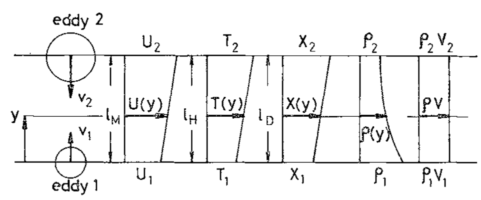

Fig. 3 Schematic diagram of eddy transfer coefficient model for turbulent transport of momentum, heat, and mass in shear flow field with large density gradients

travel. The volume of the traveling eddies is inversely proportional to the local density of fluid. The volume transported per unit area and unit time by the traveling eddies can be considered to be proportional to the transverse component of the turbulent velocity v. Hence

$$\rho_1 v_1 = \rho_2 v_2 = \rho v = \text{constant} \quad (4)$$

everywhere in the transverse direction.

By taking into account the change in volume of the traveling eddies, the turbulent transport fluxes of momentum, heat, and a chemical species may be expressed as

$$\tau_t=\rho_2U_2v_2-\rho_1U_1v_1=\rho v(U_2-U_1)$$
$$=\rho v l_M\frac{dU}{dy}=\rho\varepsilon_M\frac{dU}{dy} \quad (5)$$

$$q_t=\rho_2Cp_2T_2v_2-\rho_1Cp_1T_1v_1=\rho vCp(T_2-T_1)$$
$$=\rho Cpvl_H\frac{dT}{dy}=\rho Cp\varepsilon_H\frac{dT}{dy} \quad (6)$$

$$j_t=\rho_2(X_2/M_2)v_2-\rho_1(X_1/M_1)v_1=\rho v((X_2/M_2)-(X_1/M_1))$$
$$=\rho vl_D\frac{d(X/M)}{dy}=\rho\varepsilon_D\frac{d(X/M)}{dy} \quad (7)$$

Here, the static pressure of the flowfield and the heat capacity of the fluid were also assumed to be constant in the model consideration. These expressions are the same in form as those for incompressible flows, but they suggest that the dynamic eddy transfer coefficients $\rho\varepsilon_M$, $\rho Cp\varepsilon_H$, $\rho\varepsilon_D$ should be used as a measure of the turbulent transport phenomena in nonisothermal variable-density flows.

The experimental transfer coefficients have been calculated by the following boundary-layer type of conservation equations:

$$\rho\varepsilon_M=\frac{1}{r(\partial U/\partial r)}\int_0^r\left(r\rho U\frac{\partial U}{\partial Z}-\frac{\partial U}{\partial r}\int_0^r r\frac{\partial\rho U}{\partial Z}dr\right)dr \quad (8)$$

$$\rho Cp\varepsilon_H=\frac{1}{r(\partial T/\partial r)}\int_0^r\left(r\rho CpU\frac{\partial T}{\partial Z}-Cp\frac{\partial T}{\partial r}\times\int_0^r r\frac{\partial\rho U}{\partial Z}dr\right)dr \quad (9)$$

$$\rho\varepsilon_D=\frac{1}{r(\partial(X/M)/\partial r)}\int_0^r\left(r\rho U\frac{\partial(X/M)}{\partial Z}-\frac{\partial(X/M)}{\partial r}\times\int_0^r r\frac{\partial\rho U}{\partial Z}dr\right)dr \quad (10)$$

The right-hand sides of the above equations have been numerically integrated by substituting the observed axial and radial distributions of velocity, temperature, and concentration.

