

Research Article

An Effective Chaos-Based Image Encryption Scheme Using Imitating Jigsaw Method

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In this paper, an efficient chaos-based image encryption scheme is proposed, which uses the imitating jigsaw method containing revolving and shifting operations. In this scheme, there are three processes in encryption: preprocessing, encryption process, and postprocessing. In the preprocessing, the original image is partitioned into 64×64 pixel image blocks and then randomly revolved and shifted under control sequences which are generated by the hyperchaotic Lorenz system whose initial conditions are calculated by original image and keys. Therefore, the preprocessing is sensitive to plain image against differential attacks. In the encryption process, the after-preprocessing image is partitioned into 32×32 pixel image blocks; next they are randomly revolved and encrypted by control sequence and key blocks which are generated by the skew tent map. In postprocessing, the after-encryption image is partitioned into 16×16 pixels' image blocks, and they are randomly revolved and shifted again under control sequences which are related with encrypted image and keys. The postprocessing further increases the diffusion characteristics. Moreover, the test experiment and security analyses are given; the results show that our proposed cryptosystem has both security and speed performance.

1. Introduction

With the rapid development of electronic technology, digital information is being applied in all the fields in the society. Digital image is becoming the most widely used information carrier in our daily life with many advantages, such as intuition and vividness. At the same time, a series of security problems of image information threaten the safety of people's life and property. Therefore, the information security of digital image has become a research focus in recent years. Because the image data has many special characteristics which are different from text structure data, such as high redundancy, strong correlation, and bulk data capacity, traditional text encryption algorithms usually have low efficiency on image encryption [1]. Therefore, the image encryption scheme must be designed by considering those characteristics.

Since the chaos theory was first proposed by Lorenz [2], many chaotic phenomena were found in many fields, such as

physics, astronomy, chemistry, biology, and medicine. In recent years, the chaotic encryption attracts more and more researcher's attention. After Matthews [3] using the chaotic system to design the first image encryption scheme, many chaotic image encryption schemes are proposed in the recent years [3–38]. Fridrich [26] gave a two-dimensional chaotic symmetric ciphers scheme. Lian et al. [27] proposed a block cipher using a chaotic standard map. At the same period, many chaotic encryption schemes were put forward, such as [19–24]. Many of these schemes have been proved to have some security problems, such as weak plaintext sensitivity, small key space, and easy statistical attack.

In the past five years, many splendid image encryption schemes were presented. Hu et al. [4] proposed a chaotic image cipher scheme by using a plaintext-related permutation mechanism. In [5], an image encryption scheme was presented by using chaos sequence to control the encoding plain image to DNA sequence, and then they were encrypted by cycle operation of DNA sequence. In [6], Hua et al.

proposed a new 2D logistic-sine-coupling map and then designed an image encryption scheme using this chaotic system. In [7], Gao et al. gave a new 2D logistic ICMIC cascade map and then proposed a bit-level image encryption algorithm based on this map. Chai et al. [8] used the memristive chaotic system, elementary cellular automata, and compressive sensing to design an image encryption cryptosystem. Ye et al. [9] proposed an image encryption scheme by using SHA-3 hash function and double Arnold chaotic maps. Luo et al. [10] designed a plain image-related cryptosystem by using the tent map to generate two ergodicity sequences to control the permutation and diffusion processes. In [11], Hua et al. proposed an image encryption scheme for medical image by using high-speed scrambling and pixel adaptive diffusion. Khelifi et al. [12] presented an encryption scheme for image data sharing in cloud. In [13], Wang and Zhang used the hyperchaotic system to design an image encryption scheme which expanded the plain image into two compound images and then diffused them at the bit level. In [14], an image encryption scheme was presented which encrypted the image from four directions by interweaving pairs of rows and columns. In [15], an image encryption scheme was designed by using rows and columns switch. In [16], Liu et al. designed a self-adaptive selective permutation and inter-intra-block feedback diffusion and then used them by designing an image encryption scheme. Wu et al. [17] proposed a color image encryption scheme using the nonlinear chaotic algorithm (NCA) map-based coupled map lattice (CML) and DNA operation. Wang et al. [18] used chaos and simulated annealing algorithm to design the image encryption scheme. Li et al. [29] proposed a plaintext related image encryption scheme based on the hyperchaotic system and hash function. In [30], Wu et al. proposed an image encryption algorithm by using Henon-Sine map and DNA approach. Ye et al. [31] proposed an image encryption scheme by using quaternion discrete fractional Hartley transform and an improved pixel adaptive diffusion. In [32], Zhu et al. use block compressive sensing and singular value decomposition embedding to balance the security, compression, and robustness. In [33], an image encryption scheme is proposed by using 2D multiple parameter fractional discrete Fourier transform and 3D Arnold transform. You et al. [34] proposed a parallel image encryption scheme and implemented by OpenCL. Ouyang et al. [35] proposed a method of impulsive synchronization of coupled delayed neural networks with actuator saturation and designed an image encryption scheme by using this method. In [36], a compressed sensing strategy based on a semitensor product was proposed and a visual security image encryption scheme was designed. In [37], an image encryption scheme was proposed by using compressive sensing and random number insertion. Zhang et al. [38] proposed a plaintext-related image encryption scheme by using perceptron-like network. Li et al. [39] proposed an efficient bit-level permutation method and designed a chaos-based color image encryption scheme. Yavuz et al. [40] proposed an effective image encryption algorithm using two independent chaotic functions and some logical operations. Yavuz [41] proposed a content sensitive dynamic function

and used this mechanism to design an image encryption scheme. Amina et al. [42] proposed an image encryption scheme using logistic-tent system. Ravichandran et al. [43] presented an image encryption scheme based on combined logistic-tent system. Lakshmi et al. [44] proposed a Hopfield attractor-based image encryption scheme using neural networks. Banu et al. [45] proposed an image encryption based on chaotic attractors on frequency domain by integer wavelet transform (IWT) and fused with deoxyribonucleic acid (DNA) sequence on the spatial domain. Alawida et al. [46] proposed a new chaotification method which combined two chaotic maps and designed an image encryption scheme using this method. Alawida et al. [47] used a deterministic finite state machine to enhance chaotic properties of tent map and proposed an image encryption scheme using this new chaotic system. Artilles et al. [48] combined the block cipher and chaotic system to design an image encryption scheme; the drawback of this scheme is less efficiency. Wang et al. [49] proposed an image encryption algorithm using logistic-dynamic mixed linear-nonlinear coupled map lattices. Luo et al. [50] presented an image encryption scheme using improved baker map and logistic map.

As we all know, a good image encryption scheme not only should have an excellent security performance but also need to have an outstanding encryption speed performance. The main contributions of our work are as follows:

- (1) We proposed an efficient image encryption scheme based on chaos theory and imitating jigsaw operation
- (2) We give a complete security analysis and performance analysis
- (3) The proposed scheme is proved to have good encryption and decryption efficiencies compared with those of others' works

The organization of this paper is as follows: in Section 2, we give a detailed description of the encryption and decryption algorithms. In Section 3, we simulate our proposed cryptosystem and use some security analyses to prove our work is secure. Section 4 concludes this paper.

2. Encryption and Decryption Algorithms

In our proposed image cryptosystem, there are three processes in both encryption and decryption algorithms: preprocessing process, encryption or decryption process, and postprocessing process. In preprocessing and postprocessing processes, we use the hyperchaotic Lorenz system to generate the control sequences of revolving and shifting image blocks. In the encryption or decryption process, we use the skew tent system to generate the control sequence of revolving image blocks and the security key for encryption or decryption.

Remark 1. In our proposed scheme, we use the hyperchaotic system and skew tent map to control the encryption process. However, the other chaotic systems can also be extended in our scheme, and the only difference is the size of the key space.

The hyperchaotic Lorenz system [51] is given by

$$\begin{cases} \dot{x} = a(y - x) + w, \\ \dot{y} = cx - y - xz, \\ \dot{z} = xy - bz, \\ \dot{w} = -yz + \gamma w, \end{cases} \quad (1)$$

where a , b , c , and γ are the parameters of the system. When $a = 10$, $b = 8/3$, $c = 28$, and $\gamma \in [-1.52, -0.06]$, the system goes into chaos. Figure 1 shows attractors of the hyperchaotic Lorenz system.

The skew tent system [52] is given by

$$t_{n+1} = \begin{cases} \frac{t_n}{p}, & t_n \in (0, p), \\ (1 - t_n)(1 - p), & t_n \in [p, 1), \end{cases} \quad (2)$$

where p is the parameter of the skew tent system. When $p \in (0, 0.5) \cup (0.5, 1)$, the system can generate a chaotic sequence $t_n \in (0, 1)$.

2.1. Encryption Algorithm. The whole encryption scheme as shown in Figure 2 has two parts: image data processing before encryption part and encryption algorithm part. The purpose of data processing before encryption is to make input images suitable for processing rules of encryption algorithms. The details are as follows:

Step 1: we supposed the inputted image **IM** is an $\mathbf{M0} \times \mathbf{N0}$ 8 bit gray image and check whether $\mathbf{M0}$ and $\mathbf{N0}$ can be divided exactly by 64.

Step 2: if $\mathbf{M0}$ cannot be divided exactly by 64, but $\mathbf{N0}$ can, then execute Step 3a. If $\mathbf{N0}$ cannot be divided exactly by 64, but $\mathbf{M0}$ can, then execute Step 3b. And, if both $\mathbf{M0}$ and $\mathbf{N0}$ cannot be divided exactly by 64, then execute Step 3c (only one of the Steps 3a, 3b, and 3c can be executed).

Step 3a: take an integer number between 0 and 63 and denote it as \mathbf{nm} , to make $\mathbf{M0} + \mathbf{nm}$ to be divided exactly by 64. Build an $\mathbf{nm} \times \mathbf{N0}$ matrix **NBM** whose each element is equal \mathbf{nm} . Combine the two matrixes **IM** and **NBM** to be a new $(\mathbf{M0} + \mathbf{nm}) \times \mathbf{N0}$ matrix, as shown in Figure 3.

