

# An Effective Filtering Algorithm to Mitigate Transient Decaying DC Offset

By: Abouzar Rahmati

Authors:

Abouzar Rahmati

Reza Adhami

IS-International Services LLC

University of Alabama in Huntsville

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# Outline

- Background
- Decaying DC removal algorithms
- Fourier transform based methods
- Proposed a fast and effective approach
- Results

# Background

- MANY protective relaying functions uses the phasors at the power system fundamental frequency.
- The phasors are estimated from the voltage and current sampled waveforms using digital filtering algorithms.
- The discrete Fourier transform (DFT)-based algorithms are the most commonly used.
- As a fault takes place, besides the fundamental component the fault signals may also include harmonics and a decaying DC component.
- Harmonics may be easily filtered out, but the decaying DC component is more difficult to deal, because it is a non-periodic signal whose frequency spectrum encompasses a wide range of frequencies.
- As a result, it may seriously affect the estimated phasor, causing undesirable oscillations about its actual value.

# Decaying DC offset

- When a fault occurs, the current signal consists of an exponentially decaying dc offset and sinusoidal components.
  - Since the exponentially decaying dc offset is a non-periodic signal and its frequency spectrum includes all frequencies, the dc offset heavily influences the accuracy and convergence speed of the phasor estimation.
  - The presence of a decaying dc offset may result in a phasor estimation error.
- ❖ Therefore, the decaying dc component should be taken into consideration when estimating the phasor of the fundamental frequency component of the fault current signal.

# Decaying DC removal Techniques

- Decaying DC component filtering from the original signal before the phasor estimation
  - Calculating the dc component offset and then removing it from the original signal
- Iterative mathematical approach using sampling data from a sinusoidal input signal
  - DFT based method
  - LES based techniques
- ❖ Most of the aforementioned algorithms mainly focus on the removal of the exponentially decaying DC offset component and **use about one cycle samples** to estimate the dc offset parameters.

# Fundamental and Decaying dc Components

- Discrete Fourier transform (DFT) is generally used to calculate the phasor of the fundamental frequency component in digital protective relays.

Fundamental and Decaying dc Components:

$$i(n\Delta t) = i_{ac}(n\Delta t) + i_{dc}(n\Delta t)$$

$$i_{ac}(n\Delta t) = \sum_{h=1}^H A_h \cos(2\pi f_h n\Delta t + \phi_h)$$

$$i_{dc}(n\Delta t) = A_{dc} e^{-\frac{n\Delta t}{\tau}}$$

# DFT based amplitude estimation

- Amplitude estimation of fundamental frequency:

$$I_{DFT} = \frac{2}{N} \sum_{n=0}^{N-1} i[n] \cdot e^{-j\frac{2\pi}{N}n}$$

$$A_{est-ac}(n) = |A_{cos}(n) + jA_{sin}(n)|$$

$$A_{cos}(n) = C_c(n) * i_{ac}(n) = \sum_{k=0}^N C_c(k) i_{ac}(n - k)$$

$$A_{sin}(n) = C_s(n) * i_{ac}(n) = \sum_{k=0}^N C_s(k) i_{ac}(n - k)$$

$$C_c(n) = \frac{2}{N} \cos\left(\frac{2\pi}{N}n\right)$$

$$C_s(n) = \frac{2}{N} \sin\left(\frac{2\pi}{N}n\right)$$

# DFT based amplitude estimation

- Sin and Cos coefficients:

$$A_{cos}(n) = \frac{2A_1}{N} \sum_{k=0}^N \cos\left(\frac{2\pi}{N}k\right) \cos\left(\frac{2\pi}{N}(n-k) + \phi_1\right)$$

$$A_{sin}(n) = \frac{2A_1}{N} \sum_{k=0}^N \sin\left(\frac{2\pi}{N}k\right) \cos\left(\frac{2\pi}{N}(n-k) + \phi_1\right)$$

- AC component amplitude:

$$A_{est-ac}(n) = \left| A_1 \left[ \left(1 + \frac{1}{N}\right) \cos\left(\frac{2\pi}{N}n + \phi_1\right) + j\left(1 - \frac{1}{N}\right) \sin\left(\frac{2\pi}{N}n + \phi_1\right) \right] \right|$$