3. Experimental Results and Discussion

The previous work[9] investigated the development of nonisothermal, compressible free jets and obtained a complete set of correlative equations that can predict the radial distributions of velocity and temperature at any axial position. In essence, the set of correlative equations should be sufficient to estimate the eddy transfer coefficients for any initial jet conditions. However, as can be seen from Eqs. (8) to (10) of the present paper, the numerical calculation process for determining the transfer coefficients includes the differentiation of the velocity and temperature distribution curves in the axial and radial directions. The main part of the error in using the set of correlative equations came from data scattering in correlating the core lengths in terms of the initial density ratio. Therefore, for precision, a different set of curve-fit equations of the following form were adopted for each run:

(Centerline decay)

$$\frac{U_o}{U_m}=A_1\frac{Z}{D}+B_1 \quad (11)$$

$$\frac{T_o-T_\infty}{T_m-T_\infty}=A_2\frac{Z}{D}+B_2 \quad (12)$$

(Radial spreading)

$$\frac{r_{1/2U}}{D}=A_3\frac{Z}{D}+B_3 \quad (13)$$

$$\frac{r_{1/2T}}{D}=A_4\frac{Z}{D}+B_4 \quad (14)$$

(Radial distribution)

$$\frac{U}{U_m}=\frac{1}{(1+0.414\eta^2)^2}\quad(0\leq\eta\leq1)$$
$$=\exp(-0.693\eta^2)\quad(\eta\geq1) \quad (15)$$

$$\frac{T-T_\infty}{T_m-T_\infty}=\frac{1}{(1+0.414\eta_T^2)^2}\quad(0\leq\eta_T\leq1)$$
$$=\exp(-0.693\eta_T^2)\quad(\eta_T\geq1) \quad (16)$$

As can be expected from Figs. 9 to 11 of the previous paper[8], the observed Prandtl eddy diffusivities for nonisothermal variable-density jets vary greatly with both the axial and radial distances as distinct from those for an incompressible jet.

The distinction may be explained by the change in volume of the traveling eddies caused by the cooling process during turbulent mixing with cold ambient air. **Figure 4** shows the measured radial distributions of the eddy transfer coefficients, where the dimensionless radial distances η and η_T are used for ε_M and ε_H, respectively.

The transfer coefficient for momentum $\rho\varepsilon_M$ remains constant over the inner region $(0\leq\eta\leq1)$ while that for heat $\rho Cp\varepsilon_H$ decreases slightly in the radial direction, even in the inner region. The axial variation is small as compared with the radial variation. According to the previous paper[8], the eddy transfer coefficient for a chemical species also remains constant, but the value is close to that for heat. These transfer coefficients decrease markedly with radial distance in the outer intermittent region ($\eta>1$ and $\eta_T>1$). There may exist an effect of natural convection on intermittent turbulent mixing.

There are few experimental data available for comparison with the present result. Corrsin and Uberoi[2] measured the local distributions of temperature and dynamic pressure in heated air jets at two initial-density ratios, i. e. $\bar{\rho}_\infty=1.05$ and 2.02. By converting their dynamic pressure data into the velocity distribution, the local variation of the eddy transfer coefficients was calculated by using Eqs. (8) and (9). **Figure 5** shows the eddy transfer coefficients based on the Corrsin and Uberoi experiment. The eddy transfer coefficient for momentum $\overline{\rho\varepsilon_M}$ when $\bar{\rho}_\infty=1.05$ has the same constant value over the inner region as the Prandtl eddy diffusivity ε_M/U_oD for constant density jet flows. It should be noted that the eddy transfer coefficient for heat $\overline{\rho Cp\varepsilon_H}$ when $\bar{\rho}_\infty=1.05$ decreases slightly in the radial direction, even in the inner region. This suggests that the radial decrease in $\rho Cp\varepsilon_H$ cannot be attributed to the radial variation in heat capacity.

As can be seen in these figures, both coefficients $\rho\varepsilon_M$ and $\rho Cp\varepsilon_H$ increase greatly with initial density ratio. The values on the jet axis were determined by taking the limit as

$$\rho\varepsilon_{Mm}=\lim_{r\to+0}\rho\varepsilon_M \quad \text{and} \quad \rho Cp\varepsilon_{Hm}=\lim_{r\to+0}\rho Cp\varepsilon_H$$

The eddy transfer coefficients on the jet axis are almost independent of the axial position in the developed region.