Step 3b: take an integer number between 0 and 63 and denote it as \mathbf{nn} , to make $\mathbf{N0} + \mathbf{nn}$ to be divided exactly by 64. Build a $\mathbf{M0} \times \mathbf{nn}$ matrix **NBN** whose each element is equal \mathbf{nn} . Combine the two matrixes **IM** and **NBN** to be a new $\mathbf{M0} \times (\mathbf{N0} + \mathbf{nn})$ matrix, as shown in Figure 3.

Step 3c: take an integer number between 0 and 63 and denote it as \mathbf{nm} , to make $\mathbf{M0} + \mathbf{nm}$ to be divided exactly by 64. Take an integer number between 0 and 63 and denote it as \mathbf{nn} , to make $\mathbf{N0} + \mathbf{nn}$ to be divided exactly by 64. Generate a new $(\mathbf{M0} + \mathbf{nm}) \times (\mathbf{N0} + \mathbf{nn})$ matrix by **IM**, **NBN**, **NBM**, and **NBT** as Figure 3, where elements in matrix **NBM** are equal to \mathbf{nm} , elements in

matrix **NBN** are equal to \mathbf{nn} , and elements in matrix **NBT** are equal to $\mathbf{nm} + \mathbf{nn}$.

Step 4: finish the data processing with outputting $\mathbf{M} \times \mathbf{N}$ image matrix **IM1**.

After data processing, we put the outputted-image matrix into encryption algorithm. There are three processes in encryption algorithm: preprocessing process, encryption process, and postprocessing process. The encryption algorithm is shown in Algorithm 1.

The detailed description of encryption algorithm is as follows:

Step 1: input the data processed image **IM1** and initial key $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_9$ into the encryption algorithm.

Step 2: the preprocessing process begins. **IM1** is partitioned as 64×64 pixels image blocks, and there are $\mathbf{BN1} = (\mathbf{M}/64) \times (\mathbf{N}/64)$ blocks; they are denoted as **preBlock** (1), **preBlock** (2), ..., **preBlock** (**BN1**).

Step 3: add all the gray values of pixels of the **IM1**, and the summation is denoted as **S1**. The initial conditions of the hyperchaotic Lorenz system are generated by equations (3)–(6):

$$\mathbf{x}_0 = (\text{mod}(\mathbf{k}_1 \times \mathbf{S1}, 80) - 40) \times \mathbf{k}_2 + \mathbf{k}_6, \quad (3)$$

$$\mathbf{y}_0 = (\text{mod}(\mathbf{k}_2 \times \mathbf{S1}, 80) - 40) \times \mathbf{k}_3 + \mathbf{k}_7, \quad (4)$$

$$\mathbf{z}_0 = (\text{mod}(\mathbf{k}_3 \times \mathbf{S1}, 80) + 1) \times \mathbf{k}_4 + \mathbf{k}_8, \quad (5)$$

$$\mathbf{w}_0 = (\text{mod}(\mathbf{k}_4 \times \mathbf{S1}, 500) - 250) \times \mathbf{k}_5 + \mathbf{k}_9, \quad (6)$$

where $\text{mod}(a, b)$ means the remainder of a/b (similarly hereinafter).

Step 4: under initial conditions which are generated in Step 3, equation (1) is iterated by using the fourth-order Runge-Kutta method with 0.002 step size. The revolving control sequence **RS1** and the shifting control sequence **XS1** are generated by equations (7) and (8), respectively:

$$\mathbf{RS1} = \text{mod}(\text{floor}((y + 100) \times 10^8, 4), \quad (7)$$

$$\mathbf{XS1} = \text{mod}(\text{floor}((x + 100) \times 10^8, \mathbf{BN1}) + 1, \quad (8)$$

Step 5: image blocks are revolved by using the following equation:

$$\mathbf{preBlock}(i) = \text{imrotate}(\mathbf{preBlock}(i), 90 \times \mathbf{RS1}(i)), \quad (9)$$

where $\text{imrotate}(P, m)$ means image P is rotated m degrees in the anticlockwise direction (similarly hereinafter) and i is the serial number of elements in the sequences **preBlock** and **RS1**.

Step 6: remove the repeated elements from **XS1** and then put the absent numbers at the end to generate a new sequence **X1**.

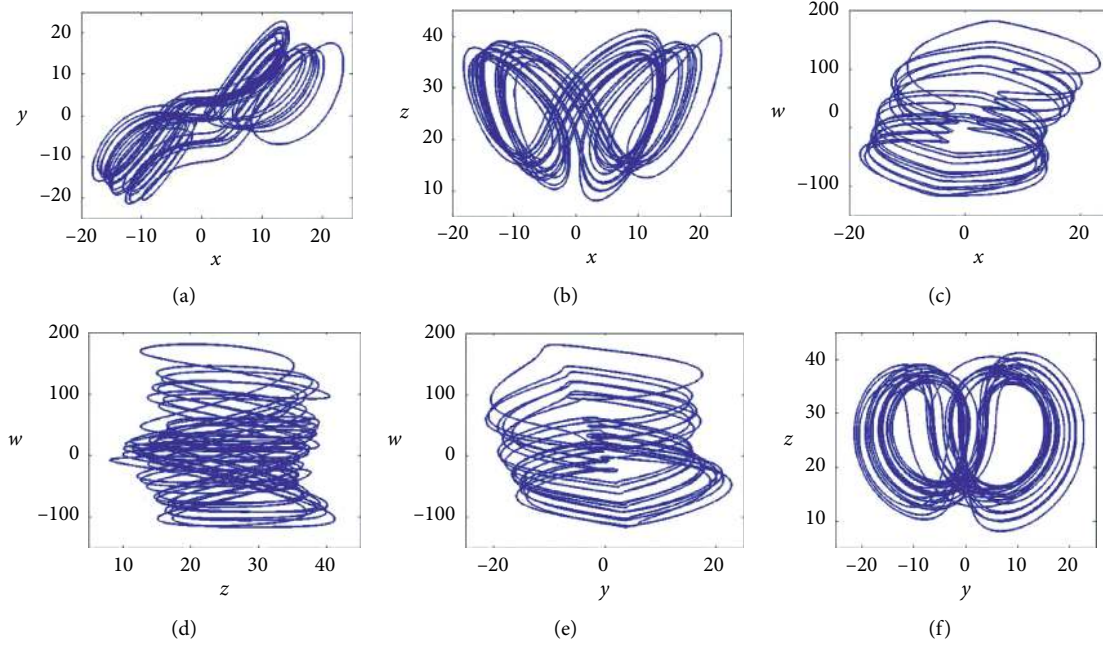


FIGURE 1: Hyperchaotic attractors in (a) x - y plane, (b) x - z plane, (c) x - w plane, (d) z - w plane, (e) y - w plane, and (f) y - z plane.

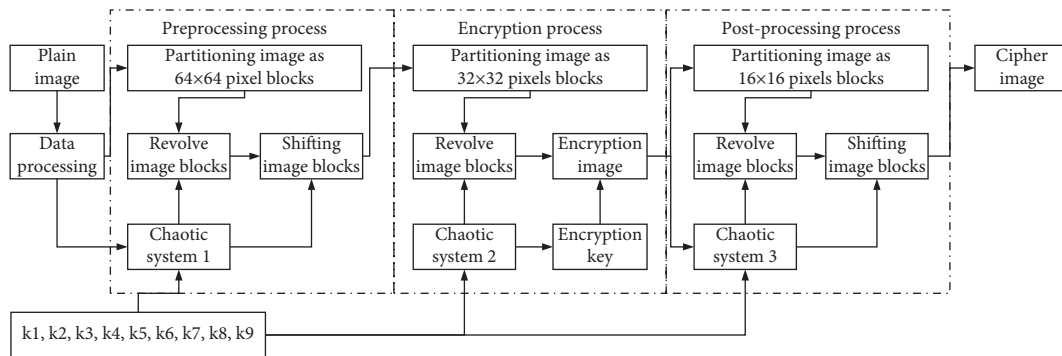


FIGURE 2: Encryption scheme.

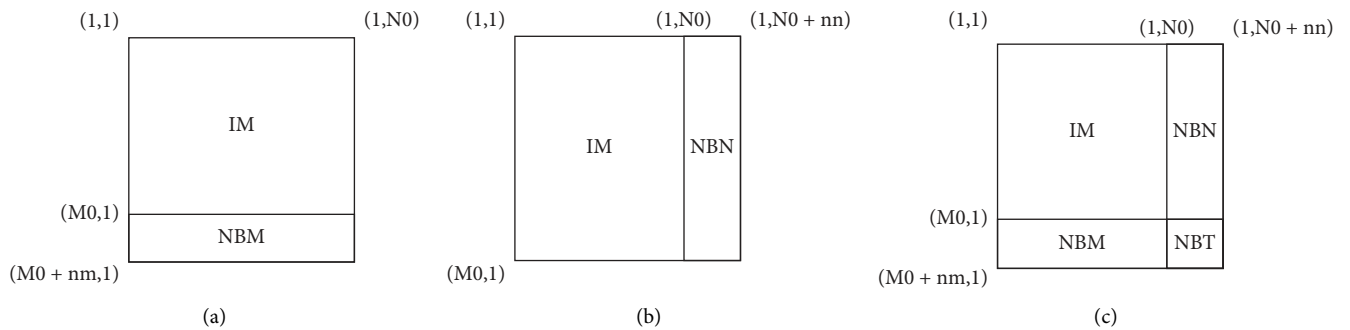


FIGURE 3: The rules of matrix combination.