# DFT based amplitude estimation

- Estimated amplitude of Decaying DC component:

$$A_{est-dc}(n) = M_{dc}\alpha^n \quad \alpha = e^{-\frac{\Delta t}{\tau}}$$

$$M_{dc} = \frac{(\alpha \cos(\frac{2\pi}{N}) - 1) + j(\alpha \sin(\frac{2\pi}{N}))}{N(2\alpha \cos(\frac{2\pi}{N}) - \alpha^2 - 1)} 2A_{dc}\alpha^{-N}$$

- Therefore, the estimated amplitude of the original signal is:

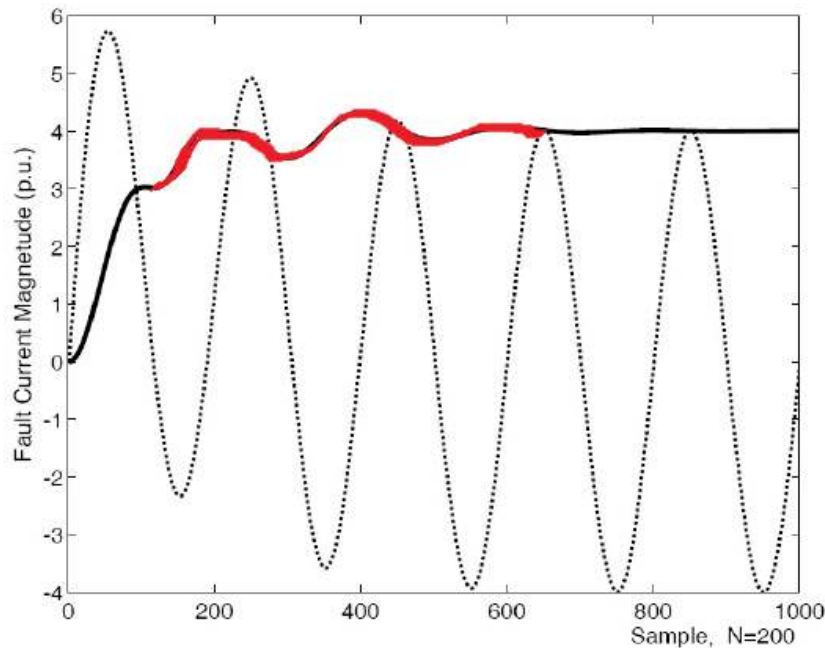
$$A_{est}(n) = A_{est-ac}(n) + A_{est-dc}(n)$$

$$|A_{est}(n)| = \sqrt{A_{est-re}^2(n) + A_{est-im}^2(n)}$$

# DFT error and oscillations

$$A_{est-re}(n) = A_1 \left(1 + \frac{1}{N}\right) \cos\left(\frac{2\pi}{N}n + \phi_1\right) + \text{Re}\{M_{dc}\}\alpha^n \rightarrow \cos\left(\frac{2\pi}{N}n + \phi_1\right)\text{Re}\{M_{dc}\}\alpha^n$$

$$A_{est-im}(n) = A_1 \left(1 - \frac{1}{N}\right) \sin\left(\frac{2\pi}{N}n + \phi_1\right) + \text{Im}\{M_{dc}\}\alpha^n \rightarrow \sin\left(\frac{2\pi}{N}n + \phi_1\right)\text{Im}\{M_{dc}\}\alpha^n$$



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# Proposed DFT Based Algorithm