These coefficients are plotted against the initial density ratio as shown in **Fig. 6**, where some measurements of the transfer coefficient for a chemical species were also plotted under an assumption of analogy between heat and mass transfer. It has been found that the two straight lines have a slope of 1/2 and that they can be expressed by

$$\overline{\rho\varepsilon_{Mm}}=0.0135\bar{\rho}_\infty^{1/2} \qquad (17)$$

$$\overline{\rho Cp\varepsilon_{Hm}}=0.0195\bar{\rho}_\infty^{1/2} \qquad (18)$$

These equations indicate that the transfer coefficients on the jet axis can be predicted by the initial jet condition (ρ_o, ρ_∞, Cp_o, U_o, D) alone. Equations (17) and (18) are not only in good agreement with Eq. (11) of the previous paper[8] but are also identical with the isothermal incompressible relations when ρ and Cp are constant. Equation (17) is also found to be in excellent agreement with the equation of Ferri *et al.*, i.e. Eq. (3). Kleinstein's constant[11] is too small to extrapolate his model to incompressible flows, although it was obtained by comparison with the axial decay data of the same Corrsin and Uberoi experiment.

Figures 7 and **8** show the radial distributions of the normalized transfer coefficients which have been averaged over the range of axial distances given in the figures. Within the experimental error, the coefficient for momentum remains constant over the inner region unless the initial density ratio is smaller than unity. When $\bar{\rho}_\infty<1$, the inverse density effect penetrates into the inner region. The coefficient for heat decreases slightly in the radial direction in the inner region even when $\bar{\rho}_\infty>1$. Owing to the small difference in velocity, the intermittent shear layer must be affected by the natural convection. This has been left as a problem for subsequent study. As aforementioned, the coefficient for heat decreases slightly in the radial direction whereas that for a chemical species remains constant.

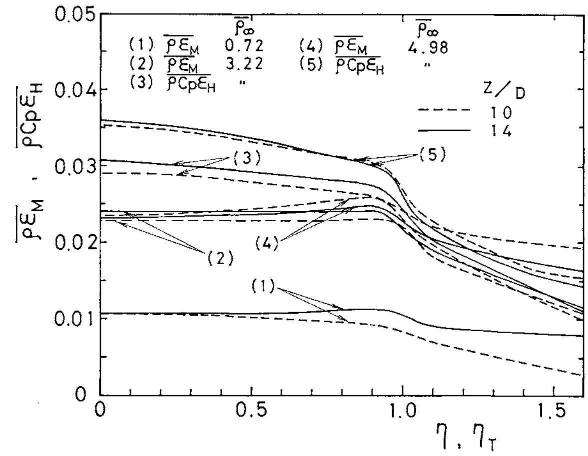

Fig. 4 Radial distributions of eddy transfer coefficients for momentum and heat

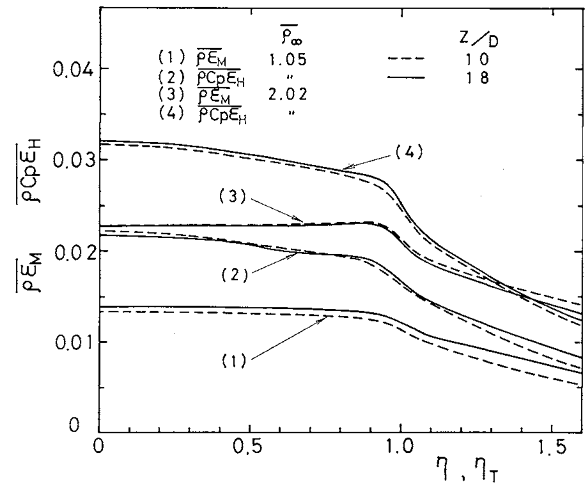

Fig. 5 Radial distributions of eddy transfer coefficients calculated from Corrsin and Uberoi's heated air jet data