Input: image **IM1** and initial key $[[k_1, k_2, \dots, k_9]]$

- (1) $[M \ N] \leftarrow \text{size}(\text{IM1})$;
- (2) Partitioning **IM1** as 64×64 pixels image blocks
 $[\text{preBlock}(1), \text{preBlock}(2), \dots, \text{preBlock}(\text{BN1})]$, where $\text{BN1} = (M/64) \times (N/64)$
- (3) $S1 \leftarrow \text{SUM}(\text{IM1})$

Generate initial conditions of the hyperchaotic Lorenz system:

$$x_0 \leftarrow (\text{mod}(k_1 \times S1, 80) - 40) \times k_2 + k_6; y_0 \leftarrow (\text{mod}(k_2 \times S1, 80) - 40) \times k_3 + k_7;$$

$$z_0 \leftarrow (\text{mod}(k_3 \times S1, 80) + 1) \times k_4 + k_8; w_0 \leftarrow (\text{mod}(k_4 \times S1, 500) - 250) \times k_5 + k_9;$$

- (4) Iterate equation (1) with x_0, y_0, z_0, w_0 .
- Generate revolving control sequence $\text{RS1} \leftarrow \text{mod}(\text{floor}((y + 100) \times 10^8), 4)$
Generate shifting control sequence $\text{XS1} \leftarrow \text{mod}(\text{floor}((x + 100) \times 10^8), \text{BN1}) + 1$
- (5) **for** $i = 1$ to BN1 **then**
 $\text{preBlock}(i) \leftarrow \text{imrotate}(\text{preBlock}(i), 90 \times \text{RS1}(i))$
end for
- (6) Generate a new sequence **X1** by removing repeated elements from **XS1** and putting the absent numbers at the end.
- (7) **for** $i = 1$ to $\text{BN1}/2$ **then**
 $\text{preBlock}(X1_i) \leftrightarrow \text{preBlock}(X1_{\text{BN1}-i+1})$
end for
- (8) Restructure **preBlock** to an image matrix **IM2**
- (9) Partitioning **IM2** as 32×32 pixels' image blocks
 $[\text{encBlock}(1), \text{encBlock}(2), \dots, \text{encBlock}(\text{BN2})]$, where $\text{BN2} = (M/32) \times (N/32)$
- (10) Generate system parameter p and initial condition t_0 of skew tent map:
 $p \leftarrow \text{mod}((k_1 + k_2 + k_3) \times k_4 + k_5, 1); t_0 \leftarrow \text{mod}((k_5 + k_6 + k_7) \times k_8 + k_9, 1)$
- (11) Iterate equation (2) with p, t_0 .
- Generate revolving control sequence $\text{RS2} \leftarrow \text{mod}(\text{floor}((t + 100) \times 10^8), 4)$
Generate key sequence $\text{KS} \leftarrow \text{mod}(\text{floor}(t \times 10^{12}), 256)$
- (12) Reshape key sequence **KS** to be a 32×32 blocks
 $[\text{keyBlock}(1), \text{keyBlock}(2), \dots, \text{keyBlock}(\text{BN2})]$
- (13) **for** $i = 1$ to BN2 **then**
 $\text{encBlock}(i) \leftarrow \text{imrotate}(\text{encBlock}(i), 90 \times \text{RS2}(i))$
end for
- (14) $\text{cBlock}(1) \leftarrow \text{encBlock}(1) \oplus \text{keyBlock}(1)$
- for** $i = 2$ to BN2 **then**
 $\text{cBlock}(i) \leftarrow \text{encBlock}(i) \oplus \text{keyBlock}(i) \oplus \text{cBlock}(i - 1)$
end for
- (15) Restructure **cBlock** to an image matrix **IM3**
- (16) Partitioning **IM3** as 16×16 pixels' image blocks:
 $[\text{postBlock}(1), \text{postBlock}(2), \dots, \text{postBlock}(\text{BN3})]$, where $\text{BN3} = (M/16) \times (N/16)$
- (17) $S2 \leftarrow \text{SUM}(\text{IM3})$
Generate initial conditions of hyper-chaotic Lorenz system:
 $x_0 \leftarrow (\text{mod}(k_6 \times S2, 80) - 40) \times k_4 + k_1; y_0 \leftarrow (\text{mod}(k_7 \times S2, 80) - 40) \times k_3 + k_2;$
 $z_0 \leftarrow (\text{mod}(k_8 \times S2, 80) + 1) \times k_2 + k_3; w_0 \leftarrow (\text{mod}(k_9 \times S2, 500) - 250) \times k_1 + k_4;$
- (18) Iterate equation (1) with x_0, y_0, z_0, w_0 .
Generate revolving control sequence $\text{RS3} \leftarrow \text{mod}(\text{floor}((y + 100) \times 10^8), 4)$
Generate shifting control sequence $\text{XS2} \leftarrow \text{mod}(\text{floor}((x + 100) \times 10^8), \text{BN3}) + 1$
- (19) **for** $i = 1$ to BN3 **then**
 $\text{postBlock}(i) \leftarrow \text{imrotate}(\text{postBlock}(i), 90 \times \text{RS3}(i))$
end for
- (20) Generate a new sequence **X2** by removing repeated elements from **XS2**, and putting the absent numbers at the end.
- (21) **for** $i = 1$ to $\text{BN3}/2$ **then**
 $\text{postBlock}(X2_i) \leftrightarrow \text{postBlock}(X2_{\text{BN3}-i+1})$
end for
- (22) Restructure **postBlock** to an image matrix **CM**

Output: cipher image **CM**

ALGORITHM 1: Encryption algorithm.

Step 7: substitute $\mathbf{preBlock}(X1_i)$ with $\mathbf{preBlock}(X1_{\mathbf{BN1}-i+1})$, where $i = 1, 2, \dots, \mathbf{BN1}/2$.

Step 8: generate a new image $\mathbf{IM2}$ by restructuring $\mathbf{preBlock}$. The preprocessing process is finished.

Step 9: The encryption process begins. $\mathbf{IM2}$ is partitioned as 32×32 pixels image blocks, and there are $\mathbf{BN2} = (M/32) \times (N/32)$ blocks; they are denoted as $\mathbf{encBlock}(1), \mathbf{encBlock}(2), \dots, \mathbf{encBlock}(\mathbf{BN2})$.

Step 10: generate system parameter \mathbf{p} and initial condition \mathbf{t}_0 of skew tent map by equations (10) and (11), respectively:

$$\mathbf{p} = \text{mod}((\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times \mathbf{k}_4 + \mathbf{k}_5, 1), \quad (10)$$

$$\mathbf{t}_0 = \text{mod}((\mathbf{k}_5 + \mathbf{k}_6 + \mathbf{k}_7) \times \mathbf{k}_8 + \mathbf{k}_9, 1), \quad (11)$$

Step 11: equation (2) was iterated under system parameter \mathbf{p} and initial condition \mathbf{t}_0 . The revolving control

sequence $\mathbf{RS2}$ and the key sequence \mathbf{KS} are generated by equations (12) and (13), respectively:

$$\mathbf{RS2} = \text{mod}(\text{floor}((\mathbf{t} + 100) \times 10^8, 4), \quad (12)$$

$$\mathbf{KS} = \text{mod}(\text{floor}(\mathbf{t} \times 10^{12}), 256). \quad (13)$$

Step 12: reshape key sequence \mathbf{KS} to be $\mathbf{BN2}$ key blocks with 32×32 and denote them as $\mathbf{keyBlock}(1), \mathbf{keyBlock}(2), \dots, \mathbf{keyBlock}(\mathbf{BN2})$.

Step 13: Revolve the image blocks by using the following equation:

$$\mathbf{encBlock}(i) = \text{imrotate}(\mathbf{encBlock}(i), 90 \times \mathbf{RS2}(i)). \quad (14)$$

Step 14: encrypt image blocks by the following equation:

$$\begin{cases} \mathbf{cBlock}(i) = \mathbf{encBlock}(i) \oplus \mathbf{keyBlock}(i), & i = 1, \\ \mathbf{cBlock}(i) = \mathbf{encBlock}(i) \oplus \mathbf{keyBlock}(i) \oplus \mathbf{cBlock}(i-1), & 2 \leq i \leq \mathbf{BN2}. \end{cases} \quad (15)$$

Step 15: generate a new image $\mathbf{IM3}$ by restructuring \mathbf{cBlock} . The encryption process is finished.

Step 16: the postprocessing process begins. $\mathbf{IM3}$ is partitioned as 16×16 pixels image blocks, and there are $\mathbf{BN3} = (M/16) \times (N/16)$ blocks; they are denoted as $\mathbf{postBlock}(1), \mathbf{postBlock}(2), \dots, \mathbf{postBlock}(\mathbf{BN3})$.

Step 17: add all the gray values of pixels of the $\mathbf{IM3}$, and the summation is denoted as $\mathbf{S2}$. The initial conditions of the hyperchaotic Lorenz system are generated by the following equations:

$$\mathbf{x}_0 = (\text{mod}(\mathbf{k}_6 \times \mathbf{S2}, 80) - 40) \times \mathbf{k}_4 + \mathbf{k}_1, \quad (16)$$

$$\mathbf{y}_0 = (\text{mod}(\mathbf{k}_7 \times \mathbf{S2}, 80) - 40) \times \mathbf{k}_3 + \mathbf{k}_2, \quad (17)$$

$$\mathbf{z}_0 = (\text{mod}(\mathbf{k}_8 \times \mathbf{S2}, 80) + 1) \times \mathbf{k}_2 + \mathbf{k}_3, \quad (18)$$

$$\mathbf{w}_0 = (\text{mod}(\mathbf{k}_9 \times \mathbf{S2}, 500) - 250) \times \mathbf{k}_1 + \mathbf{k}_4. \quad (19)$$

Step 18: under initial conditions which are generated in Step 17, equation (1) is iterated by using the fourth-order RungeKutta method with 0.002 step size. The revolving control sequence $\mathbf{RS3}$ and the shifting control sequence $\mathbf{XS2}$ are generated by equations (20) and (21), respectively:

$$\mathbf{RS3} = \text{mod}(\text{floor}((\mathbf{y} + 100) \times 10^8, 4), \quad (20)$$

$$\mathbf{XS2} = \text{mod}(\text{floor}((\mathbf{x} + 100) \times 10^8, \mathbf{BN3}) + 1 \quad (21)$$

Step 19: image blocks are revolved by the following equation:

$$\mathbf{postBlock}(i) = \text{imrotate}(\mathbf{postBlock}(i), 90 \times \mathbf{RS3}(i)). \quad (22)$$

Step 20: remove the repeated elements from $\mathbf{XS2}$ and then put the absent numbers at the end to generate a new sequence $\mathbf{X2}$.