- Even and Odd samples sets

## ❖ AC part:

$$A_{est-ac}^e(n) = A_1 \left[ \left(1 + \frac{1}{N}\right) \cos\left(\frac{2\pi}{N}2n + \phi_1\right) + j\left(1 - \frac{1}{N}\right) \sin\left(\frac{2\pi}{N}2n + \phi_1\right) \right]$$

$$A_{est-ac}^o(n) = A_1 \left[ \left(1 + \frac{1}{N}\right) \cos\left(\frac{2\pi}{N}(2n + 1) + \phi_1\right) + j\left(1 - \frac{1}{N}\right) \sin\left(\frac{2\pi}{N}(2n + 1) + \phi_1\right) \right]$$

$$A_{est-ac}^{e-o}(n) = \underbrace{A_{est-ac}^e(n)}_{\text{even samples}} - \underbrace{A_{est-ac}^o(n)}_{\text{odd samples}}$$

$$A_{est-ac}^{e-o}(n) = A_1 \left[ \left(1 + \frac{1}{N}\right) \left( \cos\left(\frac{2\pi}{N}2n + \phi_1\right) - \cos\left(\frac{2\pi}{N}2n + \phi_1 + \frac{2\pi}{N}\right) \right) \right. \\ \left. + j\left(1 - \frac{1}{N}\right) \left( \sin\left(\frac{2\pi}{N}2n + \phi_1\right) - \sin\left(\frac{2\pi}{N}2n + \phi_1 + \frac{2\pi}{N}\right) \right) \right]$$

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# Proposed DFT Based Algorithm

- Estimated amplitude for AC part:

$$f = 50Hz$$

$$f_s = 10kHz \rightarrow N = 200 \Rightarrow 1 + \frac{1}{N} \approx 1$$

$$|A_{est-ac}^{e-o}(n)| = 2A_1 \sin \frac{\pi}{N}$$

$$A_1 = \frac{|A_{est-ac}^{e-o}(n)|}{2 \sin \frac{\pi}{N}}$$

# Proposed Estimated Amplitude

## ❖ DC part

$$A_{est-dc}^e(n) = M_{dc}\alpha^{2n}$$

$$A_{est-dc}^o(n) = M_{dc}\alpha^{(2n+1)}$$

$$A_{est-dc}^{e-o}(n) = M_{dc}(1 - \alpha)\alpha^{2n}$$

Therefore, the expression for difference between the even and odd samples in the original signal is:

$$A_{est}^{e-o}(n) = A_{est-ac}^{e-o}(n) + A_{est-dc}^{e-o}(n)$$

$$A_1 \simeq \frac{|A_{est}^{e-o}(n)|}{2 \sin \frac{\pi}{N}}$$

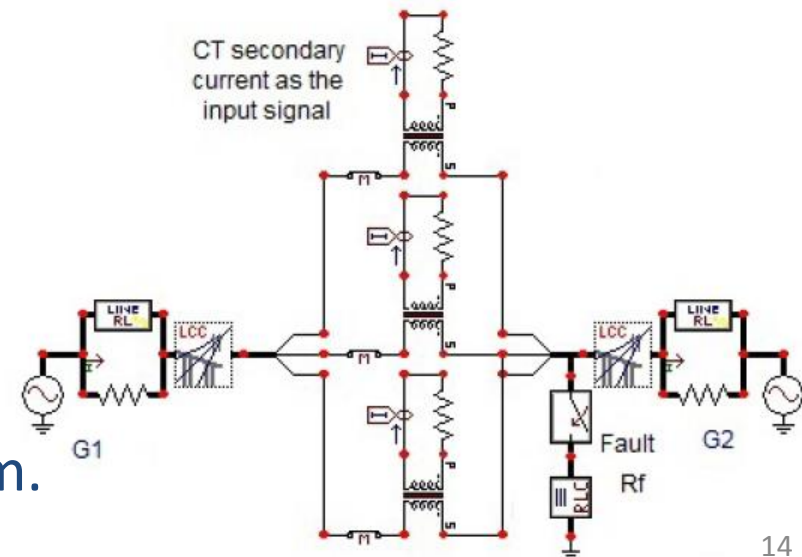
# Performance evaluation

- Two signals are defined to compare the effectiveness of the proposed algorithm and conventional DFT method.

$$i_1(t) = 1.1e^{-\frac{t}{0.05}} + \sin 100\pi t$$

$$i_2(t) = 1.1e^{-\frac{t}{0.6}} \cos(40\pi t) + \sin(100\pi t)$$

- The EMTP software is used to generate the fault currents to evaluate the proposed algorithm.