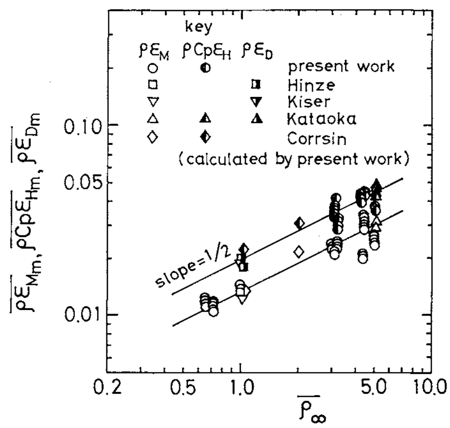

Fig. 6 Variation in eddy transfer coefficients on jet axis with initial density ratio

The present model, consisting of Eqs. (17) and (18)

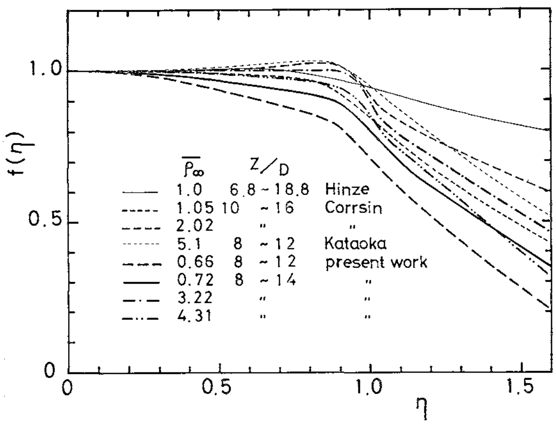

Fig. 7 Radial distributions of eddy transfer coefficient for momentum normalized with centerline values

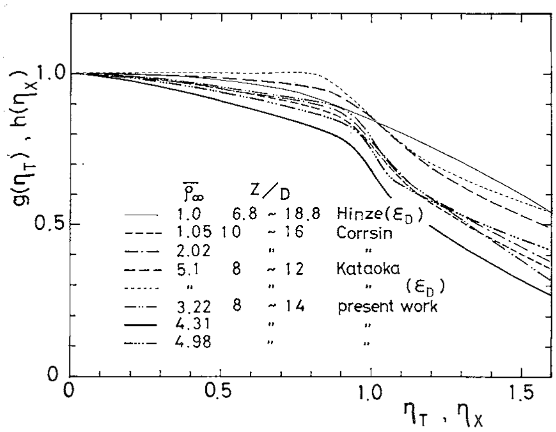

Fig. 8 Radial distributions of eddy transfer coefficients for heat and a chemical species normalized with centerline values

with Figs. 7 and 8, is useful over a wide range of initial density ratio ($0.66 \leq \bar{\rho}_\infty \leq 5.1$) to predict local values of the eddy transfer coefficients in the developed region. The turbulent Prandtl number $\varepsilon_M/\varepsilon_H$ is about 0.69 over the experimental range of the intial density ratio. This value is very close to the molecular Prandtl number of air.

Conclusion

A generalized model of eddy transfer coefficients has been experimentally obtained for nonisothermal variable-density jets. All the transfer coefficients in the developed region are proportional to the square root of the initial density ratio, but do not greatly vary in the axial direction. In the inner region, the measured transfer coefficients for momentum and a chemical species become constant, while that for heat decreases slightly in the radial direction. In the outer region, all transfer coefficients decrease rapidly with the radial distance owing to intermittent entrainment of the ambient stationary fluid. The present model can predict local eddy transfer coefficients over the range of initial density ratio from 0.66 to 5.1 if the initial jet condition alone is specified at a nozzle exit.