Step 21: substitute $\mathbf{postBlock}(X2_i)$ with $\mathbf{postBlock}(X2_{\mathbf{BN3}-i+1})$, where $i = 1, 2, \dots, \mathbf{BN3}/2$.

Step 22: generate cipher image \mathbf{CM} by restructuring $\mathbf{postBlock}$. The postprocessing process is finished.

Step 23: output the cipher image, and the encryption algorithm is finished.

2.2. Decryption Algorithm. The decryption scheme as shown in Figure 4 has two parts as encryption: the decryption algorithm part and data processing part. The decryption algorithm is the inverse process of the encryption algorithm and also has three processes: preprocessing process, decryption process, and postprocessing process. The decryption algorithm is shown in Algorithm 2.

The detail description of the decryption algorithm as follows:

Step 1: input cipher image $\mathbf{CM1}$ and initial key $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_9$ into decryption algorithm, and the cipher image is supposed to be $M \times N$ 8 bit gray image.

Note: inputted security keys are defined $\text{ask}_1, \text{ask}_2, \dots, \text{ask}_9 \in (0, 1)$, which are used to generate parameter and initial values of the hyperchaotic Lorenz system and skew tent map. To avoid weak key problem [53], we require each input key to be 15 decimal places.

Step 2: the preprocessing process begins. Partition $\mathbf{CM1}$ as 16×16 pixels image blocks, and there are $\mathbf{BN4} =$

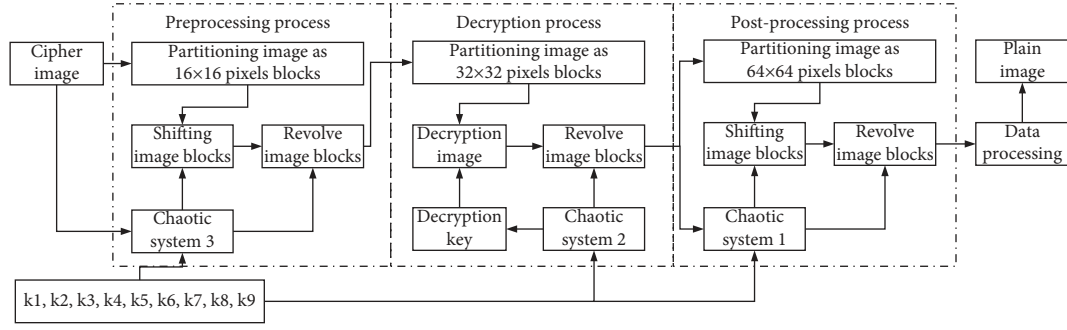


FIGURE 4: Decryption scheme.

Input: cipher image $CM1$ and initial key $[[k_1, k_2, \dots, k_9]]$

- (1) $[M \ N] \leftarrow \text{size}(CM1)$;
- (2) Partitioning $CM1$ as 16×16 pixels image blocks:
 $[\text{preCBlock}(1), \text{preCBlock}(2), \dots, \text{preCBlock}(BN4)]$, where $BN4 = (M/16) \times (N/16)$
- (3) $S3 \leftarrow \text{SUM}(CM1)$
 Generate initial conditions of the hyperchaotic Lorenz system:
 $x_0 \leftarrow (\text{mod}(k_6 \times S3, 80) - 40) \times k_4 + k_1$; $y_0 \leftarrow (\text{mod}(k_7 \times S3, 80) - 40) \times k_3 + k_2$
 $z_0 \leftarrow (\text{mod}(k_8 \times S3, 80) + 1) \times k_2 + k_3$; $w_0 \leftarrow (\text{mod}(k_9 \times S3, 500) - 250) \times k_1 + k_4$
- (4) Iterate equation (1) with x_0, y_0, z_0, w_0 .
 Generate revolving control sequence $RS4 \leftarrow \text{mod}(\text{floor}((y + 100) \times 10^8, 4)$
 Generate shifting control sequence $XS3 \leftarrow \text{mod}(\text{floor}((x + 100) \times 10^8, BN4) + 1$
- (5) Generate a new sequence $X3$ by removing repeated elements from $XS3$ and putting the absent numbers at the end.
- (6) **for** $i = 1$ to $BN4/2$ **then**
 $\text{preCBlock}(X3_i) \leftrightarrow \text{preCBlock}(X3_{BN4-i+1})$
end for
- (7) **for** $i = 1$ to $BN4$ **then**
 $\text{preCBlock}(i) \leftarrow \text{imrotate}(\text{preCBlock}(i), -90 \times RS4(i))$
end for
- (8) Restructure preCBlock to an image matrix $CM2$
- (9) Partitioning $CM2$ as 32×32 pixels' image blocks:
 $[\text{decCBlock}(1), \text{decCBlock}(2), \dots, \text{decCBlock}(BN5)]$, where $BN5 = (M/32) \times (N/32)$
- (10) Generate system parameter p and initial condition t_0 of skew tent map:
 $p \leftarrow \text{mod}((k_1 + k_2 + k_3) \times k_4 + k_5, 1)$; $t_0 \leftarrow \text{mod}((k_5 + k_6 + k_7) \times k_8 + k_9, 1)$
- (11) Iterate equation (2) with p, t_0 .
 Generate revolving control sequence $RS5 \leftarrow \text{mod}(\text{floor}((t + 100) \times 10^8, 4)$
 Generate key sequence $KS1 \leftarrow \text{mod}(\text{floor}(t \times 10^{12}), 256)$
- (12) Reshape key sequence $KS1$ to be a 32×32 blocks
 $[\text{keyCBlock}(1), \text{keyCBlock}(2), \dots, \text{keyCBlock}(BN5)]$
- (13) **for** $i = BN5$ to 2 step -1 **then**
 $\text{pBlock}(i) \leftarrow \text{de cCBlock}(i) \oplus \text{keyCBlock}(i) \oplus \text{pBlock}(i-1)$
end for
 $\text{pBlock}(1) \leftarrow \text{de cCBlock}(1) \oplus \text{keyCBlock}(1)$
- (14) **for** $i = 1$ to $BN5$ **then**
 $\text{de cCBlock}(i) \leftarrow \text{imrotate}(\text{de cCBlock}(i), -90 \times RS5(i))$
end for
- (15) Restructure pBlock to an image matrix $CM3$
- (16) Partitioning $CM3$ as 64×64 pixels' image blocks:
 $[\text{postCBlock}(1), \text{postCBlock}(2), \dots, \text{postCBlock}(BN6)]$, where $BN6 = (M/64) \times (N/64)$
- (17) $S4 \leftarrow \text{SUM}(CM3)$
 Generate initial conditions of the hyperchaotic Lorenz system:
 $x_0 \leftarrow (\text{mod}(k_1 \times S4, 80) - 40) \times k_2 + k_6$; $y_0 \leftarrow (\text{mod}(k_2 \times S4, 80) - 40) \times k_3 + k_7$;
 $z_0 \leftarrow (\text{mod}(k_3 \times S4, 80) + 1) \times k_4 + k_8$; $w_0 \leftarrow (\text{mod}(k_4 \times S4, 500) - 250) \times k_5 + k_9$;
- (18) Iterate equation (1) with x_0, y_0, z_0, w_0 .
 Generate revolving control sequence $RS6 \leftarrow \text{mod}(\text{floor}((y + 100) \times 10^8, 4)$
 Generate shifting control sequence $XS4 \leftarrow \text{mod}(\text{floor}((x + 100) \times 10^8, BN6) + 1$

(19) Generate a new sequence **X4** by removing repeated elements from **XS4** and putting the absent numbers at the end.
(20) **for** $i = 1$ to $\text{BN6}/2$ **then**
 postCBlock (X4_i) \leftrightarrow **postCBlock** ($\text{X4}_{\text{BN6}-i+1}$)
end for
(21) **for** $i = 1$ to BN6 **then**
 postCBlock (i) \leftarrow **imrotate** (**postCBlock** (i), $-90 \times \text{RS6}(i)$)
end for
(22) Restructure **postCBlock** to an image matrix **PM**
Output: plain image **PM**

ALGORITHM 2: Decryption algorithm.

$(M/16) \times (N/16)$ blocks; and they are denoted as **preCBlock** (1), **preCBlock** (2), . . . , **preCBlock** (**BN4**).

