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Abstract

The efficiency of the Simultaneous Analysis and Design (SAND) approach[1-6] in the minimum weight optimization of structural systems subject to strength and displacement constraints as well as size side constraints is investigated. SAND allows for an optimization to take place in one single operation as opposed to the more traditional and sequential Nested Analysis and Design (NAND) method[2], where analyses and optimizations alternate. Thus, SAND has the advantage that the stiffness matrix is never factored during the optimization, retaining its original sparsity. One of SAND's disadvantages is the increase in the number of design variables and in the associated number of constraint gradient evaluations. If SAND is to be an acceptable player in the optimization field, it is essential to investigate the efficiency of the method and to present a possible cure for any inherent deficiencies.

1. Introduction

For structural systems, modeled by the finite element method in a minimum weight optimization, the SAND approach adds the displacements to the set of design variables, with the global stiffness equations added as nonlinear equality constraints. This results in a larger set of design variables and constraints, and a considerably larger related number of constraint gradient evaluations.

In the late 1960s, Fox and Schmit[3] tried to integrate the traditional two-step structural optimization (NAND) approach by employing conjugate gradient minimization techniques (CG) for solving linear structural analysis problems. The CG method was not effective when dealing with the equality constraints associated with the equilibrium equations because the stiffness matrix for a finite-element model is normally ill-conditioned. Haftka[4] employed preconditioned conjugate gradient techniques and element-by-element (EBE) formulations in simultaneous analysis and design. His research showed that the element-by-element approximate inverse of the stiffness matrix can be used to speed convergence by an order of magnitude. In the 1980s, Smaoui and Schmit[5] extended their research into the optimization of geometrically nonlinear structures using a generalized reduced gradient algorithm (GRG). Their work showed that the algorithm can detect and guard against system as well as element elastic instabilities based on only the equilibrium information. Finally, Ringertz[6] conducted research on optimization of structures with nonlinear response using the sequential quadratic programming (SQP) method. In his research, two different formulations were used for the equilibrium constraints (relaxation of all versus a few equilibrium constraints). From the mentioned research, it becomes clear that the formulation of the equilibrium constraints associated with structural equilibrium and the algorithms to deal with these constraints are very important to the efficiency of SAND.

2. SAND Methodologies

In the present work, the efficiency of the SAND approach in solving the larger system of design variables and constraints as compared to NAND is evaluated relative to the gain from combining analysis and optimization into a single step.

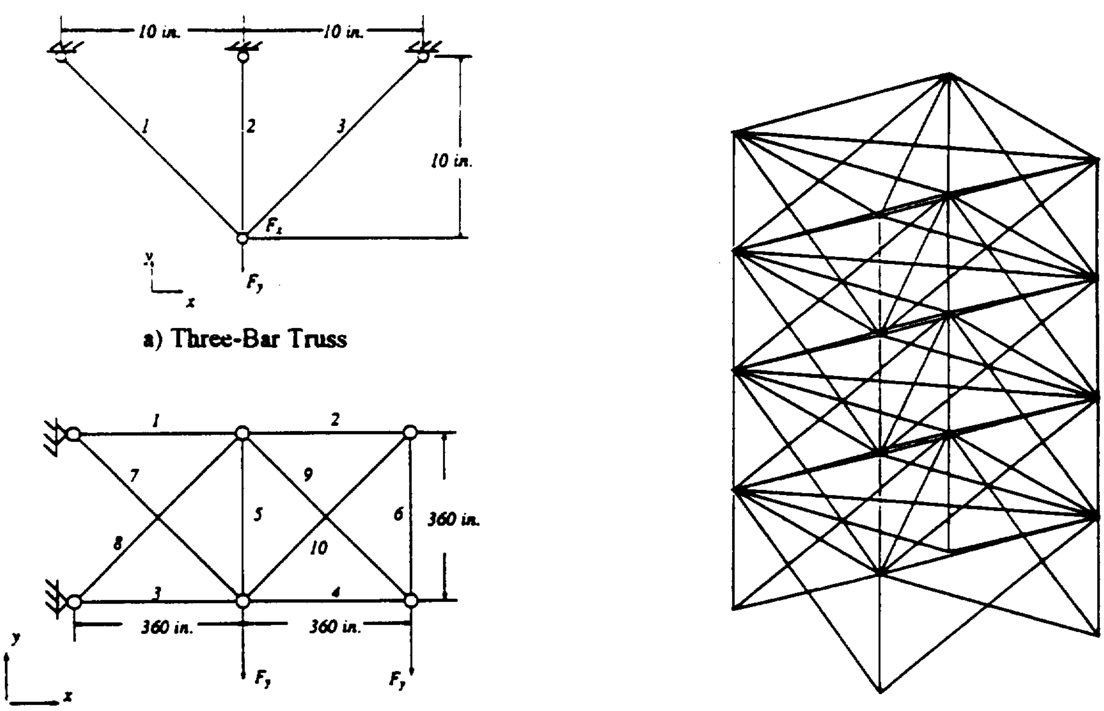
To avoid a second drawback of the SAND method, namely the potential for ill-conditioned matrices arising from the use of the global stiffness equations, a mixed force/displacement FEM approach can be applied (SAND-MM). In this strategy, the element stiffness equations are used as nonlinear equality constraints instead of the global stiffness equations, in conjunction with the linear nodal force equilibrium equations. This adds the element forces as variables to the system.