Nomenclature

A_1–A_4	= experimental constants	[—]
B_1–B_4	= experimental constants	[—]
b	= width of mixing region	[m]
Cp	= heat capacity at constant pressure	[J/kg·K]
D	= diameter of circular convergent nozzle	[m]
f	= radial distribution function of eddy transfer coefficient for momentum $=\rho\varepsilon_M/(\rho\varepsilon_M)_m$	[—]
g	= radial distribution function of eddy transfer coefficient for heat $=\rho Cp\varepsilon_H/(\rho Cp\varepsilon_H)_m$	[—]
h	= radial distribution function of eddy transfer coefficient for a chemical species $=\rho\varepsilon_D/(\rho\varepsilon_D)_m$	[—]
i	= mass flux	[kmol/m²·s]
K_F	= mixing rate parameter of Ferri *et al.* model	[—]
K_p	= mixing rate parameter of Prandtl's model	[—]
l	= mixing length	[m]
M	= average molecular weight	[kg/kmol]
q	= heat flux	[J/m²·s]
Re_o	= jet Reynolds number $=U_oD/\nu$	[—]
r	= radial distance from jet axis	[m]
T	= temperature	[K]
U	= time-averaged axial velocity	[m/s]
v	= transverse component of turbulent velocity (R.M.S. value)	[m/s]
X	= concentration in mole fraction	[—]
y	= distance in transverse direction	[m]
Z	= axial distance from nozzle exit	[m]
ε	= eddy diffusivity	[m²/s]
η	= dimensionless radial distance $= r/r_{1/2U}$	[—]
η_T	= dimensionless radial distance $= r/r_{1/2T}$	[—]
η_X	= dimensionless radial distance $= r/r_{1/2X}$	[—]
ν	= kinematic viscosity	[m²/s]
ρ	= density	[kg/m³]
$\overline{\rho Cp\varepsilon_H}$	$=\rho Cp\varepsilon_H/\rho_o Cp_o U_o D$	[—]
$\overline{\rho\varepsilon_D}$	$=\rho\varepsilon_D/\rho_o U_o D$	[—]
$\overline{\rho\varepsilon_M}$	$=\rho\varepsilon_M/\rho_o U_o D$	[—]
$\bar{\rho}_\infty$	$=\rho_\infty/\rho_o$	[—]
τ	= momentum flux	[N/m²]

⟨Subscripts⟩

D	= mass transport
H	= heat transport
M	= momentum transport
m	= quantity on jet axis
o	= quantity at nozzle exit
T	= temperature
t	= turbulent transport
U	= velocity
X	= concentration
$1/2M$	= position where $\rho U=(\rho_m U_m+\rho_\infty U_\infty)/2$
$1/2T$	= position where $T=(T_m+T_\infty)/2$
$1/2U$	= position where $U=U_m/2$
$1/2X$	= position where $X=(X_m+X_\infty)/2$
∞	= quantity in the ambient surroundings

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NONUNIFORM PRESSURE DIFFUSION OF A TERNARY GAS MIXTURE IN A CIRCULAR TUBE IN THE TRANSITION REGION

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A new, explicit equation for nonuniform pressure diffusion of a ternary gas mixture in a circular tube is obtained. The theory employed is based on the authors' existing kinetic model of four-sided local velocity distributions. The diffusion equation shows that transport of the individual components is expressed by a simple summation of two separated terms of diffusional and hydrodynamic flows. Validity of the equation is shown by comparison of the fluxes with those measured for a He-Ne-Ar mixture through a diffusion bridge. The diffusion equation gives a physical explanation for striking phenomena at low pressures, such as Hoogschagen's nonequimolar counter-diffusion at a uniform pressure, diffusio-molecular pressure effect, velocity slip at the channel wall, and Knudsen's minimum in volume flow.

The concept of "diffusional momentum slip" is newly proposed. Use of this concept is effective in deriving the solutions to transport of gases at low pressures directly from the continuum theory.

Introduction

Among many existing theoretical models of diffusion of a gas mixture at low pressures, outstanding are the dusty gas model[3,4], phenomenological theory[17,19], uniform and nonuniform gas kinetic theories[1,18], etc.[12,14,20]. Use of these theories lead directly to the diffusion equations which cover the whole pressure from Knudsen to ordinary diffusion regimes. Recently, Nakano *et al.* derived an explicit diffusion equation for a multi-component gas mixture with solid-gas reactions by introducing a new concept of the dynamic effective diffusivity[15,16]. In our previous paper, we presented a mathematical and phenomenological theory for diffusion of a binary mixture on the basis of the concept of locally uniform distributions[10]. We here develop the theory to include the effect of an additional third component and deduce the explicit equation for diffusion of a ternary mixture in a circular

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