Step 3: add all the gray values of pixels of the **CM1**, and the summation is denoted as **S3**. The initial conditions of the hyperchaotic Lorenz system are generated by equations (23)–(26):

$$x_0 = (\text{mod}(\mathbf{k}_6 \times \mathbf{S3}, 80) - 40) \times \mathbf{k}_4 + \mathbf{k}_1, \quad (23)$$

$$y_0 = (\text{mod}(\mathbf{k}_7 \times \mathbf{S3}, 80) - 40) \times \mathbf{k}_3 + \mathbf{k}_2, \quad (24)$$

$$z_0 = (\text{mod}(\mathbf{k}_8 \times \mathbf{S3}, 80) + 1) \times \mathbf{k}_2 + \mathbf{k}_3, \quad (25)$$

$$\mathbf{w}_0 = (\text{mod}(\mathbf{k}_9 \times \mathbf{S3}, 500) - 250) \times \mathbf{k}_1 + \mathbf{k}_4. \quad (26)$$

Step 4: under initial conditions which are generated in Step 3, equation (1) is iterated by using the fourth-order Runge-Kutta method with 0.002 step size. The revolving control sequence **RS4** and the shifting control sequence **XS3** are generated by equations (27) and (28), respectively:

$$\mathbf{RS4} = \text{mod}(\text{floor}((y + 100) \times 10^8, 4), \quad (27)$$

$$\mathbf{XS3} = \text{mod}(\text{floor}((x + 100) \times 10^8, \text{BN4}) + 1 \quad (28)$$

Step 5: remove the repeated elements from **XS3** and then put the absent numbers at the end to generate a new sequence **X3**.

Step 6: substitute **preCBlock** (X3_i) with **preCBlock** ($\text{X3}_{\text{BN4}-i+1}$), where $i = 1, 2, \dots, \text{BN4}/2$.

Step 7: image blocks are revolved by the following equation:

$$\begin{cases} \mathbf{pBlock}(i) = \mathbf{de\,cBlock}(i) \oplus \mathbf{keyCBlock}(i), & i = 1, \\ \mathbf{pBlock}(i) = \mathbf{de\,cBlock}(i) \oplus \mathbf{keyCBlock}(i) \oplus \mathbf{pBlock}(i-1), & 2 \leq i \leq \text{BN5}. \end{cases} \quad (34)$$

Step 14: revolve the image blocks by using the following equation:

$$\mathbf{de\,cBlock}(i) = \text{imrotate}(\mathbf{de\,cBlock}(i), -90 \times \mathbf{RS5}(i)). \quad (35)$$

$$\mathbf{preCBlock}(i) = \text{imrotate}(\mathbf{preCBlock}(i), -90 \times \mathbf{RS4}(i)). \quad (29)$$

Step 8: generate a new image **CM2** by restructuring **preCBlock**. The preprocessing process is finished.

Step 9: the decryption process begins. Partition **CM2** as 32×32 pixels image blocks, and there are $\text{BN5} = (M/32) \times (N/32)$ blocks; denote them as **decCBlock** (1), **decCBlock** (2), . . . , **decCBlock** (**BN5**).

Step 10: generate system parameter **p** and initial condition \mathbf{t}_0 of skew tent map by equations (30) and (31), respectively:

$$\mathbf{p} = \text{mod}((\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times \mathbf{k}_4 + \mathbf{k}_5, 1), \quad (30)$$

$$\mathbf{t}_0 = \text{mod}((\mathbf{k}_5 + \mathbf{k}_6 + \mathbf{k}_7) \times \mathbf{k}_8 + \mathbf{k}_9, 1). \quad (31)$$

Step 11: iterate equation (2) under system parameter **p** and initial condition \mathbf{t}_0 . The revolving control sequence **RS5** and the key sequence **KS1** are generated by equations (32) and (33), respectively:

$$\mathbf{RS5} = \text{mod}(\text{floor}((\mathbf{t} + 100) \times 10^8, 4), \quad (32)$$

$$\mathbf{KS1} = \text{mod}(\text{floor}(\mathbf{t} \times 10^{12}), 256). \quad (33)$$

Step 12: reshape key sequence **KS1** to be **BN5** key blocks with 32×32 and denote them as **keyCBlock** (1), **keyCBlock** (2), . . . , **keyCBlock** (**BN5**).

Step 13: decrypt the image blocks by the following equation:

Step 15: generate a new image **CM3** by restructuring **pBlock**. The decryption process is finished.

Step 16: the preprocessing process begins. Partition **CM3** as 64×64 pixels image blocks, and there are $\text{BN6} = (M/64) \times (N/64)$ blocks; denote them as

postCBlock (1), **postCBlock** (2), . . . , **postCBlock** (**BN6**).

Step 17: add all the gray values of pixels of the **CM3**, and the summation is denoted as **S4**. The initial conditions of the hyperchaotic Lorenz system are generated by the following equations:

$$\mathbf{x}_0 = (\text{mod}(\mathbf{k}_1 \times \mathbf{S4}, 80) - 40) \times \mathbf{k}_2 + \mathbf{k}_6, \quad (36)$$

$$\mathbf{y}_0 = (\text{mod}(\mathbf{k}_2 \times \mathbf{S4}, 80) - 40) \times \mathbf{k}_3 + \mathbf{k}_7, \quad (37)$$

$$\mathbf{z}_0 = (\text{mod}(\mathbf{k}_3 \times \mathbf{S4}, 80) + 1) \times \mathbf{k}_4 + \mathbf{k}_8, \quad (38)$$

$$\mathbf{w}_0 = (\text{mod}(\mathbf{k}_4 \times \mathbf{S4}, 500) - 250) \times \mathbf{k}_5 + \mathbf{k}_9. \quad (39)$$

Step 18: under initial conditions which are generated in Step 18, iterate equation (1) by using the fourth-order RungeKutta method with 0.002 step size. The revolving control sequence **RS6** and the shifting control sequence **XS4** are generated by equations (7) and (8), respectively:

$$\mathbf{RS6} = \text{mod}(\text{floor}((\mathbf{y} + 100) \times 10^8, 4), \quad (40)$$

$$\mathbf{XS4} = \text{mod}(\text{floor}((\mathbf{x} + 100) \times 10^8, \mathbf{BN6}) + 1 \quad (41)$$

Step 19: remove the repeated elements from **XS4** and then put the absent numbers at the end to generate a new sequence **X4**.

Step 20: substitute **postCBlock**(**X4_i**) with **postCBlock**(**X4_{BN6-i+1}**), where $i = 1, 2, \dots, \mathbf{BN6}/2$.

Step 21: revolve the image blocks by the following equation:

$$\mathbf{postCBlock}(i) = \text{imrotate}(\mathbf{postCBlock}(i), -90 \times \mathbf{RS6}(i)). \quad (42)$$

Step 22: generate plain image **PM** by restructuring **postCBlock**. The postprocessing process is finished.

Step 23: output image matrix **PM**, and the decryption algorithm is finished.

After decryption algorithm part, we also need to run data processing to get the plain image. The purpose of this section is to remove the pixels when added by data processing before the encryption part; the detailed description is as follows:

Step 1: check the relationship of three pixels' value which are located in (1, N), (M, 1), and (M, N). We denote these three pixel value as **nn**, **nm**, and **nt**, respectively. If **nt** is not equal to **nn**, **nm**, or **nn + nm**, then execute Step 2a. If **nt** is equal to **nn**, then execute Step 2b. If **nt** is equal to **nm**, then execute Step 2c. If **nt** is equal to **nn + nm**, then execute Step 2d (only one of the Steps 2a, 2b, 2c, and 2d can be executed)

Step 2a: do nothing with **PM**, and the matrix **DPM** will be equal to **PM**.

Step 2b: check some elements' value in matrix **PM** which locate from rows **1** to **M** and columns **N-nn + 1** to **N**. If all those elements' value are equal to **nn**, then matrix **DPM** is equal to a part of matrix **PM** from rows **1** to **M** and columns **1** to **N-nn**. Otherwise, the matrix **DPM** is equal to **PM**.

Step 2c: check some elements' value in matrix **PM** which locate from rows **M-nm + 1** to **M** and columns **1** to **N**. If all those elements' value are equal to **nm**, then matrix **DPM** is equal to a part of matrix **PM** from rows **1** to **M-nm** and columns **1** to **N**. Otherwise, the matrix **DPM** is equal to **PM**.

Step 2d: check some elements' value in matrix **PM** which locate from rows **M-nm + 1** to **M** and columns **N-nn + 1** to **N**. If all those elements' value are equal to **nm + nn**, then matrix **DPM** is equal to a part of matrix **PM** from rows **1** to **M-nm** and columns **1** to **N-nn**. Otherwise, the matrix **DPM** is equal to **PM**.

Step 3: output the plain image **DPM**, and the whole decryption scheme is finished.

3. Simulation and Security Analysis

In this section, we evaluated our proposed scheme by software simulation running in MATLAB 2015b. The system parameters which are given in equation (1) are $a = 10$, $b = 8/3$, $c = 28$, and $\gamma = -1$, and the inputted keys are $k1 = 0.132312432152369$, $k2 = 0.723357645512358$, $k3 = 0.234342634695632$, $k4 = 0.432313435498574$, $k5 = 0.832312114474569$, $k6 = 0.642312345436958$, $k7 = 0.532345341689841$, $k8 = 0.932378647316581$, and $k9 = 0.232373784321657$. The test images are 512×512 pixels with 8 bit gray level, and the simulation results are shown in Figure 5. In following subsections, many commonly used security analyses will be discussed.

3.1. Key Space Analysis. The key space of a cryptosystem is the very important factor on security when brute-force attack is happening. In our scheme, the cryptosystem needs input $k1, k2, k3, k4, k5, k6, k7, k8$, and $k9 \in (0, 1)$ as security key. If we supposed the change step of each inputted key are 10^{-15} , then the total key space is calculated as $S = (10^5)^9 \approx 2^{455}$, which is larger enough on security [4–6].

3.2. Differential Attack. To resist differential attack, an image cryptosystem is required to have a good plaintext sensitivity. If the inputted image has only one-bit changed, the corresponding encrypted image is totally different, then the cryptosystem is considered to have enough plaintext sensitivity to resist differential attack. To measure the difference between two image, NPCR (number of pixels change rate) and UACI (unified average changing intensity) are put forward [8–11].