Finally, a variation of this method (SAND-LMM) applies the element equilibrium equations in their local element form as nonlinear equality constraints and uses the nodal displacements for each element as design variables rather than the global displacements. This approach requires that the displacements be set equal for all elements attached to the same nodal point, resulting in additional linear constraint equations.

For complex structures and the associated large and sparse matrices, the execution times of the optimization code for SAND-MM and SAND-LMM can become excessive due to the even larger number of constraint gradient evaluations. To improve performance, the Kreisselmeier-Steinhauser function[7] or some other norm, such as the max norm, can be used to decrease the computational effort by reducing the nonlinear equality constraint system to a smaller set or even to a single combined constraint equation. As opposed to the standard SAND approach, questions related to the performance of the optimization under the action of the chosen norm need to be answered.

3. Results

To date, the standard three-bar and ten-bar trusses have been investigated, with some additional results obtained for a 72-bar truss (Figure 1). As optimizers, the two codes NPSOL[8] and MINOS[9] were used. NPSOL is based on a sequential quadratic programming (SQP) algorithm, but intended for full matrix systems. In contrast, MINOS uses a quasi-Newton method for both dense and sparse matrix systems. The finite element related input to the optimizers was automatically generated from the input decks of such standard FEM/optimization codes as NASTRAN or ASTROS with the stiffness matrices, at present, extracted from the FEM code ANALYZE.



b) Ten-Bar Truss

c) 72-Bar Truss [4]

Figure 1. Various Truss Structures used for Simultaneous Analysis and Design

3.1 Results from NPSOL

For the standard SAND approach, running NPSOL at NASA Langley Research Center first, correct results were obtained for all three trusses although convergence became slow for the 72-bar truss. When the mixed method strategy (SAND-MM) was used with NPSOL, correct results were still obtained, but the execution times became excessive, especially for the larger trusses, due to the large number of required constraint gradient evaluations. Using the KS function with NPSOL for the three- and ten-bar trusses, the computational effort dropped considerably, but the optimization seemed to become much less robust (see results for the three-bar truss in Table 1). There is also an indication (rows 2 and 3 of Table 1), that a prudent choice of the inequality constraints (here: the stresses in the bars as a function of the element forces rather than the displacements) can reduce the computational effort.

The NPSOL results in Tables 2 and 3 obtained at the University of Oklahoma (OU) behave essentially in the same way, with the additional SAND-LMM computations requiring the largest work space and longest CPU time, but resulting in a smaller number of constraint gradient evaluation sets than the equivalent SAND-MM approach.

Finally, in Table 4, additional NPSOL based optimizations are shown for the ten-bar truss and the 72-bar truss. Here, two general input programs were coded and used which are able to construct

Table 1. Efficiency of Methodology in Optimization of Three-Bar Truss by NPSOL at NASA LaRC

Method (NPSOL)	Stress as a function of	Work Space	Iters.	# of Function Evaluations	# of Gradient Evaluations	Accuracy of Results
SAND	displacements	1796	10	11	11	exact
SAND-MM	displacements	6196	583	1733	1733	exact
SAND-MM	element forces	6688	113	283	113	exact
SAND-MM KS	displacements	4516	100	171	171	~ 1% off

Table 2 Efficiency Comparison in Optimization of Three-Bar Truss by NPSOL and MINOS at OU

Method	**Stress as a function of**	**Results by NPSOL**				**Results by MINOS**			
		Work Space	Major Iters.	# of Non-lin. Constr. Evals.	Time	Work Space	Major Iters.	# of Non-lin. Constr. Evals.	Time
SAND	displace-ments	2144	15	42	2.0	25760	22	253	2.0
SAND-MM	displace-ments	6196	585	1733	33.4	25760	16	229	2.7
SAND-MM	element forces	6688	120	296	7.7	25760	43	1130	3.1