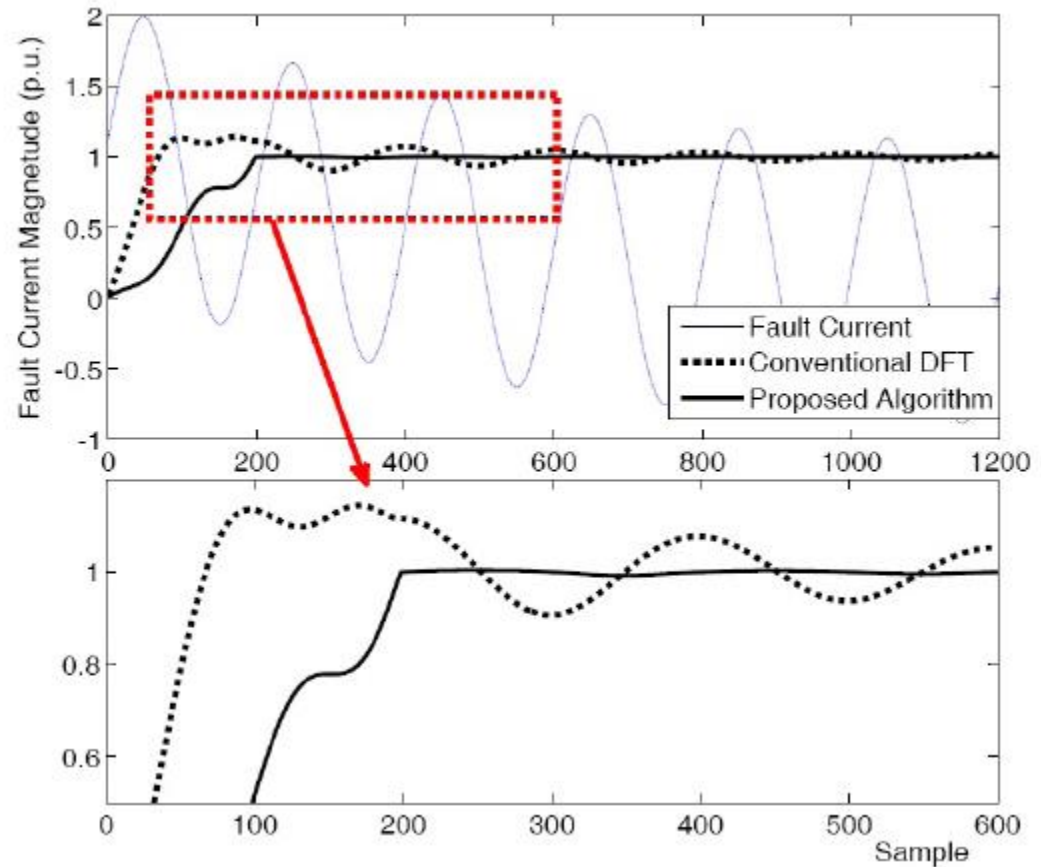


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# Performance evaluation

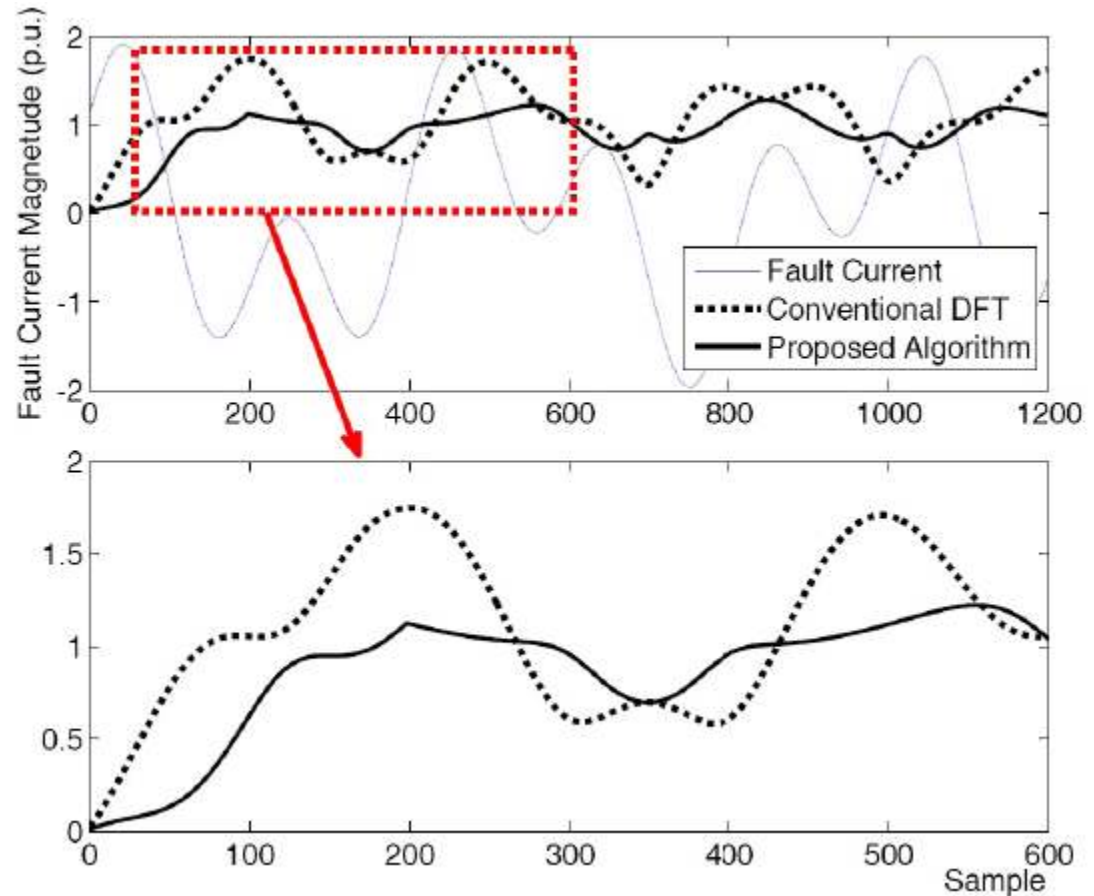
- Defined Test Case

- $i_1(t) = 1.1e^{-\frac{t}{0.05}} + \sin 100\pi t$