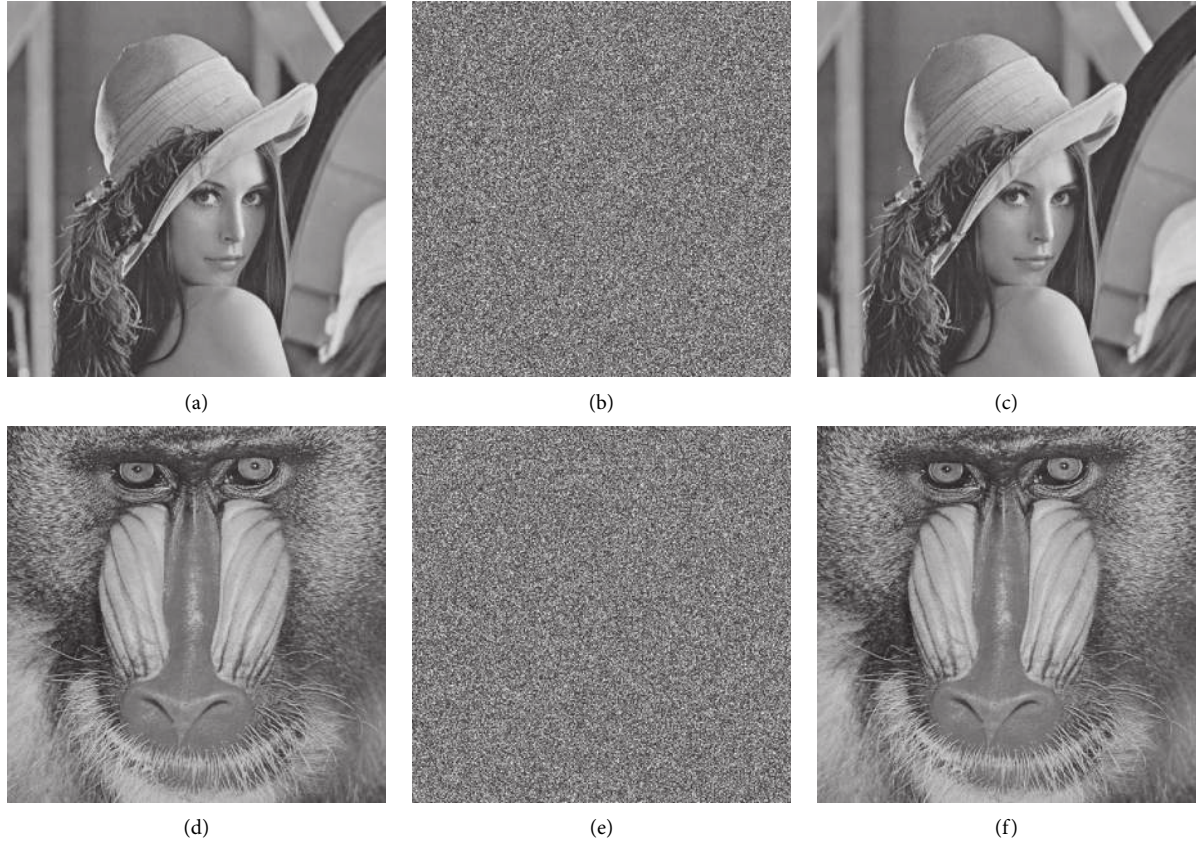


FIGURE 5: The simulation results of encryption and decryption: (a, d) plain images; (b, e) cipher images; (c, f) decrypted images.

The NPCR and UACI are given by

$$\text{NPCR} = \frac{\sum_{i=1}^M \sum_{j=1}^N D(i, j)}{M \times N} \times 100\%,$$

$$D(i, j) = \begin{cases} 0, & C_1(i, j) = C_2(i, j), \\ 1, & C_1(i, j) \neq C_2(i, j), \end{cases}$$

$$\text{UACI} = \frac{1}{M \times N} \left(\sum_{i=1}^M \sum_{j=1}^N \frac{|C_1(i, j) - C_2(i, j)|}{255} \right) \times 100\%, \quad (43)$$

where $C_1(i, j)$ and $C_2(i, j)$ represent two cipher-images obtained from encrypting two one-pixel different images. M and N represent the height and width of images, respectively.

The NPCR means the percentage of different pixels at same position between two corresponding encrypted images which are obtained by two images with one-bit difference, and the UACI represents the average intensity of the difference between two same position's pixels which are obtained from two cipher images.

In order to evaluate whether the NPCR and UACI images passed the tests, the critical values of NPCR and UACI are defined [54, 55]. For a significance level α , a critical NPCR score \mathcal{N}_α^* is obtained by

$$\mathcal{N}_\alpha^* = \frac{Q - \Phi^{-1}(\alpha) \sqrt{QH}}{Q + 1}, \quad (44)$$

where H is the total number of pixels in an image and Q represents the largest allowed pixel value in the image. An image encryption scheme can be considered to pass the NPCR if the obtained NPCR is larger than \mathcal{N}_α^* .

The critical UACI interval $(\mathcal{U}_\alpha^{*-}, \mathcal{U}_\alpha^{*+})$ can be calculated by

$$\begin{cases} \mathcal{U}_\alpha^{*-} = \mu_{\mathcal{U}} - \Phi^{-1}\left(\frac{\alpha}{2}\right)\sigma_{\mathcal{U}}, \\ \mathcal{U}_\alpha^{*+} = \mu_{\mathcal{U}} + \Phi^{-1}\left(\frac{\alpha}{2}\right)\sigma_{\mathcal{U}}, \end{cases} \quad (45)$$

where

$$\begin{aligned} \mu_{\mathcal{U}} &= \frac{Q + 2}{3Q + 3}, \\ \sigma_{\mathcal{U}} &= \sqrt{\frac{(Q + 2)(Q^2 + 2Q + 3)}{18(Q + 1)^2QH}}. \end{aligned} \quad (46)$$

If the obtained UACI falls into range $(\mathcal{U}_\alpha^{*-}, \mathcal{U}_\alpha^{*+})$, the corresponding encryption algorithm is considered to have a high security level. For images with size 512×512 , when we

set the significance level $\alpha = 0.05$, the critical value of NPCR is $\mathcal{N}_{0.05}^* = 99.5893\%$ and $(\mathcal{W}_{0.05}^-, \mathcal{W}_{0.05}^{*+}) = (33.3730\%, 33.5541\%)$.

In this section, the NPCR and UACI are calculated by two cipher images obtained by encrypting two images, the one obtained by random changing one pixel of another. We tested NPCR and UACI 100 times, and the results are shown in Table 1. According to the results in Table 1, it is no doubt that our cryptosystem is plaintext sensitive enough on resisting the differential attack.

3.3. Statistical Analysis

3.3.1. Histogram Analysis. If the histogram of the corresponding cipher image shows uniform distribution, the cryptosystem has good ability on resisting the statistical attack. In other words, there is no obvious statistical distinction of the count of pixels in each gray level. The distribution of plain images' histograms and cipher images' histograms is shown in Figure 6. We use the chi-squared test to evaluate the uniformity of encrypted image's histogram [43, 56]. When we set the significance level $\alpha = 0.05$, the chi-squared test results of cipher images is given in Table 2. The histogram analysis result shows that our proposed cryptosystem has good diffused property on resisting the statistical attack.

3.3.2. Correlation Coefficient. As we all know, the adjacent pixels in plain image have a strong correlation. However, there is less correlation among adjacent pixels in cipher image [5–10].

The correlation coefficient is given by

$$r_{ab} = \frac{\text{cov}(a, b)}{\sqrt{D(a)D(b)}}, \quad (47)$$

where a and b are two adjacent pixels gray values, and

$$\begin{aligned} E(a) &= \frac{1}{N} \sum_{i=1}^N a_i, \\ D(a) &= \frac{1}{N} \sum_{i=1}^N (a_i - E(a))^2, \\ \text{cov}(a, b) &= \frac{1}{N} \sum_{i=1}^N (a_i - E(a))(b_i - E(b)). \end{aligned} \quad (48)$$

In this test, 10000 pairs of adjacent pixels are randomly selected for calculating the correlation coefficient. The correlation coefficient results are shown in Table 2, and the correlation distribution of image "Lena" is shown in Figure 7 and Table 3

3.4. Key Sensitivity Analysis. The key sensitivity analysis is considered to be a measurement of the diffusion property of the cryptosystem. In our proposed scheme, there are nine inputted keys: $k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8$, and $k_9 \in (0, 1)$.

In this test, the plain image 'Lena' is first time encrypted with initial keys which are assigned at the beginning of Section 3, the second time encrypted with modifying $k_1' = k_1 + 10^{-15}$, and the third time encrypted with modifying $k_2' = k_2 + 10^{-15}$. And then, the differences between the ciphers encrypted by modified keys and initial keys are figured. The test result is shown in Figure 8, and the NPCR and UACI between ciphers generated by different keys are shown in Table 4. The result of test clearly shows that our proposed cryptosystem has good key sensitivity on resisting the exhaustive attack.

3.5. Information Entropy Analysis

3.5.1. Global Information Entropy. The global information entropy is used to indicate the uncertainty degree of image information [9, 57]. The global information entropy can be calculated by

$$H(s) = \sum_{i=0}^{2^K-1} P(s_i) \log_2 \frac{1}{P s_i}, \quad (49)$$

where K is the image bit depth, e.g., $K = 8$, for an 8 bit gray image, and $P(s_i)$ means the probability of s_i . The ideal case of 8 bit gray image information entropy is $H(s) = 8$ bits. The entropy test result is shown in Table 5. According to the test results, our entropies are very close to the ideal value 8. Thus, our cryptosystem shows good performance on resisting the entropy attack.

3.5.2. Local Shannon Entropy. Global Shannon entropy reflects the total randomness of image, and it has certain limitations in some occasions. Therefore, local Shannon entropy (LSE) was proposed by Wu et al. [58, 59]. To measure local entropy, we first randomly select k non-overlapping image blocks B_1, B_2, \dots, B_k with T_B pixels from image I ; then, the LSE is given by

$$\overline{H}_{k, T_B}(I) = \sum_{i=1}^k \frac{H(B_i)}{k}, \quad (50)$$

where $H(B_i)$ is the Shannon entropy of image block B_i and can be calculated by equation (49).