Table 3 Efficiency Comparison in Optimization of Ten-Bar Truss by NPSOL and MINOS at OU

Method	**Stress as a function of**	**Results by NPSOL**				**Results by MINOS**			
		Work Space	Major Iters.	# of Non-lin. Constr. Evals.	Time	Work Space	Major Iters.	# of Non-lin. Constr. Evals.	Time
SAND	displace-ments	14352	7	12	3.5	36408	14	109	2.5
SAND-MM	displace-ments	57848	12	120	5.8	330584	13	173	3.8
SAND-MM	element forces	61888	49	49	20.7	333512	15	2476	6.0
SAND-LMM	displace-ments	128312	31	75	45.8				

Table 4. Optimization of Trusses using General Input to NPSOL at OU

Truss	Method (NPSOL)	Stress as a Function of	Work Space	Major Iterations	# of Non-lin. Constr. Evals.	Time
10 Bar	SAND	displacements	14352	10	18	3.3
	SAND-MM	displacements	88688	19	36	16.2
72 Bar	SAND	displacements	427392	21	32	69.1

all required linear and nonlinear constraints and their gradients for arbitrary two-dimensional and three-dimensional structures, respectively, for the SAND and SAND-MM approaches. During the code development, a special effort was made to reduce the numbers of do-loop iterations in all subroutines which are called repeatedly during every major iteration of the optimization. The general program based on the SAND method worked very well for both structures in terms of the quality of the results and the time required to obtain optimum solutions. Because of a tremendous increase in the number of nonlinear constraints and design variables in the SAND-MM method, CPU times to obtain optimum 72-bar truss results increased considerably as compared to the SAND method. Also, the optimization was found to be highly dependent on the selection of the initial design variables. No optimum solution could be achieved for the 72-bar truss by SAND-MM to date.

3.2 Results from MINOS

As an alternate approach, the code MINOS for the optimization of dense and sparse matrix systems was applied to the problem at the University of Oklahoma in lieu of the Kreisselmeier-Steinhauser function. This approach was intended to address the large number of unnecessary matrix calculations arising from the sparse matrices in SAND, SAND-MM, and SAND-LMM. Direct comparisons were run for the three- and ten-bar trusses. Results (Tables 2 and 3) show that the code MINOS has the advantage in problem solving time, although it requires more work and time to prepare the input deck when compared to the input for NPSOL. Thus, effort will be saved by using MINOS when a problem needs to run repeatedly as the input deck needs to be prepared only once.

Table 5. Efficiency Comparison between Treating Matrix as Sparse and Dense

Truss	**Method**	**Matrix Treated as Sparse Matrix**				**Matrix Treated as Dense Matrix**			
		Work Space	Major Iters.	# of Non-lin. Constr. Evals.	Time	Work Space	Major Iters.	# of Non-lin. Constr. Evals.	Time
3 Bar	SAND	25760	22	253	2.0	25760	22	344	2.1
	SAND-MM	25760	16	229	2.7	25760	16	229	2.8
10 Bar	SAND	36408	14	109	2.5	37272	14	139	2.7
	SAND-MM	330584	13	173	3.8	335480	11	117	3.9

Since MINOS can handle a matrix system as either sparse or dense, comparisons were run for the three- and ten-bar trusses, treating the matrices in these two ways. Only very slightly shorter solution times were obtained for the sparse matrix solution procedure (Table 5). This seems to indicate that the advantage of MINOS may be due in larger part to the choice of optimizer in the code than to the sparse matrix features. It should also be mentioned that the matrix systems for the three- and ten-bar trusses are actually quite dense in SAND and SAND-MM.

4. Present and Future Work

For the sucessful optimizations, the well-known optimal values from the literature were obtained for the objective functions, with the exception of the KS-approximation results in Table 1, which were off by about 1%, as might be expected. At present, various KS formulations are being introduced and run on both codes for the three- and ten-bar trusses. Issues such as scaling, the selection and/or grouping of constraints for the KS approach, as well as formulation issues will be investigated. Additional cases for the general SAND-LMM methodology are being run by NPSOL, using both the case specific and general input approaches. At the same time, a general input strategy for arbitrary two- and three-dimensional structures is being developed for

MINOS which will be run for the ten- and 72-bar trusses. Finally, it is understood that the present small sample of problems with mostly a limited number of D.o.F. does not yet allow one to draw overall conclusions as to the performance of SAND. Therefore, more investigations of larger models, such as the 72-bar truss, are indicated, especially utilizing the sparse matrix features in MINOS.

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