# Performance evaluation

- $i_2(t) = 1.1e^{-\frac{t}{0.6}} \cos(40\pi t) + \sin(100\pi t)$





# Convergence analysis

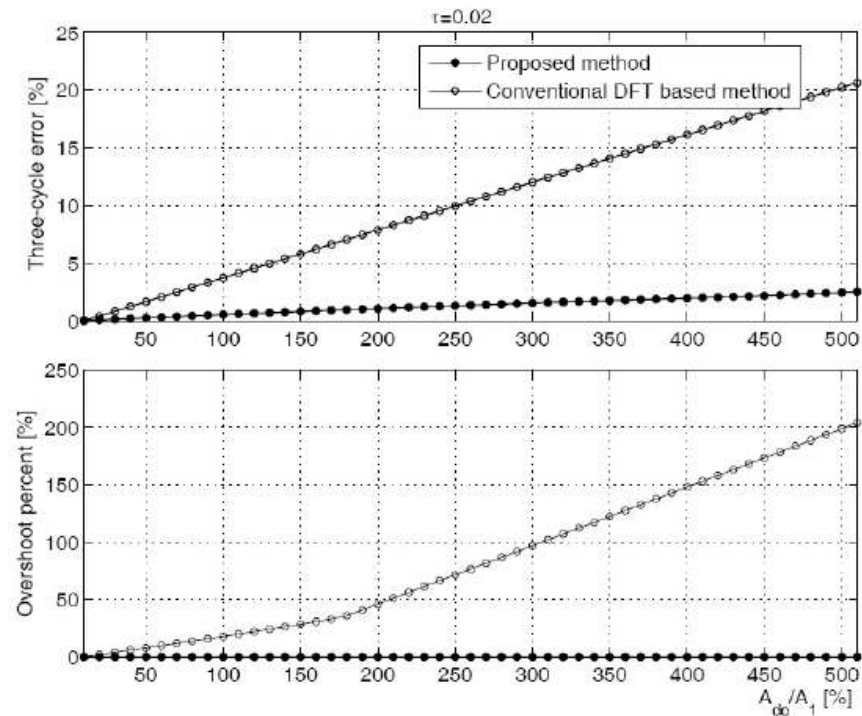
The theoretical input amplitude is computed and compared to the amplitude generated by the proposed method.