For our tests, we select parameters $(k, T_B) = (30, 1936)$, and then the ideal value of LSE is 7.902469317. The significance $\alpha = 0.05$, and we consider the tests passed when the test LSE values fall into (7.901901305, 7.903037329). The LSE test results are shown in Table 6.

3.6. NIST SP800-22 Tests. In our scheme, control sequences are generating by the hyperchaotic Lorenz system (HCLS) and skew tent map (STM). Therefore, the randomness of iterative sequences which generated by these two chaotic system is directly affecting the performance of the image cryptosystem. Here, we use SP800-22 [60] to test the randomness of binary sequences generated by the hyperchaotic Lorenz system and skew tent map. The test results are shown in Table 7.

TABLE 1: The results of NPCR and UACI.

Image	NPCR (%)				$\mathcal{N}_{0.05}^*$ 99.5893	UACI (%)			$\mathcal{U}_{0.05}^{**} = 33.5541$
	Min	Max	Average	Std.		Min	Max	Average	
Lena	99.5935	99.6326	99.6161	0.0123	Pass	33.3918	33.5717	33.4709	Pass
Baboon	99.5943	99.6342	99.6109	0.0129	Pass	33.3932	33.5913	33.4792	Pass
Pepper	99.5912	99.6391	99.6090	0.0121	Pass	33.3985	33.5574	33.4664	Pass
Airplane	99.5962	99.6323	99.6095	0.0131	Pass	33.3805	33.5353	33.4593	Pass
Boat	99.5931	99.6399	99.6101	0.0147	Pass	33.3746	33.5546	33.4654	Pass

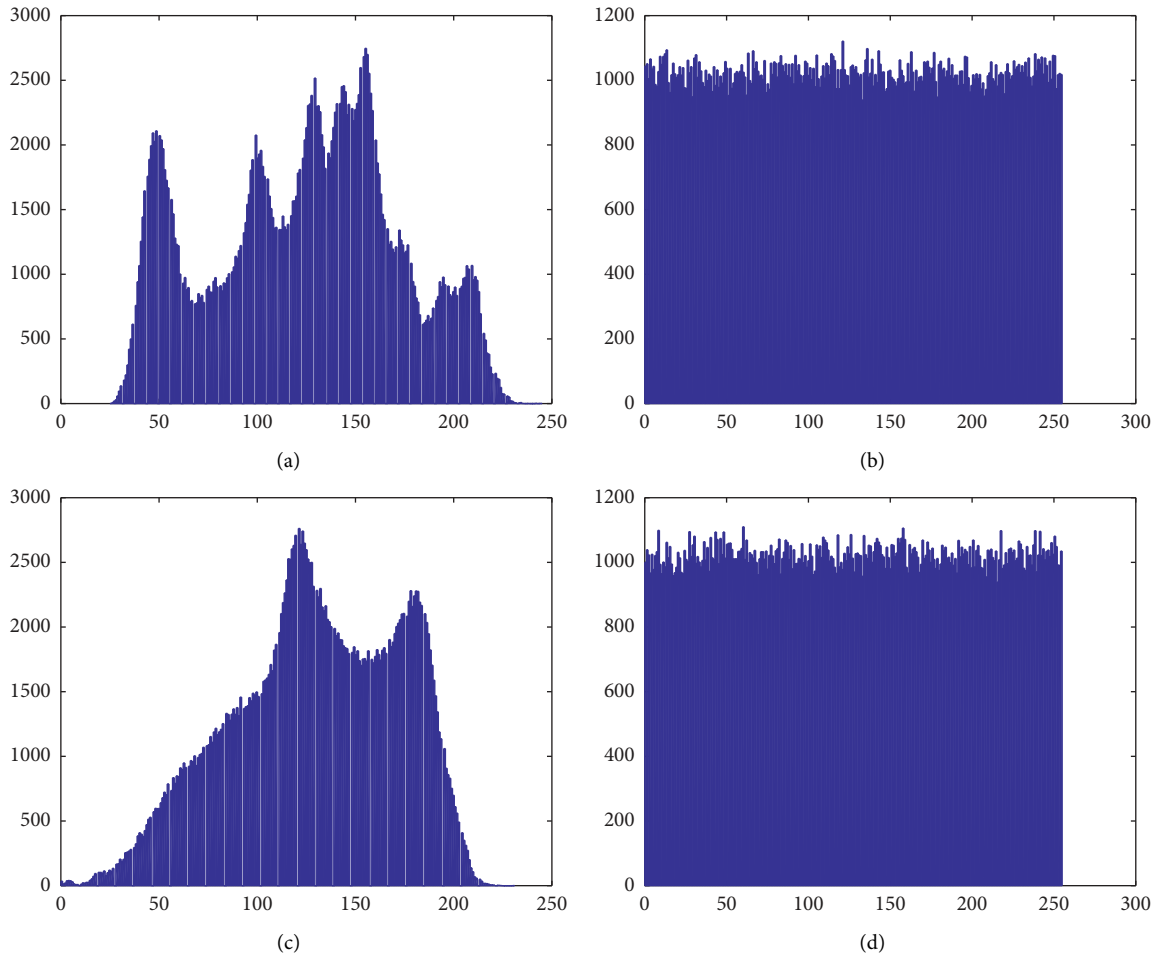


FIGURE 6: Histograms of (a, c) plain images "Lena" and "Baboon" and (b, d) their corresponding cipher images.

TABLE 2: Histogram uniformity evaluation by the chi-squared test.

Images	P value	Decision ($H=0$ or 1) $\alpha = 0.05$
Lena	0.1888141	0; Accept
Baboon	0.1117246	0; Accept

3.7. *Performance Analysis and Comparisons.* The simulation and tests are implemented in MATLAB R2015b, which is worked on a MacBook with Inter (R) Core i7, CPU 1.4 GHz, and 16 GB memory, and the software running system is macOS (High Sierra 10.13.1). As we all known, there are many factors affecting the speed of encryption and

decryption except algorithm itself, such as operating system, hardware environment, programming language, and code optimization. Thus, in this performance comparison, we not only give the comparison of time cost but also give the comparison of other correlation factors. Furthermore, we also give the comparison of encryption throughput and

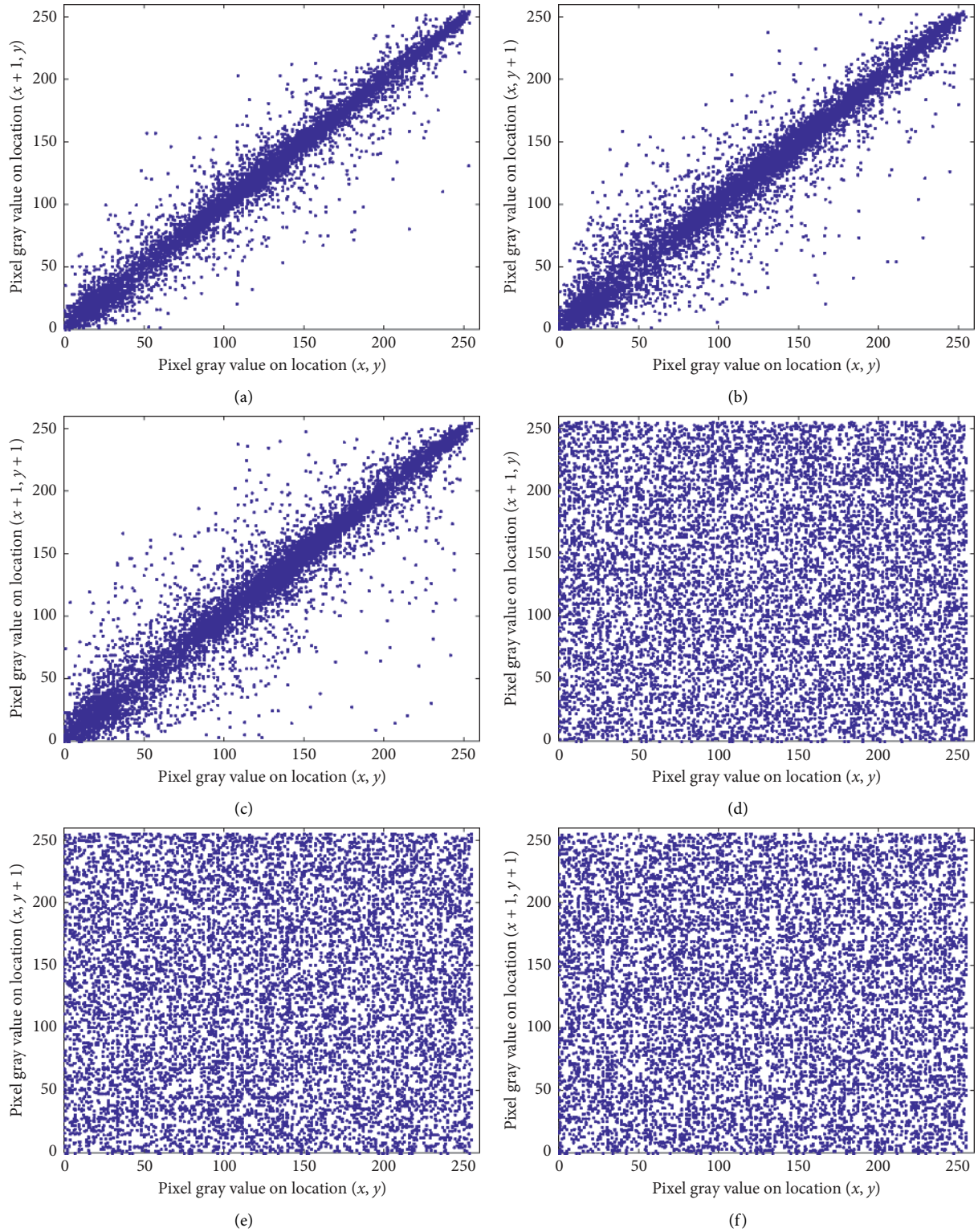


FIGURE 7: Correlation distributions of the (ac) plain image and (df) cipher image. (a) Figure 4(d) in the horizontal direction, (b) Figure 4(e) in the vertical direction, (c) Figure 4(f) in the diagonal direction.

TABLE 3: Correlation coefficients.

Images	Horizontal		Vertical		Diagonal	
	Plain	Cipher	Plain	Cipher	Plain	Cipher
Lena	0.9845	-0.0013	0.9699	0.0005	0.9585	0.0016
Baboon	0.7608	0.0032	0.8618	0.0012	0.7238	-0.0004

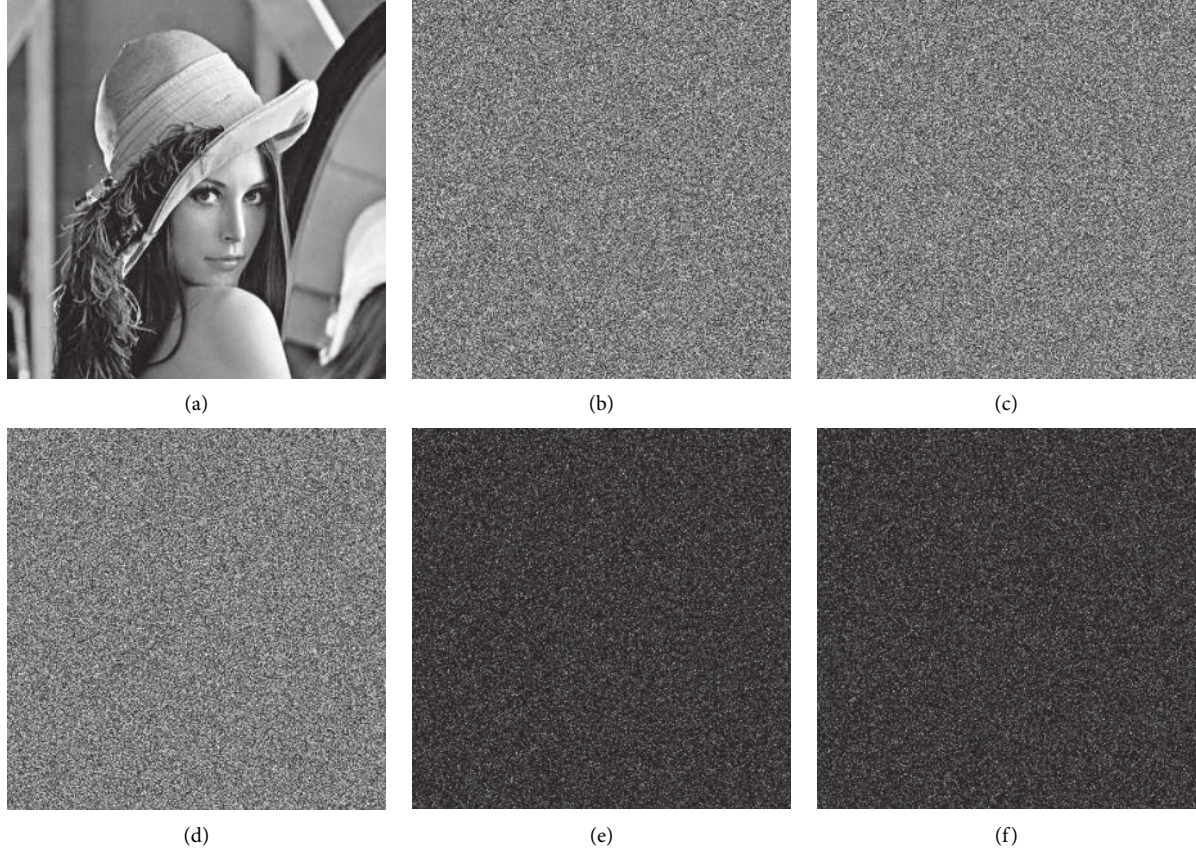


FIGURE 8: Key sensitivity analysis in encryption. (a) Original image. (b) Encrypted image with initial keys. (c) Encrypted image with modifying $k1^* = k1 + 10^{-15}$. (d) Cipher image with modifying $k2^* = k2 + 10^{-15}$. (e) Image of $|b-c|$. (f) image of $|b-d|$.

TABLE 4: Results of NPCR and UACI between ciphers.

Ciphers	NPCR (%)	$\mathcal{N}_{0.05}^* = 99.5893$	UACI (%)	$\mathcal{U}_{0.05}^{*-} = 33.3730$ $\mathcal{U}_{0.05}^{*+} = 33.5541$
Figures 8(b) and 8(c)	99.6063	Pass	33.4369	Pass
Figures 8(b) and 8(d)	99.6120	Pass	33.4494	Pass

TABLE 5: The global entropy test results.

Images	Lena	Baboon	Pepper	Airplane	Boat
Entropy	7.9993420	7.9993005	7.9993824	7.9993269	7.9994635

TABLE 6: The local Shannon entropy test results.

Images	Lena	Baboon	Pepper	Airplane	Boat
LSE	7.90278317	7.90294534	7.90219342	7.90265212	7.90301576
(7.901901305, 7.903037329)	Pass	Pass	Pass	Pass	Pass

TABLE 7: NIST SP800-22 test results.

Test items	HCLS <i>P</i> value	Result ≥0.01	STM <i>P</i> value	Result ≥0.01
Frequency	0.43304	Pass	0.32631	Pass
Block frequency	0.93008	Pass	0.88263	Pass
Runs	0.86392	Pass	0.75258	Pass
Longest runs	0.97571	Pass	0.89365	Pass
Binary matrix rank	0.50984	Pass	0.71245	Pass
Discrete Fourier transform	0.16365	Pass	0.32123	Pass
Nonoverlapping template	0.28953	Pass	0.12545	Pass
Overlapping template	0.57548	Pass	0.36585	Pass
Maurer’s universal statistical	0.38361	Pass	0.45852	Pass
Linear complexity	0.63612	Pass	0.23562	Pass
Serial*	0.13252	Pass	0.31142	Pass
Approximate entropy	0.83427	Pass	0.63253	Pass
Cumulative sums*	0.99253	Pass	0.85652	Pass
Random excursions*	0.71533	Pass	0.35262	Pass
Random excursions variant*	0.56579	Pass	0.45253	Pass

*The average values of multiple tests.

TABLE 8: Speed comparison.

Algorithms	CPU speed (GHz)	Memory size (GB)	Program language	Encryption time (ms) (512 × 512)	Encryption throughput (MBps)	Number of cycles
Our work	1.4	16	Matlab	138	1.811	1682
Ref [61]	2.5	4	Matlab	613	0.408	5846
Ref [42]	3.0	8	C	139	1.799	1591
Ref [62]	2.7	16	C	166	1.506	1710
Ref [28]	3.0	8	C	390	0.641	4463
Ref [6]	2.6	8	Matlab	484	0.517	4800
Ref [63]	2.4	8	C	203	1.232	1859
Ref [41]	2.4	8	C	85	2.941	778

TABLE 9: Security comparison.

Algorithms	Cipher correlation coefficients			Global entropy	Local entropy	Key space	Plaintext sensitivity	
	Horizontal	Vertical	Diagonal				NPCR (%)	UACI (%)
Our work	-0.0013	0.0005	0.0016	7.9993	7.902783	2^{455}	99.61	33.47
Ref. [28]	0.0010	0.0054	0.0056	7.9992	-	2^{520}	99.60	33.50
Ref. [15]	-0.0331	0.0057	0.0169	7.9972	-	2^{90}	99.61	33.40
Ref. [10]	0.0064	0.0078	0.0029	7.9993	-	$10^{15} \times 2^{256}$	99.59	33.46
Ref. [18]	0.0026	0.0019	0.0028	7.9992	7.902552	2^{640}	99.60	33.48
Ref. [40]	-0.02887	0.01459	0.03658	7.9993	-	2^{582}	99.62	33.41
Ref. [41]	0.001987	0.00449	-0.0087	7.9993	7.901558	2^{572}	99.63	33.48
Ref. [42]	0.0119	0.0092	0.0013	7.9993	-	$10^{30} \times 2^{384}$	99.64	33.61
Ref. [44]	0.003	0.0018	0.0006	7.997	7.9024	10^{112}	99.62	33.44
Ref. [45]	0.00193	0.0018	0.00342	7.998	-	10^{238}	99.68	33.47
Ref. [46]	-0.0017	-0.0084	-0.0019	7.9975	7.9051	2^{312}	99.62	33.50
Ref. [47]	-0.000026	-0.00031	-0.021	-	7.9028	-	99.96	33.69
Ref. [49]	0.004851	-0.0020	-0.00270	7.9993	7.902481	2^{250}	99.565	33.450
Ref. [50]	0.00058	0.00061	-0.0003	7.9974	-	2^{532}	99.60	33.49

number of cycles to make a more intuitive performance description. The speed analysis result is shown in Table 8, and the security comparisons with other works are shown in Table 9. The speed test and the comparisons show that our proposed cryptosystem not only has good speed on encryption but also has good security performance.

4. Conclusions

We proposed an efficient chaos-based image encryption scheme using the jigsaw method which contains two operations of image blocks: revolving and shifting. In the proposed encryption scheme, the preprocessing makes sure

our scheme has enough plain image sensitivity on resisting differential attack. The encryption process guarantees our cryptosystem has full diffusion on resisting the statistical analysis. The postprocessing process further increases the diffusion characteristic. In addition, some common security analyses and speed comparisons show that our image cryptosystem has both security and speed performance.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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