The comparison is based on the computation of the Three-Cycle Error (TCE) which is the average error in the first three cycles.

$$TCE = \frac{f\Delta t}{3} \sum_{i=1}^{\frac{3}{f\Delta t}} \frac{|A_{1e}(i) - A_1(i)|}{A_1(i)} \times 100\%$$

# Convergence analysis

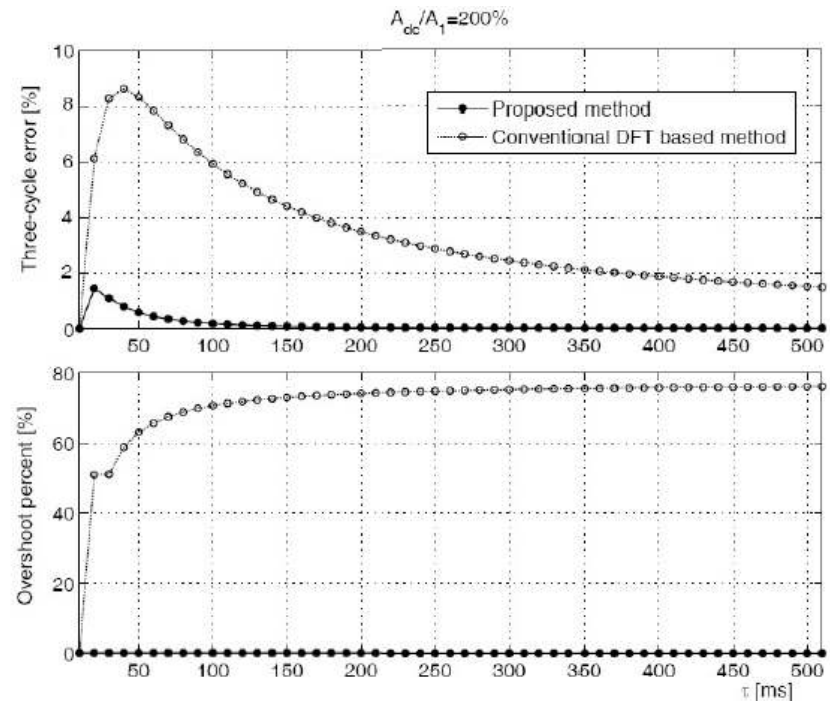
- TCE and overshoot percentages for an input signal combined with a decaying dc component with time constant  $\tau = 0.02$  S.



The effect of initial amplitude of the decaying dc component on the TCE and overshoot percentages.

# Convergence analysis

- The initial amplitude of the decaying dc component is 200% of the fundamental component.
- The TCE in the proposed method is much lower than the DFT method and its overshoot percentage is equal to zero for all time constants.



Time constant effect of decaying dc component on the TCE and overshoot percentag.

# Conclusion

- ✓ This paper has successfully developed a new DFT based algorithm to mitigate the decaying dc component in the phasor measurement.
- ✓ In general, to reduce the decaying dc component, one must estimate the decaying DC parameters. This process takes time and needs complicated calculations. However, in phasor domain, using the even and odd samples we have shown that there is no need to such estimation.
- ✓ The method is performed without loss of accuracy.
- ✓ Both theoretical and simulation results show that the convergence of the proposed algorithm to the final value is faster and has a better accuracy than the conventional DFT method.
- ✓ The DFT conventional algorithm reaches to the final value after 4-6 cycles. However, the proposed algorithm reaches the same value after 1-2 cycles with high accuracy and thus, the full-cycle DFT performance has greatly been improved.

# Questions ?

## Thank You